1 Recursive Competitive Equilibrium and Asset Pricing in a Pure Exchange Economy

Consider the following pure exchange economy. The stochastic aggregate endowment of the single consumption good is given by $z \in Z$, a finite set. z follows a Markov process with transition Matrix π and associated stationary distribution Π.

There is a Lucas tree that pays dividends d in every period equal to a fraction θ of the aggregate endowment, that is

$$d(z) = \theta z.$$

The total number of (perfectly divisible) outstanding shares in the Lucas tree is constant over time at 1, and the current owner of one share of the Lucas tree is entitled to the dividend $d(z)$. A household that purchased one share of the Lucas tree yesterday can sell the share for a price (ex dividend) p and in addition gets a dividend $d(z)$. There are no other assets in this economy. The remaining part of the aggregate endowment $(1 - \theta)z$ is paid to households as labor income.

1. Suppose for this question that $Z = \{z_l, z_h\}$ with Markov transition matrix is given by

$$\pi = \begin{pmatrix} 0.2 & 0.8 \\ 0.2 & 0.8 \end{pmatrix}.$$

Determine the (set of) invariant distribution(s) associated with this Markov transition matrix.

2. Suppose the economy is populated by an infinitely lived representative agent with lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t).$$

with $\beta \in (0, 1)$ and U satisfies the usual properties. Define a recursive competitive equilibrium for this economy. Hint: first determine and clearly state what the aggregate state variables are.

3. Now suppose that the process for z is *iid* with probabilities $\pi(z)$, for $z \in Z = \{z_1, \ldots, z_n\}$. Furthermore, let

$$U(c) = \frac{c^{1-\sigma}}{1 - \sigma}$$

with $\sigma > 0$. For $\sigma = 1$, it is understood that $U(c) = \ln(c)$. Derive the Euler equation of the representative household (in its recursive formulation). Note again that the Lucas tree is the *only* asset in this economy.

4. Use the result in the previous question to compute the equilibrium price of the Lucas tree for each aggregate state of the world.

5. How would your result in question 4 change if, in addition to the Lucas tree, an entire set of Arrow securities were traded in this economy?

6. Now suppose that the economy still lasts forever, but is populated by $I+1$ overlapping generations. Each generation lives for $I+1$ periods and then dies for sure. Newborn households start their life with no assets and households born at date have lifetime utility function

$$E_t \sum_{i=0}^{I} \beta^i U(c_{t+i}^i)$$

where c_{t+i}^i is consumption of generation t in period i of their life (and thus calender time $t+i$). The *only* asset is the Lucas tree. Households of age i receive labor income $\varepsilon_i(1-\theta)z$, where $\varepsilon_i \geq 0$ is the age-specific share of labor income, and $\sum_{i=0}^{I} \varepsilon_i = 1$. In addition, households receive income from the dividends and the sale of shares of the Lucas tree that they might have bought in the previous period. Define a recursive competitive equilibrium. Hint: first determine and clearly state what the aggregate state variables are.