Estimation with Aggregate Shocks

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Abstract

Aggregate shocks affect most households' and firms' decisions. Using three stylized models we show that inference based on cross-sectional data alone generally fails to correctly account for decision making of rational agents facing aggregate uncertainty. We propose an econometric framework that overcomes these problems by explicitly parametrizing the agents' inference problem relative to aggregate shocks. Our framework and examples illustrate that the cross-sectional and time-series aspects of the model are often interdependent. Estimation of model parameters in the presence of aggregate shock requires, therefore, the combined use of cross-sectional and time series data. We provide easy-to-use formulas for test statistics and confidence intervals that account for the interaction between the cross-sectional and time-series variation.

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1 Introduction

An extensive body of economic research suggests that aggregate shocks have important effects on households' and firms' decisions. Consider for instance the oil shock that hit developed countries in 1973. A large literature has provided evidence that this aggregate shock triggered a recession in the United States, where the demand and supply of non-durable and durable goods declined, inflation grew, the unemployment rate increased, and real wages dropped.

The profession has generally adopted one of the following three strategies to deal with aggregate shocks. The most common strategy is to assume that aggregate shocks have no effect on households' and firms' decisions and, hence, that aggregate shocks can be ignored. Almost all papers estimating discrete choice dynamic models or dynamic games are based on this premise. Examples include Keane and Wolpin (1997), Bajari, Bankard, and Levin (2007), and Eckstein and Lifshitz (2011). We show that, if aggregate shocks are an important feature of the data, ignoring them generally leads to inconsistent parameter estimates. The second approach is to add in a linear fashion time dummies to the model in an attempt to capture the effect of aggregate shocks on the estimation of the parameters of interest, as was done for instance in Runkle (1991) and Shea (1995). We clarify that, if the econometrician does not account properly for aggregate shocks, the parameter estimates will generally be inconsistent even if the actual realizations of the aggregate shocks are observed. The linear addition of time dummies, therefore, fails to solve the problem.[1] The last strategy is to fully specify how aggregate shocks affect individual decisions jointly with the rest of the structure of the economic problem. Using this approach, the econometrician can obtain consistent estimates of the parameters of interest. We are aware of only one paper that uses this strategy, Lee and Wolpin (2010). Their paper is primarily focused on the estimation of a specific empirical model, and they do not address the broader question of which statistical assumptions and what type of data requirements are needed more generally to obtain consistent estimators when aggregate shocks are present. Moreover, as we argue later on, in Lee and Wolpin's (2010) paper there are issues with statistical inference and efficiency.

[1] In the Euler equation context, Chamberlain (1984) uses examples to argue that, when aggregate shocks are present but disregarded, the estimated parameters are generally inconsistent. His examples make clear that generally time dummies do not solve the problems introduced by the presence of aggregate shocks.

The previous discussion reveals that there is no generally agreed upon econometric framework for statistical inference in models where aggregate shocks have an effect on individual decisions. The purpose of this paper is to provide such a general econometric framework. We show that inference based on cross-sectional data alone generally fails to correctly account for decision making of rational agents facing aggregate uncertainty. By parametrizing aggregate uncertainty and explicitly accounting for it when solving for the agents decision problem, we are able to offer an econometric framework that overcomes these problems. We advocate the combined use of cross-sectional and time series data, and we develop simple-to-use formulas for test statistics and confidence intervals that enable the combined use of time series and cross-sectional data.

We proceed in three steps. In Section 2, we introduce the general identification problem by examining a general class of models with the following two features. First, each model in this class is composed of two submodels. The first submodel includes all the cross-sectional features, whereas the second submodel is composed of all the time-series aspects. As a consequence, the parameters of the model can also be divided into two groups: the parameters that characterize the cross-sectional submodel and the parameters that enter the time-series submodel. The second feature is that the two submodels are linked by a vector of aggregates shocks and by the parameters that govern their dynamics. Individual decision making thus depends on aggregate shocks.

Given the interplay between the two submodels, aggregate shocks have complicated effects on the estimation of the parameters of interest. To better understand those effects, in the second step, we present three examples of the general framework that illustrate the complexities generated by the existence of the aggregate shocks. Section 3 considers a model of portfolio choices with aggregate shocks and shows that, if only cross-sectional variation is used, the estimates of the model parameters are biased and inconsistent. It also shows that to obtain unbiased and consistent estimates it is necessary to combine cross-sectional and time-series variation.

In Section 4, as a second example, we study the estimation of firms' production functions when aggregate shocks affect firms' decisions. This example shows that there are exceptional cases in which model parameters can be estimated using only repeated cross-sections if time dummies are skillfully used and not simply added as time intercepts. Specifically, we show that the method proposed by Olley and Pakes (1996) for the estimation of production functions can be modified with the proper inclusion of time dummies to account for the effect of aggregate shocks. The results of Section 4 are of independent interest since the estimation of firms' production functions is an important topic in industrial organization and aggregate shocks have significant effects in most markets.

In Section 5 we discuss as our last example a general equilibrium model of education and labor supply decisions. The portfolio example has the quality of being simply. But, because of its simplicity, it generates a one-directional relationship between the time-series and cross-sectional submodels: the variables and parameters of the time-series model affect the variables and parameters of the cross-sectional submodel, but the opposite is not true. As a result, the parameters of the time-series submodel can be estimated without knowing the cross-sectional parameters. However, this is not generally the case. In the majority of situations, the link between the two submodels is bi-directional. The advantage of the general-equilibrium example is that it produces a bi-directional relationship we can use to illustrate the complexity of the effect of aggregate shocks on parameter estimation.

The examples make clear that generally the best approach to consistently estimate the parameters of the investigated models is to combine cross-sectional data with a long time-series of aggregate variables.[2] As the last step, in Section 6 we provide easy-to-use formulas that can be employed to derive test statistics and confidence intervals for parameters estimated by combining those two data sources. The underlying asymptotic theory, which is presented in the companion paper Hahn, Kuersteiner, and Mazzocco (2016), is highly technical due to the complicated interactions that exists between the two-submodels. It is therefore surprising that the formulas necessary to perform inference take simple forms that are easy to adopt. We conclude the section, by illustrating using the general equilibrium model discussed in Section 5 how the formulas can be computed in concrete cases.

In addition to the econometric literature that deals with inferential issues, our paper also contributes to a growing literature whose objective is the estimation of general equilibrium models. Some examples of papers in this literature are Heckman and Sedlacek (1985), Heckman, Lochner, and Taber (1998), Lee (2005), Lee and Wolpin (2006), Gemici and Wiswall (2011), Gillingham,

[2] An alternative method would be to combine cross-sectional and time-series variation by using panel data. Panel data, however, are generally too short to achieve consistency, whereas long time-series data are easier to find for most of the variables that are of interest to economists. More on this at the end of Section 5.

Iskhakov, Munk-Nielsen, Rust, and Schjerning (2015). Aggregate shocks are a natural feature of general equilibrium models. Without them those models have the unpleasant implication that all aggregate variables can be fully explained by observables and, hence, that errors have no effects on those variables. Our general econometric framework makes this point clear by highlighting the impact of aggregate shocks on parameter estimation and the variation required in the data to estimate those models. More importantly, our results provide easy-to-use formulas that can be used to perform statistical inference in a general equilibrium context.

2 The General Identification Problem

This section introduces in a general form the identification problem generated by the existence of aggregate shocks. It follows closely Section 2 in our companion paper Hahn, Kuersteiner, and Mazzocco (2016). We consider a class of models with four main features. First, the model can be divided into two parts. The first part encompasses all the static aspects of the model and will be denoted with the term *cross-sectional submodel*. The second part includes the dynamic aspects of the aggregate variables and will be denoted with the term *time-series submodel*. Second, the two submodels are linked by the presence of a vector of aggregate shocks ν_t and by the parameters that govern their dynamics. Third, the vector of aggregate shocks may not be observed. If that is the case, it is treated as a set of parameters to be estimated. Lastly, the parameters of the model can be consistently estimated only if a combination of cross-sectional and time-series data are available.

We now formally introduce the general model. The variables that characterize the model can be divided into two vectors $y_{i,t}$ and z_s. The first vector $y_{i,t}$ includes all the variables that characterize the cross-sectional submodel, where i describes an individual decision-maker, a household or a firm, and t a time period in the cross-section.[3] The second vector z_s is composed of all the variables associated with the time-series model. Accordingly, the parameters of the general model can be divided into two sets, β and ρ. The first set of parameters β characterizes the cross-sectional submodel, in the sense that, if the second set ρ was known, β and ν_t can be con-

[3] Even if the time subscript t is not necessary in this subsection, we keep it here for notational consistency because later we consider the case where longitudinal data are collected.

sistently estimated using exclusively variation in the cross-sectional variables $y_{i,t}$. Similarly, the vector ρ characterizes the time-series submodel meaning that, if β and ν_t were known, those parameters can be consistently estimated using exclusively the time series variables z_s. There are two functions that relate the cross-sectional and time-series variables to the parameters. The function $f\left(y_{i,t}\middle|\,\beta,\nu_t,\rho\right)$ restricts the behavior of the cross-sectional variables conditional on a particular value of the parameters. Analogously, the function $g\left(z_s\middle|\,\beta,\rho\right)$ describes the behavior of the time-series variables for a given value of the parameters. An example is a situation in which (i) the variables $y_{i,t}$ for $i=1,\ldots,n$ are i.i.d. given the aggregate shock ν_t, (ii) the variables z_s correspond to (ν_s,ν_{s-1}), (iii) the cross-sectional function $f\left(y_{i,t}\middle|\,\beta,\nu_t,\rho\right)$ denotes the log likelihood of $y_{i,t}$ given the aggregate shock ν_t, and (iv) the time-series function $g\left(z_s\middle|\,\beta,\rho\right)=g\left(\nu_s\middle|\,\nu_{s-1},\rho\right)$ is the log of the conditional probability density function of the aggregate shock ν_s given ν_{s-1}. In this special case the time-series function g does not depend on the cross-sectional parameters β.

We assume that our cross-sectional data consist of $\{y_{i,t},\ i=1,\ldots,n\}$, and our time series data consist of $\{z_s,\ s=\tau_0+1,\ldots,\tau_0+\tau\}$. For simplicity, we assume that $\tau_0=0$ in this section.

The parameters of the general model can be estimated by maximizing a well-specified objective function.[4] Since in our case the general framework is composed of two submodels, a natural approach is to estimate the parameters of interest by maximizing two separate objective functions, one for the cross-sectional model and one for the time-series model. We denote these criterion functions by $F_n\left(\beta,\nu_t,\rho\right)$ and $G_\tau\left(\beta,\rho\right)$. In the case of maximum likelihood these functions are simply $F_n\left(\beta,\nu_t,\rho\right)=\frac{1}{n}\sum_{i=1}^{n}f\left(y_{i,t}\middle|\,\beta,\nu_t,\rho\right)$ and $G_\tau\left(\beta,\rho\right)=\frac{1}{\tau}\sum_{s=1}^{\tau}g\left(z_s\middle|\,\beta,\rho\right)$. The use of two separate objective functions is helpful in our context because it enables us to discuss which issues arise if only cross-sectional variables or only time-series variables are used in the estimation.[5]

In the class of models we consider, the identification of the parameters requires the joint use of cross-sectional and time-series data. Specifically, the objective function F of the cross-sectional model evaluated at the cross-sectional parameters β and aggregate shocks ν takes the same value for any feasible set of time-series parameters ρ. Similarly, the objective function G of the time-

[4]Our discussion is motivated by Newey and McFadden's (1994) unified treatment of maximum likelihood and GMM as extremum estimators.

[5]Note that our framework covers the case where the joint distribution of (y_{it},z_t) is modelled. Considering the two components separately adds flexibility in that data is not required for all variables in the same period.

series model evaluated at the time-series parameters and aggregate shocks takes the same value for any feasible set of cross-sectional parameters. In our class of models, however, all the parameters of interest can be consistently estimated if cross-sectional data are combined with time-series data.

The next three sections consider special cases of the type of models described above.

3 Example 1: Portfolio Choice

We now present a model of portfolio choices that illustrates the economic relevance of the general class of models introduced in Section 2.

Consider an economy that, in each period t, is populated by n households. These households are born at the beginning of period t, live for one period, and are replaced in the next period by n new families. The households living in consecutive periods do not overlap and, hence, make independent decisions. Each household is endowed with deterministic income and has preferences over a non-durable consumption good $c_{i,t}$. The preferences can be represented by Constant Absolute Risk Aversion (CARA) utility functions which take the following form: $U(c_{i,t}) = -e^{-\delta c_{i,t}}$. For simplicity, we normalize income to be equal to 1.

During the period in which households are alive, they can invest a share of their income in a risky asset with return $u_{i,t}$. The remaining share is automatically invested in a risk-free asset with a return r that does not change over time. At the end of the period, the return on the investment is realized and households consume the quantity of the non-durable good they can purchase with the realized income. The return on the risky asset depends on aggregate shocks. Specifically, it takes the following form: $u_{i,t} = \nu_t + \epsilon_{i,t}$, where ν_t is the aggregate shock and $\epsilon_{i,t}$ is an i.i.d. idiosyncratic shock. The idiosyncratic shock, and hence the heterogeneity in the return on the risky asset, can be interpreted as differences across households in transaction costs, in information on the profitability of different stocks, or in marginal tax rates. We assume that $\nu_t \sim N(\mu, \sigma_\nu^2)$, $\epsilon_{i,t} \sim N(0, \sigma_\epsilon^2)$, and hence that $u_{i,t} \sim N(\mu, \sigma^2)$, where $\sigma^2 = \sigma_\nu^2 + \sigma_\epsilon^2$.

Household i living in period t chooses the fraction of income to be allocated to the risk-free asset $\alpha_{i,t}$ by maximizing its life-time expected utility:

$$\max_{\widetilde{\alpha}} \; E\left[-e^{-\delta c_{i,t}}\right]$$
$$s.t. \quad c_{i,t} = \widetilde{\alpha}\left(1+r\right) + \left(1-\widetilde{\alpha}\right)\left(1+u_{i,t}\right), \tag{1}$$

where the expectation is taken with respect to the return on the risky asset. It is straightforward to show that the household's optimal choice of $\alpha_{i,t}$ is given by[6]

$$\alpha^*_{i,t} = \alpha = \frac{\delta\sigma^2 + r - \mu}{\delta\sigma^2}. \tag{2}$$

We will assume that the econometrician is mainly interested in estimating the risk aversion parameter δ.

We now consider an estimator that takes the form of a population analog of (2), and study the impact of aggregate shocks on the estimator's consistency when an econometrician works only with cross-sectional data. Our analysis reveals that such an estimator is inconsistent, due to the fact that cross-sectional data do not contain information about aggregate uncertainty. Our analysis makes explicit the dependence of the estimator on the probability distribution of the aggregate shock and points to the following way of generating a consistent estimator of δ. Using time series variation, one can consistently estimate the parameters pertaining to aggregate uncertainty. Those estimates can then be used in the cross-sectional model to estimate the remaining parameters.[7]

Without loss of generality, we assume that the cross-sectional data are observed in period $t = 1$. The econometrician observes data on the return of the risky asset $u_{i,t}$ and on the return of the risk-free asset r. We assume that in addition he also observes a noisy measure of the share of resources invested in the risky assets $\alpha_{i,t} = \alpha + e_{i,t}$, where $e_{i,t}$ is a zero-mean measurement error. We therefore have that $y_i = (u_{i1}, \alpha_{i1})$. We make the simplifying assumption that the aggregate shock is observable to econometricians and that the time-series variables only include the aggregate shock, i.e. $z_t = \nu_t$. Because $\mu = E\left[u_{i1}\right]$, $\sigma^2 = \mathrm{Var}\left(u_{i1}\right)$, and $\alpha = E\left[\alpha_{i1}\right]$, if only cross-sectional

[6] See the appendix.

[7] Our model is a stylized version of many models considered in a large literature interested in estimating the parameter δ using cross-sectional variation. Estimators are often based on moment conditions derived from first order conditions (FOC) related to optimal investment and consumption decisions. Such estimators have similar problems, which we discuss in Appendix A.2.

variation is used for estimation, we would have the following method-of-moments estimators of those parameters:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} u_{i1} = \bar{u}, \qquad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(u_{i1} - \bar{u}\right)^2, \qquad \text{and} \qquad \hat{\alpha} = \frac{1}{n}\sum_{i=1}^{n}\alpha_{i1}.$$

The econometrician can then use equation (2) to write the risk aversion parameter as $\delta = (\mu - r)/\left(\sigma^2\left(1-\alpha\right)\right)$ and estimate it using the sample analog $\hat{\delta} = (\hat{\mu} - r)/\left(\hat{\sigma}^2\left(1-\hat{\alpha}\right)\right)$.

In the presence of the aggregate shock ν_t, those estimators can be written as

$$\begin{aligned}
\hat{\mu} &= \frac{1}{n}\sum_{i=1}^{n} u_{i1} = \nu_1 + \frac{1}{n}\sum_{i=1}^{n}\epsilon_{i1} = \nu_1 + o_p\left(1\right), \\
\hat{\sigma}^2 &= \frac{1}{n}\sum_{i=1}^{n}\left(u_{i1} - \bar{u}\right)^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\epsilon_{i1} - \bar{\epsilon}\right)^2 = \sigma_\epsilon^2 + o_p\left(1\right), \\
\hat{\alpha} &= \alpha + \frac{1}{n}\sum_{i=1}^{n} e_{i1} = \alpha + o_p\left(1\right),
\end{aligned}$$

which implies that δ will be estimated to be

$$\hat{\delta} = \frac{\nu_1 + o_p\left(1\right) - r}{\left(\sigma_\epsilon^2 + o_p\left(1\right)\right)\left(1 - \alpha + o_p\left(1\right)\right)} = \frac{\nu_1 - r}{\sigma_\epsilon^2\left(1-\alpha\right)} + o_p\left(1\right). \tag{3}$$

Using Equation (3), we can study the properties of the proposed estimator $\hat{\delta}$. If there were no aggregate shock in the model, we would have $\nu_1 = \mu$, $\sigma_\nu^2 = 0$, $\sigma_\epsilon^2 = \sigma^2$ and, therefore, $\hat{\delta}$ would converge to δ, a nonstochastic constant, as n grows to infinity. It is therefore a consistent estimator of the risk aversion parameter. In the presence of the aggregate shock, however, the proposed estimator has different properties. If one conditions on the realization of the aggregate shock ν, the estimator $\hat{\delta}$ is inconsistent with probability 1, since it converges to $\frac{\nu_1 - r}{\sigma_\epsilon^2(1-\alpha)}$ and not to the true value $\frac{\mu - r}{(\sigma_\nu^2 + \sigma_\epsilon^2)(1-\alpha)}$. If one does not condition on the aggregate shock, as n grows to infinity, $\hat{\delta}$ converges to a random variable with a mean that is different from the true value of the risk aversion parameter. The estimator will therefore be biased and inconsistent. To see this, remember that $\nu_1 \sim N\left(\mu, \sigma_\nu^2\right)$. As a consequence, the unconditional asymptotic distribution of $\hat{\delta}$ takes the following form:

$$\hat{\delta} \rightarrow N\left(\frac{\mu - r}{\sigma_\epsilon^2\left(1-\alpha\right)}, \left(\frac{1}{\sigma_\epsilon^2\left(1-\alpha\right)}\right)^2 \sigma_\nu^2\right) = N\left(\delta + \delta\frac{\sigma_\nu^2}{\sigma_\epsilon^2}, \frac{\sigma_\nu^2}{\left(\sigma_\epsilon^2\left(\alpha - 1\right)\right)^2}\right),$$

which is centered at $\delta + \delta\dfrac{\sigma_\nu^2}{\sigma_\epsilon^2}$ and not at δ, hence the bias.

We are not the first to consider a case in which the estimator converges to a random variable. Andrews (2005) and more recently Kuersteiner and Prucha (2013) discuss similar scenarios. Our example is remarkable because the nature of the asymptotic randomness is such that the estimator is not even asymptotically unbiased. This is not the case in Andrews (2005) or Kuersteiner and Prucha (2013), where in spite of the asymptotic randomness the estimator is unbiased.[8]

As mentioned above, there is a simple explanation for our result: cross-sectional variation is not sufficient for the consistent estimation of the risk aversion parameter if aggregate shocks affect individual decisions.[9] To make this point transparent, observe that, conditional on the aggregate shock, the assumptions of this section imply that y_i has the following distribution

$$y_i|\,\nu_1 \sim N\left(\begin{bmatrix} \nu_1 \\ \dfrac{\delta\left(\sigma_\nu^2+\sigma_\epsilon^2\right)+r-\mu}{\delta\left(\sigma_\nu^2+\sigma_\epsilon^2\right)} \end{bmatrix}, \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_e^2 \end{bmatrix}\right), \tag{4}$$

Using (4), it is straightforward to see that the cross-sectional likelihood is maximized for any arbitrary choice of the time-series parameters $\rho = (\mu, \sigma_\nu^2)$, as long as one chooses δ that satisfies the following equation:

$$\frac{\delta\left(\sigma_\nu^2+\sigma_\epsilon^2\right)+r-\mu}{\delta\left(\sigma_\nu^2+\sigma_\epsilon^2\right)} = \alpha.$$

As a result, the cross-sectional parameters μ and σ_ν^2 cannot be consistently estimated by maximizing the cross-sectional likelihood and, hence, δ cannot be consistently estimated using only cross-sectional data.

A solution to the problem discussed in this section is to combine cross-sectional variables with time-series variables. In this case, one can consistently estimate (μ, σ_ν^2) by using the time-series

[8]Kuersteiner and Prucha (2013) also consider cases where the estimator is random and inconsistent. However, in their case this happens for different reasons: the endogeneity of the factors. The inconsistency considered here occurs even when the factors (i.e., aggregate shocks) are strictly exogenous.

[9]As discussed in the introductory section, a common practice to account for the effect of aggregate shocks is to include time dummies in the model. The portfolio example clarifies that the addition of time dummies does not solve the problem generated by the presence of aggregate shocks. The inclusion of time dummies is equivalent to the assumption that the aggregate shocks are known. But the previous discussion indicates that, using exclusively cross-sectional data, the estimator $\hat{\delta}$ is biased and inconsistent even if the aggregate shocks are known. An unbiased and consistent estimator of δ can only be obtained if the distribution of the aggregate shocks is known, which is feasible only by exploiting the variation contained in time-series data.

of aggregate data $\{z_t\}$. Consistent estimation of $(\delta, \sigma_\epsilon^2, \sigma_e^2)$ can then be achieved by plugging the consistent estimators of (μ, σ_ν^2) in the correctly specified cross-section likelihood (4).

The example presented in this section is a simplified version of the general class of models introduced in Section 2, since the relationship between the cross-sectional and time-series submodels is simple and one-directional. The variables and parameters of the time-series submodel affect the cross-sectional submodel, but the cross-sectional variables and parameters have no impact on the time-series submodel. As a consequence, the time-series parameters can be consistently estimated without knowing the cross-sectional parameters. In more complicated situations, such as general equilibrium models, where aggregate shocks are a natural feature, the relationship between the two submodels is generally bi-directional. In Section 5, we present a general-equilibrium with that type of relationship. But before considering that case, we study a situation in which the effect of aggregate shocks can be accounted for with the proper use of time dummies.

4 Example 2: Estimation of Production Function

In the previous section, we presented an example that illustrates the complicated nature of identification in the presence of aggregate shocks. The example highlights that generally there is no simple method for estimating the class of models considered in this paper. Estimation requires a careful examination of the interplay between the cross-sectional and time-series submodels. In this section, we consider an example showing that there are exceptions to this general rule. In the case we analyze, identification of a model with aggregate shocks can be achieved using only cross-sectional data provided that time dummies are skillfully employed. We will show that the naive practice of introducing additive time dummies is not sufficient to deal with the effects generated by aggregate shocks. But the solution is simpler than the general approach we adopted to identify the parameters of the portfolio model.

The example we consider here is a simplified version of the problem studied by Olley and Pakes (1996) and deals with an important topic in industrial organization: the estimation of firms' production functions. A profit-maximizing firm j produces in period t a product $y_{j,t}$ employing a production function that depends on the logarithm of labor $l_{j,t}$, the logarithm of capital $k_{j,t}$, and a productivity shock $\omega_{j,t}$. It takes the following functional form:

$$y_{j,t} = \beta_0 + \beta_l l_{j,t} + \beta_k k_{j,t} + \omega_{j,t} + \eta_{j,t} \tag{5}$$

where $\eta_{i,t}$ is a measurement error.

Capital and labor are optimally chosen by the firm, jointly with the new investment in capital $i_{j,t}$, by maximizing a dynamic profit function subject to constraints that determine how capital accumulated over time.[10] In the model proposed by Olley and Pakes (1996), firms are heterogeneous in their age and can choose to exit the market. In this section, we will abstract from age heterogeneity and exit decisions because they make the model more complicated without adding more insight on the effect of aggregate shocks on the estimation of production functions.

A crucial feature of the model proposed by Olley and Pakes (1996) and of our example is that the investment decision in period t is a function of the current stock of capital and productivity shock, i.e.

$$i_{j,t} = i_t\left(\omega_{j,t}, k_{j,t}\right). \tag{6}$$

Olley and Pakes (1996) do not allow for aggregate shocks, but in this example we consider a situation in which the productivity shock at t is the sum of an aggregate shock ν_t and of an i.i.d. idiosyncratic shock $\varepsilon_{j,t}$, i.e.

$$\omega_{j,t} = \nu_t + \varepsilon_{j,t}. \tag{7}$$

We will assume that the firm observes the realization of the aggregate shock and, separately, of the i.i.d. shock.

We first describe how the production function (5) can be estimated when aggregate shocks are not present, the method proposed by Olley and Pakes (1996). We then discuss how that method has to be modified with the appropriate use of time dummies if aggregate shocks affect firms' decisions.

The main problem in the estimation of the production function (5) is that the productivity shock is correlated with labor and capital, but not observed by the econometrician. To deal with that issue, Olley and Pakes (1996) use the result that the investment decision (6) is strictly increasing in the productivity shock for every value of capital to invert the corresponding function

[10] For details of the profit function and the accumulation equation for capital, see Olley and Pakes (1996).

and solve for the productivity shock, which implies

$$\omega_{j,t} = h_t\left(i_{j,t}, k_{j,t}\right). \tag{8}$$

One can then replace the productivity shock in the production function using the equation (8) to obtain

$$y_{j,t} = \beta_l l_{j,t} + \phi_t\left(i_{j,t}, k_{j,t}\right) + \eta_{j,t}, \tag{9}$$

where

$$\phi_t\left(i_{j,t}, k_{j,t}\right) = \beta_0 + \beta_k k_{j,t} + h_t\left(i_{j,t}, k_{j,t}\right). \tag{10}$$

The parameter β_l and the function ϕ can then be estimated by regressing $y_{j,t}$ on $l_{j,t}$ and a polynomial in $i_{j,t}$ and $k_{j,t}$. Equivalently, β_l is identified by

$$\beta_l = \frac{E\left[\left(l_{j,t} - E\left[l_{j,t}\middle| i_{j,t}, k_{j,t}\right]\right)\left(y_{j,t} - E\left[y_{j,t}\middle| i_{j,t}, k_{j,t}\right]\right)\right]}{E\left[\left(l_{j,t} - E\left[l_{j,t}\middle| i_{j,t}, k_{j,t}\right]\right)^2\right]}. \tag{11}$$

To identify the parameter on the logarithm of capital β_k observe that

$$E\left[y_{i,t+1} - \beta_l l_{j,t+1}\middle| k_{j,t+1}\right] = \beta_0 + \beta_k k_{j,t+1} + E\left[\omega_{j,t+1}\middle| \omega_{j,t}\right] = \beta_0 + \beta_k k_{j,t+1} + g\left(\omega_{j,t}\right), \tag{12}$$

where the first equality follows from $k_{j,t+1}$ being determined conditional on $\omega_{j,t}$. The shock $\omega_{j,t} = h_t\left(i_{j,t}, k_{j,t}\right)$ is not observed but, using equation (10), can be written in the following form:

$$\omega_{j,t} = \phi_t\left(i_{j,t}, k_{j,t}\right) - \beta_0 - \beta_k k_{j,t}, \tag{13}$$

where ϕ_t is known from the first-step estimation. Substituting for $\omega_{j,t}$ into the function $g\left(.\right)$ in equation (12) and letting $\xi_{j,t+1} = \omega_{j,t+1} - E\left[\omega_{j,t+1}\middle| \omega_{j,t}\right]$, equation (12) can be written as follows:

$$y_{i,t+1} - \beta_l l_{j,t+1} = \beta_k k_{j,t+1} + g\left(\phi_t - \beta_k k_{j,t}\right) + \xi_{j,t+1} + \eta_{j,t}. \tag{14}$$

where β_0 has been included in the function $g\left(.\right)$. The parameter β_k can then be estimated by using the estimates of β_l and ϕ_t obtained in the first step and by minimizing the sum of squared residuals in the previous equation employing a kernel or a series estimator for the function g.

We now consider the case in which aggregate shocks affect the firm's decisions and analyze how the model parameters can be identified using only cross-sectional variation. The introduction of aggregate shocks changes the estimation method in two main ways. First, the investment decision is affected by the aggregate shock and takes the following form:

$$i_{j,t} = i_t\left(\nu_t, \varepsilon_{j,t}, k_{j,t}\right).$$

where ν_t and $\varepsilon_{j,t}$ enter as independent arguments because the firm observes them separately. Second, all expectation are conditional on the realization of the aggregate shock since in the cross-section there is no variation in that shock and only its realization is relevant.

It is straightforward to show that, if the investment function is strictly increasing in the productivity shock $\omega_{j,t}$ for all capital levels, it is also strictly increasing in ν_t and $\varepsilon_{j,t}$ for all $k_{j,t}$. Using this result, we can invert $i_t(.)$ to derive $\varepsilon_{j,t}$ as a function of the aggregate shock, investment, and the stock of capital, i.e.

$$\varepsilon_{j,t} = h_t\left(\nu_t, i_{j,t}, k_{j,t}\right).$$

The production function can therefore be rewritten in the following form:

$$\begin{aligned} y_{j,t} &= \beta_0 + \beta_l l_{j,t} + \beta_k k_{j,t} + \nu_t + \varepsilon_{j,t} + \eta_{j,t} \\ &= \beta_l l_{j,t} + \left[\beta_0 + \beta_k k_{j,t} + \nu_t + h_t\left(\nu_t, i_{j,t}, k_{j,t}\right)\right] + \eta_{j,t} \\ &= \beta_l l_{j,t} + \phi_t\left(\nu_t, i_{j,t}, k_{j,t}\right) + \eta_{j,t}. \end{aligned} \tag{15}$$

If β_l is estimated using repeated cross-sections and the method developed for the case with no aggregate shocks, the estimated coefficient will generally be biased because the econometrician does not account for the aggregate shock and its correlation with the firm's choice of labor. There is, however, a small variation of the method proposed earlier that produces unbiased estimates of β_l, as long as $\varepsilon_{j,t}$ is independent of $\eta_{j,t}$. The econometrician should regress $y_{j,t}$ on $l_{j,t}$ and a polynomial in $i_{j,t}$ and $k_{j,t}$ where the polynomial is interacted with time dummies. It is this atypical use of time dummies that enables the econometrician to account for the effect of aggregate shocks on firms' decisions. The β_l can therefore be identified by

$$\beta_l = \frac{E\left[\left(l_{j,t} - E\left[l_{j,t} \middle| i_{j,t}, k_{j,t}, \nu_t = \bar{\nu}\right]\right)\left(y_{j,t} - E\left[y_{j,t} \middle| i_{j,t}, k_{j,t}, \nu_t = \bar{\nu}\right]\right)\right]}{E\left[\left(l_{j,t} - E\left[l_{j,t} \middle| i_{j,t}, k_{j,t}, \nu_t = \bar{\nu}\right]\right)^2\right]}. \tag{16}$$

Observe that the expectation operator in the previous equation is defined with respect to a probability distribution function that includes the randomness of the aggregate shock ν_t. But, when one uses cross-sectional variation, ν_t is fixed at its realized value. As a consequence, the distribution is only affected by the randomness of ε_{it}.

For the estimation of β_k, observe that

$$\begin{aligned} & E\left[y_{i,t+1} - \beta_l l_{j,t+1} \right| k_{j,t+1}, \nu_{t+1} = \bar{\nu}'] \\ & = \beta_0 + \beta_k k_{j,t+1} + E\left[\nu_{t+1} + \varepsilon_{j,t+1} \right| , \nu_{t+1} = \bar{\nu'}, \nu_t = \bar{\nu}, \varepsilon_{j,t}] \\ & = \beta_0 + \beta_k k_{j,t+1} + \bar{\nu}' + E\left[\varepsilon_{j,t+1} \right| \nu_t = \bar{\nu}, \varepsilon_{j,t}] \\ & = \beta_0 + \beta_k k_{j,t+1} + \bar{\nu}' + g_t\left(\varepsilon_{j,t} \right) \end{aligned} \tag{17}$$

where the first equality follows from $k_{j,t+1}$ being known if ν_t and $\varepsilon_{j,t}$ are known and the last equality follows from the inclusion of the aggregate shock $\nu_t = \bar{\nu}$ in the function $g_t(.)$.

The only variable of equation (17) that is not observed is $\varepsilon_{j,t}$. But remember that

$$\varepsilon_{j,t} = h_t\left(\nu_t, i_{j,t}, k_{j,t} \right) = \phi_t\left(\nu_t, i_{j,t}, k_{j,t} \right) - \beta_0 - \beta_k k_{j,t} - \nu_t.$$

We can therefore use the above expression to substitute for $\varepsilon_{j,t}$ in equation (17) and obtain

$$\begin{aligned} & E\left[y_{i,t+1} - \beta_l l_{j,t+1} \right| k_{j,t+1}, \nu_{t+1} = \bar{\nu}'] \\ & = \beta_0 + \beta_k k_{j,t+1} + \bar{\nu}' + g_t\left(\phi_t\left(\nu_t, i_{j,t}, k_{j,t} \right) - \beta_0 - \beta_k k_{j,t} - \bar{\nu} \right) \\ & = \beta_k k_{j,t+1} + g_{t,t+1}\left(\phi_t - \beta_k k_{j,t} \right), \end{aligned}$$

where in the last equality β_0, $\bar{\nu}$, and $\bar{\nu}'$ have been included in the function $g_{t,t+1}(.)$. Hence, if one defines $\xi_{j,t+1} = \varepsilon_{j,t+1} - E\left[\varepsilon_{j,t+1} \right| \nu_t = \bar{\nu}, \varepsilon_{j,t}]$, the parameter β_k can be estimated using the following equation:

$$y_{i,t+1} - \beta_l l_{j,t+1} = \beta_k k_{j,t+1} + g_{t,t+1}\left(\phi_t - \beta_k k_{j,t} \right) + \xi_{j,t+1} + \eta_{j,t+1}. \tag{18}$$

But notice that the approach without aggregate shocks cannot be applied directly to equation (18) because the function $g(.)$ depends on time t and $t+1$ aggregate shocks. With aggregate shocks a different function $g(.)$ must be estimated for each period. This can be achieved by replacing $g(.)$ with a polynomial interacted with time dummies.

The previous discussion indicates that firms' production functions can be estimated using only cross-sectional data as long as the functions ϕ and g are estimated period by period. In practice, both functions are often estimated by low degree polynomials. Our analysis indicates that if the coefficients of these polynomials are interacted with time dummies the estimation of production functions will generally be robust to the presence of aggregate shocks.

We conclude by drawing attention to three important features of the example considered in this section. First, in order to deal with the effect of aggregate shocks, we had to carefully examine the meaning of seemingly straightforward objects such as the expectation operator E. We also had to impose assumptions on the information set of the firms, namely that the firm observes the current aggregate shock. Lastly, the time dummies must be interacted with the polynomials. The standard practice of simply adding time dummies as separate intercepts for each time period does not solve the issues introduced by aggregate shocks.

5 Example 3: A General Equilibrium Model

In this section, we consider as a third example a general equilibrium model of education and labor supply decisions in which aggregate shocks influence individual choices. This example provides additional insight on the effect of aggregate shocks on the estimation of model parameters because, differently from the portfolio example, it considers a case in which the relationship between the cross-sectional and time-series models is bi-directional: the cross-sectional parameters cannot be identified without knowledge of the time-series parameters and the time-series parameters cannot be identified without knowing the cross-sectional parameters. In principle, we could have used as a general example a model proposed in the general equilibrium literature such as the model developed in Lee and Wolpin (2006). We decided against this alternative because in those models the effect of the aggregate shocks and the relationship between the cross-sectional and time-series submodels is complicated and therefore difficult to describe. Instead, we have decided to develop a model that is sufficiently general to generate an interesting relationship between the two submodels, but at the same time is sufficiently stylized for this relationship to be easy to describe and understand.

The main objective of the model we develop is to evaluate the effect of aggregate shocks on the education decisions of young individuals and on their subsequent labor supply decisions when of working-age. For that purpose, we consider an economy in which in each period $t \in T$ a young and a working-age generation overlap. Each generation is composed of a measure N_t of individuals who are endowed with preferences over a non-durable consumption good and leisure. The preferences of individual i are represented by a Cobb-Douglas utility function $U^i(c,l) = \left(c^{\sigma}l^{1-\sigma}\right)^{1-\gamma_i} \Big/ (1-\gamma_i)$, where the risk aversion parameter γ_i is a function of the observable variables $x_{i,t}$, the unobservable variables $\xi_{i,t}$, and a vector of parameters ς, i.e. $\gamma_i = \gamma\left(x_{i,t}, \xi_{i,t} \middle| \varsigma\right)$. Both young and working-age individuals are endowed with a number of hours $\mathcal{T}$ that can be allocated to leisure or to a productive activity. In each period t, the economy is hit by an aggregate shock ν_t whose conditional probability $P\left(\nu_{t+1} \middle| \nu_t\right)$ is given by $\log \nu_{t+1} = \varrho \log \nu_t + \eta_t$. We will assume that η_t is normally distributed with mean 0 and variance ω^2. The aggregate shock affects the labor market in a way that will be established later on.

In each period t, young individuals are endowed with an exogenous income $y_{i,t}$ and choose the type of education to acquire. They can choose either a flexible type of education F or a rigid type of education R. Working-age individuals with flexible education are affected less by adverse aggregate shocks, but they have lower expected wages. The two types of education have identical cost $C_e < y_{i,t}$ and need the same amount of time to acquire $\mathcal{T}_e < \mathcal{T}$. Since young individuals have typically limited financial wealth, we assume that there is no saving decision when young and that any transfer from parents or relatives is included in non-labor income $y_{i,t}$. We also abstract from student loans and assume that all young individuals can afford to buy one of the two types of education. As a consequence, the part of income $y_{i,t}$ that is not spent on education will be consumed. At each t, working-age individuals draw a wage offer $w_{i,t}^F$ if they have chosen the flexible education when young and a wage offer $w_{i,t}^R$ otherwise. They also draw a productivity shock $\varepsilon_{i,t}$ which determines how productive their hours of work are in case they choose to supply labor. We assume that the productivity shock is known to the individual, but not to the econometrician. Given the wage offer and the productivity shock, working-age individuals choose how much to work $h_{i,t}$ and how much to consume. If a working-age individual decides to supply $h_{i,t}$ hours of work, the effective amount of labor hours supplied is given by $\exp\left(\varepsilon_{i,t}\right) h_{i,t}$. We will also assume that $E\left[\exp\left(\varepsilon_{i,t}\right)\right] = 1$.

The economy is populated by two types of firms to whom the working-age individuals supply labor. The first type of firm employs only workers with education F, whereas the second type of firm employs only workers with education R. Both use the same type of capital K. The labor demand functions of the two types of firms are exogenously given and take the following form:

$$\ln H_t^{D,F} = \alpha_0 + \alpha_1 \ln w_t^F,$$

and

$$\ln H_t^{D,R} = \alpha_0 + \alpha_1 \ln w_t^R + \ln \nu_t,$$

where $H^{D,E}$ is the total demand for *effective labor*, with $E = F, R$, $\alpha_0 > 0$, and $\alpha_1 < 0$. We assume that the two labor demands have identical slopes for simplicity. These two labor demand functions allow us to introduce in the model the feature that workers with a more flexible education are affected less by aggregate shocks such as business cycle shocks. The wage for each education group is determined by the equilibrium in the corresponding labor market. It will therefore generally depend on the aggregate shock.

The description of the model implies that there is only one source of uncertainty in the economy, the aggregate shock, and two sources of heterogeneity across individuals, the risk aversion parameter and the productivity shock.

We can now introduce the problem solved by an individual of the young generation. In period t, young individual i chooses consumption, leisure, and the type of education that solve the following problem:

$$\max_{c_{i,t}, l_{i,t}, c_{i,t+1}, l_{i,t+1}, S} \frac{\left(c_{i,t}^{\sigma} l_{i,t}^{1-\sigma}\right)^{1-\gamma_i}}{1-\gamma_i} + \beta \int \frac{\left(c_{i,t+1}^{\sigma} l_{i,t+1}^{1-\sigma}\right)^{1-\gamma_i}}{1-\gamma_i} dP\left(\nu_{t+1} | \nu_t\right) \tag{19}$$

$$s.t. \quad c_{i,t} = y_{i,t} - C_e \quad \text{and} \quad l_{i,t} = \mathcal{T} - \mathcal{T}_e$$

$$c_{i,t+1} = w_{i,t+1}^S\left(\nu_{t+1}\right) \exp\left(\varepsilon_{i,t+1}\right)\left(\mathcal{T} - l_{i,t+1}\right) \quad \text{for every } \nu_{t+1}.$$

Here, $w_{i,t+1}^S\left(\nu_{t+1}\right)$ denotes the wage rate of individual i in the second period, which depends on the realization of the aggregate shock ν_{t+1} and the education choice $S = F, R$. The wage rate is per unit of the effective amount of labor hours supplied and is determined in equilibrium. The problem solved by a working-age individual takes a simpler form. Conditional on the realization of the aggregate shock ν_t and on the type of education S chosen when young, in period t, individual i of the working-age generation chooses consumption and leisure that solve the following problem:

$$\max_{c_{i,t}, l_{i,t}} \frac{\left(c_{i,t}^{\sigma} l_{i,t}^{1-\sigma}\right)^{1-\gamma_i}}{1-\gamma_i} \tag{20}$$

$$s.t. \ c_{i,t} = w_{i,t}^S\left(\nu_t\right) \exp\left(\varepsilon_{i,t}\right)\left(\mathcal{T} - l_{i,t}\right).$$

We will now solve the model starting from the problem of a working-age individual. Using the first order conditions of problem (20), it is straightforward to show that the optimal choice of consumption, leisure, and hence labor supply for a working-age individual takes the following form:

$$c^*_{i,t} = \sigma w_t\left(\nu_t, S\right)\exp\left(\varepsilon_{i,t}\right)\mathcal{T}, \tag{21}$$

$$l^*_{i,t} = \left(1-\sigma\right)\mathcal{T}, \tag{22}$$

$$h^*_{i,t} = \mathcal{T} - l_{i,t}\left(\nu_{i,t}\right) = \sigma\mathcal{T}.$$

The supply of effective labor is therefore equal to $\sigma\exp\left(\varepsilon_{i,t}\right)\mathcal{T}$. Given the optimal choice of consumption and leisure, conditional on the aggregate shock, the value function of a working-age individual with education S can be written as follows:

$$V_{i,t}\left(S,\nu_t\right) = \frac{\left[\left(\sigma w^S_{i,t}\left(\nu_t\right)\exp\left(\varepsilon_{i,t}\right)\mathcal{T}\right)^{\sigma}\left(\left(1-\sigma\right)\mathcal{T}\right)^{1-\sigma}\right]^{1-\gamma_i}}{1-\gamma_i}, \quad \text{S = F, R.}$$

Given the value functions of a working-age individual, we can now characterize the education choice of a young individual. This individual will choose education F if the expectation taken over the aggregate shocks of the corresponding value function is greater than the analogous expectation conditional on choosing education R:

$$\int V_{i,t}\left(F,\nu_{t+1}\right)dP\left(\nu_{t+1}\left|\nu_t\right.\right) \geq \int V_{i,t}\left(R,\nu_{t+1}\right)dP\left(\nu_{t+1}\left|\nu_t\right.\right). \tag{23}$$

Before we can determine which factors affect the education choice, we have to derive the equilibrium in the labor market. We show in the appendix that the labor market equilibrium is characterized by the following two wage equations:

$$\ln w^F_{i,t} = \frac{\ln n^F_t + \ln\sigma + \ln\mathcal{T} - \alpha_0}{\alpha_1} + \varepsilon_{i,t}, \tag{24}$$

$$\ln w^R_{i,t} = \frac{\ln n^R_t + \ln\sigma + \ln\mathcal{T} - \alpha_0 - \ln\nu_t}{\alpha_1} + \varepsilon_{i,t}, \tag{25}$$

where $w^F_{i,t}$ and $w^R_{i,t}$ are the individual wages observed in sectors F and R and n^F_t and n^R_t are the measures of individuals that choose education F and R. We can now replace the equilibrium wages inside inequality (23) and analyze the education decision of a young individual. To simplify the discussion, we will assume that $\varepsilon_{i,t}$ is independent of $\xi_{i,t}$, thereby eliminating sample selection issues in the wage equations. In the appendix, we show that, if the risk aversion parameter γ is greater than or equal to 1, a young individual chooses the flexible type of education if the following inequality is satisfied:

$$\frac{\sigma\omega^2}{2\alpha_1} \geq \frac{1}{1-\gamma\left(x_{i,t},\xi_{i,t}|\,\varsigma\right)}\log\left(\frac{n_t^F}{n_t^R}\right)+\frac{\varrho\log\nu_t}{1-\gamma\left(x_{i,t},\xi_{i,t}|\,\varsigma\right)}. \tag{26}$$

If $\gamma < 1$, the inequality is reversed and the young individual chooses the flexible education if

$$\frac{\sigma\omega^2}{2\alpha_1} < \frac{1}{1-\gamma\left(x_{i,t},\xi_{i,t}|\,\varsigma\right)}\log\left(\frac{n_t^F}{n_t^R}\right)+\frac{\varrho\log\nu_t}{1-\gamma\left(x_{i,t},\xi_{i,t}|\,\varsigma\right)}. \tag{27}$$

These inequalities provide some insight about the educational choice of young individuals.[11] They are more likely to choose the flexible education which insures them against aggregate shocks if the variance of the aggregate shock is larger, if they are more risk averse, if the aggregate shock at the time of the decision is lower as long as $\varrho > 0$, and if the elasticity of the wage for the rigid education with respect to the aggregate shock is less negative.

Similarly to the first example, we can classify some of the variables and some of the parameters as belonging to the cross-sectional submodel and the remaining to the time-series submodel. The cross-sectional variables include consumption $c_{i,t}$, leisure $l_{i,t}$, individual wages $w_{i,t}^F$ and $w_{i,t}^R$, the variable determining the educational choice $D_{i,t}$, the amount of time an individual can divide between leisure and productive activities $\mathcal{T}$, and the variables that enter the risk aversion parameter $x_{i,t}$. The time-series variables are composed of the aggregate shock ν_t, the measure of young individuals choosing the two types of education n^F and n^R, and the aggregate equilibrium wages in the two sectors $w_t^F = E\left[w_{it}^F\right]$ and $w_t^R = E\left[w_{it}^F\right]$.[12] We want to stress the difference between individual wages and aggregate wages. Individual wages are typically observed in panel data or repeated cross-sections whose time dimension is generally short, whereas aggregate wages are available in longer time-series of aggregate data. The cross-sectional parameters consist of the relative taste for consumption σ and the parameters of the wage equations α_0 and α_1, whereas the time-series parameters include the two parameters governing the evolution of the aggregate shock ϱ and ω^2, the parameters defining the risk aversion ς, and the discount factor β. The discount

[11] Equations (26) and (27) can also be used to illustrate the problems of achieving consistent estimation using cross-sectional variation alone. See the appendix for details.

[12] The expectation operator E corresponds to the expectation taken over the distribution of cross sectional variables.

factor is notoriously difficult to estimate. For this reason, in the rest of the section we will assume it is known.

We can now consider the estimation of the parameters of interest. Parameters in this model can be consistently estimated exploiting both cross-sectional and time-series data. We assume that the (repeated) cross-sectional data include $\bar{n}_1$ and $\bar{n}_2$ i.i.d. observations $\left(w_{i,t}^F, c_{i,t}^*, l_{i,1}^*\right)$ for individuals with $S = F, R$ from two periods $t = 1, 2$. We also assume that the time-series data span $t = 1, \ldots, \tau$, and consists of $\left(n_t^F, w_t^F, n_t^R, w_t^R\right)$.

We first discuss how α_1 can be consistently estimated with a large number of individuals using the wage equation for the flexible education (24). Using equation (24), we can consistently estimate α_1 by $\widehat{\alpha}_1$ which solves

$$\frac{1}{\bar{n}_1}\sum_{i=1}^{\bar{n}_1} \ln w_{i,1}^F - \frac{1}{\bar{n}_2}\sum_{i=1}^{\bar{n}_2} \ln w_{i,2}^F = \frac{1}{\widehat{\alpha}_1}\left(n_1^F - n_2^F\right).$$

Observe that this can be done because ε_t and γ are assumed to be independent of each other, which implies that there is no sample selectivity problem. Second, the consumption and leisure choices of working-age individuals (21) and (22) can be used to consistently estimate σ by $\widehat{\sigma}$ which solves

$$\frac{1}{\bar{n}_1}\sum_{i=1}^{\bar{n}_1} \frac{c_{i,1}^*}{l_{i,1}^*} = w_1^F \frac{\widehat{\sigma}}{1-\widehat{\sigma}}.$$

Third, with α_1 consistently estimated, it is straightforward to show that the aggregate shock in period t can be consistently estimated for $t = 1, \ldots, \tau$ using the following equation:

$$\widehat{\ln \nu_t} = \widehat{\alpha}_1 \left(\ln w_t^F - \ln w_t^R\right) - \left(\ln n_t^F - \ln n_t^R\right). \tag{28}$$

The parameters ϱ and ω^2 can then be consistently estimated by the time-series regression of the following equation:

$$\widehat{\log \nu_{t+1}} = \varrho \widehat{\log \nu_t} + \eta_t. \tag{29}$$

This is the step where we use the time-series variation. The only parameters left to estimate are the ς defining the individual risk aversion $\gamma\left(x_{i,t}, \xi_{i,t} \middle| \varsigma\right)$. If the distribution of ξ is parametrically specified, those parameters can be consistently estimated by MLE using cross-sectional variation on the educational choices and inequalities (26) and (27). Note that we were able to consistently estimate ς only because ϱ and ω^2 had been previously estimated using time-series variation. Hence, the bi-directional relationship between the cross-sectional and time-series submodels.

The previous discussion illustrates the importance of combining cross-sectional data with a long time-series of aggregate data. One alternative would be to exploit panel data in which the time-series dimension of the panel is sufficiently long. This alternative has, however, a potential drawback. In Appendix C, we argue that a "long panel" approach is mathematically equivalent to time-series analysis where T goes to infinity, while n stays fixed. The standard errors in the long panel analysis should, therefore, be based exclusively on the time series variation. This discussion has an implication for the standard errors computed in Lee and Wolpin's (2006) (see their footnote 37). Donghoon Lee, in private communication, kindly informed us that their standard errors do not account for the noise introduced by the estimation of the time series parameters, i.e. the standard errors reported in the paper assume that the time-series parameters are known or, equivalently, fixed at their estimated value. We also note that since almost all panel data sets have limited time-series dimension, using this alternative approach would lead to imprecise estimates.

When cross-sectional data are combined with long time-series of aggregate data, the standard formulas for the computation of test statistics and confidence intervals are no longer valid. In the next section, we provide new easy-to-use formulas that can be employed for coefficients estimated by combining those two data sources. The formal derivation of those formulas is contained in the companion paper Hahn, Kuersteiner, and Mazzocco (2016).

6 Standard Errors

In this section, after the derivation of the formulas required for the computation of test statistics and confidence intervals of coefficients estimated using a combination of time-series and cross-sectional data, we will explain how they can be employed in concrete cases using, as an example, the general equilibrium model developed in the previous section.

6.1 Formulas for Test Statistics and Confidence Intervals

The asymptotic theory underlying the estimators obtained from the combination of the two data sources considered in this paper is complex. It is based on a new central limit theorem that requires a novel martingale representation. Given its complexity, the theory is presented in a separate paper (Hahn, Kuersteiner and Mazzocco (2016)). However, the mechanical implementation of the formulas required for the computation of test statistics and confidence intervals is straightforward. In the rest of this subsection we provide the step-by-step description of how those formulas can be calculated.

The computation starts with the explicit characterization of the "moments"that identify the parameters. Let $\theta = (\beta, \nu_1, ..., \nu_T)$ and denote with $f_{\theta,i}(\theta, \rho)$ and $g_{\rho,t}(\beta, \rho)$ the i-th and t-th moments used in the identification of the cross-sectional and time-series parameters. Suppose that our estimator can be written as the solution to the following system of equations:

$$\sum_{i=1}^{n} f_{\theta,i}\left(\hat{\theta}, \hat{\rho}\right) = 0, \tag{30}$$

$$\sum_{t=\tau_0+1}^{\tau_0+\tau} g_{\rho,t}\left(\hat{\beta}, \hat{\rho}\right) = 0. \tag{31}$$

Formulas can then be calculated using the following steps:

1. Let $\phi = (\theta', \rho')'$ be the vector of parameters.

2. Let

$$\mathbf{A} = \begin{bmatrix} \hat{A}_{y,\theta} & \hat{A}_{y,\rho} \\ \hat{A}_{\nu,\theta} & \hat{A}_{\nu,\rho} \end{bmatrix},$$

with

$$\hat{A}_{y,\theta} = n^{-1} \sum_{i=1}^{n} \frac{\partial f_{\theta,i}\left(\hat{\theta}, \hat{\rho}\right)}{\partial \theta'}, \qquad \hat{A}_{y,\rho} = n^{-1} \sum_{i=1}^{n} \frac{\partial f_{\theta,i}\left(\hat{\theta}, \hat{\rho}\right)}{\partial \rho'},$$

$$\hat{A}_{\nu,\theta} = \tau^{-1} \sum_{t=\tau_0+1}^{\tau_0+\tau} \frac{\partial g_{\rho,t}\left(\hat{\beta}, \hat{\rho}\right)}{\partial \theta'}, \qquad \hat{A}_{\nu,\rho} = \tau^{-1} \sum_{t=\tau_0+1}^{\tau_0+\tau} \frac{\partial g_{\rho,t}\left(\hat{\beta}, \hat{\rho}\right)}{\partial \rho'}.$$

3. Let

$$\hat{\Omega}_y = \frac{1}{n} \sum_{i=1}^{n} f_{\theta,i}\left(\hat{\theta}, \hat{\rho}\right) f_{\theta,i}\left(\hat{\theta}, \hat{\rho}\right)'$$

and

$$\hat{\Omega}_v = \frac{1}{n} \sum_{i=1}^{n} g_{\rho,t}\left(\hat{\theta}, \hat{\rho}\right) g_{\rho,t}\left(\hat{\theta}, \hat{\rho}\right)'$$

4. Let

$$W = \begin{bmatrix} \frac{1}{n}\hat{\Omega}_y & 0 \\ 0 & \frac{1}{\tau}\hat{\Omega}_\nu \end{bmatrix}$$

5. Calculate

$$\mathbf{V} = \mathbf{A}^{-1} W \left(\mathbf{A}'\right)^{-1}$$

and use it as the "variance" (not the asymptotic variance) of the estimator. For instance, if one is interested in the 95% confidence interval of the first component of ϕ, it can be written as $\hat{\phi}_1 \pm 1.96\sqrt{\mathbf{V}_{1,1}}$

6.2 Formulas Applied to the General Equilibrium Model

To apply the five steps described in the previous subsection to the general equilibrium model, we only have to derive the moment conditions used in its estimation $f_{\theta,i}(\theta, \rho)$ and $g_{\rho,t}(\beta, \rho)$.

For simplicity of notation, we assume that $\bar{n}_1 = \bar{n}_2 = n$. Also, we denote by $F_{i,t}$ a dummy variable that takes the value 1 if the flexible type of education is chosen and 0 otherwise. From the discussion in Section 5, it follows directly that the moments employed in the estimation of α_1 and σ take the following form:

$$\sum_i \left(\ln w_{i,1}^F - \ln w_{i,2}^F - \frac{1}{\alpha_1}\left(n_1^F - n_2^F\right) \right) = 0$$

and

$$\sum_i \left(\frac{c_{i,1}^*}{l_{i,1}^*} - w_1^F \frac{\sigma}{1-\sigma} \right) = 0.$$

For the estimation of the parameters ρ and ω^2, equation (29) implies that the OLS estimator of ϱ and the corresponding estimator for ω^2 solve:

$$\frac{1}{\tau} \sum_t \widehat{\log \nu_t} \left(\widehat{\log \nu_{t+1}} - \widehat{\varrho} \widehat{\log \nu_t} \right) = 0$$

and

$$\frac{1}{\tau} \sum_t \left(\widehat{\log \nu_{t+1}} - \widehat{\varrho} \widehat{\log \nu_t} \right)^2 = \widehat{\omega}^2.$$

Replacing for $\widehat{\log \nu_{t+1}}$ and $\widehat{\log \nu_t}$ using equation(28), we obtain the following two moment conditions:

$$\sum_t \begin{pmatrix} \alpha_1 \left(\ln w_t^F - \ln w_t^R\right) \\ -\left(\ln n_t^F - \ln n_t^R\right) \end{pmatrix} \left(\begin{pmatrix} \alpha_1 \left(\ln w_{t+1}^F - \ln w_{t+1}^R\right) \\ -\left(\ln n_{t+1}^F - \ln n_{t+1}^R\right) \end{pmatrix} - \varrho \begin{pmatrix} \alpha_1 \left(\ln w_t^F - \ln w_t^R\right) \\ -\left(\ln n_t^F - \ln n_t^R\right) \end{pmatrix} \right) = 0$$

$$\sum_t \left(\left(\begin{pmatrix} \alpha_1 \left(\ln w_{t+1}^F - \ln w_{t+1}^R\right) \\ -\left(\ln n_{t+1}^F - \ln n_{t+1}^R\right) \end{pmatrix} - \varrho \begin{pmatrix} \alpha_1 \left(\ln w_t^F - \ln w_t^R\right) \\ -\left(\ln n_t^F - \ln n_t^R\right) \end{pmatrix} \right)^2 - \omega^2 \right) = 0$$

To derive the moments employed in the estimation of the risk aversion parameters, we need an exact expression for $\gamma\left(x_{i,t}, \xi_{i,t} | \varsigma\right)$ and select a distribution for $\xi_{i,t}$, which require some assumptions. We will consider the case $1-\gamma \geq 0$. Similar arguments can be used if $1-\gamma < 0$. To derive a close-form for $\gamma\left(x_{i,t}, \xi_{i,t} | \varsigma\right)$, inequality (26) can alternatively be written as[13]

$$\gamma\left(x_{i,t}, \xi_{i,t} | \varsigma\right) \leq 1 - \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t\right).$$

Suppose we consider the following parameterization for $\gamma\left(x_{i,t}, \xi_{i,t} | \varsigma\right)$:

$$\gamma\left(x_{i,t}, \xi_{i,t} | \varsigma\right) = G\left(x_{i,t}'\varsigma + \xi_{i,t}\right),$$

where G is some monotonically increasing function bounded above by 1 and $\xi_{i,t} \sim N(0,1)$. We can then conclude that an individual chooses the flexible education ($F_{i,t} = 1$) if and only if

$$x_{i,t}'\varsigma + \xi_{i,t} \leq G^{-1}\left(1 - \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t\right)\right)$$

or, equivalently,

$$\xi_{i,t} \leq G^{-1}\left(1 - \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t\right)\right) - x_{i,t}'\varsigma.$$

The probability that $F_{i,t} = 1$ is therefore given by the following expression:

$$\Phi\left[G^{-1}\left(1 - \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t\right)\right) - x_{i,t}'\varsigma\right],$$

where Φ and ϕ denote the CDF and PDF of a $N(0,1)$.

Despite the complicated nature of the probability, the whole expression is linear in ς implying that it is a special case of a textbook probit. The First Order Condition (FOC) derived from the

[13] For $1-\gamma \geq 0$, the inequality (26) can alternatively be written as

$$\frac{\sigma\omega^2}{2\alpha_1} \geq \frac{\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t}{1 - \gamma\left(x_{i,t}, \xi_{i,t} | \varsigma\right)}$$

Using $1 - \gamma\left(x_{i,t}, \xi_{i,t} | \varsigma\right) \geq 0$, we get

$$1 - \gamma\left(x_{i,t}, \xi_{i,t} | \varsigma\right) \geq \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t\right)$$

or

$$\gamma\left(x_{i,t}, \xi_{i,t} | \varsigma\right) \leq 1 - \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t\right).$$

maximization of that probability with respect to ς takes the form

$$\sum_i \frac{F_{i,t} - \Phi\left[G^{-1}\left(1 - \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t\right)\right) - x_{i,t}'\varsigma\right]}{\Phi\left[G^{-1}\left(1 - \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t\right)\right) - x_{i,t}'\varsigma\right]\left\{1 - \Phi\left[G^{-1}\left(1 - \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t\right)\right) - x_{i,t}'\varsigma\right]\right\}} \times$$
$$\times \phi\left[G^{-1}\left(1 - \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho \log \nu_t\right)\right) - x_{i,t}'\varsigma\right] x_{i,t} = 0$$

Using the equation

$$\ln \nu_t = \alpha_1\left(\ln w_t^F - \ln w_t^R\right) - \left(\ln n_t^F - \ln n_t^R\right),$$

and defining

$$Z_t\left(\alpha_1, \sigma, \varrho, \omega^2\right) \equiv G^{-1}\left(1 - \frac{2\alpha_1}{\sigma\omega^2}\left(\log\left(\frac{n_t^F}{n_t^R}\right) + \varrho\left(\begin{array}{c} \alpha_1\left(\ln w_t^F - \ln w_t^R\right) \\ -\left(\ln n_t^F - \ln n_t^R\right) \end{array}\right)\right)\right),$$

we can rewrite the first order condition as follows:

$$\sum_i \frac{F_{i,t} - \Phi\left[Z_t\left(\alpha_1, \sigma, \varrho, \omega^2\right) - x_{i,t}'\varsigma\right]}{\Phi\left[Z_t\left(\alpha_1, \sigma, \varrho, \omega^2\right) - x_{i,t}'\varsigma\right]\left\{1 - \Phi\left[Z_t\left(\alpha_1, \sigma, \varrho, \omega^2\right) - x_{i,t}'\varsigma\right]\right\}} \phi\left[Z_t\left(\alpha_1, \sigma, \varrho, \omega^2\right) - x_{i,t}'\varsigma\right] x_{i,t} = 0,$$

which corresponds to the moments used in the estimation of the risk aversion parameters.

Assuming without loss of generality that only the cross sections at $t = 1$ and $t = 2$ are used in the estimation, the previous discussion implies that the vectors of moments $f_{\theta,i}$ and $g_{\rho,t}$ take the following form:

$$f_{\theta,i}(\theta, \rho) = \left[\begin{array}{c} \ln w_{i,1}^F - \ln w_{i,2}^F - \frac{1}{\alpha_1}\left(n_t^F - n_{t-1}^F\right) \\ \frac{c_{i,1}^*}{l_{i,1}^*} - w_1^F \frac{\sigma}{1-\sigma} \\ \frac{F_{i,1} - \Phi\left[Z_1\left(\alpha_1, \sigma, \varrho, \omega^2\right) - x_{i,1}'\varsigma\right]}{\Phi\left[Z_1\left(\alpha_1, \sigma, \varrho, \omega^2\right) - x_{i,1}'\varsigma\right]\left\{1 - \Phi\left[Z_1\left(\alpha_1, \sigma, \varrho, \omega^2\right) - x_{i,1}'\varsigma\right]\right\}} \phi\left[Z_1\left(\alpha_1, \sigma, \varrho, \omega^2\right) - x_{i,1}'\varsigma\right] x_{i,1} \end{array}\right],$$

and

$$g_{\rho,t}(\beta, \rho) = \left[\begin{array}{c} \left(\begin{array}{c} \alpha_1\left(\ln w_t^F - \ln w_t^R\right) \\ -\left(\ln n_t^F - \ln n_t^R\right) \end{array}\right)\left(\left(\begin{array}{c} \alpha_1\left(\ln w_{t+1}^F - \ln w_{t+1}^R\right) \\ -\left(\ln n_{t+1}^F - \ln n_{t+1}^R\right) \end{array}\right) - \varrho\left(\begin{array}{c} \alpha_1\left(\ln w_t^F - \ln w_t^R\right) \\ -\left(\ln n_t^F - \ln n_t^R\right) \end{array}\right)\right) \\ \left(\left(\begin{array}{c} \alpha_1\left(\ln w_{t+1}^F - \ln w_{t+1}^R\right) \\ -\left(\ln n_{t+1}^F - \ln n_{t+1}^R\right) \end{array}\right) - \varrho\left(\begin{array}{c} \alpha_1\left(\ln w_t^F - \ln w_t^R\right) \\ -\left(\ln n_t^F - \ln n_t^R\right) \end{array}\right)\right)^2 - \omega^2 \end{array}\right],$$

and the parameter estimates of the general equilibrium model are the solution to the following system of equations:

$$n^{-1/2} \sum_{i=1}^{n} f_{\theta,i}\left(\hat{\theta}, \hat{\rho}\right) = 0,$$

$$\tau^{-1/2} \sum_{t=\tau_0+1}^{\tau_0+\tau} g_{\rho,t}\left(\hat{\beta}, \hat{\rho}\right) = 0.$$

We are now ready to describe the five steps required in the computation of test statistics and confidence intervals for the general equilibrium model. As a first step, let $\theta = \beta = (\alpha_1, \sigma, \varsigma)$ and $\rho = (\varrho, \omega^2)$. Observe that the aggregate shock is not in the set of estimated parameters, since the general equilibrium model implies that $\ln \nu_t = \alpha_1 \left(\ln w_t^F - \ln w_t^R\right) - \left(\ln n_t^F - \ln n_t^R\right)$. In the second, third, and fourth steps compute the matrices A, $\hat{\Omega}_y$, $\hat{\Omega}_v$, and W using the vectors of moments $f_{\theta,i}$ and $g_{\rho,t}$ derived above. In the last step, calculate the variance matrix $V = A^{-1}W\left(\mathbf{A}'\right)^{-1}$.

7 Summary

Using a general econometric framework and three examples we have shown that generally, when aggregate shocks are present, model parameters cannot be identified using cross-sectional variation alone. Identification of those parameters requires the combination of cross-sectional and time-series data. When those two data sources are jointly used, standard formulas for the computation of test statistics and confidence intervals are no longer valid. We provide new easy-to-use formulas that account for the interaction between the time-series and cross-sectional data. Our results are expected to be helpful for the econometric analysis of rational expectations models involving individual decision making as well general equilibrium models.

References

[1] Ackerberg, D.A., K. Caves, and G. Frazer (2015): "Identification Properties of Recent Production Function Estimators," Econometrica 83, 2411–2451.

[2] Andrews, D.W.K. (2005): "Cross-Section Regression with Common Shocks," Econometrica 73, pp. 1551-1585.

[3] Aldous, D.J. and G.K. Eagleson (1978): "On mixing and stability of limit theorems," The Annals of Probability 6, 325–331.

[4] Arellano, M., R. Blundell, and S. Bonhomme (2014): "Household Earnings and Consumption: A Nonlinear Framework," unpublished working paper.

[5] Bajari, Patrick, Benkard, C. Lanier, and Levin, Jonathan (2007): "Estimating Dynamic Models of Imperfect Competition," Econometrica 75, 5, pp. 1331-1370.

[6] Billingsley, P. (1968): "Convergence of Probability Measures," John Wiley and Sons, New York.

[7] Brown, B.M. (1971): "Martingale Central Limit Theorems," Annals of Mathematical Statistics 42, pp. 59-66.

[8] Campbell, J.Y., and M. Yogo (2006): "Efficient Tests of Stock Return Predictability," Journal of Financial Economics 81, pp. 27–60.

[9] Chamberlain, G. (1984): "Panel Data," in Handbook of Econometrics, eds. by Z. Griliches and M. Intriligator. Amsterdam: North Holland, pp. 1247-1318.

[10] Chan, N.H. and C.Z. Wei (1987): "Asymptotic Inference for Nearly Nonstationary AR(1) Processes," Annals of Statistics 15, pp.1050-1063.

[11] Dedecker, J and F. Merlevede (2002): "Necessary and Sufficient Conditions for the Conditional Central Limit Theorem," Annals of Probability 30, pp. 1044-1081.

[12] Duffie, D., J. Pan and K. Singleton (2000): "Transform Analysis and Asset Pricing for Affine Jump Diffusions," Econometrica, pp. 1343-1376.

[13] Durrett, R. (1996): "Stochastic Calculus," CRC Press, Boca Raton, London, New York.

[14] Eagleson, G.K. (1975): "Martingale convergence to mixtures of infinitely divisible laws," The Annals of Probability 3, 557–562.

[15] Feigin, P. D. (1985): "Stable convergence of Semimartingales," Stochastic Processes and their Applications 19, pp. 125-134.

[16] Gagliardini, P., and C. Gourieroux (2011): "Efficiency in Large Dynamic Panel Models with Common Factor," unpublished working paper.

[17] Gemici, A., and M. Wiswall (2014): "Evolution of Gender Differences in Post-Secondary Human Capital Investments: College Majors," International Economic Review 55, 23–56.

[18] Gillingham, K., F. Iskhakov, A. Munk-Nielsen, J. Rust, and B. Schjerning (2015): "A Dynamic Model of Vehicle Ownership, Type Choice, and Usage," unpublished working paper.

[19] Hahn, J., and G. Kuersteiner (2002): "Asymptotically Unbiased Inference for a Dynamic Panel Model with Fixed Effects When Both n and T are Large," Econometrica 70, pp. 1639–57.

[20] Hahn, J., and W.K. Newey (2004): "Jackknife and Analytical Bias Reduction for Nonlinear Panel Models," Econometrica 72, pp. 1295–1319.

[21] Hall, P., and C. Heyde (1980): "Martingale Limit Theory and its Applications," Academic Press, New York.

[22] Hansen, B.E. (1992): "Convergence to Stochastic Integrals for Dependent Heterogeneous Processes," Econometric Theory 8, pp. 489-500.

[23] Heckman, J.J., L. Lochner, and C. Taber (1998), "Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents," Review of Economic Dynamics 1, pp. 1-58.

[24] Heckman, J.J., and G. Sedlacek (1985): "Heterogeneity, Aggregation, and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market," Journal of Political Economy, 93, pp. 1077-1125.

[25] Jacod, J. and A.N. Shiryaev (2002): "Limit Theorems for stochastic processes," Springer Verlag, Berlin.

[26] Kuersteiner, G.M., and I.R. Prucha (2013): "Limit Theory for Panel Data Models with Cross Sectional Dependence and Sequential Exogeneity," Journal of Econometrics 174, pp. 107-126.

[27] Kuersteiner, G.M and I.R. Prucha (2015): "Dynamic Spatial Panel Models: Networks, Common Shocks, and Sequential Exogeneity," CESifo Working Paper No. 5445.

[28] Kurtz and Protter (1991): "Weak Limit Theorems for Stochastic Integrals and Stochastic Differential Equations," Annals of Probability 19, pp. 1035-1070.

[29] Lee, D. (2005): "An Estimable Dynamic General Equilibrium Model of Work, Schooling and Occupational Choice," International Economic Review 46, pp. 1-34.

[30] Lee, D., and K.I. Wolpin (2006): "Intersectoral Labor Mobility and the Growth of Service Sector," Econometrica 47, pp. 1-46.

[31] Lee, D., and K.I. Wolpin (2010): "Accounting for Wage and Employment Changes in the U.S. from 1968-2000: A Dynamic Model of Labor Market Equilibrium," Journal of Econometrics 156, pp. 68–85.

[32] Magnac, T. and D. Thesmar (2002): "Identifying Dynamic Discrete Decision Processes," Econometrica 70, pp. 801–816.

[33] Magnus, J.R. and H. Neudecker (1988): "Matrix Differential Calculus with Applications in Statistics and Econometrics," Wiley.

[34] Mikusheva, A. (2007): "Uniform Inference in Autoregressive Models," Econometrica 75, pp. 1411–1452.

[35] Murphy, K. M. and R. H. Topel (1985): "Estimation and Inference in Two-Step Econometric Models," Journal of Business and Economic Statistics 3, pp. 370 – 379.

[36] Olley, G.S., and A. Pakes (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," Econometrica 64, pp. 1263 – 1297.

[37] Peccati, G. and M.S. Taqqu (2007): "Stable convergence of generalized L^2stochastic integrals and the principle of conditioning," Electronic Journal of Probability 12, pp.447-480.

[38] Phillips, P.C.B. (1987): "Towards a unified asymptotic theory for autoregression," Biometrika 74, pp. 535-547.

[39] Phillips, P.C.B. (1988): "Regression theory for near integrated time series," Econometrica 56, pp. 1021-1044.

[40] Phillips, P.C.B. and S.N. Durlauf (1986): "Multiple Time Series Regression with Integrated Processes," Review of Economic Studies 53, pp. 473-495.

[41] Phillips, P.C.B. and H.R. Moon (1999): "Linear Regression Limit Theory for Nonstationary Panel Data," Econometrica 67, pp. 1057-1111.

[42] Phillips, P.C.B. (2014): "On Confidence Intervals for Autoregressive Roots and Predictive Regression," Econometrica 82, pp. 1177–1195.

[43] Renyi, A (1963): "On stable sequences of events," Sankya Ser. A, 25, 293-302.

[44] Rootzen, H. (1983): "Central limit theory for martingales via random change of time," Department of Statistics, University of North Carolina, Technical Report 28.

[45] Runkle, D.E. (1991): "Liquidity Constraints and the Permanent-Income Hypothesis: Evidence from Panel Data," Journal of Monetary Economics 27, pp. 73–98.

[46] Shea, J. (1995):"Union Contracts and the Life-Cycle/Permanent-Income Hypothesis," American Economic Review 85, pp. 186–200.

[47] Tobin, J. (1950): "A Statistical Demand Function for Food in the U.S.A.," Journal of the Royal Statistical Society, Series A. Vol. 113, pp.113-149.

[48] Wooldridge, J.M. and H. White (1988): "Some invariance principles and central limit theorems for dependent heterogeneous processes," Econometric Theory 4, pp.210-230.

A Discussion for Section 3

A.1 Proof of (2)

The maximization problem is equivalent to

$$\max_{\alpha} \; -e^{-\delta(\alpha(1+r)+(1-\alpha))} E\left[e^{-\delta(1-\alpha)u_{i,t}}\right].$$

Since $-\delta\,(1-\alpha)\,u_{i,t} \sim N\left(-\delta\,(1-\alpha)\,\mu, \delta^2\,(1-\alpha)^2\,\sigma^2\right)$, we have

$$E\left[e^{-\delta(1-\alpha)u_{i,t}}\right] = e^{-\delta(1-\alpha)\mu + \frac{\delta^2(1-\alpha)^2\sigma^2}{2}},$$

and the maximization problem can be rewritten as follows:

$$\max_{\alpha} \; -e^{-\delta\left(\alpha(1+r)+(1-\alpha)(1+\mu)-\frac{\delta(1-\alpha)^2\sigma^2}{2}\right)}.$$

Taking the first order condition, we have,

$$0 = -\delta\left(r - \mu + \sigma^2\delta - \alpha\sigma^2\delta\right)$$

from which we obtain the solution

$$\alpha = \frac{1}{\sigma^2\delta}\left(r - \mu + \sigma^2\delta\right).$$

A.2 Euler Equation and Cross Section

Our model in Section 3 is a stylized version of many models considered in a large literature interested in estimating the parameter δ using cross-sectional variation. Estimators are often based on moment conditions derived from first order conditions (FOC) related to optimal investment and consumption decisions. We illustrate the problems facing such estimators.

Assume a researcher has a cross-section of observations for individual consumption and returns $c_{i,t}$ and $u_{i,t}$. The population FOC of our model[14] takes the simple form $E\left[e^{-\delta c_{i,t}}\left(r - u_{i,t}\right)\right] = 0$. A

[14] We assume $\delta \neq 0$ and rescale the equation by $-\delta^{-1}$.

just-identified moment based estimator for δ solves the sample analog $n^{-1}\sum_{i=1}^{n} e^{-\hat{\delta}c_{i,t}}\left(r-u_{i,t}\right)=0$. It turns out that the probability limit of $\hat{\delta}$ is equal to $\left(\nu_t-r\right)/\left(\left(1-\alpha\right)\sigma_\epsilon^2\right)$, i.e., $\hat{\delta}$ is inconsistent.

We now compare the population FOC a rational agent uses to form their optimal portfolio with the empirical FOC an econometrician using cross-sectional data observes:

$$n^{-1}\sum_{i=1}^{n} e^{-\delta c_{i,t}}\left(r-u_{i,t}\right)=0.$$

Noting that $u_{i,t}=\nu_t+\epsilon_{i,t}$ and substituting into the budget constraint

$$c_{i,t}=1+\alpha r+\left(1-\alpha\right)u_{i,t}=1+\alpha r+\left(1-\alpha\right)\nu_t+\left(1-\alpha\right)\epsilon_{i,t}$$

we have

$$\begin{aligned} n^{-1}\sum_{i=1}^{n} e^{-\delta c_{i,t}}\left(r-u_{i,t}\right) &= n^{-1}\sum_{i=1}^{n} e^{-\delta\left(1+\alpha r+\left(1-\alpha\right)\nu_t\right)-\delta\left(1-\alpha\right)\epsilon_{i,t}}\left(r-\nu_t-\epsilon_{i,t}\right) \\ &= e^{-\delta\left(1+\alpha r+\left(1-\alpha\right)\nu_t\right)}\left(\left(r-\nu_t\right)n^{-1}\sum_{i=1}^{n} e^{-\delta\left(1-\alpha\right)\epsilon_{i,t}}-n^{-1}\sum_{i=1}^{n} e^{-\delta\left(1-\alpha\right)\epsilon_{i,t}}\epsilon_{i,t}\right). \end{aligned} \tag{32}$$

Under suitable regularity conditions including independence of $\epsilon_{i,t}$ in the cross-section it follows that

$$n^{-1}\sum_{i=1}^{n} e^{-\delta\left(1-\alpha\right)\epsilon_{i,t}}=E\left[e^{-\delta\left(1-\alpha\right)\epsilon_{i,t}}\right]+o_p\left(1\right)=e^{\frac{\delta^2\left(1-\alpha\right)^2\sigma_\epsilon^2}{2}}+o_p\left(1\right) \tag{33}$$

and

$$n^{-1}\sum_{i=1}^{n} e^{-\delta\left(1-\alpha\right)\epsilon_{i,t}}\epsilon_{i,t}=E\left[e^{-\delta\left(1-\alpha\right)\epsilon_{i,t}}\epsilon_{i,t}\right]+o_p\left(1\right)=-\delta\left(1-\alpha\right)\sigma_\epsilon^2 e^{\frac{\delta^2\left(1-\alpha\right)^2\sigma_\epsilon^2}{2}}+o_p\left(1\right). \tag{34}$$

Taking limits as $n\to\infty$ in (32) and substituting (33) and (34) then shows that the method of moments estimator based on the empirical FOC asymptotically solves

$$\left(\left(r-\nu_t\right)+\delta\left(1-\alpha\right)\sigma_\epsilon^2\right)e^{\frac{\delta^2\left(1-\alpha\right)^2\sigma_\epsilon^2}{2}}=0. \tag{35}$$

Solving for δ we obtain

$$\operatorname{plim}\hat{\delta}=\frac{\nu_t-r}{\left(1-\alpha\right)\sigma_\epsilon^2}.$$

This estimate is inconsistent because the cross-sectional data set lacks cross sectional ergodicity, or in other words does not contain the same information about aggregate risk as is used by rational agents. Therefore, the empirical version of the FOC is unable to properly account for aggregate risk and return characterizing the risky asset. The estimator based on the FOC takes the form of an implicit solution to an empirical moment equation, which obscures the effects of cross-sectional non-ergodicity. A more illuminative approach uses our modelling strategy in Section 2.

On the other hand, it is easily shown using properties of the Gaussian moment generating function that the population FOC is proportional to

$$E\left[e^{-\delta(1-\alpha)u_{i,t}}\left(r-u_{i,t}\right)\right]=\left(r-\mu+\delta\left(1-\alpha\right)\sigma^{2}\right)e^{-\delta(1-\alpha)\mu+\frac{\delta^{2}(1-\alpha)^{2}\sigma^{2}}{2}}=0. \tag{36}$$

The main difference between (33) and (34) lies in the fact that σ_v^2 is estimated to be 0 in the sample and that $\nu_t \neq \mu$ in general. Note that (36) implies that consistency may be achieved with a large number of repeated cross sections, or a panel data set with a long time series dimension. However, this raises other issues discussed later in Section C.

B Details of Section 5

B.1 Proof of Inequalities (26) and (27)

In the proof we will drop the i subscripts for notational purposes. We can rewrite (23) as follows:

$$\begin{aligned}&\int\frac{\left[\left(\sigma w_{t+1}^{F}\left(\nu_{t+1}\right)\exp\left(\varepsilon_{t+1}\right)T\right)^{\sigma}\left(\left(1-\sigma\right)T\right)^{1-\sigma}\right]^{1-\gamma}}{1-\gamma}dP\left(\nu_{t+1}\left|\nu_{t}\right.\right)\\ &\geq\int\frac{\left[\left(\sigma w_{t+1}^{R}\left(\nu_{t+1}\right)\exp\left(\varepsilon_{t+1}\right)T\right)^{\sigma}\left(\left(1-\sigma\right)T\right)^{1-\sigma}\right]^{1-\gamma}}{1-\gamma}dP\left(\nu_{t+1}\left|\nu_{t}\right.\right),\end{aligned}$$

As a consequence, education F is chosen if

$$\psi\left(\gamma,\nu_{t}\right)\equiv\int\left[\left(w_{t+1}^{F}\left(\nu_{t+1}\right)\right)^{\sigma}\right]^{1-\gamma}dP\left(\nu_{t+1}\left|\nu_{t}\right.\right)-\int\left[w_{t+1}^{R}\left(\nu_{t+1}\right)^{\sigma}\right]^{1-\gamma}dP\left(\nu_{t+1}\left|\nu_{t}\right.\right)\geq 0 \tag{37}$$

We rewrite the value function of an old individual with education F

$$V_{t}\left(F,\nu_{t}\right)=\frac{\left[\left(\left(\frac{n^{F}\sigma T\exp\left(\varepsilon_{t}\right)}{e^{\alpha_{0}}}\right)^{1/\alpha_{1}}\sigma T\right)^{\sigma}\left(\left(1-\sigma\right)T\right)^{1-\sigma}\right]^{1-\gamma}}{1-\gamma}$$

Likewise, the value function of an old individual with education R takes a similar form:

$$V_{t}\left(R,\nu_{t}\right)=\frac{\left[\left(\left(\frac{n^{R}\sigma T\exp\left(\varepsilon_{t}\right)}{e^{\alpha_{0}}\nu_{t}}\right)^{1/\alpha_{1}}\sigma T\right)^{\sigma}\left(\left(1-\sigma\right)T\right)^{1-\sigma}\right]^{1-\gamma}}{1-\gamma}$$

We can now describe the solution to the problem of a young worker. Given the assumptions, optimal consumption and leisure in the first period can be easily computed to be:

$$c_t^* = y_t - C_e,$$
$$l_t^* = T - T_e.$$

This implies that current consumption and leisure are independent of the education choice and of the aggregate shock. As a consequence, the current utility will also be independent of the education choice and of the aggregate shock. The education choice will therefore only depend on the utility when old. Specifically, the individual will choose education F if

$$\int V_t(F, \nu_{t+1})\, dP(\nu_{t+1}|\nu_t) \geq \int V_t(R, \nu_{t+1})\, dP(\nu_{t+1}|\nu_t).$$

Write

$$V_t(F, \nu_{t+1}) = \frac{\left[\left(\left(\frac{n^R \sigma T}{e^{\alpha_0}}\right)^{1/\alpha_1} \sigma T\right)^{\sigma} ((1-\sigma)T)^{1-\sigma}\right]^{1-\gamma}}{1-\gamma} \exp(\sigma(1/\alpha_1)(1-\gamma)\varepsilon_{t+1})$$

$$V_t(R, \nu_{t+1}) = \frac{\left[\left(\left(\frac{n^R \sigma T}{e^{\alpha_0}}\right)^{1/\alpha_1} \sigma T\right)^{\sigma} ((1-\sigma)T)^{1-\sigma}\right]^{1-\gamma}}{1-\gamma} \exp(\sigma(1/\alpha_1)(1-\gamma)\varepsilon_{t+1}) \left(\nu_{t+1}^{-\sigma(1/\alpha_1)(1-\gamma)}\right)$$

We see that education F is chosen if and only if

$$\frac{\left[\left(\left(\frac{n^F \sigma T}{e^{\alpha_0}}\right)^{1/\alpha_1} \sigma T\right)^{\sigma} ((1-\sigma)T)^{1-\sigma}\right]^{1-\gamma}}{1-\gamma} \exp(\sigma(1/\alpha_1)(1-\gamma)\varepsilon_{t+1}) \geq$$
$$\frac{\left[\left(\left(\frac{n^R \sigma T}{e^{\alpha_0}}\right)^{1/\alpha_1} \sigma T\right)^{\sigma} ((1-\sigma)T)^{1-\sigma}\right]^{1-\gamma}}{1-\gamma} \exp(\sigma(1/\alpha_1)(1-\gamma)\varepsilon_{t+1})\, E_t\left[\nu_{t+1}^{-\sigma(1/\alpha_1)(1-\gamma)}\right]$$

(We made use of the assumption that ε_{t+1} is known to the workers.) This is equivalent to

$$\left(n^F\right)^{\sigma(1-\gamma)/\alpha_1} \geq \left(n^R\right)^{\sigma(1-\gamma)/\alpha_1} E_t\left[\nu_{t+1}^{-\sigma(1-\gamma)(1/\alpha_1)}\right] \qquad \text{if} \qquad 1-\gamma \geq 0 \tag{38}$$

and to

$$\left(n^F\right)^{\sigma(1-\gamma)/\alpha_1} < \left(n^R\right)^{\sigma(1-\gamma)/\alpha_1} E_t\left[\nu_{t+1}^{-\sigma(1-\gamma)(1/\alpha_1)}\right] \qquad \text{if} \qquad 1-\gamma < 0 \tag{39}$$

Because $\log \nu_{t+1} = \rho \log \nu_t + \eta_t$, or

$$\nu_{t+1} = \nu_t^{\rho} \exp(\eta_t)$$

we can write

$$E_t\left[\nu_{t+1}^{-\sigma(1-\gamma)(1/\alpha_1)}\right] = E_\eta\left[\left(\nu_t^{\rho} \exp(\eta_t)\right)^{-\sigma(1-\gamma)(1/\alpha_1)}\right] = \nu_t^{-\rho\sigma(1-\gamma)(1/\alpha_1)} E\left[\exp\left(-\sigma(1-\gamma)(1/\alpha_1)\eta_t\right)\right]$$

where $E_\eta[\cdot]$ denotes the integral with respect to η_t alone. The assumption that $\eta_t \sim N(0, \omega^2)$ allows us to write

$$E\left[\exp\left(-\sigma(1-\gamma)(1/\alpha_1)\eta_t\right)\right] = \exp\left(\frac{(\sigma\omega(1-\gamma)(1/\alpha_1))^2}{2}\right)$$

recognizing that the expectation on the left is nothing but the moment generating function of $N(0, \omega^2)$ evaluated at $-\sigma(1-\gamma)(1/\alpha_1)$. Therefore, we have

$$E_t\left[\nu_{t+1}^{-\sigma(1-\gamma)(1/\alpha_1)}\right] = \nu_t^{-\rho\sigma(1-\gamma)(1/\alpha_1)} \exp\left(\frac{(\sigma\omega(1-\gamma)(1/\alpha_1))^2}{2}\right) \tag{40}$$

Consider first the case $1-\gamma \geq 0$. Combining (38) and (40), we can rewrite the decision as

$$\left(n^F\right)^{\sigma(1-\gamma)/\alpha_1} \geq \left(n^R\right)^{\sigma(1-\gamma)/\alpha_1} \nu_t^{-\rho\sigma(1-\gamma)(1/\alpha_1)} \exp\left(\frac{(\sigma\omega(1-\gamma)(1/\alpha_1))^2}{2}\right).$$

Taking logs, we obtain

$$\frac{\sigma(1-\gamma)}{\alpha_1}\log n^F \geq \frac{\sigma(1-\gamma)}{\alpha_1}\log n^R - \rho\frac{\sigma(1-\gamma)}{\alpha_1}\log \nu_t + \frac{(\sigma\omega(1-\gamma))^2}{2\alpha_1^2}$$

Dividing by σ and multiplying by $\alpha_1 < 0$, we conclude that the decision is equivalent to

$$(1-\gamma)\log n^F \leq (1-\gamma)\log n^R - \rho(1-\gamma)\log \nu_t + \frac{\sigma(1-\gamma)^2\omega^2}{2\alpha_1}$$

or

$$-\rho(1-\gamma)\log \nu_t + \frac{\sigma(1-\gamma)^2\omega^2}{2\alpha_1} \geq (1-\gamma)\log\left(\frac{n^F}{n^R}\right).$$

which proves inequality (26). If $1-\gamma < 0$, following the same steps, we have

$$-\rho(1-\gamma)\log \nu_t + \frac{\sigma(1-\gamma)^2\omega^2}{2\alpha_1} < (1-\gamma)\log\left(\frac{n^F}{n^R}\right),$$

which proves inequality (27).

Remark 1 *Equations 26 and 27 hints at the problem of identification based on cross section variation alone. This is because the cross section variation does not identify ρ, which implies that γ is not identified as a consequence. Suppose that γ has a multinomial distribution, i.e., it has a finite support $\gamma_1^*, \ldots, \gamma_M^*$. Let ρ^* denote the true value of ρ. Let $\gamma_m(\rho)$ be defined by*

$$-\rho \log \nu_t + (1 - \gamma_m(\rho)) \frac{\sigma\omega^2}{2\alpha_1} = -\rho^* \log \nu_t + (1 - \gamma_m^*) \frac{\sigma\omega^2}{2\alpha_1}$$

More precisely, let

$$\gamma_m(\rho) = 1 - \frac{\rho \log \nu_t - \rho^* \log \nu_t + (1 - \gamma_m^*) \frac{\sigma\omega^2}{2\alpha_1}}{\frac{\sigma\omega^2}{2\alpha_1}}$$

We then have

$$-\rho \log \nu_t + (1 - \gamma_m(\rho)) \frac{\sigma\omega^2}{2\alpha_1} = -\rho^* \log \nu_t + (1 - \gamma_m^*) \frac{\sigma\omega^2}{2\alpha_1}$$

for range of possible values of ρ, and the education choice implied by $(\rho, \gamma_1(\rho), \ldots, \gamma_M(\rho))$ is identical to the one implied by the true value $(\rho^*, \gamma_1^*, \ldots, \gamma_M^*)$ of the parameter. The parameter $(\rho^*, \gamma_1^*, \ldots, \gamma_M^*)$ is not identified.

B.2 Proof of (24) and (25)

Note that individual heterogeneity is completely summarized by the vector $\chi_t \equiv (\varepsilon_t, \gamma)$, which means that we can define the labor supply $h_t^F(\chi)$ and $h_t^R(\chi)$ for each type χ of workers. We assume that the mass of individuals such that $(\varepsilon_t, \gamma) \in A$ for some $A \subset R^2$ is given by $N_t \int_A G(d\chi)$, where G is a joint CDF. For simplicity, we assume that G is such that the first and second components are independent of each other. We also assume that $\int \exp(\varepsilon_t) G(d\chi) = 1$.

The labor markets for both types of education must be in equilibrium. To determine the equilibrium, remember that the individual educational choice is summarized by the function $\psi(\gamma, \nu_{t-1})$, which was defined in inequality (37) and describes the values of the risk aversion parameter and of the aggregate shocks for which an individual chooses a particular education. Specifically, an individual choose education F if $\psi(\gamma, \nu_{t-1}) > 0$. We can now introduce the equilibrium condition for education F. It takes the following form:

$$H_t^{D,F} = N_t \int_{E=F} h_t^F(\chi) G(d\chi) = N_t \sigma \mathcal{T} \int_{\psi(\gamma, \nu_{t-1}) \geq 0} \exp(\varepsilon_t) G(d\chi)$$
$$H_t^{D,R} = N_t \int_{E=R} h_t^F(\chi) G(d\chi) = N_t \sigma \mathcal{T} \int_{\psi(\gamma, \nu_{t-1}) < 0} \exp(\varepsilon_t) G(d\chi)$$

Using independence between γ and ε as well as $\int \exp(\varepsilon_t) G(d\chi) = 1$, we can write

$$\begin{aligned}\int_{\psi(\gamma,\nu_t)\geq 0} \exp(\varepsilon_t) G(d\chi) &= \left(\int_{\psi(\gamma,\nu_{t-1})\geq 0} G(d\chi)\right)\left(\int \exp(\varepsilon_t) G(d\chi)\right) \\ &= \int_{\psi(\gamma,\nu_{t-1})\geq 0} G(d\chi) \\ &= \text{Fraction of workers in Sector } F\end{aligned}$$

so we can write $H_t^{D,F} = n_t^F \sigma T$, where n^F is the mass/measure of individuals that chose education F. Taking logs, we have:

$$\ln H_t^{D,F} = \ln n_t^F + \ln \sigma + \ln T,$$

Substituting for $H_t^{D,F}$, we obtain the following equilibrium condition:

$$\alpha_0 + \alpha_1 \ln w_t^F = \ln n_t^F + \ln \sigma + \ln T,$$

Solving for $\ln w_t^F$, we have the log equilibrium wage:

$$\left(z_t^F \equiv\right) \quad \ln w_t^F = \frac{\ln n_t^F + \ln \sigma + \ln T - \alpha_0}{\alpha_1}.$$

This wage is for the unit of effective labor. Because the worker i provides $\sigma \exp(\varepsilon_t) T$ of effective labor, his recorded earning is $\sigma \exp(\varepsilon_t) T \exp\left(\frac{\ln n_t^F + \ln \sigma + \ln T - \alpha_0}{\alpha_1}\right)$. Because he works for σT hours, his wage for the labor is $\exp(\varepsilon_t) \exp\left(\frac{\ln n_t^F + \ln \sigma + \ln T - \alpha_0}{\alpha_1}\right)$; we will assume that the cross section "error" consist of n IID copies of ε_t, i.e.,the observed log equilibrium individual wage follows:

$$\ln w_{it}^F = \frac{\ln n_t^F + \ln \sigma + \ln T - \alpha_0}{\alpha_1} + \varepsilon_{it}.$$

Similarly, the equilibrium condition for education R has the following form:

$$H_t^{D,R} = n_t^R \sigma T,$$

where n^R is the mass/measure of individuals that chose education R. Substituting for $H_t^{D,R}$ and solving for $\ln w_t^R$, we obtain the following equilibrium wage for R:

$$\left(z_t^R \equiv\right) \quad \ln w_t^R = \frac{\ln n_t^R + \ln \sigma + \ln T - \alpha_0 - \ln \nu_t}{\alpha_1}.$$

By the same reasoning, the observed log equilibrium wage would look like

$$\ln w_{it}^R = \frac{\ln n_t^R + \ln \sigma + \ln T - \alpha_0 - \ln \nu_t}{\alpha_1} + \varepsilon_{it}.$$

C Long Panels?

Our proposal requires access to two data sets, a cross-section (or short panel) and a long time series of aggregate variables. One may wonder whether we may obtain an estimator with similar properties by exploiting panel data sets in which the time series dimension of the panel data is large enough.

One obvious advantage of combining two sources of data is that time series data may contain variables that are unavailable in typical panel data sets. For example the inflation rate potentially provides more information about aggregate shocks than is available in panel data. We argue with a toy model that even without access to such variables, the estimator based on the two data sets is expected to be more precise, which suggests that the advantage of data combination goes beyond availability of more observable variables.

Consider the alternative method based on one long panel data set, in which both n and T go to infinity. Since the number of aggregate shocks ν_t increases as the time-series dimension T grows, we expect that the long panel analysis can be executed with tedious yet straightforward arguments by modifying ideas in Hahn and Kuersteiner (2002), Hahn and Newey (2004) and Gagliardini and Gourieroux (2011), among others.

We will now illustrate potential problem with the long panel approach with a simple artificial example. Suppose that the econometrician is interested in the estimation of a parameter γ that characterizes the following system of linear equations:

$$\begin{aligned} q_{i,t} &= x_{i,t}\frac{\gamma}{\omega} + \nu_t + \varepsilon_{i,t} \qquad i = 1, \ldots, n;\ t = 1, \ldots, T, \\ \nu_t &= \omega \nu_{t-1} + u_t. \end{aligned}$$

The variables $q_{i,t}$ and $x_{i,t}$ are observed and it is assumed that $x_{i,t}$ is strictly exogenous in the sense that it is independent of the error term $\varepsilon_{i,t}$, including all leads and lags. For simplicity, we also assume that u_t and $\varepsilon_{i,t}$ are normally distributed with zero mean and that $\varepsilon_{i,t}$ is i.i.d. across both i and t. We will denote by δ the ratio γ/ω.

In order to estimate γ based on the panel data $\{(q_{i,t}, x_{i,t}), i = 1, \ldots, n;\ t = 1, \ldots, T\}$, we can adopt a simple two-step estimator of γ. In a first step, the parameter δ and the aggregate shocks ν_t are estimated using an Ordinary Least Square (OLS) regression of $q_{i,t}$ on $x_{i,t}$ and time dummies.

In the second step, the time-series parameter ω is estimated by regressing $\widehat{\nu}_t$ on $\widehat{\nu}_{t-1}$, where $\widehat{\nu}_t$, $t = 1, \ldots, T$, are the aggregate shocks estimated in the first step using the time dummies. An estimator of γ can then be obtained as $\widehat{\delta}\widehat{\omega}$.

The following remarks are useful to understand the properties of the estimator $\widehat{\gamma} = \widehat{\delta}\widehat{\omega}$. First, even if ν_t were observed, for $\widehat{\omega}$ to be a consistent estimator of ω we would need T to go to infinity, under which assumption we have $\widehat{\omega} = \omega + O_p\left(T^{-1/2}\right)$. This implies that it is theoretically necessary to assume that our data source is a "long" panel, i.e., $T \rightarrow \infty$. Similarly, $\hat{\nu}_t$ is a consistent estimator of ν_t only if n goes to infinity. As a consequence, we have $\hat{\nu}_t = \nu_t + O_p\left(n^{-1/2}\right)$. This implies that it is in general theoretically necessary to assume that $n \rightarrow \infty$.[15] Moreover, if n and T both go to infinity, $\widehat{\delta}$ is a consistent estimator of δ and $\widehat{\delta} = \delta + O_p\left(n^{-1/2}T^{-1/2}\right)$. All this implies that

$$\widehat{\gamma} = \widehat{\delta}\widehat{\omega} = \left(\delta + O_p\left(\frac{1}{\sqrt{nT}}\right)\right)\left(\omega + O_p\left(\frac{1}{\sqrt{T}}\right)\right) = \delta\omega + O_p\left(\frac{1}{\sqrt{T}}\right) = \gamma + O_p\left(\frac{1}{\sqrt{T}}\right).$$

The $O_p\left(n^{-1/2}T^{-1/2}\right)$ estimation noise of $\widehat{\delta}$, which is dominated by the $O_p\left(T^{-1/2}\right)$, is the term that would arise if ω were not estimated. The term reflects typical findings in long panel analysis (i.e., large n, large T), where the standard errors are inversely proportional to the square root of the number $n \times T$ of observations. The fact that it is dominated by the $O_p\left(T^{-1/2}\right)$ term indicates that the number of observations is effectively equal to T, i.e., the long panel should be treated as a time series problem for all practical purposes.

This conclusion has two interesting implications. First, the sampling noise due to cross-section variation should be ignored and the "standard" asymptotic variance formulae should generally be avoided in the panel data analysis when aggregate shocks are present. We note that Lee and Wolpin's (2006, 2010) standard errors use the standard formula that ignores the $O_p\left(T^{-1/2}\right)$ term. Second, since in most cases the time-series dimension T of a panel data set is relatively small, despite the theoretical assumption that it grows to infinity, estimators based on panel data will generally be more imprecise than may be expected from the "large" number $n \times T$ of observations.[16]

[15] For $\widehat{\omega}$ to have the same distribution as if ν_t were observed, we need n to go to infinity faster than T or equivalently that $T = o(n)$. See Heckman and Sedlacek (1985, p. 1088).

[16] This raises an interesting point. Suppose there is an aggregate time series data set available with which consistent estimation of γ is feasible at the standard rate of convergence. Also suppose that the number of observations there, say τ, is a lot larger than T. If this were the case, we should probably speculate that the panel data analysis is strictly dominated by the time series analysis from the efficiency point of view.