

Cole's Problem:

Consider the following stochastic optimal labor-tax model. There is a representative household with preferences given by

$$E \left\{ \sum_t \beta^t [\alpha \log(c) + (1 - \alpha) \log(1 - l)] \right\}$$

over consumption (c) and labor (l). Households produce output with labor every period, and their productivity level can be either high or low. One unit of labor produces $z(s)$ units of output, where

$$s \in \{h, l\}, \text{ and } z(h) > z(l).$$

We will assume that states s are i.i.d. over time. There is a government which chooses a sequence of tax rates on output in order to finance a fixed level of government spending g in every period. There is no way to transfer physical goods between periods - that is, there is no capital or storage technology.

Let $s^t = \{s_1, \dots, s_t\}$ denote the realized sequence of shocks. The standard constraints are given as follows.

The resource constraint:

$$z(s^t)l(s^t) = c(s^t) + g(s^t),$$

the household's budget constraint:

$$c(s^t) + b(s^t) \leq (1 - \tau(s^t))z(s^t)l(s^t) + R(s^t)b(s^{t-1}),$$

the government's budget constraint:

$$g + R(s^t)b(s^{t-1}) \leq \tau(s^t)z(s^t)l(s^t) + b(s^t),$$

where R denotes the gross return on government debt b .

Cole's Problem continued:

a) Define the household's problem. Define a competitive equilibrium. Discuss whether markets are complete in this economy given that we only have a single bond being traded.

b) Show that if the household's budget constraint and the resource constraint hold, then the government's budget constraint holds. Derive the conditions that characterize a solution to the household's problem. Show how these conditions can be used to determine the only prices in the model - $R(s^t)$ - and the only policy parameter - τ - in terms of the allocation. Use these results to derive a set of conditions involving only physical quantities (except initial debts R_0b_0) which characterize the set of allocations which are consistent with there being an associated competitive equilibrium.

c) Construct the Ramsey problem for this economy and use it to characterize the optimal allocation that is consistent with there being a competitive equilibrium for this economy. How does this allocation depend upon R_0b_0 ?

d) Assume that the probability of the good shock ($s = h$) was π and that of the bad shock ($s = b$) was $1 - \pi$. Does the optimal allocation change as we change π , and if so how? To what extent does this optimal allocation depend upon the past history of shocks - that is, does $[c(s^t), l(s^t)]$ depend upon s_t, s_{t-1}, \dots ? What would happen if good and bad states became persistent?