

**Cole's Problem:** Consider the following monetary model.

*Households:* There is a single representative household with preferences over consumption

( $c$ ) and leisure ( $l$ ) given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t); \quad 0 < \beta < 1.$$

The household supplies capital  $k$  and labor  $n$  to a firm, so that income is  $(1 + r_t)k_t + w_t n_t$ .

The household also brings  $M_t$  units of money into the period. Let  $P_t$  be the price level in period  $t$  and define  $m_t = M_t/P_t$ . The household's budget constraint is then

$$(1 + r_t)k_t + w_t n_t + m_t = c_t + k_{t+1} + m_{t+1}(1 + \pi_{t+1})$$

where  $1 + \pi_{t+1} = P_{t+1}/P_t$ . Money is held to facilitate consumption transactions. The transactions technology is such that  $s_t$  units of time are required to purchase  $c_t$  given real money balances  $m_t$  :

$$s_t = g(c_t, m_t); \quad g_c > 0, \quad g_m < 0.$$

*Firms:* Firms produce output in competitive markets using labor and capital, both of which are hired from the households on spot markets. Firms have access to a standard CRTS production function  $f(k_t, n_t)$ . The depreciation rate is  $0 < \delta < 1$ . (I'm assuming that depreciation is netted out of firm output.)

*Questions:*

- A. Define the households problem and characterize it's solution.
- B. Define a competitive equilibrium. Assume that the money supply is constant. What is the system of equations that characterize this equilibrium.
- C. Is money neutral in this economy? Prove your answer using the system of equation that define a competitive equilibrium.

D. Would money still be neutral if the transactions technology used nominal money balances i.e.,  $s_t = g(c_t, M_t)$ ? Explain the intuition for your result.