1 Cole's Problem May 2015

Consider the following private information tax problem. Households are distinguished by their skill level θ_t and produce output according to $y_t = \theta_t l_t$. Skills evolve stochastically over time, and are drawn from a finite set Θ, with some probability Π_Θ on Θ^T. Households learn their skill realization as of time t at the being of period t. Both a household's skill level and their labor effort are private information, but y_t is observable.

An allocation are mappings (consumption) $c_t(\theta^T)$ and (output) $y_t(\theta^T)$, which map to R_+ and are measurable w.r.t. θ^t (and hence cannot depend upon the future). The output requirement implies the household's labor effort is $y_t(\theta^T)/\theta_t$. All households have the same preferences and their payoff, given this notation is

$$E\left\{\sum_{t=0}^{T}\beta^t\left[u(c_t(\theta^T)) - v(y_t(\theta^T)/\theta_t(\theta^T))\right]\right\}$$

This is a small open economy which can borrow and lend at gross interest rate R. So the economy faces the *budget constraint*

$$\sum_{\theta^T\in\Theta^T}\sum_{t=0}^{T}R^{-t}\left[c_t(\theta^T) - y_t(\theta^T)\right]\Pi_\Theta(\theta^T).$$

A) Write down the social planning problem and use it to show what a complete information optimal tax program would look like for a utilitarian planner.

B) Given that productivity and labor effort are private information what additional constraint(s) would we need to place on our planning problem to take account of this?

C) Derive the inverse Euler equation result. Explain what this means for our consumption allocation over time relative to the complete information case.