1   Cole's Problem August 2015

Consider the following model of insurance with limited enforcement and complete informa-
tion. There is an isolated village in which each household can grow food. For simplicity
assume that there is an infinite number of these households, all of which are ex ante identi-
cal. A household's production of food in period t is given by y_t and follows an i.i.d. process
in which $y_t \in Y$, and the probability for output is given by $\Pi(y_t)$. We will assume that
$Y = \{y_1, ..., y_N\}$ is a fixed finite set with $y_i < y_{i+1}$ and $y_1 > 0$. For simplicity, assume that
the unconditional mean of output is 1, and note that with an infinite number of villagers
this is the per capita output level of the village with probability one.

Assume that the village suffers from a terrible rat problem, so food is not storable. The
households have preferences of stochastic consumption streams given by

$$E\left\{\sum_{t=0}^{\infty} \beta^t u(c_t)\right\},$$

where u is strictly concave.

Originally the villagers lived a life of isolation and potential desolation in which each
household just ate the food it grew each period. Call this autarky. Then one day a bright
spark (i.e. smart person) had the idea of creating an insurance arrangement in which the
villagers would mutually insure each other. People in the village were at first very skeptical,
but the bright spark pointed out that any villager who violated the agreement could be cast
out and forced back into autarky forever. Discussion immediately went then to figuring out
what was the best possible insurance arrangement.

A) Naturally the starting point was complete insurance. Under what conditions could
complete insurance be sustained as an equilibrium outcome for the villagers? Discuss the
circumstances under which a household might want to violate the insurance arrangement.
Discuss how the curvature of u would factor into your answer. Discuss also how the stochastic
process for y_t factors in.

B) Assume that complete insurance was not possible. Should the villagers given up and
settle for autarky, or is there something better that they could attain? Be sure to fully justify
your answer.

C) Assume that complete insurance was possible. But, before things got started a Bank
from a far away town decided to set up a branch office in the village. Assume that the
branch office would not make any loans, but that the villagers could safely save at the gross
real rate R where $R > 0$ and $\beta R < 1$ to keep things bounded. A meeting of the village was
quickly called to determine whether or not this would affect things and if so how. Please
give your *thoughtful* opinion on this matter.