Microeconomic Theory II
Preliminary Examination
August 16, 2010

The exam is worth 120 points in total.

There are 3 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means proving that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

1. **(20 points)** Consider a competitive firm with a single output production function $f(x)$, where $x \in \mathbb{R}_+^K$ is the amount of $K > 1$ goods used as inputs. The price of output is one, and the price vector for the inputs is $w \in \mathbb{R}_{++}^K$, so the total cost of input is $w \cdot x$. The firm is under a budget constraint so that there is a maximum amount B on the total cost of inputs.

   Suppose f is continuous and satisfies the following "local productiveness condition": For all $x = (x_1, ..., x_K)$ and $\varepsilon > 0$, there is some $x' = (x_1', \ldots, x_K')$, such that $\max_{k=1,\ldots,K} |x_k' - x_k| < \varepsilon$ and $f(x') > f(x)$. Assume the firm maximizes output, let $y^*(w, B)$ denote this output, i.e.,

   $$y^*(w, B) = \sup\{f(x) : w \cdot x \leq B\}.$$

   (a) Show that there exists some $x^* \in \mathbb{R}_+^K$ such that $y^*(w, B) = f(x^*)$ and $w \cdot x^* = B$, i.e., the budget constraint is binding. [10 points]

   (b) Show that $y^*(w, B)$ is quasi-convex in (w, B) (note that we have *not* assumed that f is concave). [10 points]

2. **(30 points)** A long-lived player, player 1 (the row player) plays the following game against a sequence of short-lived players, the column player:

	L	R
T	$2, 0$	$3, 1$
B	$0, 2$	$4, 0$

   The long-lived player's payoffs are the average discounted value, with discount factor δ. Each short-lived player lives only one period and receives a payoff only from the period in which he plays. Histories are public, so that the short-lived player in period t knows the entire history of actions up to t (the game has perfect monitoring).

   (a) Describe a strategy profile that yields a payoff of 3 to the long-lived player, and uses Nash reversion to punish deviations. For what values of the discount factor is this profile a subgame perfect equilibrium? [15 points]

   (b) Describe a simple pure-strategy profile that yields a payoff of 3 to the long-lived player and which is a subgame perfect equilibrium for lower discount factors than the bound identified in part 2(a). [15 points]

Question 3 is on the next page.

3. **(70 points)** Consider a firm making wage offers to a worker over time. There are two periods, and the worker can accept a wage offer in both, either, or neither periods. Both worker and firm discount at rate $\delta \in (0,1)$. Denote the worker's reservation wage by $r > 0$, so that the worker's payoff is given by

$$e_1(w_1 - r) + \delta e_2(w_2 - r),$$

where w_t is the wage offered in period t, and $e_t = 1$ if the worker accepts the wage offer in period t and $e_t = 0$ otherwise. Similarly, the firm's payoff is

$$e_1(A - w_1) + \delta e_2(A - w_2).$$

The value of A is common knowledge. There is incomplete information about the worker's reservation wage.

The game is as follows: In the first period, the firm announces a wage $w_1 \in \mathbb{R}_+$. Nature then determines the worker's reservation wage (type) according to the probability distribution that assigns probability $\frac{1}{2}$ to $r = r_H$ and probability $\frac{1}{2}$ to $r = r_L$. The worker's reservation wage is the same in both periods. The worker learns his type (but the firm does not) and then decides whether to accept the wage offer or not. In the second period, the firm again announces a wage offer $w_2 \in \mathbb{R}_+$ (knowing whether the worker had accepted or not in the first period) and the worker then decides whether to accept or not. (Nature moves after the firm so that each first period wage offer w_1 leads to a true subgame.)

Assume $r_H - r_L > A - r_H > 0$. Restrict attention to pure strategies.

(a) Describe the worker's and firm's (extensive form) strategies of the two period game. What conditions should a pure strategy profile satisfy in order to be a perfect Bayesian equilibrium? [20 points]

(b) Consider a subgame starting at $w_1 \in [(1-\delta)r_L + \delta r_H, r_H]$. Describe a separating perfect Bayesian equilibrium of this subgame (naturally, the separation occurs in the first period). [15 points]

(c) Consider now subgames starting at $w_1 \in [(1-\delta)r_L + \delta r_H, r_H)$. Note that now $w_1 \neq r_H$. Is there a pooling equilibrium in these subgames (i.e., an equilibrium in which both types of worker behave the same way in period 1)? **Hint:** remember that $r_H - r_L > A - r_H$. [15 points]

(d) Suppose $w_1 \geq r_H$. Describe the pooling perfect Bayesian equilibrium in this subgame. Why is this the unique pure strategy equilibrium when $w_1 > r_H$? [10 points]

(e) Suppose that the worker always accepts when indifferent. Suppose there is a minimum wage floor $\overline{w} = (1-\delta)r_L + \delta r_H$ on first period wages. What is the first period equilibrium wage $w_1 \geq \overline{w}$? [10 points]