Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

August 2, 2015

Instructions

This exam has 4 questions and a total of 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly if you want partial credit.

Good luck!

1. (25 pts) Each day a worker consumes leisure, ℓ, and income, x, and has a strictly increasing utility function $u(\ell, x)$. Leisure is measured in hours. She works for the remaining $L = 24 - \ell$ hours, at wage w. The income she consumes must thus satisfy $x \leq wL$. She has differentiable demand functions.

   (a) (15 pts) Prove that if leisure is an inferior good for this consumer, then her labor supply curve must be upward sloping, i.e., $\widehat{L}'(w) \geq 0$.

   (b) (10 pts) Prove that if leisure is a normal good for this consumer, then her labor supply curve might slope down. In particular, show that $\widehat{L}'(w) < 0$ is possible even if her utility function is increasing and quasiconcave.

2. (25 pts) In this problem you first prove a result comparing the riskiness of gambles, and then a result comparing the value of information to different decision makers.

   (a) (10 pts) Let $\hat{x} < x < y < \hat{y}$ and $p \in (0, 1)$. Let g be the gamble yielding x with probability p and y with probability $1 - p$. Similarly, let $\hat{g}$ be the gamble yielding $\hat{x}$ with probability p and $\hat{y}$ with probability $1-p$. Assume g and $\hat{g}$ have the same expected value. Rigorously prove the following.

   Lemma. Any risk averse expected utility maximizer strictly prefers g to $\hat{g}$. (Assume the EU maximizer has a strictly increasing, strictly concave C^1 utility function, $v : \mathbb{R} \to \mathbb{R}$.)

   (b) (15 pts) Axel is a newsboy who must choose whether to open his newsstand today. His profit will be 0 if he keeps it closed. If he opens it, his profit will be the random variable

   $$\pi = \begin{cases} -15 & \text{with probability } p \\ 35 & \text{with probability } 1 - p \end{cases},$$

   where $p \in (0, 1)$. Axel is risk neutral, and so maximizes expected profit.

   Before Axel decides whether to open his newsstand, he is able to purchase perfect information about whether his profit from opening will be $\pi = -15$ or $\pi = 35$. Let I_A be the maximum amount Axel is willing to pay for this information.

   Barb also owns a newsstand. She faces the same environment as Axel. Barb, however, is risk averse: she has a C^1 Bernoulli utility function for money, v, which is strictly increasing and strictly concave. She too can choose to purchase the perfect information about π. Let I_B be the maximum amount Barb is willing to pay for this information.

   Recall that $p_B \in (0, 1)$ exists such that, if she does not acquire the information, Barb's optimal decision is to open her newsstand iff $p < p_B$. Assuming $p < p_B$, show that $I_B > I_A$.

   (**Hint:** consider using the Lemma in (a), even if you were unable to prove it.)

3. (25 pts) Consider an exchange economy with three agents and three commodities. The initial endowments for the three agents are $\omega_1 = (1, 1, 0)$, $\omega_2 = (1, 0, 1)$, and $\omega_3 = (0, 1, 1)$. The agents have the same utility function, $U(x, y, z) = xyz$.

   (a) (6 pts) Show that any interior core allocation, $w = (w^1, w^2, w^3)$, of this economy must have the form $w^1 = (a, a, a)$, $w^2 = (b, b, b)$, and $w^3 = (c, c, c)$. That is, each agent's consumption bundle in a core allocation must have equal quantities of the three commodities.

(b) (6 pts) Show that there is no core allocation in which some agent receives nothing. That is, $w^i = (0, 0, 0)$ is not part of a core allocation. (**Hint:** the answer to (a) is useful here.)

(c) (6 pts) Part (b) only shows that no agent can get 0 of each commodity in a core allocation. Are there boundary core allocations in which an agent gets a positive quantity of some commodity and 0 of another? Explain.

(d) (7 pts) What is the maximum quantity agent 3 can get of the commodities in any core allocation in which agents 1 and 2 get the same bundle, (a, a, a)?

4. (25 pts) Consider an economy with one private good and one public good. The public good can be produced from the private good according to the production function $y = f(x) = x$, where y is the public good that can be produced using x units of the private good. There are n agents. Agent h's initial endowment is $(e^h, 0)$. (That is, no agent has an initial endowment of the public good.) Agent h's utility function is $u^h(x^h, y) = x^h + \theta^h \ln y$, $h = 1, ..., n$.

(a) (10 pts) Calculate the Lindahl equilibrium taxes and public good level for the n-person economy, assuming that the Lindahl outcome is interior.

(b) (15 pts) Suppose $n = 3$ and $(e^h, \theta^h) = (1, 1)$ for $h = 1, 2, 3$. Find all Nash equilibria of the voluntary provision game, and the corresponding quantities of public good.