Economics 702, Dirk Krueger

1 The Stochastic Neoclassical Growth Model with Many Countries

Consider the stochastic neoclassical growth model with N countries of equal size (which you may normalize to 1 for each country), often also called the IRBC (International Real Business Cycle) model. Let i index a specific country. In each country there is a representative household with preferences over $\{c_t^i, l_t^i\}$ represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t U^i(c_t^i, l_t^i).$$

The representative household is endowed with one unit of time in every period and with initial capital stock k_0^i, and owns the capital stock in any competitive equilibrium considered below. In each country the production technology is given by

$$z_t^i \left(k_t^i\right)^{\alpha} \left(l_t^i\right)^{1-\alpha}.$$

Furthermore denote by $z_t = (z_t^1, \ldots, z_t^N)$ the N-dimensional vector of country-specific technology shocks, and by $z^t = (z_0, z_1, \ldots, z_t)$ the history of these shocks. Assume that the stochastic process of technology shocks $\{z_t\}$ follows a finite Markov chain with transition matrix $\pi(z_{t+1}|z_t)$ and associated invariant distribution Π. Finally, assume that the capital stock (in each country) depreciates at rate δ and denote the initial world aggregate capital stock by K_0.

1. Suppose that $N = 2$ and thus $z_t = (z_t^1, z_t^2)$. Suppose further that $z_t^i \in \{\zeta_1, \zeta_2\}$ can take two values for each country, and that for each country the z_t^i draws are independently and identically distributed (both across time *and* across countries) with $prob(\zeta_1) = prob(\zeta_2) = 0.5$. Write down the Markov transition matrix $\pi(z_{t+1}|z_t)$.

2. Let $N = 1$. State the social planner problem of the one country in recursive formulation. Clearly identify the state and the control variables.

3. Now let $N > 1$ and assume that consumption and capital are perfectly movable across countries even within the period. Labor is country-specific and thus labor input from country i cannot be used in country $j \neq i$. State the world social planner problem, for given Pareto weights $(\theta^i)_{i=1}^N$. Clearly identify the state and control variables and make sure you write the problem with the *minimal* set of state variables.

4. Now assume that it takes one period to reallocate capital across countries, that is, if $(k^i)'$ is chosen today, it only can be used in production in country i tomorrow. Repeat question 3.

5. Again let $N = 1$. Define a recursive competitive equilibrium.

6. Now let $N > 1$ and use the same assumptions as in question 4. Define a recursive competitive equilibrium for the world economy.