Economics 702, Dirk Krueger

1 The Neoclassical Growth Model with Productive Government Expenditures (Barro 1990)

Consider the neoclassical growth model. A representative firm produces output according to the production function

$$Y_t = F(K_t, N_t, G_t)$$

where G_t is government expenditures. Capital depreciates at a constant rate $\delta \geq 0$. The aggregate resource constraint reads as

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t + G_t$$

where C_t is aggregate consumption

Government expenditures are exogenous and follow the following deterministic process:

$$G_t = \rho G_{t-1}$$

where ρ is a parameter. *Households as well as the social planner take the sequence $\{G_t\}$ as given and beyond their control.* Assume that $G_0 > 0$ is a given initial condition.

There is a large number of identical households with total mass equal to 1. Each household is endowed with k_0 units of capital and one unit of time in every period. The household has preferences over individual consumption streams $\{c_t\}_{t=0}^{\infty}$ and labor streams $\{n_t\}_{t=0}^{\infty}$ representable by the lifetime utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_t, n_t).$$

1. State the social planner problem recursively. Clearly identify the state and control variables.

2. Use the first order condition and the envelope condition to derive the intratemporal and the intertemporal optimality condition (Euler equation) of the social planner problem. You can ignore corner solutions.

3. Let the government levy lump-sum taxes T_t to finance the government expenditures. Assume that the budget is balanced in every period, so that

   $$G_t = T_t$$

   Thus after-tax income is given by $w_t n_t - T_t + r_t k_t$, where w_t is the wage, r_t the rental rate of capital and k_t the capital owned by the household. Define a recursive competitive equilibrium.

4. Use the first order condition and the envelope condition to derive the intratemporal and the intertemporal optimality condition (Euler equation) that the representative household faces. You can ignore corner solutions.

5. Suppose that
$$F(K_t, N_t, G_t) = K_t^{\alpha} N_t^{1-\alpha} G_t^{\phi}$$
For the purpose of this question you can assume that there exists a unique competitive equilibrium. Is this equilibrium Pareto efficient? How does your answer depend on the parameter values for (β, α, ϕ). You don't need to provide a formal proof, but use the answers to 2. and 4. to explain your answer, and give some intuition for your answer.

6. Now suppose instead of a lump-sum tax, the government is restricted to use a proportional labor income tax, such that the budget constraint of the government reads as
$$G_t = \tau_t w_t N_t$$
and household after tax income is given by $(1 - \tau_t) w_t n_t + r_t k_t$. Repeat question 5.