Economics 702, Dirk Krueger

1 The Stochastic Neoclassical Growth Model with Many Countries

Consider the stochastic neoclassical growth model with N countries of equal size (which you may normalize to 1 for each country), often also called the IRBC (International Real Business Cycle) model. Let i index a specific country. In each country there is a representative household with preferences over $\{c_t^i, l_t^i\}$ represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t U^i(c_t^i, l_t^i).$$

The representative household is endowed with one unit of time in every period and with initial capital stock k_0^i, and owns the capital stock in any competitive equilibrium considered below. In each country the production technology is given by

$$z_t^i \left(k_t^i\right)^{\alpha} \left(l_t^i\right)^{1-\alpha}.$$

Furthermore denote by $z_t = (z_t^1, \ldots, z_t^N)$ the N-dimensional vector of country-specific technology shocks, and by $z^t = (z_0, z_1, \ldots, z_t)$ the history of these shocks. Assume that the stochastic process of technology shocks $\{z_t\}$ follows a finite Markov chain with transition matrix $\pi(z_{t+1}|z_t)$ and associated invariant distribution Π. Finally, assume that the capital stock (in each country) depreciates at rate δ and denote the initial world aggregate capital stock by K_0 and the initial capital stock in each country (owned by the representative household in the respective country) as k_0^i. Assume that consumption is perfectly movable across countries even within the period. Labor is country-specific and thus labor input from country i cannot be used in country $j \neq i$ at all. It takes one period to reallocate capital across countries, that is, if $(k^i)'$ is chosen today, it only can be used in production in country i tomorrow.

1. Suppose that $N = 2$ for the rest of the question and thus $z_t = (z_t^1, z_t^2)$. Suppose further that $z_t^i \in \{\zeta_l, \zeta_h\}$ can take two values for each country. Suppose that z_t^1 and z_t^2 are independent over time with $prob(\zeta_l) = prob(\zeta_h) = 0.5$, but *perfectly positively correlated* across countries.

Write down the Markov transition matrix $\pi(z_{t+1}|z_t)$ and the (set of) invariant distributions associated with this matrix.

2. Repeat question 1, but now assume that z_t^1 and z_t^2 are *perfectly negatively correlated* across countries.

3. State the world social planner problem, for given Pareto weights $(\theta^i)_{i=1}^2$, in sequential form.

4. State the world social planner problem, for given Pareto weights $(\theta^i)_{i=1}^2$, in recursive form. Clearly identify the state and control variables and make sure you write the problem with the *minimal* set of state variables.

5. Describe briefly how to construct the solution of the sequential social planners problem from the recursive problem. What guarantees that this construction delivers the correct solution to the sequential problem?

6. Define a recursive competitive equilibrium.

7. For simplicity assume that $k_0^1 = k_0^2$ and assume that the productivity shocks are perfectly positively correlated across the two countries (as in question 1). Also assume that

$$U^i(c_t^i, l_t^i) = u(c_t^i) + v(l_t^i)$$

that is, utility functions are identical across countries and separable between consumption and labor. Characterize as fully as possible the Arrow-Debreu equilibrium allocation (you do not need to define it first).