1 Cole's Problem

Consider a version of our Ramsey tax model in which there are a unit measure of each of two types of household indexed by $i = 1, 2$, which each supply their own specific kind of labor. The household's preferences are given by

$$\sum_t \beta^t u(c_t, 1 - l_{it}),$$

where $1 - l_{it}$ is the level of leisure, and $c_t, l_{it}, 1 - l_{it} \geq 0$. The household's budget constraint is given by

$$\sum_t p_t\{(1 - \tau_{l,i,tt})w_{it}l_{it} + (1 - \tau_{k,t})q_t k_t\}$$
$$= \sum_t p_t \left\{c_t + k_{t+1} - (1 - \delta)k_t\right\}.$$

The firm uses both kinds of labor in production, and it's problem is given by

$$\max \sum p_t \left[f(k_t, l_{1t}, l_{2t}) - q_t k_t - w_{1t}l_{1t} - w_{2t}l_{2t}\right].$$

Note that we have allowed the tax rate on these two types of labor to be different.
A) Define a competitive equilibrium and develop a set of conditions to characterize it.
B) Construct the implementability condition(s) for the government and with it, the government's optimization problem. Be sure to think carefully about the set of conditions you need to ensure that any feasible allocation can be supported as a competitive equilibrium.
C) Derive the optimality conditions that characterize the optimal policy, and try to characterize the labor tax rates and the level of capital taxation in the steady state.
D) Would your steady state capital taxation result be changed if we constrained the labor tax rates to be the same? Be sure to carefully explain your answer.