Microeconomic Theory II
Preliminary Examination
University of Pennsylvania

August 13, 2012

The exam is worth 120 points in total.

There are 4 questions. Do all questions.

Start each question in a new book, clearly labeled.

Fully justify all answers and show all work (in particular, describing an equilibrium means providing a full description and proving that it has the desired properties).

Label all diagrams clearly.

Write legibly.

If you need to make additional assumptions, state them clearly.

Good luck!

1. **(20 points)** Mr. A has a complete and transitive preference ordering $\succeq_A$ over monetary lotteries $\tilde{x}$ that is monotone in the following sense: $\delta_x \succ_A \delta_y$ for all $x > y$, where δ_x and δ_y are the degenerate lotteries that put probability one on the amounts x and y, respectively.

   (a) (1 pt) Define what it means for Mr. A to be strictly risk averse.

   (b) (2 pts) Define Mr. A's certainty equivalent c_A for a given non-degenerate lottery $\tilde{x}$.

   (c) (3 pts) Assume Mr. A is strictly risk averse. For the $\tilde{x}$ and c_A from (b), what can you say about the relationship between c_A and $\mathbb{E}\tilde{x}$? Prove your answer.

   Now assume Mr. A has a Bernoulli utility function, u_A. Ms. B also has a Bernoulli utility function, u_B. Both functions are twice differentiable, with $u'_A(x) > 0$ and $u'_B(x) > 0$ for all $x \in \mathbb{R}$.

   (d) (1 pts) Define their Arrow-Pratt coefficients of absolute risk aversion, $R_i(x)$ for $i = A, B$.

   (e) (13 pts) Assume Mr. A is more risk averse than Ms. B, in the sense that $R_A(x) > R_B(x)$ for all $x \in \mathbb{R}$. Letting $\tilde{x}$ be a non-degenerate lottery, and c_A and c_B be their certainty equivalents for it, prove that $c_A < c_B$.

2. **(40 points)** Sheila would like to hire Bruce to undertake a task which she values at $v > 0$. Bruce incurs a disutility of $c > 0$ in completing the task. Suppose the nature of the bargaining between Bruce and Sheila is that Sheila can make a take-it-or-leave-it offer of a wage $w \in \mathbb{R}_+$ to Bruce, which Bruce can accept or reject. If an offer of w is accepted, Sheila's payoff is $v - w$ and Bruce's payoff is $w - c$. If Sheila does not make an offer (which without loss of generality, we can treat as offering a wage of 0), or Bruce rejects the offer, then both receive a payoff of 0.

   (a) Suppose $v \neq c$. Describe the subgame perfect pure strategy equilibria of this game. (Your answer will depend on the parameters v and c.) [5 points]

   For the remainder of the question, set $v = 5$ and $c = 4$. Bruce can undertake an investment that incurs a cost of 1. This investment reduces Bruce's disutility of the task to 2.

(b) Suppose that Bruce can only undertake this investment *after* he has accepted Sheila's offer. What is the extensive form of the game (a clearly labelled diagram is sufficient; as usual, be sure to include the payoffs at the terminal nodes)? Describe the unique subgame perfect equilibrium of this game. Why is it unique? [10 points]

(c) Suppose now that Bruce can only undertake this investment *before* Sheila makes her offer to Bruce, and that the investment is *public*, i.e., Sheila knows whether the investment has been undertaken before making a wage offer. What is the extensive form of the game (again, a clearly labelled diagram is sufficient)? Describe the unique subgame perfect equilibrium of this game. [10 points]

(d) Finally, suppose that Bruce can only undertake this investment *before* Sheila makes her offer to Bruce, but that the investment is *private*, i.e., Sheila does not know whether the investment has been undertaken before making a wage offer.

   i. Prove that there are two wages, w and w', such that in any subgame perfect (pure strategy or not) equilibrium, Sheila either offers w or w'. What are the values of w and w'? [**Hint:** The analysis in part 2(c) is helpful in determining w and w'. As a first step, prove that Sheila will never offer a wage that is rejected for sure by Bruce. What is the resulting lower bound on equilibrium wage offers?] [10 points]

   ii. Given the analysis of part 2((d))i, we can analyze the game as follows: Sheila chooses between wages w and w' not knowing of Bruce's investment choice. For the purposes of this question, **assume** that, conditional on Bruce's investment and wage offer, Bruce accepts when indifferent. This yields a 2×2 matrix game. What is it, and what are the equilibria? [5 points]

Question 3 is on the next page.

3. **(30 points)** Consider the following repeated prisoners' dilemma: In each period, player i chooses an action $a_i \in \{E, S\}$ and payoffs are

	E	S
E	$2, 2$	$-1, 8$
S	$8, -1$	$0, 0$

.

The common discount factor is denoted by $\delta < 1$ and payoffs in the repeated game are evaluated using average discounted values.

(a) Suppose the infinitely repeated game has perfect monitoring. Construct a two state automaton that for large δ is a pure strategy subgame perfect equilibrium, and yields an average discounted payoff to each player larger than 3. Prove that the automaton has the desired properties. [10 points]

(b) Suppose now the infinitely repeated game has imperfect public monitoring (as usual, the payoff matrix describes the ex ante payoffs to the players). A player's action choices are not observed by the other player. There is a public signal $y \in \{\underline{y}, \bar{y}\}$ of the chosen action profile, whose realization is determined by the distribution

$$\Pr\{\bar{y} \mid a_1 a_2\} = \begin{cases} p, & \text{if } a_1 a_2 = EE, \\ q, & \text{if } a_1 a_2 \neq EE, \end{cases}$$

with $0 < q < p < 1$.

i. Can a player receive a payoff greater than 2 in any pure strategy perfect public equilibrium? Why or why not? [10 points]

ii. If grim trigger (begin in EE, and continue with EE till the first observation of $\underline{y}$, then play SS forever) is a perfect public equilibrium, it implies strictly positive payoffs to both players. Characterize the values of the parameters p, q, and δ under which grim trigger is a pure strategy perfect public equilibrium. [10 points]

Question 4 is on the next page.

4. **(30 points)** Agents 1 and 2 are working together in a partnership. If the agents exert effort e_1 and e_2, then the total output produced is

$$y = 2\alpha(e_1 + e_2) - 2e_1e_2,$$

where $\alpha > 0$ is a technological parameter determining marginal productivity of effort, and the negative term e_1e_2 reflects a negative productive externality between the two agents. The output is evenly divided between the two agents. The cost of effort to agent i is e_i^2. Thus, the payoff to agent i is given by

$$u_i(\alpha, e_1, e_2) = \alpha(e_1 + e_2) - e_1e_2 - e_i^2.$$

The productivity of effort α is uncertain. Nature first determines α, assigning probability p to $\alpha = \alpha_L$ and complementary probability $1-p$ to $\alpha = \alpha_H > \alpha_L$. Agent 1, knowing α, first chooses her effort e_1 from the interval $[0, 2]$. Agent 2, after observing agent 1's effort choice e_1 (but not observing α), then chooses his effort e_2 from the interval $[0, 2]$.

(a) Define a pure strategy profile for this game. Define a pure strategy perfect Bayesian equilibrium (PBE) (if you prefer, interpret PBE as almost perfect Bayesian equilibrium). Be precise in the restrictions imposed on behavior. [5 points]

(b) What further restrictions are implied by the requirement that a PBE be separating? In particular, is the behavior of any type of agent 1 pinned down in a separating PBE? If so, which one and how? [9 points]

(c) Suppose $\alpha_L = 3$ and $\alpha_H = 4$. There is a separating PBE in which agent 1 chooses $e_1 = \frac{3}{2}$ when $\alpha = 4$. Describe it and prove it is a separating PBE. [8 points]

(d) Does the equilibrium from part 4(c) pass the intuitive criterion? Why or why not? If not, describe a separating PBE that does (you do not need prove that the new PBE passes the intuitive criterion). [8 points]