Local Public Good Provision, Myopic Voting, and Mobility

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Abstract

Few empirical strategies have been developed that investigate public provision under majority rule while taking explicit account of the constraints implied by mobility of households. Previous empirical work by Epple, Romer, and Sieg (2001) has focused on necessary conditions that observed expenditures, housing prices, and tax rates had to satisfy in a myopic voting equilibrium. The results reported in that paper suggest that myopic voting behavior is not consistent with the data. This finding is puzzling, especially given the prominence that myopic voting plays in the theoretical literature. The goal of this paper is to clarify these results and improve our understanding of the main limitations of myopic voting. We develop a new empirical approach which allows us to impose all restrictions that arise from locational equilibrium models with myopic voting simultaneously on the data generating process. We can then analyze how close myopic models come in replicating the main regularities about expenditures, taxes, sorting by income and housing observed in the data. We find that our baseline myopic model performs reasonably well in explaining variation in housing expenditures and educational expenditures across communities, but it performs poorly in explaining variation in tax rates. An extension of the model that incorporates peer effects fits all dimensions of the data reasonably well.

JEL classification: C51, H31, R12

1 Introduction

Models of interjurisdictional equilibrium take as their starting point the idea that households are (at least potentially) mobile. Communities may differ according to their levels of public good provision, tax rates (usually property tax rates), and local housing market conditions. Each household takes these factors into account in choosing a community. If local public good provision is decentralized, for example via local majority rule, then in each community, the level of public goods will depend on characteristics (tastes, endowments) of the community's residents. Households will sort among communities according tastes and endowments, so that households with similar preferences for local public goods will tend to live in the same community. Because the population of each community is endogenous, the set of households who live in the community and the decisive voters in the community are jointly determined in equilibrium.

An important issue in modeling this interaction of locational and political decisions has to do with what it is that voters take into account. In particular, when voting, and thereby collectively determining the level of public good provision within a community, do voters take into consideration the interaction among housing market equilibrium, mobility and public good provision? Almost all models of locational equilibrium rest on the assumption that voters are myopic: voters in each community ignore all effects of migration. Under this assumption, voters treat the populations of the communities as fixed and believe that the distribution of households across communities is not affected by a change in public good provision.

Few empirical strategies have been developed that investigate public provision under majority rule while taking explicit account of the constraints implied by mobility of households. Epple, Romer, and Sieg (2001) (ERS) developed an equilibrium framework that allows one to make a variety of assumptions about voter sophistication. They found little empirical support for the hypothesis that myopic voting behavior is consistent with the data. This finding deserves further scrutiny, especially given the prominence that myopic voting plays in the theoretical literature.[1]

ERS used an approach that focused on necessary conditions that allocations must satisfy in equilibrium. Here we take a different approach. To better understand the empirical implications of the myopic voting assumption, we need to impose all restrictions that arise from the locational equilibrium model simultaneously. We can then ask whether our model can replicate some of the main stylized facts about expenditures, taxes, sorting by income, and housing. This approach thus differs significantly from the previous work of ERS. In this paper, we completely specify a generic locational equilibrium model, characterize its equilibrium properties, and solve for equilibria for different parameter values.

We estimate the parameters of the model by matching the observed outcomes to those predicted by our model. In contrast, most previous empirical work has either ignored locational equilibrium or, more recently, has relied on partial solution algorithms of the type suggested in Epple and Sieg (1999) (ES) and Sieg, Smith, Banzhaf, and Walsh (2004).[2] Using full solution techniques allows us to impose all relevant restrictions that arise from locational equilibrium models simultaneously on the data generating process. This allows us to evaluate the main questions of interest: which dimensions of the data are explained reasonably well by a myopic voting model and which ones are not? This approach is only feasible if we can easily compute equilibria for models with large numbers of communities. We therefore provide a careful discussion of computational issues and provide a new algorithm that allows us to compute equilibria for a model with a large number of communities. This algorithm is also appealing since it exploits local uniqueness of equilibrium, a result which has not been established in prior research.

The data set we use in this paper includes the communities that constitute the Boston Metropolitan Area. Massachusetts is convenient to study because cities and school districts are coterminous. Hence a single residential tax rate applies within a community's boundary.

[1] See for example Epple, Filimon, and Romer (1984), Nechyba (1997a), Nechyba (1997b), and Fernandez and Rogerson (1998).

[2] The most notable exception is Ferreyra (2003) who estimates the parameters of a model with public and private schooling using a full solution algorithm.

We therefore avoid problems that may arise due to overlapping jurisdictions. Property taxes are also the primary source of local revenues in Massachusetts, which avoids the need to model other revenue sources. Our data set is from the 1980 US Census. This time period predates a Massachusetts law that restricts property taxation (usually referred to as Proposition $2\frac{1}{2}$). This law, which was passed in 1981, limited property tax rates to two-and-a-half percent (after some adjustment period). Since many jurisdictions had property taxes in the period leading up to 1981 that were higher than the limits set in Proposition $2\frac{1}{2}$, the law imposed for all practical purposes a binding constraint on these communities. We model the political process within each community as unconstrained choices determined by majority rule and we estimate the parameters of the model using data prior to Proposition $2\frac{1}{2}$.

There are 92 communities in our data set, and one of the main contributions of this paper is that it provides a method for estimating full equilibrium for a large number of communities. Moreover, we do this in a context when households differ in two dimensions: income and taste for the local public goods. To accomplish this, we also developed a solution algorithm that makes it feasible to solve for equilibrium rapidly a large number of times in the process of estimating the model's parameters.

Our main findings indicate that the simple myopic model considered in this paper fits the observed distribution of households by income across communities well. It can generate a distribution of local expenditures that fits the data well. On the other hand, the myopic model does not fit the observed tax rates. So the model performs well in explaining the locational pattern of households and less well in explaining the pattern of local political choices. We, therefore, explore plausible extensions of our baseline model. We find that an extension of the myopic voting model, which also allows for peer effects in the production of educational quality, fits the data much better.

The rest of the paper is organized as follows. Section 2 reviews the theoretical model on which the analysis is based. Section 3 introduces a parameterization of the model. Section 4 discusses existence and uniqueness of equilibrium. It also discusses computational issues.

The estimation strategy is developed in Section 5. Section 6 discusses the data and Section 7 reports the main empirical findings. Section 8 concludes the paper. Appendices A and B provide details about the computation of equilibrium and some technical matters.

2 The Theoretical Framework

2.1 A Baseline Model

The theoretical framework is that of ERS and we reproduce it here.

The economy consists of a continuum of households, C, living in a metropolitan area. Throughout the paper we will refer to a household as the decision-making unit, though for variety we will sometimes also use the terms "individual", "voter", and "agent" to mean the same thing. The homogeneous land in the metropolitan area is divided among J communities, each of which has fixed boundaries. Jurisdictions may differ in the amount of land contained within their boundaries. We also assume that households behave as price-takers. A household that lives in community j has preferences defined over a local public good, g, a local housing good, h, and a composite private good, b. Let p denote the relative gross-of-tax price of a unit of housing services in community j, p^h the net-of-tax price, and let y be the household's endowment of the composite private good. Households pay taxes that are levied on the consumption of housing services. Let t be an *ad valorem* tax on housing in community j. Households differ in their endowed income, y, and in a taste parameter, α, which reflects the household's valuation of the public good. The continuum of households, C, is implicitly described by the joint distribution of y and α. We assume that this distribution has a continuous density, $f(\alpha, y)$, with respect to Lebesgue measure. We refer to a household with taste parameter α and income y as (α, y).

The preferences of a household are represented by a utility function, $U(\alpha, g, h, b)$, which is strictly quasi-concave and twice differentiable in its arguments. Households maximize their utility with respect to the budget constraint, which is given by:

$$(1+t)\, p^h\, h \;=\; y \;-\; b \tag{1}$$

and choose their preferred location of residence by comparing maximum attainable utility levels among communities. We can represent the preferences of a household by specifying the indirect utility function. Let

$$V(\alpha, g, p, y) \;=\; U(\alpha, g, h(p, y, \alpha), y - p\, h(p, y, \alpha)) \tag{2}$$

denote the indirect utility function of a household, where $p = (1+t)\ p^h$.[3] We assume that the indirect utility function satisfies standard single-crossing properties. In particular, indifference curves in the (g, p) plane have slopes increasing in y for given α and increasing in α for given y.

Let $C_j \subset C$ denote the population living in community j. The set of border households between communities j and $j+1$ is characterized by the following expression:

$$V(\alpha, g_j, p_j, y) \;=\; V(\alpha, g_{j+1}, p_{j+1}, y) \tag{3}$$

This boundary indifference condition defines loci $y_j(\alpha)$. The single crossing properties imply that the population, C_j living in community j is thus given by

$$C_j \;\;=\;\; \{(\alpha, y)|\; y_{j-1}(\alpha) \leq y \leq y_j(\alpha)\} \tag{4}$$

We also assume that the budget of community j must be balanced.[4] This implies that:

$$t\, p^h \int_{C_j} h(p, y, \alpha)\;\; f(\alpha, y)\; dy\; d\alpha \Big/ P(C_j) \;=\; c(g) \tag{5}$$

[3]Here we anticipate a simplification adopted in our empirical analysis. Preferences are assumed separable in g and (h, b) so that housing demand does not depend on g.

[4]The analysis can be extended to incorporate lump sum transfers, for example, from the state government to the local governments.

where $c(g)$ is the cost per household of providing g and

$$n_j = P(C_j) = \int_{C_j} f(\alpha, y)\, dy\, d\alpha \tag{6}$$

is the size of community j.

Furthermore, we assume that housing is produced from land and non-land factors with constant returns to scale, so that housing per household is given by:

$$H_j = h(A_j, Z_j) \tag{7}$$

where A_j is the fixed amount of land area in community j and Z_j is a mobile factor used in production. Assume that p_z is the same in all communities. Profit maximization by price-taking producers implies that the per-household housing supply function is given by:

$$H_j^s = H_j^s(p_j, t_j) \tag{8}$$

Total housing demand is given by:

$$H_j^d(g_j, p_j, t_j) = \int_{C_j} h(p_j, y, \alpha)\, f(\alpha, y)\, dy\, d\alpha \tag{9}$$

Following most previous positive studies in the literature, we assume that the pair (t, g) in each community is chosen by majority rule. In each community, voters take the (t, g) pairs in all other communities as given when making their decisions. One can make a variety of assumptions about voter sophistication regarding anticipation of the way changes in the community's own (t, g) pair affect the community's housing prices and migration into or out of the community. For example, voters might take the community's net-of-tax price and the community tax base as given, and then deduce from the budget constraint the link between gross-of-tax price and expenditures on local public goods. This is the simplest and most commonly adopted approach (Epple, Filimon, and Romer, 1984).[5] Alternatively, voters in

[5]Fernandez and Rogerson (1996) provide a formalization of the timing of moving and voting that ratio-

a community might take the (t, g) pairs in other communities as given and then predict how changes in their community's tax and expenditure policy will affect the price of housing in their community.[6] The myopic model is used extensively both in theoretical models and in empirical analysis. Our focus in this paper is on investigating in our equilibrium framework whether the myopic voting assumption provides a good fit of the data.

The community budget constraint, housing market clearing, and perceived migration effects define a locus of (g, p) pairs that determine the government-services possibility frontier, i.e. GPF $= \{g(t), p(t) \; | t \in R^{+}\}$. For given tax and expenditure policies in other communities, a point on the GPF that cannot be beaten in a majority vote is a majority equilibrium. Let $y_j(\alpha)$ be the implicit function defined by equation (3). Consider a point (g_j^*, p_j^*) on community j's GPF, and let $\tilde{y}_j(\alpha)$ define a set of voters who weakly prefer (g_j^*, p_j^*) to any other (g_j, p_j) on the GPF. It follows that (g_j^*, p_j^*) is a majority voting equilibrium for the given GPF if

$$\int_0^{\infty} \int_{y_{j-1}(\alpha)}^{\tilde{y}_j(\alpha)} f(\alpha, y)\, dy\, d\alpha \;=\; \frac{1}{2} \int_0^{\infty} \int_{y_{j-1}(\alpha)}^{y_j(\alpha)} f(\alpha, y)\, dy\, d\alpha \tag{10}$$

Note that $\tilde{y}_j(\alpha)$ defines a locus of pivotal voters.[7]

In order to characterize pivotal voters in a community, we need to derive an expression for the slope of the GPF. Recall that the GPF is defined as the locus of (g_j, p_j) such that housing markets are in equilibrium:

$$F_j(g_j, p_j, t_j) \;=\; H_j^d(g_j, p_j, t_j) \;-\; H_j^s(p_j, t_j) \;=\; 0 \tag{11}$$

and the community budget is balanced:

$$G_j(g_j, p_j, t_j) \;=\; c(g_j) \;-\; p_j \, \frac{t_j}{1 + t_j} \, \frac{H_j^d(g_j, p_j, t_j)}{n_j} \;=\; 0 \tag{12}$$

nalizes this assumption on the part of the voters.

[6]This approach is developed in Epple and Romer (1991) and also adopted in Epple and Platt (1998).

[7]A formal proof of a similar result is in Epple and Platt (1998) and the same argument applies in this model.

given the perceived migration effects. Totally differentiating (11) and (12) and solving for dp_j/dg_j yields:

$$\frac{dp_j}{dg_j}\Big|_{GPF} = -\frac{\frac{G_{jg}}{G_{jt}} - \frac{F_{jg}}{F_{jt}}}{\frac{G_{jp}}{G_{jt}} - \frac{F_{jp}}{F_{jt}}} \tag{13}$$

The right-hand side of (13) does not have a simple closed form solution in general.

If voters are myopic, they ignore all effects of migration; i.e., voters treat the population boundaries of the communities as fixed. Hence, voters believe that the distribution of households across communities is not affected by a change in public good provision. Furthermore, if voters also treat the housing demand as fixed when voting, then we obtain the simple myopic voting model:

$$\frac{dp_j}{dg_j}\Big|_{MV} = \frac{c'(g_j)}{H_j/n_j} \tag{14}$$

The right hand side of equation (14) gives the slope of the GPF as perceived by a myopic voter. This is equivalent to the assumption that when voting, each resident of the community takes the net-of-tax price of housing, community population, and the aggregate housing demand as fixed. The main technical advantage of the myopic voting model is that the slope of the GPF is basically a function of only two variables: the marginal cost of providing the public good and the housing demand. This formulation is implicit in all prior empirical work estimating demand functions for local public goods and traces to the pioneering work by Barr and Davis (1966) and Bergstrom and Goodman (1973).

To summarize, voters in each community decide about the level of provision of the public good, g, and the tax level, t. Mobility among communities is costless, and in equilibrium every household lives in his or her preferred community. Having specified all components of a (generic) equilibrium model, we define an intercommunity equilibrium as follows:

Definition 1 *An* **intercommunity equilibrium** *consists of a set of communities, $\{1, ..., J\}$; a continuum of households, C; a distribution, P, of household characteristics*

α and y; and a partition of C across communities $\{C_1, ..., C_J\}$, such that every community has a positive population, i.e. $0 < n_j < 1$; a vector of prices and taxes, $(p_1^*, t_1^*, ..., p_J^*, t_J^*)$; an allocation of public goods, $(g_1^*, ..., g_J^*)$; and an allocation, (h^*, b^*), for every household (α, y), such that:

1. *Every household (α, y), living in community j maximizes its utility subject to the budget constraint:*[8]

$$\begin{aligned}(h^*, b^*) &= \arg\max_{(h,b)} U(\alpha, g_j^*, h, b) \\ &\quad s.t.\ p_j^* h = y - b\end{aligned}$$

2. *Each household lives in one community and no household wants to move to a different community, i.e. for a household living in community j, the following holds:*

$$V(\alpha, g_j^*, p_j^*, y) \geq \max_{i \neq j} V(\alpha, g_i^*, p_i^*, y) \tag{15}$$

3. *The housing market clears in every community:*

$$\int_{C_j} h^*(p_j^*, y, \alpha)\ f(\alpha, y)\ dy\ d\alpha = H_j^s(\frac{p_j^*}{1 + t_j^*}) \tag{16}$$

4. *The budget of every community is balanced:*

$$\frac{t_j^*}{1 + t_j^*}\, p_j^* \int_{C_j} h^*(p_j^*, y, \alpha)\ f(\alpha, y)\ dy\ d\alpha \Big/ n_j = c(g_j^*) \tag{17}$$

5. *There is a myopic voting equilibrium in each community: Over all levels of (g_j, t_j) that are perceived to be feasible allocations by the voters in community j, at least half of the voters prefer (g_j^*, t_j^*) over any other feasible (g_j, t_j).*

[8] Strictly speaking, all statements only have to hold for almost every household; deviations of behavior of sets of households with measure zero are possible.

2.2 Extensions

From the analysis in ERS, we know that the simple myopic voting model discussed in the previous section is unlikely to fit all relevant dimensions of the data. We, therefore, also consider an extension of the model to incorporate peer effects. Peer effects may be important to understand sorting of households among communities for a variety of different reasons. First, one of the most important local public goods is educational quality. Educating economically disadvantaged children may be more costly than educating children from higher income families. Second, social interactions may enhance or detract from student performance. Third, parental involvement in local schools may be a function of household income. Finally, peer effects may pick up other differences in the quality of public good provision that are not related to education. For example, peer effects may be used to proxy for (unobserved) differences in public safety, since the propensity to commit crime is often inversely related to income

In our extended model, we therefore distinguish between the quality of local public good provision denoted by q, publicly provided public goods (i.e. expenditures per household), g, and a measure of peer quality, denoted by $\bar{y}$. We assume that the quality of public good provision satisfies an index assumption and can be expressed as $q = q(g, \bar{y})$. Finally, we assume that peer quality can be measured by the mean income in a community, which is given by:

$$\bar{y}_j \;=\; \int_{C_j} y\, f(\alpha, y)\, dy\, d\alpha \Big/ n_j \tag{18}$$

It is straightforward to generalize the theoretical model presented in the previous section to account for peer effects. For example, household preferences in the extended model are now defined as

$$V(\alpha, q(g, \bar{y}), p, y) \;=\; U(\alpha, q(g, \bar{y}), h(p, y, \alpha), y - p\, h(p, y, \alpha)) \tag{19}$$

Similarly, we can modify the other elements of the model and define a locational equilibrium with peer effects.[9]

3 A Parameterization

Since we are interested in empirical and computational analysis, it is necessary to parameterize the model. Let the joint distribution of $\ln(\alpha)$ and $\ln(y)$ be bivariate normal. The means of the distribution are denoted by $\mu_{\ln(y)}$ and $\mu_{\ln(\alpha)}$. The variances are $\sigma^2_{\ln(y)}$ and $\sigma^2_{\ln(\alpha)}$, and the correlation is denoted by λ. Furthermore, assume that the indirect utility function is given by:

$$V(q,p,y,\alpha) \;=\; \left\{\alpha\, q^{\rho} \;+\; \left[e^{\frac{y^{1-\nu}-1}{1-\nu}}\; e^{-\frac{Bp^{\eta+1}-1}{1+\eta}}\right]^{\rho}\right\}^{\frac{1}{\rho}} \tag{20}$$

where the quality index q is given by:

$$q_j \;=\; g_j \left(\frac{\bar{y}_j}{\bar{y}}\right)^{\phi} \tag{21}$$

and $\rho < 0,\ \alpha > 0,\ \eta < 0,\ \nu > 0,\ \phi \geq 0$ and $B > 0$. $\bar{y}$ is mean income in the population. We assume that while α can vary across households, $\nu,\ \eta,\ \rho,\ B$ and ϕ are the same for all agents. Roy's Identity applied to equation (20) implies that the individual housing demand function can be written as $h(p_j, y) = B\ p^{\eta}\ y^{\nu}$. Given the utility function above, the locus of households indifferent between communities j and $j+1$ can be written as:

$$\ln(\alpha) \;-\; \rho \left(\frac{y^{1-\nu}-1}{1-\nu}\right) \;=\; \ln\left(\frac{Q_{j+1}-Q_j}{q_j^{\rho}-q_{j+1}^{\rho}}\right) \;\equiv\; K_j \tag{22}$$

[9]The role of peer effects in choice of schools and jurisdictions has been the subject of increasing theoretical research. Recent research includes Benabou (1993, 1996), Caucutt (2002), deBartolome (1990), Durlauf (1996), Epple and Romano (1998, 2003), Nechyba (1999, 2000). Ferreyra (2003) has incorporated peer effects in an econometric model of multi-district equilibrium. Bayer, McMillan, and Reuben (2003) also estimate a sorting model with peer effects. There is also a burgeoning empirical literature seeking to document the presence and magnitude of peer effects. See Epple and Romano (2003, footnote 12) for a brief summary and references. We emphasize that the model that we propose is consistent either with the presence of peer effects or with a preference for high-income neighbors. We use the term "peer effects" as a convenient shorthand.

where

$$Q_j = e^{-\rho \frac{B p_j^{\eta+1} - 1}{1+\eta}} \tag{23}$$

The first-order condition of the voting problem can be expressed as:

$$\ln(\alpha) \; - \; \rho \left(\frac{y^{1-\nu} - 1}{1-\nu} \right) \;\; = \;\; L_j \tag{24}$$

where the intercept, L_j, is given by

$$L_j \;\; = \;\; \ln \left[\frac{B \; e^{-\rho \frac{B p_j^{\eta+1} - 1}{1+\eta}} \; p_j^{\eta} \; \frac{dp_j}{dg_j}}{q_j^{\rho-1} \frac{\partial q_j}{\partial g_j}} \right] \tag{25}$$

We also assume that the production function for housing is Cobb-Douglas with exponents s and $1-s$ on land and non-land inputs respectively. Hence we obtain the housing supply function:

$$H_j^s(p_j, t_j) \;\; = \;\; A_j \left(\frac{p_j}{1+t_j} \right)^{\psi} \tag{26}$$

where $\psi = (1-s)/s$ and units of Z are chosen such that p_z is scaled conveniently to equal $(1-s)$. The cost function is linear:

$$c(g_j) \;\; = \;\; g_j \tag{27}$$

Thus the 11 parameters of the model are $\mu_{\ln(y)}, \mu_{\ln(\alpha)}, \sigma_{\ln(y)}, \sigma_{\ln(\alpha)}, \lambda, \rho, \eta, \nu, B,\ \phi$, and ψ. In much of what follows, we assume that peer effects are absent ($\phi = 0$), so that $q = g$. We return to more explicit consideration of peer effects at the end of section 7.

4 Existence, Uniqueness, and Computation of Equilibrium

If household preferences satisfy single-crossing properties, the existence of an intercommunity equilibrium has been shown in somewhat simpler versions of this model, *e.g.* models without taste variation and peer effects considered in Epple, Filimon, and Romer (1993) (EFR). Moreover, we believe the strategy used to prove existence of equilibrium in EFR can be modified in a straightforward way to establish existence of equilibrium in this model. In EFR existence follows under a set of regularity assumptions by applying a fixed point argument on a mapping of the community boundaries. In that model, there is no taste heterogeneity and communities consist of an interval of the type $[y_{j-1}, y_j]$. In this model, community boundaries are characterized by the slope of the boundary indifference curve and the community specific intercept K_j. Instead of defining a mapping on the y_j's, we can define a similar mapping on the K_j's. Under the similar regularity conditions used in EFR, we conjecture that this alternative mapping will also have a fixed point.[10]

Equilibria cannot be computed analytically. Instead we rely on numerical algorithms to find them.[11] Computing an equilibrium for models without peer effects requires us to solve a system of $3J$ nonlinear equations: J housing market equations, J budget equations, and J equations characterizing pivotal voters.[12] Thus computation of equilibria only exploits necessary conditions that equilibria must satisfy. Once the algorithm has found such an allocation, one still needs to make sure that all second order conditions are satisfied.[13]

From the perspective of empirical analysis, land that is available for residential use in communities is not easily measured. By contrast, community populations are measured with a relatively high degree of accuracy. Hence our approach in this paper is to take observed community populations as equilibrium outcomes from the model that we have described.

[10] The only technical difficulty which arises in the extension of the proof in EFR is that we need to guarantee that the housing consumption is also monotonically increasing in the community rank. In EFR this condition is trivially satisfied. Here it requires some additional assumptions.

[11] In Appendix A, we describe in detail how to compute equilibria numerically for models without peer effects.

[12] Adding peer effects to the model implies that we need to add J equations to the system.

[13] An appendix which derives the second order conditions is available upon request from the authors.

Following ES, we take these populations as measured without error. This permits us to focus on household location and voting in our empirical analysis. More formally, we focus on allocations that satisfy the J budget equalities, the J equations characterizing pivotal voters, and $J-1$ equations that constrain the observed populations to equal the predicted population sizes. These allocations are equilibria in the following sense. For any allocation that satisfies the $3J-1$ equations above, there exist housing supply functions for each community such that housing markets are in equilibrium.[14] An interesting extension, which we do not pursue in this paper, is to investigate the housing supply implied by the model.

The algorithm to compute equilibria takes observed community populations and the hierarchy of communities (ordered by mean income) as known (i.e. observed in the data). One advantage of this approach is that we can prove uniqueness of the equilibrium that gives rise to a given set of community populations and ordering of communities, taking voters to be myopic. We have the following result:

Proposition 1 *Given a set of equilibrium community populations, the associated equilibrium ordering of communities, and myopic voters, the equilibrium is unique.*

A proof is given in Appendix B.

This uniqueness result is useful in justifying our estimation approach. Estimation is based on a full solution approach, i.e. at each parameter vector we compute the equilibrium of the model and match the predicted equilibrium to the one observed in the data. If the equilibrium were not unique, we would need to compute all equilibria at each parameter vector and find the one that matches the data the best. Proposition 1 establishes that at each parameter vector, there is only one equilibrium that is consistent with the observed community sizes and the observed hierarchy.

[14] Since we are going from a system with $3J$ equations in $3J$ unknowns to a system with $3J-1$ equations in $3J$ unknowns, one equilibrium variable is not determined. We solve this problem by normalizing the price of housing in the lowest community to be equal to 1.

5 Estimation

The estimation procedure can be implemented in two stages. The first stage uses the model's implications regarding locational equilibrium, while the second stage incorporates voting equilibrium. We will briefly describe the first stage of the estimation procedure implemented in Epple and Sieg (1999), which we apply in this paper. We have made parametric assumptions on the joint distribution of income and tastes for the population of the metropolitan area and the indirect utility function of the households. With these assumptions, the model determines a joint distribution of income and taste parameters for every community. If the model is evaluated at the correct parameter values, the difference between the empirical quantiles of the income distributions observed in the data and the quantiles predicted by the model should be small. This provides the rationale for the first stage of the estimation.

Equation (22) implies that quantiles of the income distribution of community j depend on (q_j, p_j) only through the community-specific intercepts K_j. We can, therefore, solve equation (6) recursively to obtain the community-specific intercepts, K_j, as a function of the parameters of the bivariate distribution of income and tastes, $(\mu_{\ln(y)}, \mu_{\ln(\alpha)}, \lambda, \sigma_{\ln(y)}, \sigma_{\ln(\alpha)})$, the parameters (ν, ρ), and the community sizes, $n_1, ..., n_J$. These community size restrictions in the estimation procedure pin down the values for the community-specific intercepts. We then estimate the parameters that are identified from community populations and income distributions by matching the quantiles of the income distributions subject to the constraint that community-specific intercepts are chosen to replicate observed community sizes.

Heterogeneity in tastes and income in the metropolitan population, together with self-selection of households into municipalities, means that income distributions will differ across municipalities in the metropolitan area. In equilibrium, the self-selection of the metropolitan population into municipalities results in boundary loci in the (α, y) plane that divide the metropolitan population into the various municipalities in the metropolitan area. The within-community income distributions that result thus depend on the shape and position of the boundary loci and on the parameters of the joint distribution of (α, y). The empirical differences in the within-community distributions of income across municipalities prove to be sufficient to identify the parameters that determine the slope and shape of the boundary loci $(\rho/\sigma_{\ln(\alpha)}, \nu)$ and the correlation between tastes and income (λ). The mean and variance of tastes are not identified in this stage because we do not exploit information on public good provision. The parameter ρ determines the slope of the indifference curve and hence affects sorting in equilibrium. Less obviously, the lack of identification of $\sigma_{\ln(\alpha)}$ also implies that we can identify only the ratio $\rho/\sigma_{\ln(\alpha)}$ in the first stage. Finally, ν determines the curvature of the boundary indifference curves and hence the composition of populations within and among communities. Identification of ν thus rests on functional form assumptions of the indirect utility function since we do not exploit housing expenditure data at this stage of the analysis.

To summarize, in the first stage of the estimation procedure the following parameters, denoted by θ_1, are identified: the mean and the standard deviation of the income distribution $(\mu_{\ln(y)}, \sigma_{\ln(y)})$, the correlation between income and tastes (λ), the income elasticity of demand for housing (ν), and the ratio of ρ to the standard deviation of the taste for public goods $(\sigma_{\ln(\alpha)})$. The estimates from this stage typically have a relatively high degree of precision because the relevant sample size is not the number of communities (J) but rather the number of households (N) sampled by the U.S. Census; i.e., the asymptotics of the first stage estimator only require N to go to infinity, for any given value of J.

That leaves us with five parameters to be estimated: the two remaining parameters of the housing demand equation η and B, as well as the mean and the standard deviation of the distribution of α and ϕ.[15] In the absence of housing price data, it is hard to estimate η. We, therefore, set $\eta = -0.3$ and conduct some sensitivity analysis to demonstrate that our main results do not depend on the choice of η. Thus, in the second stage, we need to estimate the following parameters $\theta_2 = (\mu_{\ln(\alpha)}, \sigma_{\ln(\alpha)}, B, \phi)$. The rest of this section focuses on the second stage of the estimator which differs from our previous work. ERS derived necessary conditions that local public expenditures, housing prices, and tax rates had to

[15]Given our computation approach, the parameter of the housing supply function ψ is not needed to compute the type of equilibria considered in this paper.

satisfy in equilibrium. That paper found that the observed equilibrium in the data did not satisfy necessary conditions implied by locational equilibrium and myopic voting. Here we follow a different approach. We compute equilibria, thus forcing our predicted outcomes to satisfy all restrictions implied by theory. We then investigate how closely our model can predict the observed outcomes.

In the estimation, we incorporate the implications of the myopic voter assumptions. These are embodied in equations (14), (24), and (25). Together they imply that the first-order condition determining the level of expenditures in community j depends on (g_j, p_j) but not directly on the property tax rate t_j. Using the parameters and $J-1$ boundary loci estimated in the first stage of the estimation and given values for $\theta = (\theta_1, \theta_2)$, we can calculate the implied equilibrium (g_j, p_j) for $j = 1, ..., J$, up to an arbitrary normalization. (We adopt the normalization $p_1 = 1$.) Note that the equilibrium (g_j, p_j) pairs can be calculated without using any information about community land areas or parameters of the housing supply function. These are the key consequences of the property of myopic voting noted above. Having calculated the (g_j, p_j) pairs for all communities, the community budget constraints can be solved to obtain tax rates. In particular, the budget constraint for community j is

$$t_j \, p_j^h \, \bar{h}_j(p_j) = g_j \tag{28}$$

where

$$\overline{h}_j(p_j) \;\; = \;\; \int_{-\infty}^{\infty} \int_{y_{j-1}(\alpha)}^{y_j(\alpha)} h(p_j, y) \, f(\alpha, y) \, dy d\alpha \Big/ n_j \tag{29}$$

is per-household housing consumption in j. This and the identity $p_j = p_j^h(1+t_j)$ imply:

$$t_j \, p_j \, \bar{h}_j(p_j) \, / \, (1+t_j) \;=\; g_j \tag{30}$$

Given (g_j, p_j) and the community boundary loci, equation (30) can be solved for each j to obtain t_j. We do not observe the price per unit of housing services, p_j^h. Hence, we calculate the per capita annualized rental value of housing consumed in each community: $R_j = p_j^h \bar{h}_j(p_j)$.

Let $t_j(\theta_2|\hat{\theta}_1)$, $g_j(\theta_2|\hat{\theta}_1)$, $R_j(\theta_2|\hat{\theta}_1)$ denote, respectively, the tax rate, expenditure level, and mean housing expenditures predicted by the model as a function of the parameters that have been estimated in the first round $\hat{\theta}_1$ and the parameters that need to be estimated in the second round θ_2. We assume that the observed levels of these three variables differ from the ones predicted by our model because of measurement error:

$$\begin{aligned} t_j &= t_j(\theta_2|\,\hat{\theta}_1) + \epsilon_j^t \\ g_j &= g_j(\theta_2|\,\hat{\theta}_1) + \epsilon_j^g \\ R_j &= R_j(\theta_2|\,\hat{\theta}_1) + \epsilon_j^R \end{aligned} \tag{31}$$

We assume that the measurement errors, $(\epsilon_j^t, \epsilon_j^g, \epsilon_j^R)$, are jointly normally distributed. For each set of trial values of θ_2, and given the first stage estimator $\hat{\theta}_1$, we solve for (g_j, t_j, R_j) in each community.[16] We can then estimate the remaining parameters of the model using a maximum likelihood estimator.[17] Obtaining these maximum likelihood estimates is the second stage of our estimation procedure.

We view the above estimation procedure as appealing for two reasons. First, it does not require estimates of the amount of land in each community that is available for residential development. Such measures are likely to be subject to substantial measurement error. Instead, we use community populations and income distributions. While these are also subject to measurement error, such measurement errors are likely to be small relative to

[16]Precise details of the computation of equilibrium for a given set of parameter values are given in Appendix A.

[17]Our model may be misspecified. One of the main objectives of the analysis is to determine and evaluate the fit of the model. This exercise is well-defined even if our model is misspecified. In that case, it makes more sense to interpret the MLE as a quasi-maximum-likelihood estimator. Basic asymptotic theory suggests that the quasi- maximum-likelihood estimator is still well defined and converges almost surely to the parameter vector that minimizes the Kullback-Leibler discrepancy which measures the distance between our class of models and the true data generating process. Moreover, the limiting distribution of the quasi MLE is still asymptotically normal.Of course, the standard formula that we use for estimating asymptotic standard errors would need to be modified in this case to account for misspecification problems. For an introduction to the theory of misspecified MLE see, for example, Gallant (1997).

errors in measuring land areas. Second, the two-stage approach permits us to exploit, in the first stage, the large sample (five percent of the metropolitan population) that is available for estimating community income boundaries and parameters of the distribution of income. The second stage then exploits the implications of household location and voting in determining community property values and local government expenditures and tax rates.

6 The Data Set

The data set used in this paper builds on the one used in ES and ERS on the Boston Metropolitan area in 1980. In addition, we also use data on residential property values in each community. Table 1 reports some descriptive statistics of the most important variables in the sample. The sample size of 92 equals the number of cities and townships in the Boston Metropolitan Area. Since a detailed discussion of the data is published in ES and ERS, we provide details here only on issues that have not been previously discussed.

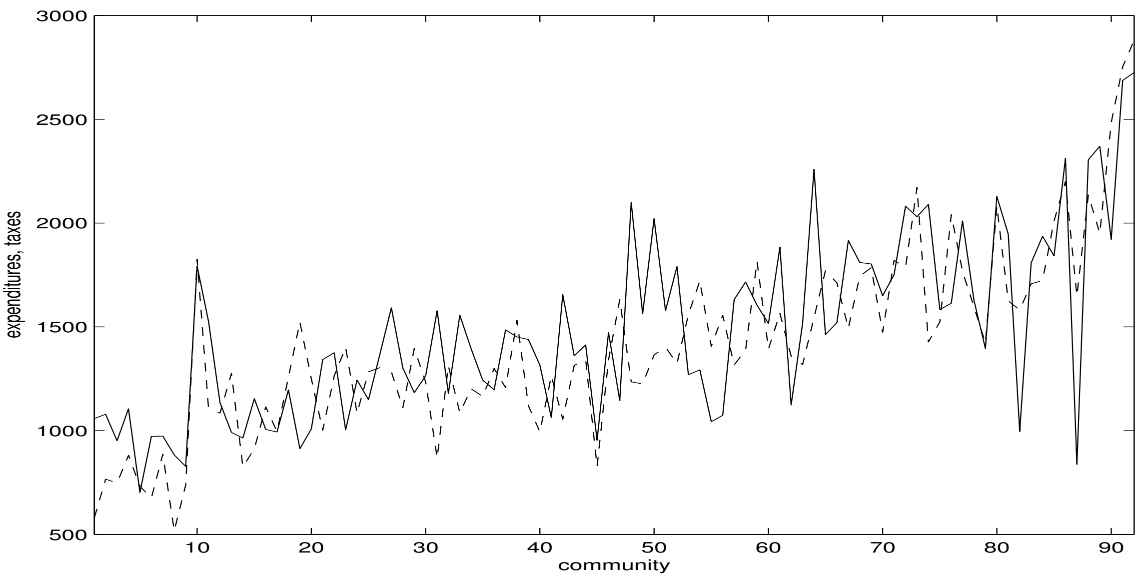
Table 1: Descriptive Statistics of the Sample

Variable	Mean	Std. Deviation
Population size	30036	59719
Number of households	10769	23335
Mean income^a	27402	8024
Median income^a	24108	6481
Education expenditure^a	1479	435
Property tax rate^b	0.031	0.009
Median property value^a	64923	21515
Median gross rent^a	314.35	58.22
Fraction of renters	0.28	0.16

Notation: ^{a} per household. ^{b} per dollar of value

First, consider the relationship of the community budget constraint to residential property tax revenue. We have plotted residential property tax revenue and educational expenditure per household in Figure 1. As the lower two lines of the plot below show, residential property tax revenue and education expenditures per household track relatively well across the communities. The correlation coefficient is 0.73. This is striking given that the data come from different sources. The expenditure data are from the U.S. Census of Governments while data for tax rates and assessed residential property tax bases are from state government sources. Thus, we will view the residential property tax as earmarked for education and property tax as the only source of revenue for education.

Figure 1: Residential Property Tax Revenue and Educational Expenditure per Household



Notation: — expenditures per household on education, - - property taxes per household

Next, recall that housing consumption in our model is framed in terms of the flow of housing services. We consider the conversion of property values to annualized implicit rental values for the 1980 Boston data. Let R be annualized implicit rent and V be the housing value. These are related by the following identity:

$$R = k_p V \tag{32}$$

where, k_p is the user-cost factor (Poterba, 1992) given by the following expression:

$$k_p = (1 - t_y)(i + t_v) + \zeta \tag{33}$$

where t_y is the income tax rate, t_v is the tax rate on property value, i is the nominal interest rate, and $\zeta = \beta + m + \delta - \pi$ where β is the risk premium for housing investments, m and δ are maintenance and depreciation costs, and π is the inflation rate. We wish to calculate implicit rents net of the property tax, so we remove t_v from the previous expression. Following Poterba, let $\zeta = -.02$ and $i = .1286$. We set $t_y = .15$. Then,

$$k_p = .85 * (.1286) - .02. \tag{34}$$

Thus, the average user-cost factor for these communities is then:

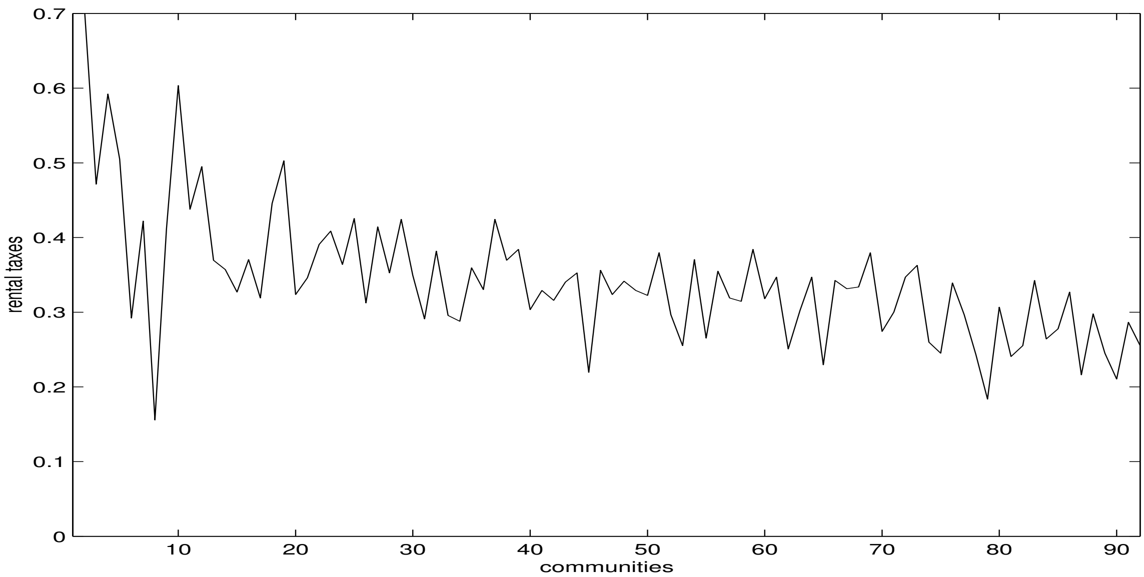
$$k_p = .85 * (.1286) - .02. = .0893. \tag{35}$$

It is natural to question whether the assessed values ("equalized residential values" or ERV) provide an adequate measure of actual property values in communities. While this cannot be answered definitely, we can check the consistency between these values and values that community residents report to the U.S. Census. We converted rents into housing values using Poterba's formula, as discussed above. We then regressed equalized residential property value per household on aggregate owner-occupied property values (Census) and the imputed values from aggregate rents (Census).[18] The overall fit is high, with $R^2 = .94$. The coefficients on each of the right-hand side variables should be 1. For owner-occupied housing, we find that the estimated coefficient is 1.039 with an estimated standard error of 0.031. Thus we fail to reject the null hypothesis that the coefficient is equal to 1. For rental housing, we find that the coefficient is equal to 0.7117 with an estimated standard error of 0.147. The null hypothesis that the coefficient equals 1 has p-value = .058.

[18] The regression results are available upon request from the authors.

We thus find that the user-cost factor seems to understate the extent to which rentals are converted to property values. If we add .05 to the user-cost factor (i.e., let $\zeta = .03$ instead of -.02), then we get a coefficient on the rental variable very close to 1 in our regression. Such an increase could be motivated by greater depreciation and maintenance costs than Poterba assumed or greater risk factor in housing investments. Alternatively, it may be that rental properties are under-assessed relative to owner-occupied properties. All in all, these regressions are overall relatively reassuring about the alternative house value measures that we have. In the empirical analysis of this paper, we use ERV as our measure of value.

Figure 2: Equalized Residential Tax on Imputed Rental Values



Finally, we also need to convert tax rates on property values to rates on annualized implicit rental values to get a tax rate on the flow of housing services. The property tax rate on implicit rental, R, is related to the property tax rate on value, V by the following identity:

$$t_r R = t_v V \tag{36}$$

Thus,

$$
\begin{aligned}
t_r &= t_v V/R = t_v/k_p = t_v/[(1-t_y)i + \zeta] \\
&= t_v/[.85 * .1286 - .02]
\end{aligned} \tag{37}
$$

Figure 2 plots the implied rates on rental expenditures.

7 Empirical Results

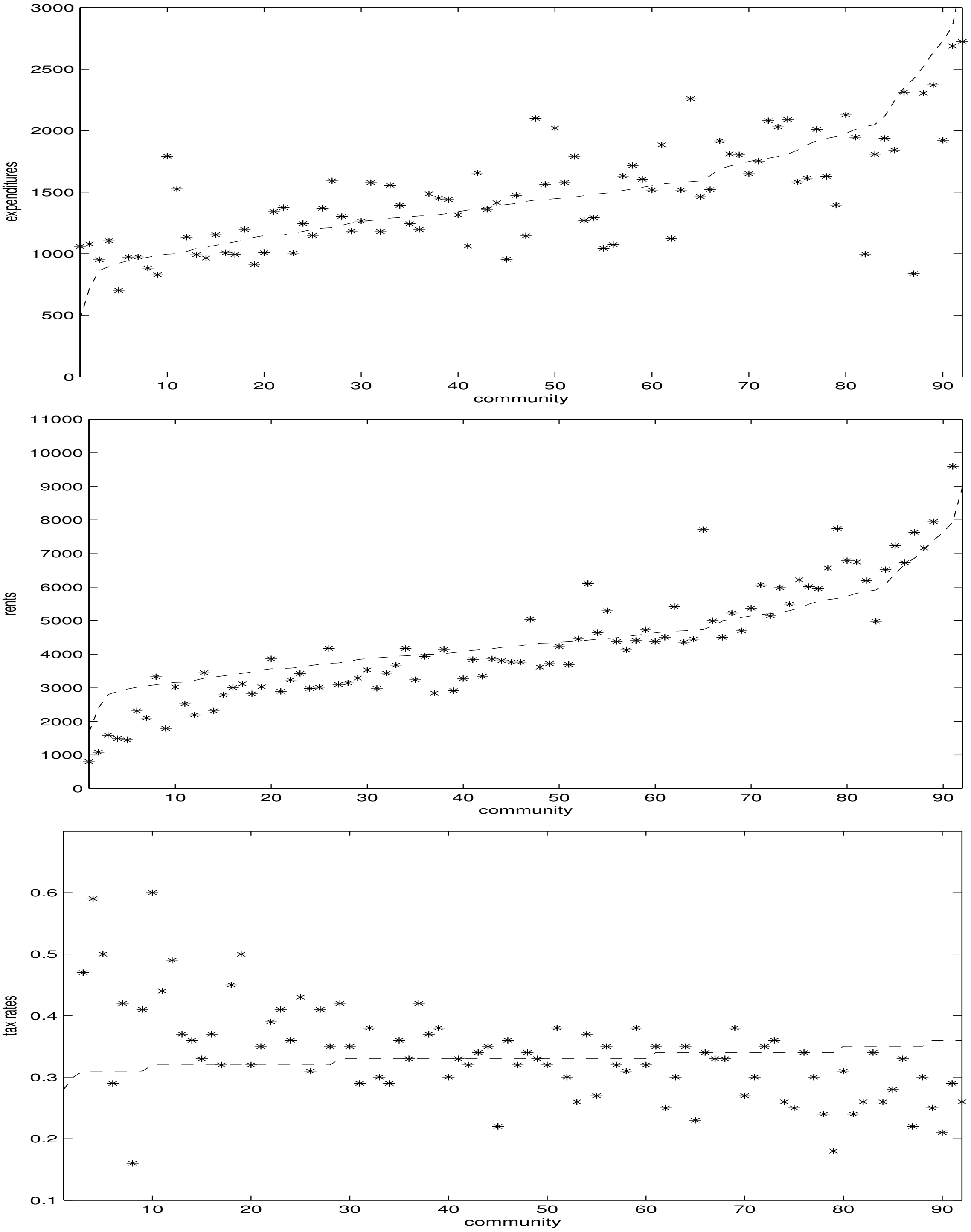
In the first stage of the estimation procedure, we match selected quantiles of the empirical income distributions of the communities with their predicted counterparts. This part of the estimation procedure is identical to the one in Epple and Sieg (1999). The mean of log income, $\mu_{\ln(y)}$, is 9.790 with an estimated standard error of 0.002. The estimate of the standard deviation of log income, $\sigma_{\ln(y)}$, is 0.755 (0.004). The correlation between income and tastes for local public goods is -0.019 (0.031). The ratio $\rho/\sigma_{\ln(\alpha)}$ is -0.283 (0.013). Finally, the income elasticity of housing demand is estimated to be 0.938 (0.026).[19] As detailed in ES, the estimated quantiles of income for the 92 communities fit the observed (Census) quantiles quite well.

In this paper, we estimate the remaining parameters by matching the observed distribution of tax rates, expenditures, and imputed rents as discussed in section 5. We set $\phi = 0$; i.e., we assume there are no peer effects. Column I of Table 2 reports the estimates for the baseline model. We find that the estimate for $\sigma_{\ln(\alpha)}$ reaches the lower boundary of 0.1, which is set in the estimation algorithm to keep $\sigma_{\ln(\alpha)}$ positive. This is our first indication that the myopic model performs poorly. With $\sigma_{\ln(\alpha)} = .1$, the implied value of ρ is given by the first-stage coefficient restriction that $\rho = -.283 * \sigma_{\ln(\alpha)} = -.0283$. Our estimate for $\mu_{\ln(\alpha)}$ is -2.623. Our estimate for B is 0.325. These estimates are not of intrinsic interest here, but they are needed to calculate the predicted equilibrium quantities of interest.[20]

[19]These estimates are reproduced from Table 1 in ES (1999).

[20]We have verified that the second-order conditions are satisfied for these estimates.

Figure 3: Observed versus predicted Expenditures, Rents, and Taxes: Baseline Model



Notation: * actual, – predicted

Table 2: Estimation Results

parameters	I baseline model	II extended model
$\mu_{\ln(\alpha)}$	-2.622	-2.643
	(0.021)	(0.017)
$\sigma_{\ln(\alpha)}$	0.1	0.1
	—	—
B	0.325	0.175
	(0.006)	(0.007)
ϕ	0.0	2.623
	—	(0.147)
likelihood function	-1360.92	-996.51
R^2 expenditures	0.680	0.739
R^2 rents	0.786	0.930
R^2 taxes	- 0.301	0.728

Estimated standard errors are given in parentheses.

Next we focus on the goodness of fit of the myopic voter model. First, figure 3 plots observed and predicted expenditures, rents, and tax rates. We find that the model fits the observed expenditure patterns reasonably well, though with some overstatement of expenditures in the poorer communities and some understatement in the higher-income communities.[21] The correlation between observed and predicted expenditures is 0.727. Imputed rents are both equilibrium housing consumption and the tax base in our model. The model somewhat over-predicts rents for poor communities and under-predicts rents for high income communities. The correlation coefficient is 0.94. Finally, we consider observed and predicted tax rates. Here the results are not favorable and point to a serious lack of fit. The over-prediction of housing values in the poorer communities and under-prediction of expenditures combine to create a severe under-prediction of tax rates in the poorer communities. Similarly, in the wealthier communities, the under-prediction of housing values and over-prediction of expenditures create an over-prediction of tax rates. Thus, while observed tax rates decrease in community rank, the model predicts tax rates increasing in community rank. The correlation between observed and predicted tax rates is -.67. This

[21] The term "expenditures" refers to local spending on education.

failure to fit tax rates is a serious shortcoming of the model. We also conducted some sensitivity analysis. Changing the price elasticity, η, from -.3 to -.5 has negligible effect on the likelihood function.

Given these findings, it is useful to consider extensions of the simple myopic voting model that may fit the data better. As we have seen in Section 2.2, we can assume that the quality of public good provision not only depends on local expenditures, but also peer effects. The parameter estimates of the extended model are shown in Column II of Table 2. We find that the estimate of ϕ is large and statistically significant. Expressing the exponent of the quality function in relative terms, our estimates imply that the peer effects are 2.5 times as important as spending. Introducing peer effects into the model specification also improves the fit of the model as documented in Figure 4. We find that a model with peer effects can not only explain expenditures, but also tax rates and tax bases (rents) reasonably well. In particular, we find that the correlation between actual and predicted tax rates is 0.747. We thus conclude that the extended myopic voting model which allows for peer effects in public good provision fits our data very well.

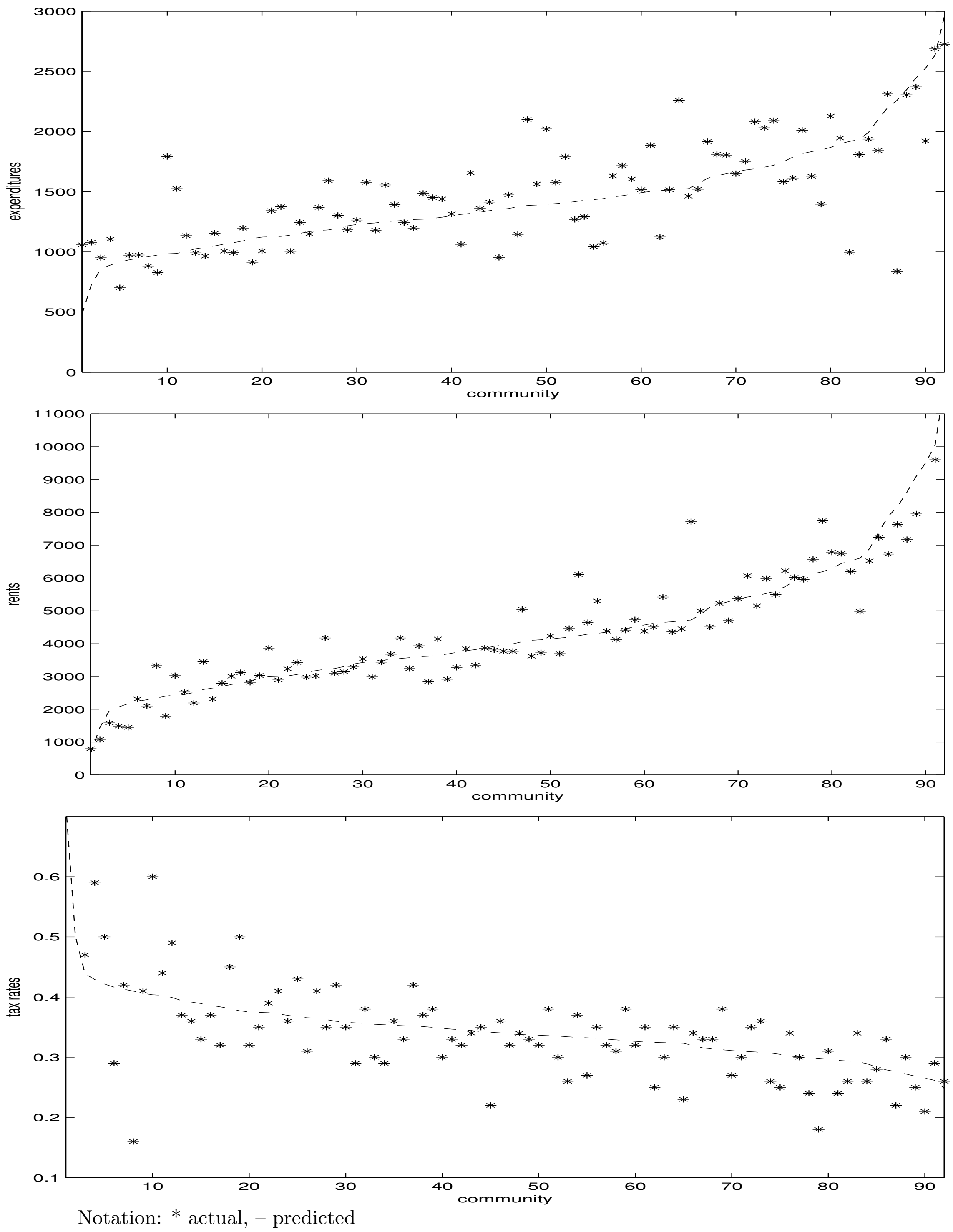
In the estimation, we treat housing prices as latent. Our model predicts that housing prices (per unit of housing consumption) vary from 1.0 in the lowest community to 5.14 in the most expensive community.[22] These price differences are similar compared to those found by quite different methods in empirical work such as Epple and Sieg (1999) and Sieg, Smith, Banzhaf, and Walsh (2002).

Our model presumes that the marginal source of funds for increasing a community's educational expenditures is the community's residential property tax. It is natural to wonder whether the poor fit of the baseline model (i.e., the model without peer effects) may be due to failure to incorporate factors that affect incentives for local property taxation.

We explored whether intergovernmental aid formulas might have embodied features that significantly affected marginal incentives for local taxation. After extensive investigation, we concluded that this is not the case. The state aid formula applicable during the period

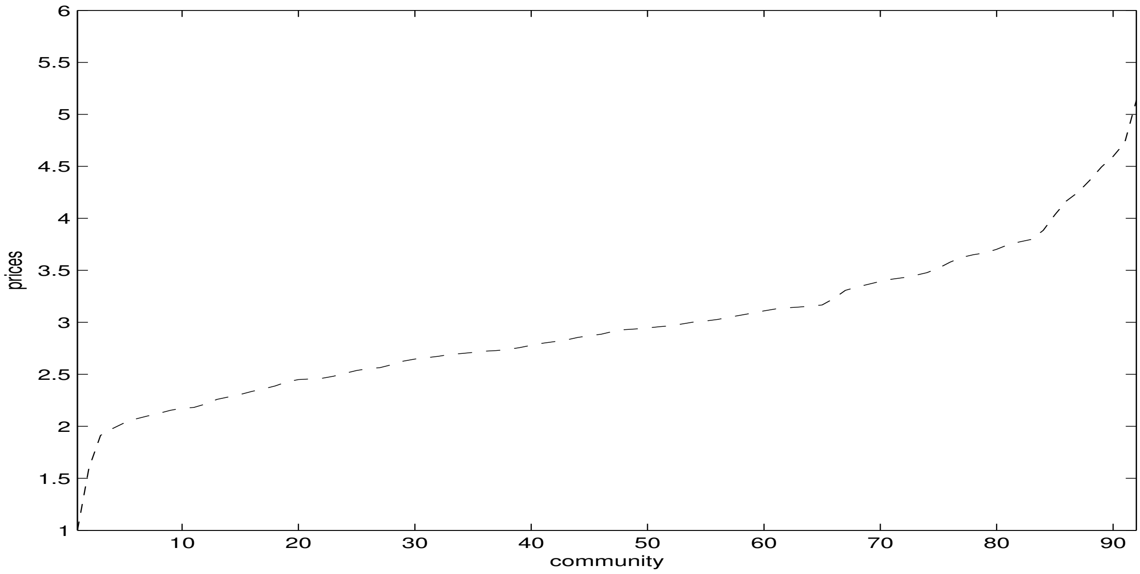
[22] In the model without peer effects prices ranged from 1 to 1.6.

Figure 4: Observed versus predicted Expenditures, Rents, and Taxes: Extended Model



Notation: * actual, – predicted

Figure 5: Housing Prices



from which our data are drawn specified aid as a function of the local property tax base and school enrollment.[23] The school enrollment variable in the formula gave higher weight to disadvantaged students. In addition, greater aid went to districts with lower property tax base per capita. Thus, both components of the aid formula had the effect of directing greater aid to lower-income municipalities. A key aspect of the formula is that, beyond a threshold, aid was not conditioned on the local tax rate or local expenditures. Thus, because it was not tied to local taxation or expenditure, the aid provided by the state would tend to induce localities to respond by lowering their tax rates. Moreover, since aid was higher to lower-income communities, the associated tax rate reduction would tend to be higher in poorer communities. Recall that a major shortcoming of the fit of the baseline model is that the model substantially underpredicts tax rates in poorer communities. If anything, incorporating state aid into the model would worsen the fit of the model to the data by inducing even lower predicted tax rates in poorer communities.[24]

[23]The relevant statute is *Acts and Resolves of Massachusetts*, 1978, ch. 367, amending ch. 70, "School Funds and State Aid for Public Schools."

[24]There is one potentially important caveat to the preceding discussion. In order to receive the full amount

We also explored a second possibility. Suppose poorer municipalities have proportionately more non-residential property than wealthier municipalities. If localities were required to impose the same tax rate on both residential and non-residential property, then poorer municipalities might have an incentive to raise the property tax rate in order to extract additional revenue from non-residential sources.[25] During the time period from which our data are drawn, Massachusetts state law did require that the same property tax rate be imposed on both types of property. However, municipalities routinely circumvented this requirement by assessing residential and non-residential properties at different rates (Bradbury, 1988). The poorest municipalities all imposed higher effective rates on non-residential than on residential property. Thus, presence of non-residential property does not appear to have created incentives for poor communities to increase tax rates on residential property.

8 Conclusions

Few empirical strategies have been developed that investigate public provision under majority rule while taking explicit account of the constraints implied by mobility of households. Epple, Romer, and Sieg (2001) focused on necessary conditions that observed expenditures, housing prices, and tax rates must satisfy in myopic voting equilibrium models. They found that the observed expenditures and (estimated) housing prices do not simultaneously satisfy boundary indifference conditions and the first order conditions implied by myopic voting. This finding is puzzling and raises some issues about the widespread use of myopic voting in theoretical models. In this paper, we have explored the implications of the myopic voter model in further detail. We have specified a generic locational equilibrium model, characterized its equilibrium properties, and derived a new algorithm to compute equilibria. Moreover, we have developed a new empirical approach that imposes all restrictions that arise from these equilibrium models on the data generating process. This allows us to study

of aid specified by the formula, a municipality was required to meet a threshold spending level from own sources. Unfortunately, the data are not sufficient to permit us to determine whether that condition was binding on any municipalities.

[25]The extent of any such incentive would depend on the relative "exportability" of taxes on residential and non-residential activities.

the goodness of fit of the model and helps us determine which dimensions of the data can or cannot be explained by myopic voting.

Our first set of findings reinforce the results in Epple, Romer, and Sieg (2001). While the myopic voting model can replicate many important stylized features of the data including the observed expenditure patterns, it yields distributions of property tax rates that differ significantly from the ones observed in the data. Our model predicts that tax rates are higher in high income communities than in low income communities. In the data, we observe the opposite: high income communities have on average lower property tax rates than poor communities. We also find that the implied mean housing expenditures are too low in the predicted equilibrium for rich communities and too high for poor communities. We, therefore, explore extensions of the simple myopic model and introduce peer effects into the model specification. Our findings are encouraging and suggest that peer effects may be important components in determining the quality of local public good provision. Moreover, the extended model fits the data much better than the simple myopic model considered in ERS. While the fit of the extended model is quite satisfactory, we note that the estimate of one of the parameters, $\sigma_{\ln \alpha}$, converges to the lower bound constraint that we impose in estimation.

Our analysis of myopic voting requires us to impose a number of additional assumptions about the shape of preferences and the demand for housing, which in principle are not a generic feature of the myopic voting model. Our results may therefore be partly due to our specification of household preferences or the distribution of household characteristics. Thus there seem to be two potential avenues which are promising for future research. First, one would try to estimate myopic voting models imposing less restrictive assumptions about household preferences and the distribution of household types. In particular, one could generalize preferences for housing allowing for additional sources of observed and unobserved heterogeneity. In our current specification, we allow for heterogeneity in the tastes for the local public good but not in the taste for housing. As one allows for more general housing demand functions, it should be easier to fit the observed pattern of housing consumption. This may help to address the most obvious limitation of the current generation of equilibrium models.

Alternatively, one can abandon the myopic voting paradigm and focus on more sophisticated voting models. Here the challenge would be to integrate the analysis of locational and voting equilibrium along the lines suggested in this paper. Moreover, we know that home ownership structure (renters vs owner-occupants) is important once we abandon the simple myopic framework. One of the key remaining challenges is to incorporate more realistic ownership structures into the empirical analysis of sophisticated voting models.

Appendix A: Computation of Equilibrium

For simplicity, consider the model without peer effects, i.e. assume that $q = g$. Equilibrium is an allocation of households across communities and goods across households such that all individuals maximize utility in choice of consumption bundles and community of residence, the housing market in each community clears, each community's budget is in balance, and each community's tax and expenditure is chosen by majority rule of residents.

Recall that the population of community j is given by:

$$P(C_j) \;=\; n^j = \int_0^\infty \int_{\alpha^{j-1}(y)}^{\alpha^j(y)} f(y,\alpha) d\alpha dy \; j = 1, ..., J \tag{38}$$

Note that n^j depends on housing prices and public good provision levels in j and in the community or pair of communities adjacent to j. Let P, P_h, G. and T be the $J \times 1$ vectors of community net- and gross-of-tax housing prices, public good provision levels, and tax rates. Let N(P,G) be the $J \times 1$ vector of functions obtained by "stacking" the n^j.

Housing demand in j is given by:

$$H_d^j(P,G) = \int_0^\infty \int_{\alpha^{j-1}(y)}^{\alpha^j(y)} h(p,y) f(y,\alpha) d\alpha dy \; j = 1, ..., J \tag{39}$$

Let H_D(P,G) be the $J \times 1$ vector of housing demands obtained by "stacking" the community housing demand functions. Likewise, let $H_S(P_h)$ be the $J \times 1$ vector of housing supply functions. Then market clearing in housing markets in all communities requires:

$$H_D(P,G) \;=\; H_S(P./(1+T)) \tag{40}$$

where ./ denotes element-by-element division, and budget balance in all communities requires:

$$T. * P_h. * H_D(P,G) \;=\; N(P,G). * G \tag{41}$$

where .* denotes element-by-element multiplication.

Next, consider voting, focusing initially on the case in which all households are renters. The necessary condition for an allocation to be a voting equilibrium is that there is a locus

$$\tilde{\alpha}^j(y) = \tilde{\alpha}(y; p^j, p^{j+1}, g^j, g^{j+1}) \tag{42}$$

satisfying

$$M(p^j, g^j, \tilde{\alpha}^j(y), y) = \left.\frac{dp}{dg}\right|_{GPF} \tag{43}$$

such that half the voters in a community are below $\tilde{\alpha}^j(y)$and half are above; hence voters below the locus comprise half the population of the community:

$$\int_0^\infty \int_{\alpha^{j-1}(y)}^{\tilde{\alpha}^j(y)} f(y,\alpha) d\alpha dy = \frac{1}{2} \int_0^\infty \int_{\alpha^{j-1}(y)}^{\alpha^j(y)} f(y,\alpha) d\alpha dy \ j = 1, ..., J \tag{44}$$

Above $\left.\frac{dp}{dg}\right|_{GPF}$is the voters' belief about how a change in g will affect their community's housing price. Thus, the form of $\left.\frac{dp}{dg}\right|_{GPF}$depends on a characterization of how voters anticipate the consequences of alternative community spending levels. The myopic case implies:

$$\left.\frac{dp}{dg}\right|_{GPF} = \frac{n^j}{H_d^j} \tag{45}$$

Let

$$\tilde{n}^j = \int_0^\infty \int_{\alpha^{j-1}(y)}^{\tilde{\alpha}^j(y)} f(y,\alpha) d\alpha dy \tag{46}$$

and let $\tilde{N}(P,G,T)$ $J \times 1$ vector of functions obtained by stacking the $\tilde{n}^j$. Thus voting equilibrium in all communities requires:

$$\tilde{N}(P,G,T) = N(P,G)/2 \tag{47}$$

Then a multi-community equilibrium is a $J \times 3$ matrix (P,G,T) satisfying (40), (41), and

(47). The latter have been written exploiting the identity P = P_h.*(1+T).

The following describes the strategy for computing equilibrium. By a standard change of variables, rewrite community population as:

$$n^j = \int_{-\infty}^{\infty} \int_{\ln(\alpha^{j-1}(y))}^{\ln(\alpha^j(y))} f(\ln y, \ln \alpha) d\alpha dy \; j = 1, ..., J \tag{48}$$

The population of communities up to and including j is:

$$N_c^j = \int_{-\infty}^{\infty} \int_{-\infty}^{\ln(\alpha^j(y))} f(\ln y, \ln \alpha) d\alpha dy \; j = 1, ..., J \tag{49}$$

The integral in (49) can be rewritten as in Appendix B of ES:

$$N_c^j = \int_{-\infty}^{\infty} f(\ln y) \left[\int_{-\infty}^{z^j(y)} \phi(\xi) d\xi \right] dy \; j = 1, ..., J \tag{50}$$

Equation (50) follows from the left-hand-side of equation (B9) of ES appendix B and $z^j(y)$ is defined following equation (B9). Next, rewrite (50) as:

$$N_c^j = \int_{-\infty}^{\infty} f(\ln y) \Phi(z^j(y)) dy \; j = 1, ..., J \tag{51}$$

where $\Phi(.)$ is the unit normal CDF.

Let y^v= [y_1, y_2,...,y_K] be a row vector of K ordinates to be used for numerical integration. Then $z_v^j = z^j(y^v)$ is a $1 \times K$ row vector of points on the locus $z^j(y)$.

Let $I^j(y)$ denote the integrand of (50):

$$I^j(y) = f(\ln y) \Phi(z^j(y)) \tag{52}$$

Using Simpson's rule, the integral in (50) will be approximated by the sum of K rectangles. Rectangle i has height $I^j(y_i)$. Let w_i be the width of the base of the rectangle, hence the area of the rectangle is $I^j(y_i)w_i$. Let W_v = [w_1, w_2,...,w_K] be the column vector vector of widths associated with y_v. Then the integral in (50) is approximated by:

$$\sum_{i=1}^{K} I^j(y_i) \cdot w_i = I^j(y^v) \cdot W_v \tag{53}$$

Let z_v^Jbe a $1 \times K$ row vector all of whose elements are the upper bound on z, z_{max}.[26] Let Z_m be the $J \times K$ matrix obtained by "stacking" the z_v^j. Let $I(y^v)$ be the $J \times K$ matrix of functions obtained by "stacking" the $I^j(y^v)$ functions:

$$I(y^v) = f(\ln y^v). * \Phi(z_m) \tag{54}$$

Stack theN_c^jinto a $J \times 1$ vector, N_c. We then have that:

$$N_c = [f(\ln y^v). * \Phi(z_m)] \cdot W_v \tag{55}$$

Let $N_c^{-J} = [0, N_c^1, N_c^2, ..., N_c^{J-1}]$. Then we have the numerically integrated counterpart to N(P,G) , the $J \times 1$ vector of community populations:

$$N(P, G) = N_c - N_c^{-J} \tag{56}$$

The housing demand function is: $h(p,y)=Bp^{\eta}y^{\nu}$. Thus, to calculate community housing demand, we need the integral of y^{ν} for each community. Proceeding as above, noting that $y^{\nu} = e^{\nu lny}$, let Y_c be defined analogously to (55):

$$Y_c = \left[e^{v \ln y}. * f(\ln y). * \Phi(z_m)\right] \cdot W_v \tag{57}$$

Let $Y_c^{-J} = [0, Y_c^1, Y_c^2, ..., Y_c^{J-1}]$, and let Y^{ν}(P,G) be:

$$Y^{\nu}(P, G) = Y_c - Y_c^{-J} \tag{58}$$

[26]The upper bound on z is infinity. For purposes of numerical integration, z_{max} can be set equal to a value sufficiently large that only a small fraction of the integral in (51) lies above z_{max}.

The $J \times 1$ vector of community housing demands is then:

$$H_D(P,G) = BP^{\eta}Y^{\nu}(P,G) \tag{59}$$

To do the computations for voting equilibrium, use L_j as defined in equation (2.18) of ERS:

$$L_j = -\rho\frac{Bp_j^{\eta+1} - 1}{\eta+1} + \ln(Bp_j^{\eta}) + \ln\left(\frac{dp_j}{dg_j}\right) - (\rho - 1)\ln g_j \tag{60}$$

In the above equation, the expression for voters' perceptions of the response of p_j to changes in g_j $\left.\left(\frac{dp_j}{dg_j}\right)\right|_{RPF}$ is substituted in place of $\left(\frac{dp_j}{dg_j}\right)$. Let $\tilde{z}^j(y)$be the locus of pivotal voters for community j. This is defined analogously to $z^j(y)$except that L_j appears in $\tilde{z}^j(y)$where K_j appears in $z^j(y)$. Define $\tilde{z}_m$ analogously to z_m. Let:

$$\tilde{N}_c = [f(\ln y^v). * \Phi(\tilde{z}_m)] \cdot W_v \tag{61}$$

Then the following vector gives the number of voter below the pivotal locus in each of the J communities:

$$\tilde{N}(P,G,T) = \tilde{N}_c - N_c^{-J} \tag{62}$$

Equations (56), (59), and (62) provide the results needed for computing the equilibrium conditions in equations (40), (41) and (47).

For the empirical analysis in this paper, we replace equation (33) with the requirement that populations implied by the model equal populations observed in the data. The population size equations yield J-1 conditions. The additional condition that we invoke is to normalize the price in community one to be equal to one.

Appendix B: Proof of Proposition 1

We prove Proposition 1 using the parameterization introduced in Section 3 and assuming that $q = g$. Given community populations and the ordering of communities, the community-specific intercepts, K_j, defined in equation (22) are uniquely determined by the recursion in ES equation (14). Given the community-specific intercepts, the boundary loci delineating communities are given by equations (22). This in turn implies that the distribution of (α, y) types in each community is known. The community-specific voting intercepts, L_j, defined in equations (25) are then uniquely determined by the recursion in ERS equation (2.19). It remains to show that, given the preceding, the internal equilibrium in each community is unique. For this, it suffices to show show that the conditions in Assumptions 7 and 8 of EFR are satisfied for voters on the pivotal locus.

Assumption 7 requires:

$$\begin{array}{ll} i) & \dfrac{\partial M}{\partial g} \leq 0 \\ ii) & \dfrac{\partial M}{\partial p} \leq 0 \\ iii) & \dfrac{\partial M}{\partial g} + \dfrac{\partial M}{\partial p} < 0 \\ iv) & \lim\limits_{g \to 0} M = 0 \\ v) & \lim\limits_{p \to \infty} M = 0 \end{array} \tag{63}$$

Assumption 8 requires:

$$\frac{\partial ph(p, y)}{\partial p} \geq 0 \text{ for all } p, y \tag{64}$$

To verify the conditions of Assumption 7, consider a voter (α, y) on the pivotal locus. Since $M(\cdot)$ is the same for all voters on the pivotal locus, it is sufficient to establish the results for an arbitrarily chosen voter on the pivotal locus. For this voter, the slope of an indirect indifference curve in the (p, g) plane is given in ES Equation (9) and reproduced here:

$$M(p,g,y,\alpha) = -\frac{\partial V(p,g,y,\alpha)/\partial g}{\partial V(p,g,y,\alpha)/\partial y} = \frac{\alpha g^{\rho-1}\left[e^{\frac{y^{1-\nu}-1}{1-\nu}}\right]^{-\rho}\left[e^{-\frac{Bp^{1+\eta}-1}{1+\eta}}\right]^{-\rho}}{Bp^{\eta}} > 0 \tag{65}$$

Differentiating M, we obtain:

$$\begin{aligned} &i) \qquad \frac{\partial M}{\partial g} = (\rho - 1)\frac{M}{g} < 0 \\ &ii) \qquad \frac{\partial M}{\partial p} = M\rho B p^{\eta} < 0 \end{aligned} \tag{66}$$

The above inequalities are implied by $\rho < 0$, $B > 0$, and $M > 0$. Condition (iii) then follows since both inequalities above are strict. Condition (iv) follows from inspection of the expression for M.

To verify condition (v), simplify notation by letting:

$$z = \alpha g^{\rho-1}\left[e^{\frac{y^{1-\nu}-1}{1-\nu}}\right]^{-\rho} \tag{67}$$

Then

$$M(p,g,y,\alpha) = -\frac{\partial V(p,g,y,\alpha)/\partial g}{\partial V(p,g,y,\alpha)/\partial y} = \frac{z e^{\rho\frac{Bp^{1+\eta}-1}{1+\eta}}}{Bp^{\eta}} \tag{68}$$

Condition (v) requires finding the limit of M as $p \to \infty$. Note that, for p sufficiently large,

$$e^{-\rho\frac{Bp^{1+\eta}-1}{1+\eta}} > \frac{-\rho B p^{1+\eta}}{1+\eta} \tag{69}$$

The preceding inequality follows from observing that the right-hand side increases linearly in $p^{1+\eta}$ while the left-hand side increases exponentially in $p^{1+\eta}$. Hence, for p sufficiently large

$$M = \frac{ze^{\rho\frac{Bp^{1+\eta}-1}{1+\eta}}}{Bp^{\eta}} = \frac{z}{e^{-\rho\frac{Bp^{1+\eta}-1}{1+\eta}}Bp^{\eta}} < \frac{z}{\frac{-\rho Bp^{1+\eta}}{1+\eta}Bp^{\eta}} = \frac{z}{\frac{-\rho B^2 p}{1+\eta}} \tag{70}$$

By observation, the limit of the last expression above approaches zero as p approaches infinity. This and $M \geq 0$ imply Condition (v).

The housing demand function implied by our indirect utility function is

$$h(p, y) = Bp^{\eta}y^{\nu} \tag{71}$$

Assumption 8 then follows from $|\eta| < 1$:

$$\frac{\partial ph(p, y)}{\partial p} = (\eta + 1)h(p, y) > 0 \tag{72}$$

Q.E.D.

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