Microeconomic Theory II
Preliminary Examination
June 7, 2010

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means proving that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

1. **(20 points)** Given a fixed gamble $\tilde{x}$ (which is a random variable) and a twice continuously differentiable and increasing utility function $u : \mathbb{R} \to \mathbb{R}$, the *sale price* $s \in \mathbb{R}$ of the gamble is defined by
$$\mathbf{E}\left[u(\tilde{x})\right] = u(s).$$
Thus, s is the minimum amount of money the person with the utility function u must be given in order to induce him to give up the gamble $\tilde{x}$. If instead he starts with zero money and no gamble, then the maximum amount he would be willing to pay for the gamble is its *buy price* $b \in \mathbb{R}$, defined by
$$\mathbf{E}\left[u(\tilde{x} - b)\right] = u(0).$$
   (a) Show first that if u exhibits constant absolute risk aversion (CARA), i.e., the coefficient of absolute risk aversion is a constant $\lambda > 0$, then u can be written as
$$u(x) = -e^{-\lambda x}$$
up to an affine transformation. Then show that CARA implies $b = s$. [10 points]

   (b) What is the relationship between b and s if u exhibits strictly decreasing absolute risk aversion, and why? [10 points]

2. **(35 points)** Two firms are competing in the market for widgets. The output of the two firms are perfect substitutes, so that the demand curve is given by $Q = \max\{\alpha - p,\ 0\}$, where p is low price. Firm i has constant marginal cost of production $0 < c_i < \alpha$, and no capacity constraints. Firms simultaneously announce prices, and the lowest pricing firm has sales equal to total market demand. The division of the market in the event of a tie (i.e., both firms announcing the same price) depends upon their costs: if firms have equal costs, then the market demand is evenly split between the two firms; if firms have different costs, the lowest cost firm has sales equal to total market demand, and the high cost firm has no sales.

   (a) Suppose $c_1 = c_2 \equiv c$ (i.e., the two firms have identical costs). Restricting attention to pure strategies, prove that there is a unique Nash equilibrium. What is it? What are firm profits in this equilibrium? [10 points]

   (b) Suppose $c_1 < c_2 < (\alpha + c_1)/2$. Still restricting attention to pure strategies, describe the set of Nash equilibria. Are any in weakly dominated strategies? [10 points]

(c) We now add an investment stage before the pricing game. At the start of the game, both firms have identical costs of c_H, but before the firms announce prices, firm 1 has the opportunity to invest in a technology that gives a lower unit cost c_L of production (where $c_L < c_H < (\alpha + c_L)/2$). This technology requires an investment of k, where $0 < k$. The acquisition of the technology is *public* before the pricing game subgame is played.

Describe the extensive form of the game. Describe a subgame perfect equilibrium in which firm 1 acquires the technology (as usual, make clear any assumptions you need to make on the parameters). Is there a subgame perfect equilibrium in which firm 1 does not acquire the technology? If not, why not? If there is, compare to the equilibrium in which firm 1 acquires the technology. [15 points]

3. **(50 points)** A worker is engaged in a long-term employment relationship with a firm in a world with no enforceable contracts. In each period, the firm chooses a wage $w \geq 0$ to pay the worker and the worker decides whether to exert effort, E, or shirk, S (these choices are made simultaneously). The worker incurs a disutility of e by exerting effort (relative to shirking). Effort choice is not observable, but does result in the production of some random *public* output $y \in \{0, \bar{y}\}$. The output y is distributed according to the distribution

$$\Pr\{\bar{y} \mid a\} = \begin{cases} p, & \text{if } a = E, \\ q, & \text{if } a = S, \end{cases}$$

where a is the worker's decision on whether to exert effort and $0 < q < p < 1$. Both worker and firm are risk neutral, with firm flow payoffs given by $y - w$ and worker flow payoffs by $w - e$ if E and w if S. The relationship has an infinite horizon, and both worker and firm discount the future using the discount factor $\delta \in (0, 1)$. Suppose $p\bar{y} - e > q\bar{y}$.

(a) Is there a perfect public equilibrium in which the worker never exerts effort? Why or why not? [5 points]

(b) Describe a perfect public equilibrium in which the worker is hired and exerts effort in every period and the wage paid depends only the last period output (make explicit any necessary bounds on model parameters). [20 points]

(c) Suppose now in each period, the firm first chooses a wage offer (including no offer) to make to the worker and the worker can accept or reject an offered wage. If the worker accepts an offered wage, she then chooses whether to exert effort. If either the firm decides not to make an offer to the worker, or the worker declines the offer, then the relationship ends with the firm receiving a value of 0, and the worker receiving a reservation value of w_0. Suppose $p\bar{y} - e > w_0 > q\bar{y}$.

i. Is there a perfect public equilibrium in which the worker never exerts effort? Why or why not? [5 points]

ii. How does your answer to parts 3(b) change? Will the worker be always indifferent between accepting or rejecting the equilibrium wage offers? Can the worker be indifferent between working and shirking? [20 points]

Question 4 is on the next page.

4. **(15 points)** Consider the game displayed in Figure 1.

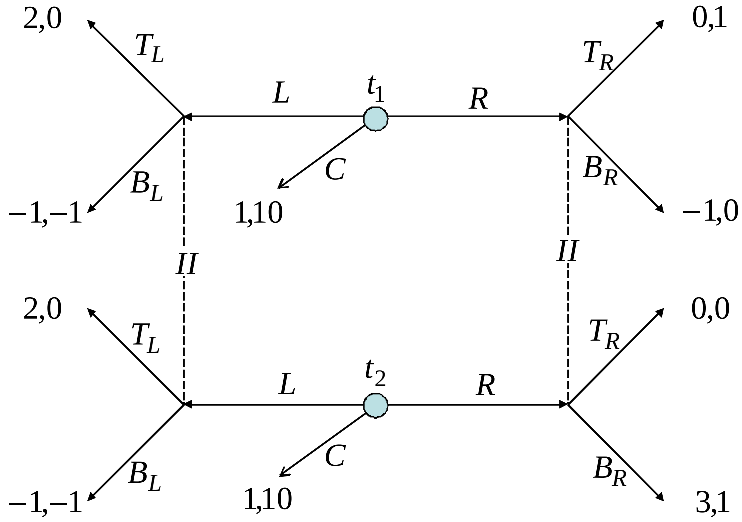

Figure 1: The game for question 4. The two types, t_1 and t_2, of player I are equally likely. Player I's payoffs are listed first in each terminal node.

(a) Is there a Bayes-Nash equilibrium in which both types of player I choose C? Is it perfect Bayes? [5 points]

(b) Describe all the pure strategy perfect Bayes equilibria. [5 points]

(c) Are all the pure strategy perfect Bayes equilibria equally plausible? Why or why not? [5 points]