Econ 706 Prelim
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Throughout, provide detailed support for all assertions. Good luck!

Consider a conditionally heteroskedastic ARCH process in the tradition of Engle et al.:

$$y_t | y_{t-1} \sim N(0,\ h_t)$$

$$h_t = 0.1 + .95\, y_{t-1}^2.$$

1. Is the process well-defined and covariance stationary? Why or why not? If not, give conditions that make it well-defined and covariance stationary. What is its unconditional mean? Unconditional variance? Unconditional skewness? Unconditional kurtosis? Unconditional ninth moment? Is it serially uncorrelated? Serially independent? Conditionally Gaussian? Unconditionally Gaussian?

2. Provide an explicit expression for the pseudo-likelihood function, making use of a conditional factorization (Schweppe decomposition). How do you handle the density of the first observation? How would you maximize the likelihood numerically using an iterative Newton algorithm? What is the benefit of Newton relative to steepest descent? How would you assess the legitimacy of the conditional normality assumption upon convergence of the Newton iterations?

3. Suppose that you are given a sample path from this process (assumed well-defined and covariance stationary) of length 5000. Using that sample path, you estimate the model by Gaussian pseudo-ML. How would you then calculate the series of estimated conditional standard deviations, $\hat{h}_t^{1/2}$? Discuss, contrast, and graph the qualitative shapes of sample paths of y_t and $\hat{h}_t^{1/2}$, and the sample autocorrelation functions of y_t, y_t^2, $y_t/\hat{h}_t^{1/2}$, and $(y_t/\hat{h}_t^{1/2})^2$.

4. What is a leverage effect, and where/why might it be important in the context of the ARCH process above ? Generalize the ARCH process above to include a leverage effect. How would you test the hypothesis of no leverage effect?

5. Compare and contrast the ARCH process above with the stochastic volatility (SV) process:

$$y_t = [\exp(h_t)]^{1/2}\, \varepsilon_t$$

$$h_t = \omega + \phi h_{t-1} + v_t,$$

where ε and ν are N(0,1), contemporaneously and serially independent at all leads and lags. In what sense are the ARCH and SV processes similar? In what sense are they fundamentally different? Transform the SV process as written above into a linear state space system with constant system parameters. What properties would the Kalman filter have for filtering h_t assuming known system parameters? What properties would Gaussian pseudo-ML of the linearized system have for estimating the system parameters?