Microeconomic Theory II
Preliminary Examination
August 3, 2015

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and **proving** that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. Two firms compete in the market for widgets. The market price is given by $A - q_1 - q_2$, where $A > 0$ is a constant and q_i is the quantity of widgets produced by firm i, $i = 1, 2$. There are no costs.

   (a) Suppose the firms choose quantities simultaneously. What is the Nash equilibrium? **[5 points]**

   Suppose now firm 2 can invest in an advertising campaign that will increase demand. In particular by investing $I > 0$, the firm can cause the demand intercept to increase from A to $A' > A$. Firm 2's decision on undertaking the advertising campaign is made publicly before the firms choose quantities.

   (b) Describe the extensive form and the appropriate equilibrium concept. For what values of I while firm 2 undertake the campaign in equilibrium? **[10 points]**

   Suppose I is such that firm 2 would undertake the campaign in equilibrium, as identified in part 1(b).

   (c) Suppose firm 2 has the option of delaying the decision on the campaign, and making it simultaneously with her quantity decision (so that firm 1 must make his quantity decision not observing firm 2's decision). Delaying however, means that the cost of the advertising campaign has increased from I to $I^D > I$. Describe the extensive form and the appropriate equilibrium concept. For what values of I^D would firm 2 delay the decision and then undertake the ad campaign? **[15 points]**

Question 2 is on the next page.

2. **[30 points]** Two partners in a law firm (with the surprising names 1 and 2) wish to recruit a law student to their firm. Either partner can ensure the student will accept the offer by calling the student and promoting the firm. However, it is costly to make this call. Assume the payoffs can be represented as follows:

	C	D
C	v_1-1, v_2-1	v_1-1, v_2
D	v_1, v_2-1	$0,0$

where Y means "call," N means "don't call," and v_i is partner i's benefit from hiring the student, $i = 1,2$. The cost of making the call is the same for both partners. Assume each partner chooses his/her action without learning the other's action.

(a) Assume partner 1's benefit v_1 is common knowledge, and $v_1 > 2$. Partner 2's benefit $v_2 \in \{\underline{v}, \overline{v}\}$ is partner 2's private information. Partner 1 assigns probability p to partner 2's benefit being $\underline{v}$ and the remaining probability to $\overline{v}$. This probability assignment is common knowledge. Assume that $0 < \underline{v} < 1 < \overline{v}$ and that $p > \frac{1}{2}$.

i. Show that partner 2 will not call in any equilibrium when his/her benefit is $\underline{v}$. **[5 points]**

ii. Show that partner 1 will call in any Bayesian Nash equilibrium. **[5 points]**

iii. Find the unique Bayesian Nash equilibrium of this game. **[5 points]**

(b) Assume v_1 and v_2 are independently drawn from a uniform distribution on $[0,2]$. Denote partner i's pure strategy as a measurable function $\sigma_i : [0,2] \to \{C, D\}$.

i. Show that the following is true in any pure strategy Bayesian Nash equilibrium (σ_1, σ_2): For any $v_i \in (0,2]$, if $\sigma_i(v_i) = C$, then $\sigma_i\left(v_i'\right) = C$ for any $v_i' > v_i$. **[5 points]**

ii. A symmetric pure strategy Bayesian Nash equilibrium (σ_1, σ_2) takes the following form: for some constant $v^* \in [0,2]$,

$$\sigma_i(v_i) = \begin{cases} C, & \text{if } v_i \geq v^*, \\ D, & \text{otherwise.} \end{cases}$$

Find v^*. **[10 points]**

Question 3 is on the next page.

3. **[35 points]** Suppose that the payoff to a firm from hiring a worker of type θ with education e at wage w is $\theta - w$.

   The utility of a worker of type θ with education e receiving a wage w is $w - c(e,\theta)$, where $c(e,\theta) = \frac{e^2}{2\theta}$.

   The worker's ability is privately known by the worker. There are at least two firms. The worker (knowing his ability) first chooses an education level $e \in \mathbb{R}_+$; firms then compete for the worker by simultaneously announcing a wage; finally the worker chooses a firm. Treat the wage determination as in class, a function $w : \mathbb{R}_+ \to \mathbb{R}_+$ determining wage as a function of education.

   The set of possible types for a worker are $\Theta = \{\theta_L, \theta_H\}$ where $\theta_L = 1$ and $\theta_H = 2$. All firms believe that either type is equally likely.

   (a) What is the set of all education levels that can be supported as a *pooling* PBE (Perfect Bayesian Equilibrium)? **[10 points]**

   (b) Describe carefully the equilibrium corresponding to the *highest* education level that can be supported as a pooling PBE. **[4 points]**

   (c) What is the set of all education levels for type θ_H that can be supported as a *separating* PBE? **[10 points]**

   (d) Describe carefully the equilibrium corresponding to the *highest* education level for type θ_H that can be supported as a separating PBE. **[4 points]**

   (e) Define the Intuitive Criterion refinement for signaling games. Argue that the equilibrium described in the previous part does not survive this refinement. Which separating equilibrium does survive? **[7 points]**

Question 4 is on the next page.

4. **[25 points]** A seller of a divisible good can manufacture any quantity $q \in \Re_+$ of the good on demand with no fixed costs and a *marginal* cost $M(q) = cq$, where c is publicly known.

   There is a single buyer interested in the good who has constant marginal value v, privately known to her. In other words, if this buyer purchases q units at price p, she has net utility $vq - p$.

   By the revelation principle, we consider direct revelation mechanisms where the seller offers quantity $q(v)$ at price $p(v)$ to a buyer who reports marginal value v. The seller believes the buyer's marginal value to be distributed on interval $[\underline{v}, \overline{v}]$ with pdf $f(\cdot)$, cdf $F(\cdot)$.

   (a) Derive conditions on $q(v), p(v)$ so that it is incentive compatible and individually rational for a buyer of any value v to truthfully report their value to the seller. **[8 points]**

   (b) Derive the quantity schedule offered in an efficient mechanism, i.e. the $q(\cdot)$ offered in an IC, IR mechanism that maximizes expected gains from trade. **[5 points]**

   (c) Derive the quantity schedule offered in the profit maximizing mechanism for the seller, i.e. the $q(\cdot)$ offered in an IC, IR mechanism that maximizes the expectation of the buyer's payment to the seller less the cost of production. Be clear about any assumptions you need to make. **[12 points]**