1 The Stochastic Neoclassical Growth Model

The social planner in the stochastic neoclassical growth model chooses stochastic consumption and capital allocations $\{c_t, k_{t+1}\}$ to solve the following maximization problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

s.t.

$$c_t + k_{t+1} = z_t k_t^{\alpha}$$

with $\beta \in (0, 1)$ and $\alpha \in (0, 1)$ being parameters. The initial endowment of capital k_0 is given, and households are endowed with one unit of time in every period. technology shock z_t follows a N-state Markov chain. Let $Z = \{z_1, z_2, \ldots z_N\}$ be the state space of the Markov chain, and let $\pi(z_{t+1}|z_t)$ denote the Markov transition matrix of the chain.

1. Let $N = 3$ and the transition matrix be given by (for this question only):

$$\pi = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0 & 0.9 & 0.1 \\ 0 & 0 & 1 \end{pmatrix}$$

where rows refer to states today, and columns to states tomorrow. Compute the set of all invariant distributions associated with π.

2. Write down the functional equation the solution to which solves the social planner's problem. State explicitly what the state variables and control variables are.

3. Write down the first order condition(s). Use the Benveniste-Scheinkman (envelope) theorem to derive an expression for the derivative of the value function with respect to the current capital stock k. Use both results to derive the Euler equation for this model.

4. Suppose the representative household owns the capital stock and makes the investment decision in every period. There are no other assets. Define a recursive competitive equilibrium.

5. Call the economy in the previous question in which households can only save by investing in the capital stock E1, and suppose that you have computed a sequential equilibrium $CE^1 = \{c_t(z^t), k_{t+1}(z^t), w_t(z^t), r_t(z^t)\}$ for this economy (I am NOT asking you to do this or to define a sequential markets equilibrium!). Now consider economy E2 in which, in addition to

capital, households can save (or borrow) by trading a full set of state-contingent Arrow securities. The Arrow securities are in zero net supply. Construct a competitive equilibrium

$$CE^2 = \{\hat{c}_t(z^t), \hat{k}_{t+1}(z^t), \hat{a}_t(z^t, z_{t+1}), \hat{w}_t(z^t), \hat{r}_t(z^t), \hat{q}_t(z^t, z_{t+1})\}$$

for economy E2 using the equilibrium CE^1 from economy 1.

6. Given Arrow securities prices, and for a fixed z^t, for all $z_{t+1} \in Z$ calculate the realized return $R_t(z^t, z_{t+1})$ on an Arrow security that pays in state $\hat{z}_{t+1}$.