Preliminary Examination 2010
Economics 702

1 The Stochastic Neoclassical Growth Model With Two Countries (25 points)

First consider the closed economy stochastic neoclassical growth model. The social planner chooses stochastic consumption and capital allocations $\{c_t, k_{t+1}\}$ to solve the following maximization problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \text{ s.t.}$$

$$c_t + k_{t+1} + \frac{1}{2}\phi\left(\frac{k_{t+1}-k_t}{k_t}\right)^2 = z_t k_t^{\alpha} + (1-\delta)k_t$$

with $\alpha, \beta \in (0,1)$ and $\delta \in [0,1]$, and $\phi \geq 0$ measures the extent to which the economy is subject to adjustment costs. The initial endowment of capital k_0 is given. The technology shock z_t follows a Markov chain with state space Z and Markov transition function $\pi(z_{t+1}|z_t)$.

1. Formulate the social planner problem recursively. Clearly state the state and control variables.

2. Now there are two countries $i = 1, 2$ that are *identical* to each other, i.e. have equal size, preferences and technologies, same initial capital stock and face the same stochastic process for technology shocks. The *realizations* of the technology shock are not necessarily identical across countries, that is, although both countries have the same Z and π, the realized values for the technology shocks at time t, z_t^1 and z_t^2 can differ from each other. Furthermore the realizations of z_t^1 and z_t^2 are independent of each other. Index country allocations by $i = 1, 2$.Now imagine a *world* social planner that maximizes the sum of both countries expected lifetime utilities, subject to a world resource constraint and the appropriate non-negativity constraints. Formulate the problem of the world social planner recursively. Clearly state the state and control variables.

3. For this part only, suppose that $\phi = 0$ and that the planner can move capital across countries instantaneously. Formulate the problem of the world social planner recursively with the minimum number of state variables. Clearly state the state and control variables.

2 The Overlapping Generations Model with Growth (35 points)

Consider an overlapping generations economy in which the representative consumer in each generation born in period t, $t = 1, 2, 3, \ldots$, lives for two periods. She values consumption of the single nonstorable good according to

$$u(c_t^t, c_{t+1}^t) = \log\left(c_t^t\right) + \beta \log\left(c_{t+1}^t\right)$$

and has strictly positive endowments of the consumption good (e_t^t, e_{t+1}^t). Endowments are growing over time, specifically

$$\begin{aligned} e_t^t &= w_1(1+g)^t \\ e_{t+1}^t &= w_2(1+g)^t \end{aligned}$$

for $t \geq 1$. Population grows at rate $n > 0$ (if the population is 1 today, it is $1 + n > 1$ tomorrow). In addition there is an initial old generation with utility function

$$u(c_1^0) = \log(c_1^0)$$

and an endowment of the consumption good $e_1^0 = w_2$. There is NO money, that is $m = 0$. Normalize the size of the initial old generation to 1. The government runs a balanced budget social security system that taxes endowments of young households at rate τ, so that after tax endowments of young households born at time t are given by $(1-\tau)e_t^t$. It pays benefits b_t to each old household at time t.

1. Use the budget constraint of the social security system to express b_t as a function of (w_1, g, n, τ).

2. For $\tau = 0$, compute the equilibrium interest rate (which happens to be constant over time), as a function of the parameters of the model.

3. Under what condition on (w_1, w_2, β, g, n) is the introduction of a small social security system Pareto-improving (in the sense that the competitive equilibrium allocation with a small social social security system Pareto dominates that with no social security system)?

4. Use your result in 3. to interpret your result in 4. (i.e. express the condition you found in 4. in terms of the equilibrium interest rate without social security, and then interpret that condition).

3 A Continuous Time Growth Model with Human Capital (40 points)

The representative household has preferences over consumption represented by

$$u(c) = \int_0^\infty e^{-\rho t} \log(c(t))dt$$

where $\rho > 0$ is a parameter. At period 0 its initial endowment of human capital is given by $h(0) > 0$ and at each instant the household has one unit of time. Human capital is accumulated according to the human capital production technology

$$\dot{h}(t) = h(t)\left[1 - s(t)\right] - \delta h(t)$$

where $\delta \in (0, 1)$ and $s(t)$ is the time devoted to market work. The consumption good is produced according to

$$c(t) = h(t)^\theta s(t)$$

with $\theta \geq 0$. Note that there is NO physical capital in this economy.

1. Write down the Hamiltonian of the social planner problem.
2. Derive the first order condition with respect to the control variable(s) and the equation(s) relating the derivative of the co-state variable(s) to the derivative of the Hamiltonian with respect to the state variable.
3. Use your results from the previous part derive a system of differential equations that characterizes the dynamics of the optimal solution to the social planner problem.
4. For what parameter values does this economy have a steady state? Compute the steady state share s of time spend with market work
5. Can this economy exhibit positive long run balanced growth in per capita consumption? If yes, give the set of parameter values for which it does. If no, show that long run positive balanced growth is impossible.