

Econ 706 Prelim
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Good luck!

1. Consider linear filtering as routinely used in econometric time series analysis. In particular, consider filtering strong white noise by the linear filter $1/(1 - .9L)$.
 - Characterize both the input and the output in the time and frequency domains.
 - Call the output y . Calculate the optimal (linear least squares) 2-step-ahead prediction of y using both the Wold approach and the Wiener-Komogorov approach. Do they agree? Why or why not?
 - Calculate the variance of y and the variance of the 2-step-ahead linear least squares prediction error of y . Which is smaller? Why?
2. Consider a first-order two-state Markovian dynamic for state transitions, as often invoked in macro-econometric models involving regime-switching. In particular, consider the transition probability matrix:

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Define, and then verify and/or calculate:

- Validity of the transition probability matrix
 - The Chapman-Kolmogorov theorem
 - Communication and reducibility
 - First and eventual transition probabilities
 - Recurrence / transience
 - Stationary probabilities
 - Time reversibility.
3. Consider simulating a realization of length N governed by (unconditional) probability density f , by the simple accept-reject method and by a more complicated accept-reject method, random-walk Metropolis-Hastings.
 - Describe each method precisely.
 - For each method, discuss whether and why it generates pseudo-independent draws. Why care?
 - For each method, discuss whether and why it requires the ability to evaluate f . Why care?

4. Consider the process with *iid* innovations:

$$\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} L \right) \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} = \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

– Write the system in multivariate Dickey-Fuller form. Need the system be integrated for the multivariate Dickey-Fuller form to exist? *Is* the system integrated?

– What is the key condition for cointegration based on your multivariate Dickey-Fuller representation? Is the system cointegrated? If so, what is the common trend?

– Does an “error-correction” representation exist? Why or why not? If so, display it, and display the “errors” that get “corrected.”