Microeconomic Theory II
Preliminary Examination
August 5, 2013

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and proving that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. **(25 points)**A consumer has wealth w that she must consume over two periods. In period 2 she has a random income shock, $\theta\tilde{y}$, where $\theta > 0$, $\mathbb{E}\tilde{y} = 0$, and $\mathbb{E}\tilde{y}^2 > 0$. Her expected utility when she chooses to save an amount x is

$$u(w - x) + \mathbb{E}v(x + \theta\tilde{y}).$$

She can save any amount, i.e., x can be any real number. The functions u and v are C^3, with strictly positive first derivatives and strictly negative second derivatives. Let $x^*(w, \theta)$ denote her optimal savings function.

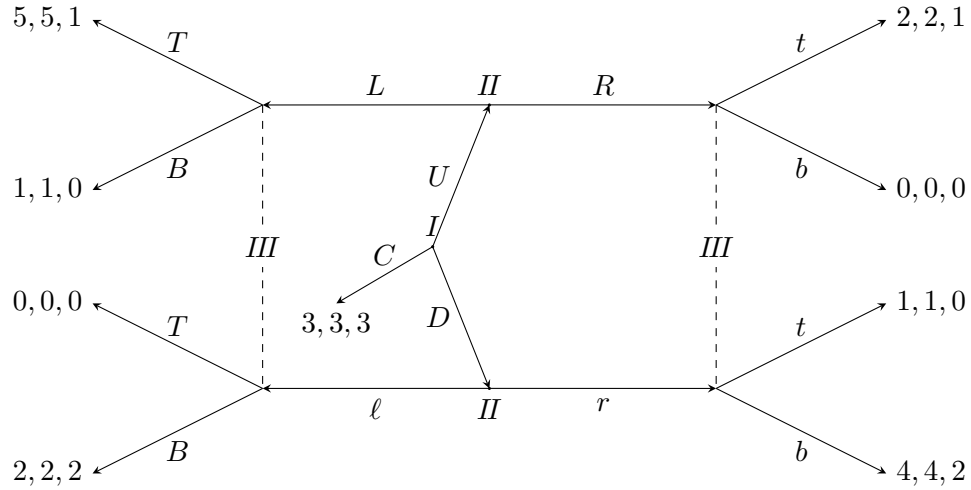
   (a) Does x^* increase or decrease in w, or can it do either depending on the utility functions? Prove your answer. **[5 points]**

   (b) Show that x^* strictly increases in θ if v exhibits nonincreasing absolute risk aversion (NIARA). **[20 points]**

2. **(20 points)** Players 1 and 2 are bargaining over \$1 using the following bargaining procedure. In each period, a biased coin determines who makes the offer, with player i selected with probability $p_i \in (0, 1)$, $p_1 + p_2 = 1$. If the offer is accepted, the offer is implemented and the game is over. If the offer is rejected, play proceeds to the next period, until agreement is reached. Players have the same discount factor $\delta \in (0, 1)$. Suppose each player is risk neutral (so that player i's utility from receiving x is given by $u_i(x) = x$). Describe a stationary subgame perfect equilibrium. Are there any other stationary subgame perfect equilibria? Describe its comparative statics (with respect to p_i, particularly near the boundaries) and provide some intuition. **[20 points]**

Question 3 is on the next page.

3. **(35 points)** Consider the following *complete information* game:



(a) There is a perfect Bayesian equilibrium in pure strategies in which player I plays C. What is it? Remember to give a complete description of the strategy profile and associated beliefs, and to prove that the profile has the required properties. **[10 points]**

(b) Suppose now that player I plays for player II as well (so that only players I and III play, with player I first choosing from $\{L, R\}$ if he had chosen U, and choosing from $\{\ell, r\}$ if he had chosen D; note that player I and II receive the same payoff at each terminal node). How does this change your answer to part 3(a)? **[5 points]**

(c) What is the (reduced) normal form of the game in part 3(b)? What is the result of iteratively deleting dominated strategies? **[10 points]**

(d) What is the result of iteratively deleting dominated strategies in the game in part 3(a)? Compare your answer to that in part 3(c). **[10 points]**

Question 4 is on the next page.

4. **(40 points)** A prospector owns a gold mine where he can dig to recover gold. His output depends on the amount of gold in the mine, denoted by x. The prospector knows the value of $x \in \{1, 3\}$. The prospector can either sell the mine to a large mining company (which is more efficient in its mining techniques) or mine it himself. The mining company does not know the value of x, assigning equal probability to $x = 1$ and $x = 3$. If the prospector mines himself, his payoff is

$$3x.$$

If the prospector sells the mine to the company at a price p, his payoff is p, while the company's payoff is

$$4x - p.$$

(a) Suppose the company makes an offer of p to the prospector, which the prospector must accept or reject (with no possibility of a counteroffer, i.e., the company's offer is take-it-or-leave-it). What are the pure strategy perfect Bayes equilibria of this game? **[10 points]**

(b) Suppose now that it is the prospector that makes the take-it-or-leave-it offer of p to the company. What are the pure strategy perfect Bayes equilibria of this game? Compare your answer to that in part 4(a). **[15 points]**

After further prospecting, the prospector discovers that he can sell off part of the mine without affecting the extraction from any part of the mine. Denote by $\lambda \in [0, 1]$ the fraction of the mine the prospector sells. If the prospector sells λ at a price p, his payoff is

$$(1 - \lambda)3x + p,$$

while the mining company's payoff is

$$\lambda 4x - p.$$

(c) Suppose the prospector makes an offer of (λ, p) to the company, which the company must accept or reject (with no possibility of a counteroffer, i.e., the prospector's offer is take-it-or-leave-it). What are the pure strategy perfect Bayes equilibria of this game? **[15 points]**