# Econ 705 : Preliminary Examination

August, 2010.

**Instructions:**

(1) All the answers should be written legibly.

(2) In giving answers, try to justify them.

(3) Even when you are not able to delineate precise conditions, try to provide a sketch of derivations and solutions as clearly as possible.

**(1)(a-d)** (40 Points) Suppose that one has i.i.d. random vectors of $\{(Y_i, X_i, Z_i, u_i)\}_{i=1}^n$ which are assumed to have been generated from the following regression models:

$$Y_i = \Phi(\beta_0 + X_i\beta_1 + Z_i\beta_2) + u_i, \ \ i = 1, 2, \cdots, n,$$

where $Y_i, X_i, Z_i$ are observed random variables, $u_i$ is an unobserved random variable and $\Phi$ is the standard normal CDF. The scalars $\beta_0$, $\beta_1$, and $\beta_2$ are unknown parameters. As for $u_i$, it is assumed that

$$\begin{aligned} \mathbf{E}\left[u_i|X_i, Z_i\right] &= 0 \text{ and} \\ \mathbf{E}\left[u_i^2|X_i, Z_i\right] &= \sigma^2 \end{aligned}$$

for some unknown constant $\sigma^2 > 0$.

**(a)** Suppose that $X_i = 2Z_i + 3$. Is $\beta_1$ identified from the model? Explain your answer.

For simplicity, we assume that it is known to the researcher that $\beta_2 = 0$ for the remaining questions (b), (c), and (d).

**(b)** Provide an estimator of $\beta_1$. Is the estimator unbiased? Explain your answer.

**(c)** Is there an estimator that has a better quality than (b) in terms of its asymptotic variance? Explain your answer.

**(d)** Suppose that one is interested in testing:

$$\begin{aligned} H_0 &: \beta_1 = 0 \text{ against} \\ H_1 &: \beta_1 \neq 0. \end{aligned}$$

Suppose that one uses a one-sided $t$-test instead of a two-sided test in testing the above hypothesis. Explain why such a procedure is not desirable.

**(2)(a-f)** (60 Points) A researcher is interested in a study of female labor participation. Let $X_i$ denote the $i$-th worker's years of education and $D_i$ the indicator of participation in the labor market. As for the participation in the labor market, the researcher considers estimating the following model:

$$D_i = \begin{cases} 1 \text{ if } 0.5 + X_i\beta > \varepsilon_i \\ 0 \text{ if } 0.5 + X_i\beta \leq \varepsilon_i \end{cases},$$

where the conditional distribution of $\varepsilon_i$ given $X_i$ is $N(0,1)$. Assume that $\{(D_i, X_i, \varepsilon_i)\}_{i=1}^n$ are i.i.d. and only $\{(D_i, X_i)\}_{i=1}^n$ are observed. The quantity $\beta$ denotes the unknown parameter.

**(a)** Write the log-likelihood function for $\beta$, and compute the asymptotic variance of the MLE $\hat{\beta}$ for $\beta$.

**(b)** Suggest appropriate moment conditions that can be used to estimate $\beta$ using GMM. (HINT: Write the model in the form of a nonlinear regression model with $D_i$ as the dependent variable.) Does the GMM require the assumption that the conditional distribution of $\varepsilon_i$ given $X_i$ is normal? Explain your answer.

**(c)** One is interested in estimating the average partial effect of education on the labor market participation:

$$\mathbf{E}\left[p'(X)\right]$$

where $p'(\cdot)$ is the first order derivative of $p(x)$ and

$$p(x) = P\left\{D_i = 1 | X_i\beta = x\beta\right\}.$$

In other words, $\mathbf{E}[p'(X)]$ measures the average change of probability of labor market participation when one obtains more education. Show that the average partial effect can be written as

$$\mathbf{E}\left[\phi\left(0.5 + X_i\beta\right)\right]\beta,$$

where $\phi$ is the standard normal PDF.

**(d)** Suggest an estimator of the average partial effect $\mathbf{E}\left[\phi\left(0.5 + X_i\beta\right)\right]\beta$ using the MLE of (a).

**(NOTE: Questions (e)-(f) are on the next page.)**

**(e)** Find the asymptotic distribution of the average partial effect estimator in (d).

**(f)** Suppose that one wants to test whether the average partial effect is zero:

$$\begin{aligned} H_0 &: \mathbf{E}\left[\phi\left(0.5+X_i\beta\right)\right]\beta = 0 \text{ against} \\ H_1 &: \mathbf{E}\left[\phi\left(0.5+X_i\beta\right)\right]\beta \neq 0. \end{aligned}$$

Suggest a test statistic and explain how one can obtain appropriate critical values.