Scale-specific risk in the consumption CAPM*

Federico M. Bandi† and Andrea Tamoni‡

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Abstract

While contemporaneous consumption growth is known not to price the cross section of stock returns, we find that suitable sub-components (or details) of consumption growth with periodicities corresponding to the business cycle do. Specifically, we disaggregate consumption growth into details with different levels of persistence and show that those corresponding to business cycle scales can explain the differences in risk premia across book-to-market and size-sorted portfolios. We argue that accounting for persistence heterogeneity in consumption is important for interpreting risk compensations in financial markets but also for capturing the joint dynamics of consumption and returns across horizons (for instance, the hump-shaped pricing ability of the covariance between ultimate consumption and returns, the hump-shaped structure of long-run risk premia as well as the decaying pattern in consumption growth predictability).

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†Johns Hopkins University and Edhec-Risk Institute.

‡London School of Economics, Finance Department.

1 Introduction

It is a basic tenet of economic theory that agents care about their consumption stream. A traditional implication, for asset valuation in financial markets, of this accepted premise is that the risk of any asset should depend on its covariance with consumption growth. Assets which pay off in adverse states of nature (those in which consumption growth is lower) are assets which should be perceived as being less risky and should therefore provide, in equilibrium, lower expected returns. Conversely, those assets whose returns are positively correlated with consumption growth are assets with inferior hedging abilities. Their demand should be lower, thereby justifying their lower prices, and their higher expected returns.

This logic, grounded in the classical Consumption CAPM (CCAPM) model of Rubinstein (1976) and Breeden (1979), is known not to be supported by economic data. Differences in expected returns across risky securities are empirically not attributable to sheer differences in the variability of the returns of these securities with respect to changes in aggregate consumption.[1] This failure has lead to alternative models of economic behavior in which a prominent role is given to suitable modifications of the conventional time-separable utility paradigm.[2]

Is the traditional CCAPM truly limited? This paper argues that aggregate consumption growth, the subject of much investigation, can be separated into a variety of components, layers, or *details*. These details operate at different frequencies, thereby representing features of the overall consumption stream with cycles of different lengths. While the covariance between *aggregate* consumption growth and individual assets' returns does not explain the cross-sectional dispersion of expected excess returns, we show that the covariance between specific *details* of the consumption growth process and assets' returns is an important determinant of risk in financial markets.

In essence, investors may not focus on very high-frequency components of the consumption process. For the purpose of asset pricing, these components may just amount to short-term noise. Lower frequency components of consumption growth with various degrees of persistence may, how-

[1]Campbell (2003) and Cochrane (2001) provide comprehensive discussions.

[2]Fundamental contributions are Abel's *external habit* as in Abel (1990), Constantinides (1990), Detemple and Zapatero (1991), Campbell and Cochrane (1999) and Mezly, Santos and Veronesi (2004), *inter alia*, Epstein and Zin's *recursive utility* as in Epstein and Zin (1989, 1991), Bansal and Yaron (2002), and Hansen, Heaton, and Li (2008), among others, and *loss aversion* as in Barberis, Huang and Santos (2001). For an interesting discussion of their unified interpretation as preference theories defined over consumption and a suitable reference factor, as well as for further extensions in an expected utility framework, we refer the interested reader to the work of Garcia, Renault, and Semenov (2004).

ever, be important drivers of risk premia. A suitable noise-to-signal extraction mechanism, along with the application of conventional pricing techniques, allows us to identify these priced components. In the size and book-to-market space represented by the traditional Fama-French portfolios, we find that the main consumption detail driving risk compensations is a business-cycle component with periodicity between 4 and 8 years. When used in isolation, this component yields a value for the coefficient of determination (R^2) close to 30% and insignificant pricing errors. The addition of a component with periodicity between 2 and 4 years increases the coefficient of determination to 42% and improves the performance of the model along a variety of additional dimensions. Taken together, these results point to the importance, for the cross-sectional pricing of financial assets, of low-frequency components of the consumption process spanning between 2 and 8 years of economic activity (the usual length of the business cycle according to Burns and Mitchell, 1946; see also the implicit taxonomy in Comin and Gertler, 2006).

Our analysis begins with a model for consumption growth $g_{t+1} = \log \frac{c_{t+1}}{c_t}$ which expresses it as $g_t = \sum_{j=1}^{J} g_t^{(j)} + \pi_t^{(J)}$, where the $g_t^{(j)}$s are scale-specific consumption details with various persistence properties and $\pi_t^{(J)}$ is a long-run trend. Each detail is associated with fluctuations between 2^{j-1} and 2^j quarters. Importantly, the shocks determining individual components are, in general, *not* aggregates of high-frequency shocks. They are, instead, scale-specific as well as time-specific. This modeling device represents an important departure from classical time series specifications (see Bandi, Perron, Tamoni and Tebaldi, 2012, BPTT henceforth), one which represents the idea that different layers of the consumption process may be the result of random shocks with different sizes and different half-lives. The separation into $J+1$ details with $J > 1$ gives us more granularity in the analysis of fluctuations with different cycles that is the case with more traditional (2-component) decompositions of the Beveridge-Nelson type. This granularity is, of course, crucial to evaluate the differential impact of various consumption details on risk premia. Write the generic asset i's excess return as $R_{t,t+1}^{ei} = \sum_{j=1}^{J} R_{t,t+1}^{ei(j)} + \eta_{t+1}^{(J)}$, where the symbols have the same interpretation as for g_t. We find that that size and book-to-market portfolios are suitably priced by $Cov\left[g_{t+1}^{(j)}, R_{t,t+1}^{ei(j)}\right]$ with $j = 4$ and $j = 5$, i.e., two sub-components of the classical aggregate covariance $Cov\left[g_{t+1}, R_{t,t+1}^{ei}\right]$, the sub-components corresponding - as pointed out earlier - with (business-cycle) fluctuations between 2 to 8 years.

In essence, we find that accounting for persistence heterogeneity in consumption is key for interpreting differences in risk compensation across assets. We also show that it is key for capturing the joint dynamics of consumption growth and returns across different time horizons. To this extent, the paper uses a variety of *metrics* intended to evaluate the pricing ability of a model in which business-cycle components of the consumption process are the main determinants of the cross-sectional dispersion in risk premia.

First, the model generates positive consumption growth autocorrelations up three lags (quarters) and largely insignificant autocorrelations thereafter. This finding is consistent with data.

Second, following Parker and Julliard (2005) who define risk in terms of covariances with respect to *ultimate* consumption ($Cov\left[g_{t,t+h+1}, R^{ei}_{t,t+1}\right]$ with h large), we show that the proposed pricing model closely reproduces their documented hump-shape pattern of R^2s with a peak corresponding to a time period between 2.5 and 3 years. Averaging, as in the definition of ultimate consumption, reveals persistent components by eliminating short-term fluctuations (BPTT, 2012, for a formal treatment). Thus, there is an important conceptual link between ultimate consumption and a data generating process, like the one we propose, in which persistent components of the consumption process with business-cycle fluctuations drive risk premia.

Third, we look at predictability of consumption growth as in Piazzesi (2001). In this context, we report the (average, across stocks) covariance between future consumption growth and current excess returns ($Cov\left[g_{t+h,t+h+1}, R^{ei}_{t,t+1}\right]$) to show that, barring short-term seasonal patterns, the component model yields the consumption predictability found in the data.

Finally, as suggested by Cochrane and Hansen (1992), we examine the equity premium at long horizons, namely the (average, across stocks) covariance between long-run consumption growth and long-run excess returns divided by the horizon ($\frac{1}{h}Cov\left[g_{t,t+h}, R^{ei}_{t,t+h}\right]$). In the data this standardized covariance is typically found to be hump-shaped: it increases up to about 2 years before decreasing monotonically. Our component model for consumption reproduces this pattern satisfactorily.

The proposed approach has several important features. First, our analysis of the pricing abilities of consumption details keeps us within the economically-appealing framework of the CCAPM in its traditional format. We are however emphasizing that risk can be thought of as being driven by individual component(s) of $Cov\left[g_{t+1}, R^{ei}_{t,t+1}\right]$ rather than by the full covariance. Having made this point, our choice not to depart from the typical paradigm with time-separable utilities is by no means a refusal of alternative utility models whose potential has been widely documented. It is, similarly, not a tacit endorsement of conventional utility specifications. It is, however, a useful way for us to push the boundaries of the classical time-separable specification and highlight the ability of consumption *itself* to price risky assets without enriching preferences. Second, through (business cycle-like) fluctuations in the consumption components, we generate *directly* (business cycle-like) fluctuations in the stochastic discount factor, a feature which was discussed by Alvarez and Jermann (2002) and Parker and Julliard (2005) as being empirically warranted and theoretically meaningful. Finally, we provide an alternative, but rather natural, channel through which hard-to-detect persistent components in the consumption process affect asset prices. The role of persistence in consumption for asset pricing has been highlighted in influential, recent work (e.g., Alvarez and Jermann, 2002, Bansal and Yaron, 2005, and Hansen, Heaton, and Li, 2008).

An interesting question to ask is whether there are subsets of frequencies over which pricing models, like the traditional CCAPM, fare satisfactorily. This question has been addressed in the time domain (Daniel and Marshall, 1997) and, as is natural, in the frequency domain (e.g., Berkowitz, 2001, and Cogley, 2001). A consistent conclusion of this line of work is that the fit of the model improves as the horizon increases, thereby providing support for the implication that asset pricing puzzles are largely short-term phenomena having to do with frictions. A related, but different, question is: at which frequency do the shocks that matter to price *short-term* returns of the kind routinely used in asset pricing tests operate? This issue has also been addressed in the time domain (e.g., Bansal and Yaron, 2005, and Hansen, Heaton and Li, 2008) as well as in the frequency domain (Dew-Becker and Giglio, 2013), and, more recently, in the joint time-scale domain (Ortu, Tamoni and Tebaldi, 2013). We operate in the time domain and provide an explicit consumption/return data generating process which expresses all processes as sums of components (and, therefore, shocks) with periodicity of different length. Our decomposition, and the resulting decomposition of the overall covariance between consumption growth and asset returns into sub-covariances (one for each scale), is explicit about the role of consumption shocks operating at different frequencies for the pricing of *short-term* returns. Importantly, since aggregation reveals low-frequency relations (and consumption betas defined on low-frequency components are shown in the paper to capture cross-sectional risk premia), the proposed framework also translates into a price formation mechanism yielding effective cross-sectional pricing for *low-frequency* returns. In this sense, our scale-based decompositions nicely tie low-frequency consumption dynamics to both the pricing of long-run returns (as in, e.g., Daniel and Marshall, 1997) and that of short-run returns (as in, e.g., Bansal and Yaron, 2005).

2 A scale-based decomposition of the CCAPM

The most classical asset pricing formula states that

$$E_t[R^i_{t,t+1} m_{t+1}] = 1, \tag{1}$$

where $m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$. In Eq. (1), $R^i_{t,t+1}$ is a generic asset i's uncertain return, $u(\cdot)$ is the period utility function defined over the consumption stream, and β is a discount factor. One implication of Eq. (1) is that, after taking dividends into account, prices are martingales (and are, therefore, unpredictable) under the natural probability measure only when investors are risk neutral. If investors are not risk neutral, prices are martingales once the true probabilities are suitably modified by the change of measure m_{t+1}.

Alternatively, in terms of risk compensations in *excess* of the risk-free asset R^f, one could write

$$E_t[(R^i_{t,t+1} - R^f) m_{t+1}] = E_t[R^{ei}_{t,t+1} m_{t+1}] = 0, \tag{2}$$

or, equivalently,

$$E_t[R^{ei}_{t,t+1}] = -\frac{Cov_t\left[m_{t+1}, R^{ei}_{t,t+1}\right]}{E_t[m_{t+1}]},$$

which is a beta representation for expected excess returns.

Under a constant relative risk aversion utility function $u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}$ and a simple approximation,[3] this representation leads to

$$E_t[R^{ei}_{t,t+1}] \underset{\text{small } \Delta c_{t+1}}{\approx} \theta Cov_t\left[\log \frac{c_{t+1}}{c_t}, R^{ei}_{t,t+1}\right] = \theta Cov_t\left[g_{t+1}, R^{ei}_{t,t+1}\right] = \theta \beta^i_t.^4 \tag{3}$$

The interpretation of Eq. (3) is standard: the risk of any asset should be measured by the covariance of the asset's return with respect to consumption growth. Assets whose returns are relatively lower in states of nature in which consumption growth is lower are perceived as riskier and should, therefore, require a higher risk compensation, leading to higher expected returns in excess of the risk-free rate.

In spite of its economic purity and its emphasis on consumption as the main driver of agent's

[3] Ignoring the discount factor, write $m_{t+1} = \frac{u'(c_{t+1})}{u'(c_t)} = \left(\frac{c_{t+1}}{c_t}\right)^\theta = \left(\frac{\Delta c_{t+1}}{c_t} + 1\right)^\theta \approx (\Delta \log c_{t+1} + 1)^\theta \approx 1 + \theta \Delta \log c_{t+1}$ when Δc_{t+1} is small.

[4] We use the convention of defining beta as a covariance, rather than as a covariance divided by the variance of the factor.

hedging decisions, this simple intuition is not validated by data. Regardless of the sampling period, a broad literature has documented that differences in average excess returns across US assets do not appear to be justifiable based on the corresponding differences in the covariance of the assets' returns with respect to consumption growth.

Using data on the 25 Fama-French (FF) portfolios sorted on size and book-to-market between 1963:Q1 and 2012:Q4 as well as data on consumption of nondurables and services, seasonally adjusted in 1996 chain weighted dollars,[5] we confirm this standard result.

Figure 1 plots the average returns on the portfolios. It shows the typical decreasing pattern across size (from small to large) with the sole (also typical) exception of the first value quintile for which the trend is largely reversed. We will return to the first value quintile in what follows. Similarly, the figure displays an increasing pattern of average portfolio returns along the book-to-market dimension, i.e., the well-known value premium.

Figure 2–Panel A plots average excess portfolio returns (on the vertical axis) against expected excess returns implied by an unconditional version of the model in Eq. (3), i.e.,

$$E[R^{ei}_{t,t+1}] = \theta Cov\left[g_{t+1}, R^{ei}_{t,t+1}\right] = \theta\beta^{i} \tag{4}$$

a traditional CCAPM. Predicted returns do not align with historical returns, the largest deviations corresponding with portfolios in the first and in the last value quintile.

Is aggregate consumption growth too coarse a measure to deliver meaningful quantities of risk and, consequently, to "imply" meaningful expected returns? We address this issue by working with multi-scale decompositions for both consumption growth and excess returns on the test assets. Since the data spans a time frame of about 50 years, our chosen lowest frequency component ($J = 6$, in the notation used above) strikes a compromise between identifiability (higher frequency details are easier to identify) and richness of the overall decomposition (the larger the number of details, the richer the decomposition). Such a component captures fluctuations between 8 years and 16 years. We will show that a large percentage of the pricing ability of consumption growth is associated with higher (than $J = 6$) frequency details. Write

$$g_{t+1} = \sum_{j=1}^{6} g^{(j)}_{t+1} + \pi^{(6)}_{t+1} \tag{5}$$

[5]For complete information on the data, we refer the interested reader to Appendix B.

and

$$R^{ei}_{t,t+1} = \sum_{j=1}^{6} R^{ei(j)}_{t,t+1} + \eta^{(6)}_{t+1}. \tag{6}$$

The components or details $\left\{g^{(j)}_{t+1}, R^{ei(j)}_{t,t+1}\right\}$ are extracted using Haar wavelets. They can be thought of as cross-sectionally uncorrelated linear autoregressive processes, with a scale specific autoregressive parameter ρ_j and scale specific shocks, on the dilated time $t - 2^j$ of each individual scale. Since j is measured in terms of quarters, each detail is associated with periodic fluctuations between 2^{j-1} and 2^j quarters. The decompositions in Eqs. (5) and Eqs. (6), along with the autoregressive dynamics of the details, translate into aggregated processes $\left\{g_{t+1}, R^{ei}_{t,t+1}\right\}$ for which a *generalized* Wold representation holds, one in which the time series are linear combinations of shocks that are both time and scale specific. This representation captures the idea that economic time series may be suitably interpreted as the result of a cascade of shocks occurring at different times and different frequencies. BPTT (2012) discuss the extraction mechanism and the properties of the extracted details and the aggregated process. A summary of the approach is in Appendix A.

Returning to the unconditional CAPM specification in Eq. (4), we notice that

$$\beta^i = \sum_{j=1}^{6} \beta^{i(j)}, \tag{7}$$

where $\beta^{i(j)} = Cov\left[g^{(j)}_{t+1}, R^{ei(j)}_{t,t+1}\right]$ provided $Cov\left[g^{(j)}_{t+1}, R^{ei(k)}_{t,t+1}\right] = 0$ with $j \neq k$. While unnecessary for our treatment, the zero condition on the cross-covariances is roughly satisfied for our data. This observation leads to a beta formulation conveniently expressed in terms of covariances between *contemporaneous* details of the consumption growth and return processes, namely

$$E[R^{ei}_{t,t+1}] = \sum_{j=1}^{6} \lambda_j \beta^{i(j)}. \tag{8}$$

Eq. (8) can now be interpreted as an "unconstrained" CCAPM model. The prices of risk (λ_j) are, in fact, not constrained, as implied by the classical formulation in Eq. (4), to be the same across components of the covariance of aggregate consumption with returns. They are, instead, treated as free parameters.

As is customary in the literature, to evaluate the relative price of risk (λ_j) associated with different scale-specific betas ($\beta^{i(j)}$), we first run time-series regressions of the type

$$R^{ei(j)}_{k2^j,k2^j+2^j} = \beta_0 + \beta^{i(j)} g^{(j)}_{k2^j,k2^j+2^j} + \epsilon_{k2^j+2^j}, \tag{9}$$

with $k \in \mathbb{Z}$. The difference here between the standard time series approach and our (scale-wise) approach is that the regressions are run on details sampled every 2^j times. BPTT (2012) refer to these sub-series of the original details defined on chronological time as *decimated* series. For each scale, the decimated series are designed to capture all relevant information about dynamics at the corresponding frequency (see Appendix A for more details).

Using the first-pass estimates $\beta^{i(j)}$, we then estimate the regression

$$E[R^{e,i}_{t,t+1}] = \lambda_0 + \sum_{j=1}^{6} \lambda_j \beta^{i(j)} + \alpha_i \tag{10}$$

with and without an intercept and, when appropriate, with zero restrictions on the λ_j's. We begin by analyzing the behavior of $\beta^{(j)}$ and λ_j across alternative scales and portfolios.

3 Scale-specific betas and lambdas

In this section we quantify first the exposures of the 25 FF portfolios returns to consumption over alternative scales, and then the corresponding prices or investors' compensations.

Figure 3 plots the betas of the 25 FF portfolios associated with different scales. Going from the highest frequency scale to our chosen lowest frequency scale ($j = 6$), we witness a rotation from size to value. In particular, the consumption detail corresponding to scale $j = 2$ (i.e., 6 months to 1 year) translates into betas which align very effectively with average returns (as reported in Figure 1) in the size dimension (higher betas for small firm portfolios, lower betas for large firm portfolios) while failing to capture the second dimension, i.e., value. The betas corresponding to lower frequency details of the consumption process ($j = 3$, 4, and 5) display improved alignment in the value space without losing their ability to capture cross-sectional variability in risk premia across size. The scale $j = 5$, among them, is the one for which the visual alignment of the betas across both dimensions appears to be the best. The value premium, in particular, is captured rather effectively at this low frequency (4 to 8 years). An important size tilt is, however, also present.

While it is not hard to conjecture that the betas corresponding to scale $j = 5$ may be the most effective in explaining cross-sectional variability in average returns, the preceding scale $j = 4$ may represent an improvement in the size space. Interestingly, such an improvement is expected to be brought about even by portfolios in the first value quintile. As documented in Figure 1, these are portfolios for which the typical decreasing trend with size increases does not hold. Consistently, at scale $j = 4$, the betas associated with the first value quintile have the same hump-shaped pattern which characterizes average historical returns.

Figure 2–Panel B plots average excess portfolio returns (on the vertical axis) against returns implied by the model in Eq. (10) with all lambdas restricted to zero with the exception of the lambda associated with a specific scale. Confirming the intuition drawn from the corresponding betas, the scale $j = 5$ appears to be the most effective in representing cross-sectional variability in risk compensations in the size/value space. Higher frequency details of the consumption growth process ($j < 5$) generate betas which, as shown, replicate dispersion in risk premia along one dimension - generally size - but fail to provide - in spite of their mild rotation to value - a justification for both.

Table 1 contains univariate (one beta at a time) estimates of the model in Eq. (10) corresponding to $j = 1, ..., 6$. In agreement with the previous visual evidence, the scale $j = 5$ yields the maximum value for the coefficient of determination (28%). The prices of risk are very significant whether or not an intercept is inserted. When introduced, the intercept is insignificant. With and without an intercept, the difference between root mean square alpha (RMSA) and mean absolute alpha (MAA) is very small. These results contrast sharply with what is found for all other scales, even for those (like $j = 2$) for which the size effect is nicely reproduced. In their case, the reported failure across the value dimension leads to poor performance. Not surprisingly, in spite of its inability to capture value effectively, the scale $j = 4$ represents a meaningful transitions from less informative scales to $j = 5$. Without an intercept, the price of risk is sensible. In this case, however, the addition of an intercept leads to an insignificant lambda and a significant constant.

We now turn to bivariate specifications in which one lambda at a time is added to the lambda corresponding to $j = 5$. Table 2 provides the results. For all models, the constant is insignificant. This effect is due to the inclusion of the 5^{th} scale and is independent of all other components. Along all dimensions, the specification with $j = 4$ and $j = 5$ outperforms the other specifications. The increase in R^2, in addition to what is yielded by the 5^{th} scale, is sizeable (from 28% to 42%). When inserting an intercept, the intercept is insignificant but the contribution of the higher frequency factor ($\beta^{(4)}$) appears to be significant. This specification has a low MAA of 1.64% per year which is comparable to the 10bp per month MAA (1.2% per year) of the Fama-French 3-factor model. The specification with $j = 1$ and $j = 3$, on the other hand, has an unreasonable sign for the price of risk attached to $\beta^{(3)}$ (i.e., a negative sign along with a positive beta). The model with $j = 6$ has a negative sign for the price of risk associated, this time, with negative values of $\beta^{(6)}$ (see Figure 3). In this sense, the 6^{th} detail resembles a liquidity or a volatility effect (see Tamoni, 2011).

Do investors fear two sources of risk occurring at different scales of time (one at frequencies of about $2 - 4$ years and the other at lower frequencies up to 8 years) and, therefore, price them differently? Or, are fluctuations evolving at business-cycle frequencies contributing equally to the demanded premium? To answer this question, we test a single-beta model in which the beta measures exposures to the fourth and the fifth consumption components:

$$E[R^{e,i}_{t,t+1}] = \lambda_0 + \lambda_{45}\left(\beta^{i(4)} + \beta^{i(5)}\right) + \alpha_i. \tag{11}$$

Table 3 reports the results. For the reader's convenience, Panel A also reports the prediction of the model with different prices of risk on the consumption's detail covariances, $\beta^{i(4)}$ and $\beta^{i(5)}$. Essentially, restricting the price of risk on the fourth and the fifth consumption components to be the same does not eliminate significant information about excess returns. In fact, the pricing errors are only between 5 and 7 basis point, per year, higher than in the unrestricted case, with an overall fit of 39%. In addition, the constant stays insignificant.[6]

In sum, there are sound economic reasons to place emphasis on both the 4^{th} and the 5^{th} scale. They capture size and value effects in different, but complementary, ways. Reflecting the rotation of the scale-specific betas from size to value as we transition from higher frequency details of the consumption process to lower frequency details, the 4^{th} scale is quite effective along the size dimension whereas, while offering an appealing size tilt, the best-performing 5^{th} scale is more effective along the value dimension. As pointed out earlier, these two contiguous scales *jointly* span the accepted length of the business cycle capturing fluctuations in consumption growth between 2 and 8 years.

[6] More formally, one can use a likelihood ratio test to examine the hypothesis implied by the restricted model against the unrestricted model. When we consider the model with no constant included, a test of over-identifying restrictions yields a statistics of 2.438 with an associated p-val of 0.12; hence we cannot reject the hypothesis that the two fluctuations, the one with half-life between two and four years, and the one with four and eight years, have the same price of risk at 10% conventional level.

We now turn to several metrics intended to evaluate the empirical importance of a component model for consumption. In this context, we provide an additional assessment of the relative contribution, to asset pricing, of shorter (2 to 4 year) cycles in consumption (as represented by $g_{t+1}^{(4)}$) in addition to the 4 to 8 year cycles represented by the main component $g_{t+1}^{(5)}$.

4 The dynamics of returns and consumption growth: some *metrics*

What are the implications of the presence of heterogeneous details in consumption growth for the joint dynamic properties of consumption and asset returns? In order to address this issue, we simulate return/consumption processes according to a component model motivated by the previous empirical analysis. We then employ the simulated process to verify the extent to which an array of stylized facts about the joint behavior of asset returns and consumption growth over horizons of different lengths are satisfied by the model.

The return details relate to the consumption details based on the specification

$$R_{t,t+1}^{i(j)} = \begin{cases} \beta^{i(j)} g_{t+1}^{(j)} & \text{for } j = 4, 5, \\ \epsilon_{i,t}^{(j)} & \text{otherwise} \end{cases},$$

where $\epsilon_t^{(j)}$ is $N(0, \sigma^{i(j)})$ and $\sigma^{i(j)}$ is chosen so as to match the variance of the component $R_{t,t+1}^{i(j)}$ at scale j for asset i. In other words, aggregate returns are generated *solely* based on two details of the consumption process along with noise components at all other scales. For consistency with a pricing model which includes the 4^{th} and the 5^{th} scale, we further impose

$$E[R_{t,t+1}^{i}] = \beta^{i(4)}\lambda_4 + \beta^{i(5)}\lambda_5, \tag{12}$$

where the betas and the lambdas are estimated from the data. Since the consumption details are mean zero, the restriction implied by Eq. (12) is imposed by simply adding a constant term to all simulated return series. We begin with the autocorrelation of consumption growth.

4.1 Autocorrelation of consumption growth

In the data, the consumption growth autocorrelation is positive and significant up to the third quarter. The values implied by the model are very plausible. Figure 4 provides the empirical autocorrelations along with 95% confidence bands and their model-implied counterparts. The first and the second quarter autocorrelations are very closely matched. The third quarter autocorrelation implied by the model is slightly lower than that in the data and outside of the corresponding confidence bands. However, it is clearly positive just like in the data. Keeping in mind that the addition of components, or details, of the consumption process may easily reconcile these small differences, a simple specification with two details (the 4^{th} and the 5^{th}) appears to capture fundamental first-order effects in the consumption dynamics.

4.2 Parker and Julliard's effects

Parker and Julliard (2005) find that consumption growth measured over long horizons (dubbed *ultimate* consumption) is an effective driver of risk premia. In other words, it leads to covariances $Cov\left[g_{t,t+h+1}, R^{ei}_{t,t+1}\right]$ which, for appropriate values of h (corresponding to about 3 years in their framework), nicely align with historical average returns in the same size/value space investigated here and, ubiquitously, in much of the literature. Parker and Julliard (2005) report a hump-shape in the coefficients of determination of their pricing model as a function of the horizon over which consumption growth is calculated. Their R^2s are monotonically increasing up to 11 quarters before decreasing steadily thereafter. For their efficient GMM estimates, the reported R^2 at the peak is around 38%.

We apply the same methodology as in Parker and Julliard (2005) to our simulated data (Figure 5(a) and 5(b)). When using only one dominant component (the 5^{th}), the hump-shape is reproduced with a peak between 9 and 10 quarters and a corresponding R^2 of about 23% (**Figure 6 a**). Adding only one additional component (the 4^{th}) preserves the location of the peak and raises the R^2 to about 29% (**Figure 6 b**). Said differently, with two details *only*, the 4^{th} and the 5^{th}, the adopted specification translates into R^2 spikes at horizons around two years and a half and very close to the 11-quarter time frame emphasized by Parker and Julliard (2005).

As illustrated formally by BPTT (2012), averaging is an effective mechanism to bring to light persistent details while eliminating contaminations with more frequent cyclical fluctuations. In this sense, the mapping between Parker and Julliard's 3-year horizon and components with cycles between 2 and 8 years is not surprising. On the one hand, *ultimate* consumption can be justified, at a fundamental level, by a data generating process, like the one we propose, in which business-cycle components of the consumption process play a dominant role in the determination of risk premia. On the other hand, the averaging implicit in the definition of ultimate consumption may provide information about the frequency at which the relevant (for asset pricing) details of the consumption process operate. Their 3-year horizon is suggestive of the importance of business-cycle fluctuations in consumption. These fluctuations are captured explicitly by our reported 4^{th} and 5^{th} detail.

4.3 Predictability of consumption growth

A related object of interest, focusing on risk rather than on its pricing, is the cumulative covariance of the portfolio returns $R^{ei}_{t,t+1}$ with respect to future consumption growth $g_{t+h,t+h+1}$, namely $Cov\left[g_{t+h,t+h+1}, R^{ei}_{t,t+1}\right]$. Figure 6 plots this quantity over time along with 95% confidence bands constructed using Newey-West standard errors (dotted lines). The contemporaneous covariance ($h = 0$) is non-zero. Its value increases up to 7 quarters. Beyond that time, the numbers decrease slightly with the horizon, but the confidence bands become larger.

The figure shows that the covariance pattern in the data is well replicated by the adopted specification. In a model with only the 5^{th} component, the implied covariances are hump-shaped, as in the data, and close to the empirical ones, but with a peak around 10 quarters (Figure 6–Panel A). Remarkably, the addition of the 4^{th} component (in Figure 6–Panel B) leads to a close replication of the trend in the empirical covariances up to 16 quarters (including the hump around 7 quarters).

As suggested by Piazzesi (2001), this long-run risk measure can be decomposed into its individual elements, i.e.,

$$Cov\left[g_{t,t+h+1}, R_{t,t+1}\right] = \sum_{i=0}^{h} Cov\left[g_{t+i+1}, R_{t,t+1}\right].$$

Figure 7 plots the individual elements under the summation sign divided by the corresponding horizon, i.e., the slopes of the cumulative covariance function in Figure 6. In the data, these slopes are positive up to horizon 8. Barring seasonal fluctuations, a model solely inclusive of the 5^{th} detail would fare quite well in replicating this pattern of covariances (Figure 7–Panel A). It would, however, yield an excessively flat structure at high frequencies. As earlier, the addition of the 4^{th} component is successful in providing a solution to this issue (Figure 7–Panel B). This component raises the value of the simulated covariances precisely where needed (namely, at short horizons), thereby closely replicating the convex pattern of the empirical covariances.

4.4 The equity premium at long horizons

Following Cochrane and Hansen (1992), we conclude this section by focusing on the long-run covariance between consumption growth and stock returns divided by the horizon, i.e.,

$$\frac{1}{h}Cov\left[g_{t,t+h}, R^{ei}_{t,t+h}\right].$$

For our data, this normalized covariance is hump-shaped with a peak around 2 years (Figure 8). Using only the 5^{th} detail gives us a hump at 3.5 years (Figure 8(a)). Introducing the 4^{th} detail improves, as before, matters, especially at short horizons (Figure 8(b)). In particular, it relocates the hump around the correct time frame. Not only does the model capture the location of the peak, it also does not predict a somewhat counterfactual high covariance of consumption growth and stock returns at long horizons, an implication of the long-run risk model or models entailing monitoring costs and heterogeneous agents in which only a fraction of households adjusts consumption over discrete intervals.

5 Scale-specific betas and long-run aggregation

In this section, we show the sense in which our proposed (risk-detection and pricing) framework is linked to classical temporal aggregation as used in the long-run asset pricing literature. To do so, we use the fact that our decomposition in Eqs. (5) and (6) satisfies the following property

$$g_{t,t+2^s} = \sum_{j=s+1}^{6} g^{(j)}_{t+2^s} + \pi^{(6)}_{t+2^s} \tag{13}$$

and

$$R^{ei}_{t,t+2^s} = \sum_{j=s+1}^{6} R^{ei(j)}_{t+2^s} + \eta^{(6)}_{t+2^s}, \tag{14}$$

where $s = 0, 1, \ldots, 5$ denotes the aggregation level.[7] This property implies that aggregation of the time series uncovers information at different scales or, more precisely, for scales that are higher

[7]When $s = 0$ we obtain again Eqs. (5) and (6).

than the one corresponding to the aggregation level (BPTT, 2012, for a thorough treatment). The property also makes apparent the link between the scaled-wise time-series regression in Eq. (9) (reported here for the reader's convenience)

$$R^{ei(s)}_{t,t+2^s} = \beta_0 + \beta^{i(s)} g^{(s)}_{t,t+2^s} + \epsilon_{t+2^s},$$

and the long-horizon regression

$$R^{ei}_{t,t+h} = \beta_{0,h} + \beta^i_h g_{t,t+h} + w_{t+h}. \tag{15}$$

Letting $h = 2^{s-1}$, and using Eqs. (13) and (14), it turns out that long-horizon regressions filter out noisy high-frequency components at scales $j < s$, thereby capturing co-movements of the components $g^{(s)}_t$ and $R^{ei,(s)}_t$. In this sense, we expect the long-horizon betas β^i_h to behave similarly to the $\beta^{i(s)}$ at scale s. The main difference is that β^i_h is also influenced by the (co-movement of the) components $g^{(j)}_t$ and $R^{ei,(j)}_t$ at higher scales $j > s$. Hence, it should reflect lower frequency fluctuations as well.

In essence, the previous discussion suggests an alternative way to study the C-CAPM. Specifically, to measure risk exposure, one may run long-horizon regressions to filter out the noisy components in consumption and returns. In order to explore this avenue, one has to choose the aggregation horizon. Our previous results document that business cycle frequencies between 2 and 8 years, i.e., the frequencies captured by the components $j = 4$ and $j = 5$, are key to explaining the cross-section of average returns on portfolio sorted by size and book-to-market. Therefore, we choose $s = 4$, so that the long-horizon regression in Eq. (15) relies on 2-years (i.e. $h = 2^{s-1} = 8$ quarters) aggregated returns and consumption growth. Since we know that, at this horizon, temporal aggregation filters out the components at scale $j = 1, 2, 3$, one may wonder how close is 2-years consumption growth to the components at scale $j = 4$ and $j = 5$? Figure 9 displays the aggregated series of consumption growth, along with the sum of its components at business-cycle scales. We observe that the two series strongly co-move, a fact also confirmed by a high correlation of 0.86. More formally, setting $s = 3$ in Eq.(13), one obtains

$$g_{t,t+2^3} - (g^{(4)}_{t,t+2^3} + g^{(5)}_{t,t+2^3}) = g^{(6)}_{t,t+2^3} + \pi^{(6)}_{t+2^s}$$

i.e., the difference between the two series is driven by a component at longer than business cycle frequencies (scale $j = 6$). When this very persistent component is relatively small in volatility, *then* aggregation does a very good job in extracting risk.

Motivated by this analysis one would expect, similarly to our scale-based CCAPM, a CCAPM model based on *long-horizon* betas to explain *short-term* returns. This is, as shown, an implication of our component-based data generating process. To this extent, we test the model

$$E[R^{e,i}_{t,t+1}] = \lambda_0 + \lambda_h \beta^i_h + \alpha_i \tag{16}$$

and compare it to

$$E[R^{e,i}_{t,t+1}] = \lambda_0 + \lambda_{45}\left(\beta^{i(4)} + \beta^{i(5)}\right) + \alpha_i.$$

Figure 10 displays the results for the scale-based CCAPM (Panel A) and for the long-horizon CCAPM (Panel B). The Figure 10-Panel B reveals that the average returns are well explained by the $\beta^{(i)}_h$'s. In particular, the model achieves an R^2 of 26%, with a mean absolute pricing error of 1.78 percent per year, while the constant λ_0 is not significant (results available upon request). These findings are comparable to the ones in Table 3-Panel B where the consumption betas are defined on business-cycle frequency components only.[8]

In sum, we have shown how one can use - and, importantly, justify - long-horizon betas to study the short-run premium. The relevant role of these betas may be induced by a component model of the type we propose. Since long-run average returns have the same structure as short-run average returns, our proposed model provides a justification for long-horizons returns on long-run betas as well. In this sense, our scale-based consumption decomposition nicely ties low-frequency consumption dynamics to both the pricing of long-run returns (as in, e.g., Daniel and Marshall, 1997) and to that of short-run returns (as in, e.g., Bansal and Yaron, 2005).

[8] Overall the main difference between the two models is driven by the small growth portfolio. When this portfolio is included, the scale-based model achieves a better fit. However, once we exclude the extreme portfolio the two models fair the same both in terms of R^2, which increases to 46%, and of pricing errors, which are now reduced to 1.54 percent per year.

6 Extensions

We now turn to the consumption-based pricing of alternative portfolios. We first examine the average returns on 10 portfolios sorted solely based on either size or book-to-market, rather than on size and book-to-market jointly. This analysis will allow us to zoom in onto the relative contribution of alternative consumption components in explaining well-known stylized facts, like the size premium (Banz, 1981) and the value premium (Stattman, 1980, and Rosenberg, Reid and Lanstein, 1985). We then turn to momentum (Jegadeesh and Titman, 1993) and portfolios constructed based on (short-term) past winners and losers. As observed by Fama and French (1996), Carhart (1997) and many others since, the momentum effect is known to represent a considerable challenge for otherwise very successful pricing models, like the Fama-French three-factor model (Fama and French, 1993). With few exceptions, consumption models have been traditionally tested along size and book-to-market dimensions only. Here, we re-evaluate momentum in the context of our pure (scale-based) consumption model. We begin with size.

6.1 Size-sorted portfolios

In agreement with Figure 3 and previous findings, the second detail of the consumption growth process is very successful in explaining the cross-sectional pricing of the 10 size-sorted portfolios (Table 4). This detail corresponds to economic fluctuations between 6 months and 1 year. Since size is the only dimension being considered, fluctuations at frequencies lower than the business cycle are sufficient to capture much of the dispersion in risk premia. With value in the picture, lower frequency details would, however, be needed to also capture this additional dimension. We will return to this issue.

When estimating the model in Eq. (9) and Eq. (10) on the second detail and with an intercept, we find a very low root mean square alpha (RMSA) and a low mean absolute alpha (MAA). The intercept is clearly statistically insignificant and the corresponding R^2 value is a sizeable 88%. The addition of a zero restriction on the intercept does not modify the pricing errors drastically. It does increase slightly the numerical value of the price of risk on the second detail (and its statistical significance) bringing it from 0.642 to 0.919.

Exploring now the remaining details, we notice that along a variety of different directions, such as the magnitude of the coefficients of determination, the significance of the intercept, the sign of the lambdas, and those of the pricing errors, they fail to perform nearly as convincingly as the second detail. The 5^{th} detail and, to a lesser extent, the 4^{th} detail have, however, a valuable size tilt as well. In light of the ability, to which we now turn, of the 5^{th} detail to capture value effectively, it is not surprising that a combination of these low-frequency details leads, as shown earlier, to quantities of risk capable of pricing classical test assets defined with respect to *both* size and book-to-market.

6.2 Book-to-market sorted portfolios

The 5^{th} detail prices the 10 book-to-market portfolios with an R^2 of 55% (Table 5). The intercept is insignificant, the slope is significant and the corresponding MAA and RMSA are small. Restricting the intercept to zero increases the pricing errors somewhat but, in light of the insignificance of the estimated intercept, not drastically. A closer look at the remaining details shows that the 3^{rd} and the 4^{th} detail perform better than the 1^{st} and the 2^{nd}. In the latter cases, the signs are incorrect and the pricing errors are large. This finding, obtained through the explicit isolation of consumption cycles spanning between 2 and 8 years, yields an economically-appealing channel through which long-run consumption risk relates positively to the value premium (Bansal and Yaron, 2004, Hansen, Heaton, and Li, 2008, Malloy, Moskowitz, and Vissing-Jorgensen, 2009, and Parker and Julliard, 2005, are recent influential contributions in this area). The implications of our assumed (return) data generating process for the link between long-run consumption and value were explored explicitly in Subsection 4.1.

While size is a "high-frequency" phenomenon nicely captured by fluctuations in consumption with 6 months to 1 year cycles, value has a marked business-cycle nature. When pricing both size and value effects, the "rotation" of the scale-specific betas from size (at low scales) to value (at high scales) leads to a data generating process and a pricing model, like the one described above, in which the high scales play an important role. These (business-cycle) scales capture the value premium effectively while still containing a useful size tilt.

6.3 Momentum-sorted portfolios

We now focus on a different, rather challenging, dimension for asset pricing tests: momentum (Table 6). Stocks with high 12-month past returns (high positive momentum) are ubiquitously found to outperform stocks with low 12-month past returns (low positive momentum). This classical evidence (Jegadeesh and Titman, 1993) has been confirmed for other asset classes, like currencies and commodities, and has been described as pervasive across countries (Asness, Moskowitz and Pedersen, 2013, for discussions).

The inability of successful pricing models to capture short-term momentum effects has generally lead to the addition of suitable momentum factors. Differently from this approach, we ask the question: can a pure consumption model price momentum-sorted portfolios? Equivalently in our framework, are the higher mean returns of higher positive momentum portfolios the result of more risk as given by a larger covariance of their returns with certain details of the consumption process?

When implementing the model in Eq. (9) and Eq. (10), the second and the third consumption detail appear to perform well along a variety of dimensions (coefficient of determination, sign of the slopes with and without a zero restriction on the intercept, similarity between the magnitude of the pricing errors with and without a zero restriction on the intercept, insignificance of the intercepts when estimated). All other component-wise models perform unsatisfactorily according to several of these metrics. In all cases, for instance, not imposing a zero restriction on the intercept flips the sign of the slope (as compared to the zero intercept specification) and yields intercept estimates which are economically rather large and statistically very significant (with t-statistics in a neighborhood of 6).

The pricing ability of the second and the third component, spanning consumption cycles between 6 months and 2 years, is of interest. The one-year horizon is, in fact, in between these two components, in the sense that the second component ends with yearly cycles while the third one begins with cycles of the same length. Motivated by this observation, we investigate the performance of a model in which the second and the third component jointly explain average returns in the cross section of momentum portfolios, namely

$$E[R^{ei}_{t,t+1}] = \lambda_0 + \lambda_2 \beta^{i(2)} + \lambda_3 \beta^{i(3)} + \alpha_i.$$

Table 7 displays the results. When focusing our attention on the model with no constant λ_0, see Panel A, the mean absolute alphas is 1.79 percent per year, a noticeable improvement as compared to the pricing errors of 2.05 and 2.56 achieved by models in which only the second or the third component are considered (see Table 6-Panel B and C). The prices of risk λ_2 and λ_3 are close in magnitude, further reinforcing the hypotheses that the relevant consumption fluctuations for these test assets have a half-life of one-year, a frequency rightly at the intersection of those captured by the second and the third component. If this is the case, a better measure of risk may be represented by $\beta^{i(2)}+\beta^{i(3)}$. Therefore, in Table 7-Panel B we investigate the performance of the following model

$$E[R^{ei}_{t,t+1}] = \lambda_0 + \lambda_{23}\left(\beta^{i(2)} + \beta^{i(3)}\right) + \alpha_i.$$

The results show that the intercept is insignificant, and that the pricing errors and the fit of the model do not worsen. For instance, the mean absolute alphas is now 1.80, only one basis point per year higher than the one obtained by allowing the coefficients on the details' covariances $\beta^{i(2)}$ and $\beta^{i(3)}$ to be different. Moreover, restricting the constant λ_0 to be zero, we obtain a price of risk of 0.649, a value nearly identical to those achieved in the model with two separately priced components.[9] Figure 11 summarizes graphically the results in Table 6 and 7. In particular, we compare the fit of models with risk measured only by β_2 (see Panel A), by β_3 (see Panel B), or by $\beta^{i(2)} + \beta^{i(3)}$ (see Panel C). In all three cases cross-sectional average returns align well with cross-sectional covariances at specific scales.[10] However, the model with only the third component requires a significant constant to explain 69% of average returns, whereas the model with only the second component does not require a constant, at the cost of a lower explanatory power equal to 42% (see again Table 6). The best fit is achieved when we proxy for risk using $\beta^{i(2)} + \beta^{i(3)}$, the constant being insignificant and the R^2 being 60%.

In sum, a single-beta model obtained with a combination of the second and the third consumption component, i.e., those components with persistence between 6 months and 2 years, may justify the momentum effect. Being that we are analyzing momentum portfolios constructed using 1-year past returns, the close mapping between relevant (for pricing) consumption horizons and momentum horizons appears intriguing.

7 Conclusions

The economic purity of the CCAPM has lead to a variety of approaches intended to reconcile the appeal of consumption-based explanations of the pricing of risky assets with well-known empirical

[9]More formally, one can test for the coefficients on β_2 and β_3 to be the same. A test of over-identifying restrictions yields a statistics of 0.0087 with an associated p-val of 0.925; hence we cannot reject the restricted model at any conventional levels.

[10]The only exception is the extreme loser portfolio.

regularities. The use of economically-motivated scaling factors in the definition of a stochastic discount factor defined with respect to consumption (Lettau and Ludvingson, 2001) or the emphasis on alternative utility specifications (and consumption dynamics) capable of suitably modifying a stochastic discount factor, again, defined over consumption (Campbell and Cochrane, 1999, Bansal and Yaron, 2002, and Hansen, Heaton, and Li, 2008, *inter alia*) are successful examples of this reconciliation in the literature.

In this paper we step back a little and take an alternative view of the same issue. We suspect that certain features (components) of the consumption process may matter for the purpose of asset pricing, whereas the relative impact of other components may be drastically lower. Said differently, if we separate the covariance between consumption growth and asset returns into sub-covariances (one for each component of the consumption process and each component of the return process), it may be the case that sub-components (i.e., sub-covariances) of the typical object of interest (the overall covariance between consumption growth and asset returns) explain the observed cross-sectional dispersion in average returns, whereas the overall covariance does not (the latter being a typical finding). From a purely mathematical standpoint, this observation solely amounts to leaving the prices of risk unrestricted across components of the overall covariance and evaluating which prices of risk could be restricted to zero in deriving a data generating process for returns or, equivalently, a consumption-based pricing model. From an economic standpoint, the observation derives from the realization that consumption cycles of different length may affect the pricing of risky assets differently. In particular, high-frequency consumption cycles may solely represent short-term noise attenuating the explanatory power of the classical consumption betas.

Consistent with this logic, after careful separation of the consumption components and using portfolios sorted based on traditional dimensions like size and book-to-market, we find that consumption risk may be defined in terms of the covariance of asset returns with consumption components with periodicity between 2 and 8 years. In other words, the cross-sectional dispersion of the risk premia of common test assets depends crucially on business-cycle fluctuations in consumption.

We show that, by zooming in onto the relevant (for pricing) layers of the consumption process, explicit separation of heterogeneous (in terms of their persistence and periodicity) consumption components leads to satisfactory quantities of risk, prices of risk, and pricing errors. While these are ubiquitous, for good economic reasons, metrics, they are not the only ones. To address this issue, we focus on suitable, alternative criteria, namely consumption growth autocorrelation, the hump-shaped pricing ability of the covariance between ultimate consumption (as defined in Parker and Julliard, 2005) and returns, the hump-shaped structure of long-run risk premia and the decaying pattern in consumption growth predictability. According to all of these metrics, a heterogeneous-component model for consumption growth fares very satisfactorily in addressing stylized facts about the joint dynamics of consumption and asset returns over time. It does so while remaining within the appealing confines of a CCAPM framework, one in which investors solely weigh different layers of the consumption process differently.

A Decomposing Time Series along the Persistence Dimension

This section shows how to decompose a time series into components with different levels of persistence. For a deeper treatment and an application to structural asset pricing model, please refer to Ortu, Tamoni and Tebaldi (2013).

Given a time series $\{g_t\}_{t\in\mathbb{Z}}$ we begin by constructing moving averages $\pi_t^{(j)}$ of size 2^j:

$$\pi_t^{(j)} = \frac{1}{2^j} \sum_{p=0}^{2^j-1} g_{t-p} \tag{A.1}$$

where $\pi_t^{(0)} \equiv g_t$. Given the choice of sample size, it is readily observed that these moving averages satisfy the iterative relation:

$$\pi_t^{(j)} = \frac{\pi_t^{(j-1)} + \pi_{t-2^{j-1}}^{(j-1)}}{2} \tag{A.2}$$

In words each element $\pi_t^{(J)}$ is the $h-$period moving average with $h = 2^J$ and time is consistently scaled by a factor 2^J. Next, we denote by $g_t^{(j)}$ the difference between moving averages of sizes 2^{j-1} and 2^j, i.e. :

$$g_t^{(j)} = \pi_t^{(j-1)} - \pi_t^{(j)} \tag{A.3}$$

Intuitively, $g_t^{(j)}$ captures fluctuations that survive to averaging over 2^{j-1} terms but disappear when the average involves 2^j terms, i.e. fluctuations with half-life in the interval $\left[2^{j-1}, 2^j\right)$. Accordingly, the moving average $\pi_t^{(j)}$ includes fluctuations whose half-life exceeds 2^j periods. From now on, we refer to the derived time series $\left\{g_t^{(j)}\right\}_{t\in\mathbb{Z}}$ as to the *component* of the original time series $\{g_t\}_{t\in\mathbb{Z}}$ with level of persistence j. Since $\pi_t^{(0)} \equiv g_t$, by summing up over j it follows immediately from (A.3) that:

$$g_t = \sum_{j=1}^{J} g_t^{(j)} + \pi_t^{(J)} \tag{A.4}$$

for any $J \geq 1$. In words, equation (A.4) decomposes the time series g_t into a sum of components with half-life belonging to a specific interval, plus a residual term that represents a long-run average.

Due to the overlap of the moving averages that define $g_t^{(j)}$, the decomposition (A.4) can lead to a biased evaluation of the persistence of the time series g_t. To address this issue we select the information in the components $g_t^{(j)}$ and $\pi_t^{(j)}$ in a suitable manner. In particular, since by definition each component $g_t^{(j)}$ is a linear combination of the realizations g_t, g_{t-1}, ..., g_{t-2^j+1}, to remove any spurious serial correlation introduced by the overlapping of the moving averages we restrict our attention to the sub-series:

$$\left\{g_t^{(j)}, t = k2^j, k \in \mathbb{Z}\right\} \tag{A.5}$$

$$\left\{\pi_t^{(j)}, t = k2^j, k \in \mathbb{Z}\right\} \tag{A.6}$$ [11]

We refer to these sub-series as to the *decimated components* at level of persistence j of the original time series. Clearly, persistence in a decimated component is not an artifact; rather, it represents an actual fluctuation of the original series with an half-life in the interval $\left[2^{j-1}, 2^j\right)$.

The process of decimation controls for spurious persistence by deleting from the components $g_t^{(j)}$ and $\pi_t^{(j)}$ *all and only* the information irrelevant to reconstruct the original time series g_t. Formally, this follows from observing that for any $J \geq 1$ one can define a linear, invertible operator $\mathcal{T}^{(J)}$ that maps the decimated components $\left\{g_t^{(j)}, t = k2^j, k \in \mathbb{Z}\right\}$, $j = 1, ..., J$ and $\left\{\pi_t^{(j)}, t = k2^j, k \in \mathbb{Z}\right\}$ into the time series $\{g_t\}_{t\in\mathbb{Z}}$. To illustrate how this works for $J = 2$ we first observe that in this case (A.1) yields:

$$\pi_t^{(2)} = \frac{g_t + g_{t-1} + g_{t-2} + g_{t-3}}{4} \tag{A.7}$$

Next we substitute (A.2) into (A.3) and let $j = 1, 2$ to obtain:

$$\begin{aligned}
g_t^{(2)} &= \frac{\pi_t^{(1)} - \pi_{t-1}^{(1)}}{2} = \frac{1}{2}\left(\frac{g_t + g_{t-1}}{2} - \frac{g_{t-2} + g_{t-3}}{2}\right) \\
g_t^{(1)} &= \frac{\pi_t^{(0)} - \pi_{t-1}^{(0)}}{2} = \left(\frac{g_t - g_{t-1}}{2}\right) \\
g_{t-2}^{(1)} &= \frac{\pi_{t-2}^{(0)} - \pi_{t-3}^{(0)}}{2} = \left(\frac{g_{t-2} - g_{t-3}}{2}\right)
\end{aligned} \tag{A.8}$$

We then consider the system obtained by stacking (A.7) on top of (A.8), which in matrix notation becomes:

$$\begin{pmatrix} \pi_t^{(2)} \\ g_t^{(2)} \\ g_t^{(1)} \\ g_{t-2}^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} g_t \\ g_{t-1} \\ g_{t-2} \\ g_{t-3} \end{pmatrix} \tag{A.9}$$

Denoting by $\mathcal{T}^{(2)}$ the (4×4) matrix in (A.9), we notice that $\mathcal{T}^{(2)}$ is orthogonal, that is $\Lambda^{(2)} \equiv \mathcal{T}^{(2)} \left(\mathcal{T}^{(2)}\right)^\top$ is diagonal. Moreover, the diagonal elements of $\Lambda^{(2)}$ are non-vanishing so that $\left(\mathcal{T}^{(2)}\right)^{-1} = \left(\mathcal{T}^{(2)}\right)^\top \left(\Lambda^{(2)}\right)^{-1}$

[11] These sub-series are defined up to a translation factor, which results in 2^j degrees of freedom. More precisely for any $h = 0, 1, \ldots, 2^j - 1$ we could sample the realizations $\left\{g_{h+k2^j}^{(j)}, k \in \mathbb{Z}\right\}$, $\left\{\pi_{h+k2^J}^{(J)}, k \in \mathbb{Z}\right\}$. The approach described in this section satisfies a translation invariant property, i.e. it is independent of the parameter h. This is why in constructing (A.5) and (A.6) we let $h = 0$ without loss of generality.

is well-defined, and hence:

$$\begin{pmatrix} g_t \\ g_{t-1} \\ g_{t-2} \\ g_{t-3} \end{pmatrix} = \left(\mathcal{T}^{(2)}\right)^{-1} \begin{pmatrix} \pi_t^{(2)} \\ g_t^{(2)} \\ g_t^{(1)} \\ g_{t-2}^{(1)} \end{pmatrix} \tag{A.10}$$

By letting t vary in the set $\{t = k2^2, k \in \mathbb{Z}\}$, equation (A.10) shows how to reconstruct uniquely the entire time series $\{g_t\}_{t\in\mathbb{Z}}$ from the decimated components $\left\{g_t^{(j)}, t = k2^j, k \in \mathbb{Z}\right\}$, $j = 1, 2$ and $\left\{\pi_t^{(2)}, t = k2^2, k \in \mathbb{Z}\right\}$.[12]

B Data

Our empirical exercise is conducted on data sampled on a quarterly frequency. The data cover the first quarter of 1963 through fourth quarter of 2012. Following earlier work (e.g. Hansen and Singleton, 1983 and Bansal and Yaron, 2004), we use data on U.S. real nondurables and services consumption per capita from the Bureau of Economic Analysis. We make the standard end-of-period timing assumption that consumption during period t takes place at the end of the period. Growth rates are constructed by taking the first difference of the corresponding log series.

The portfolios employed in our empirical tests sort firms on dimensions that lead to cross-sectional dispersion in measured risk premia. We first consider a 5×5 two-way sort on market capitalization and book-to-market resulting in 25 portfolios (see Fama and French (1993)). We also consider one-way sorted portfolio. The particular characteristics that we consider are firms' market value, book-to-market ratio, and past returns (momentum). Data on returns from these portfolio sorts are obtained from Ken French's web site at Dartmouth college.[13] Portfolios comprise stocks listed on NYSE, AMEX and NASDAQ. Returns on value weighted portfolios are used, but results are very similar when using equal weighted portfolios. The market is the value-weight return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) and the excess returns are with respect to the one-month Treasury bill rate (from Ibbotson Associates). The returns on equity and the risk-free rate are aggregated to a quarterly level by multiplying returns within a quarter.

[12]For an extension of this procedure to any $J \geq 2$ and a recursive algorithm for the construction of the matrix $\mathcal{T}^{(J)}$, associated to an arbitrary level of persistence J, see e.g. Mallat, 1989.

[13]Details on how these portfolios are formed are available from http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

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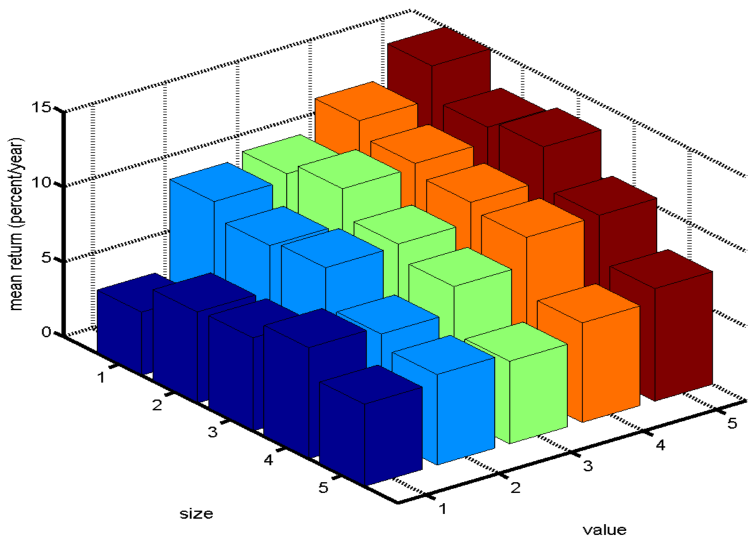
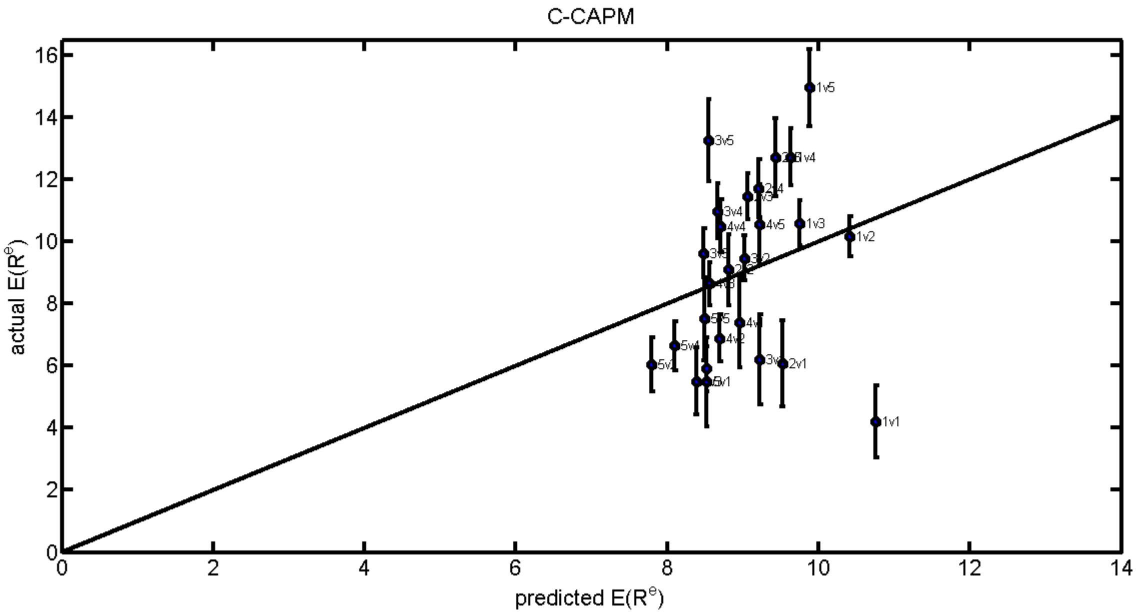
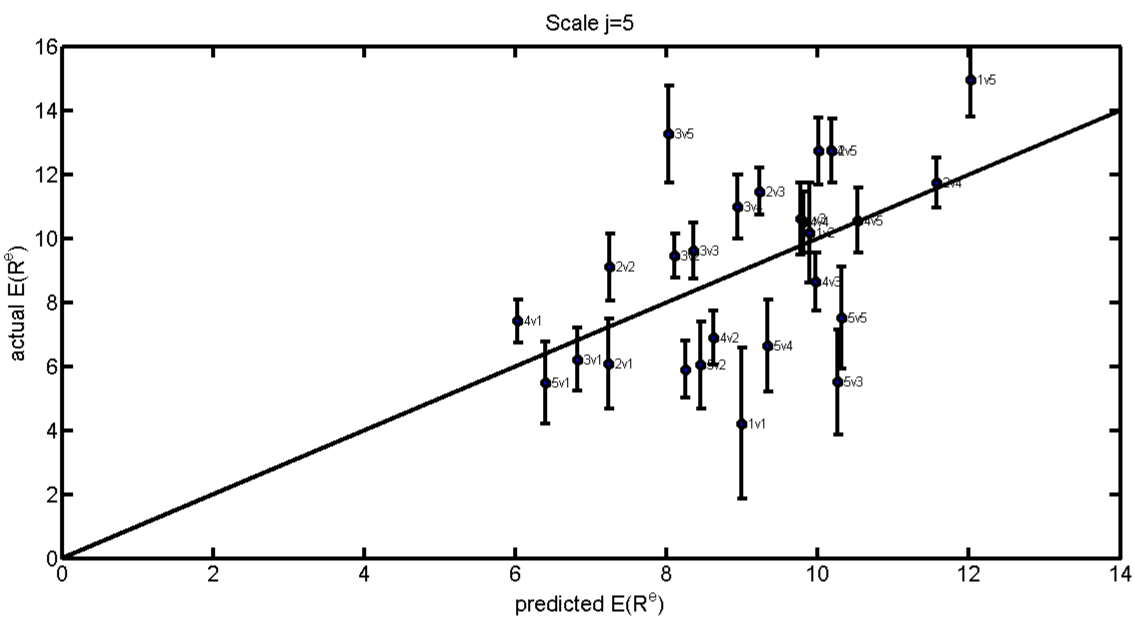


Figure 1: Average realized returns of the 25 Fama-French portfolios sorted on Size and Book-to-Market.



(a) Standard C-CAPM



(b) Scale $j = 5$

Figure 2: **Cross-Sectional Fit**. **Panel A**: The figure plots fitted versus average actual excess returns (% per year) of standard consumption-capm model for the 25 size and book-to-market portfolios. **Panel B**: The figure plots fitted and average returns when the priced factors is the consumption component at scale $j = 5$.

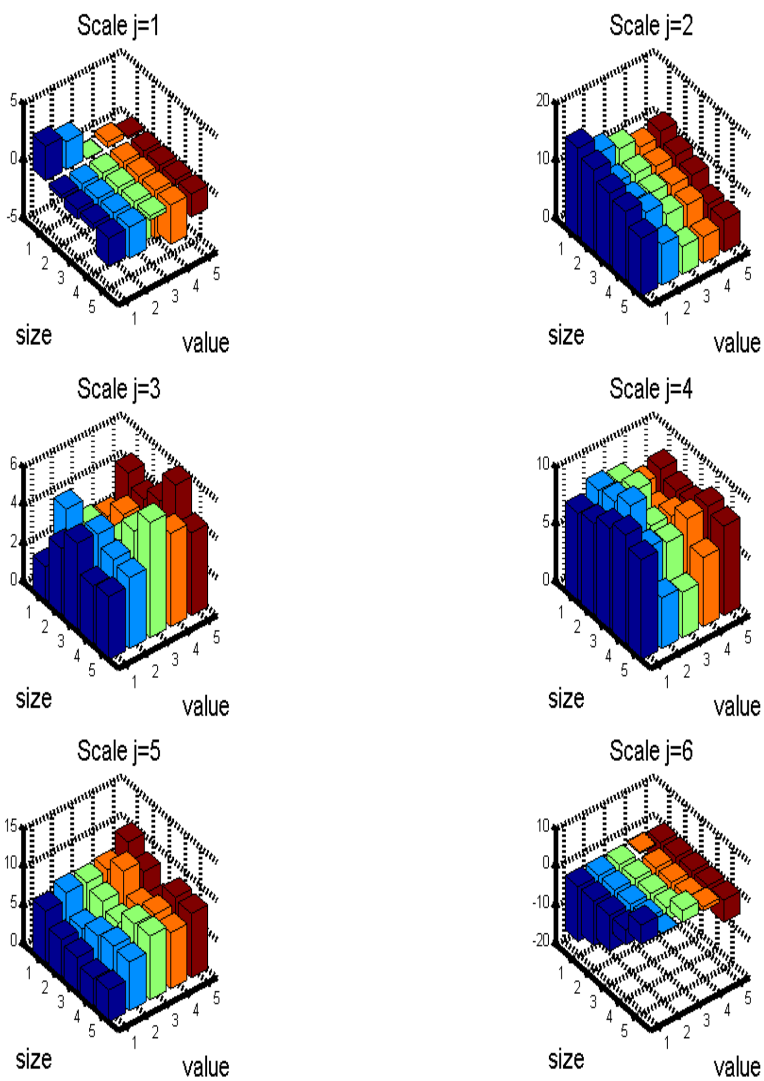


Figure 3: Betas $\beta_i^{(j)}$ for portfolios $i = 1, \ldots, N$. Each Panel refers to a scale $j = 1, \ldots, J$.

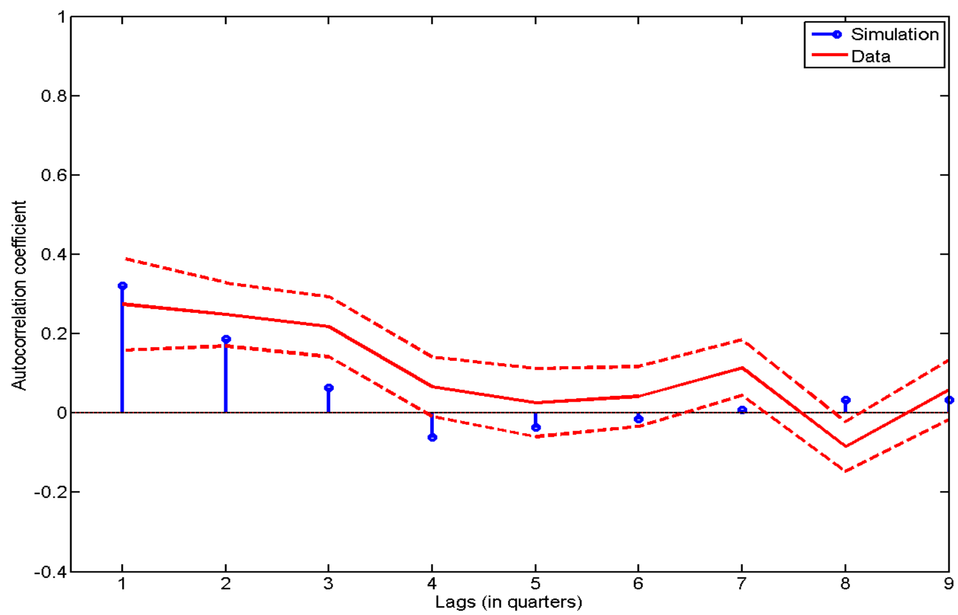
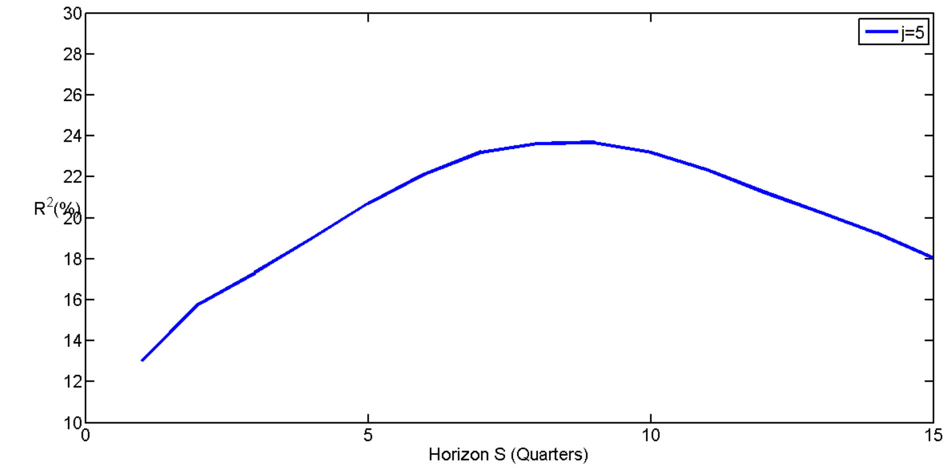
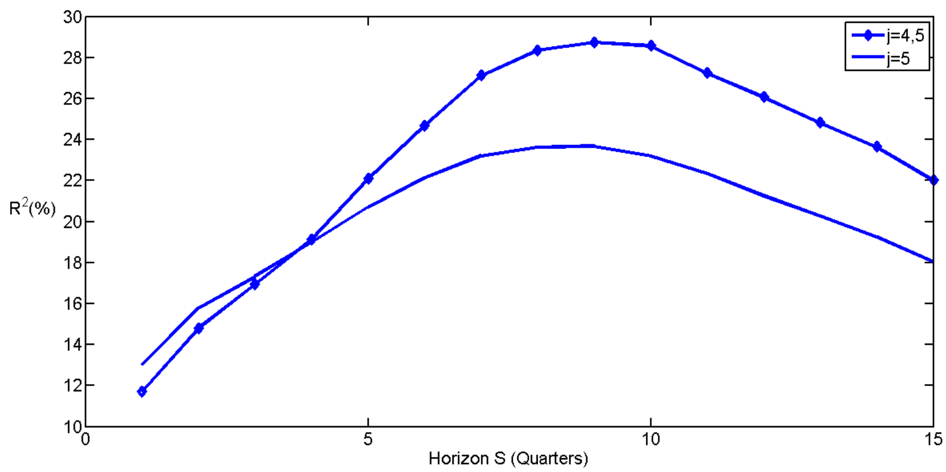


Figure 4: Autocorrelation of consumption growth in the model.

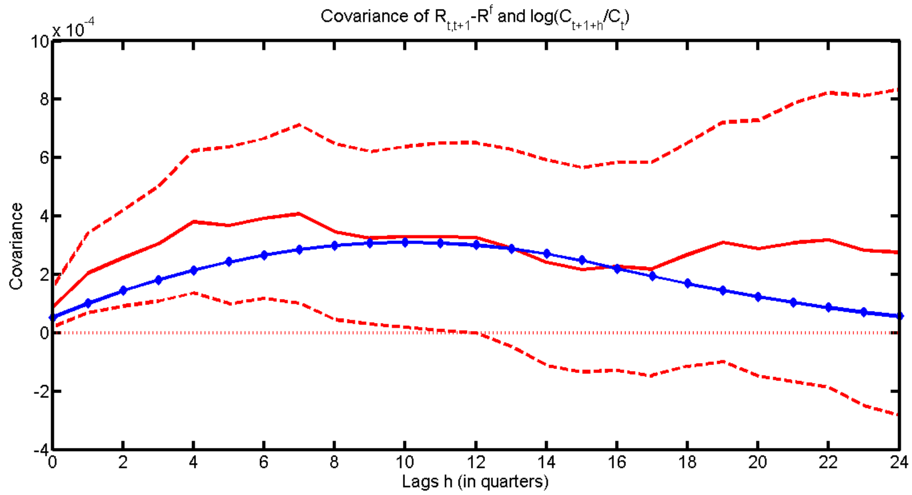


(a) Scale $j = 5$

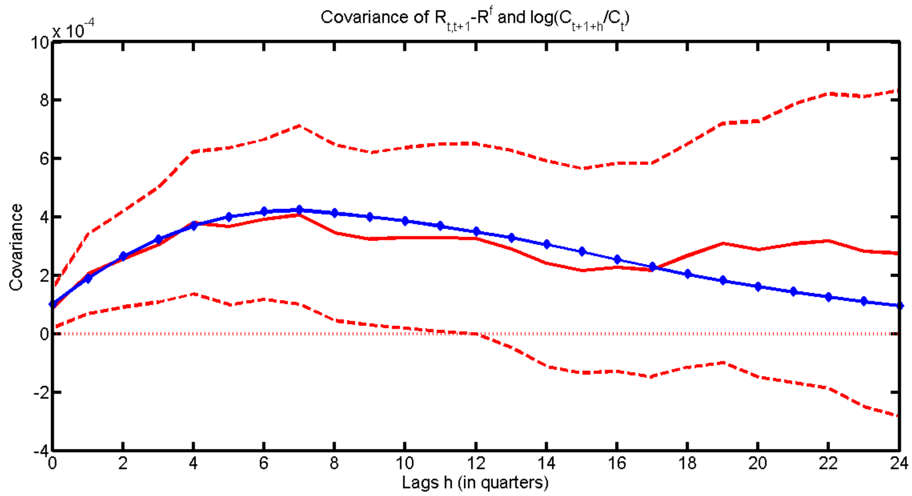


(b) Scale $j = 4, 5$

Figure 5: Average (across portfolios) of R^2 obtained from cross-sectional regressions a-la Parker and Julliard.

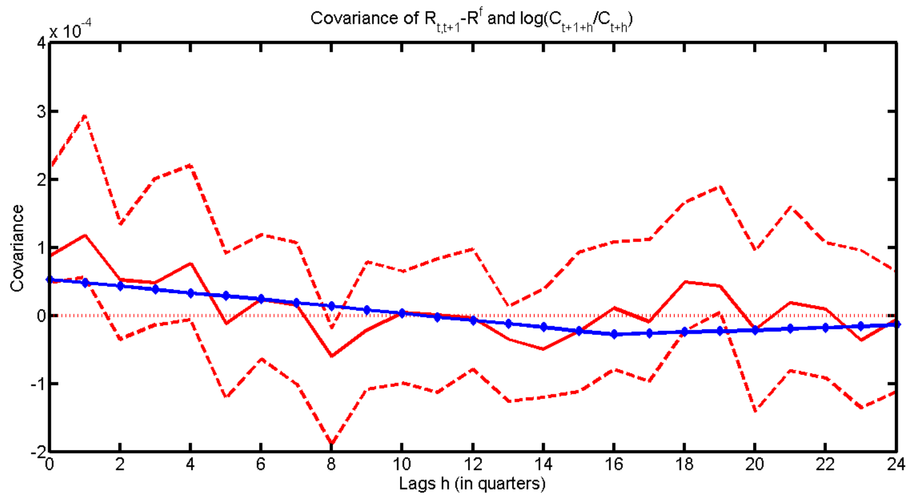


(a) Scale $j = 5$

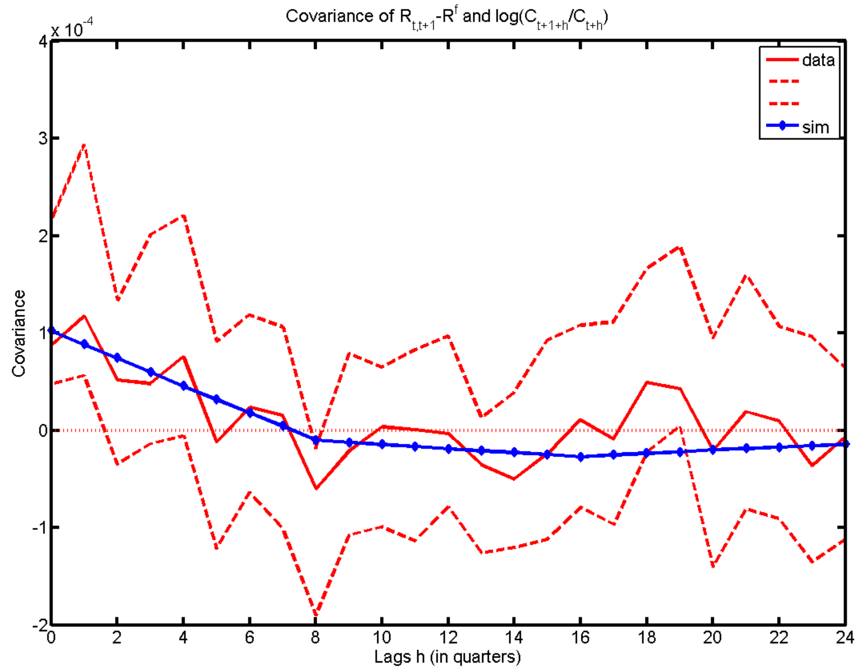


(b) Scale $j = 4, 5$

Figure 6: Covariance of $\log(\frac{C_{t+h+1}}{C_t})$ and $R^{ei}_{t,t+1}$ divided by h (model with solid diamonds, $j = 5$ and $j = 4, 5$).

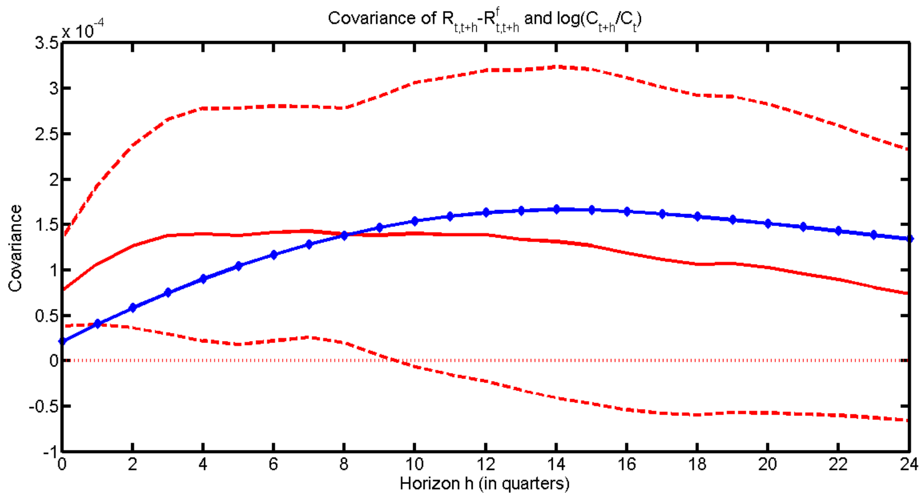


(a) Scale $j = 5$

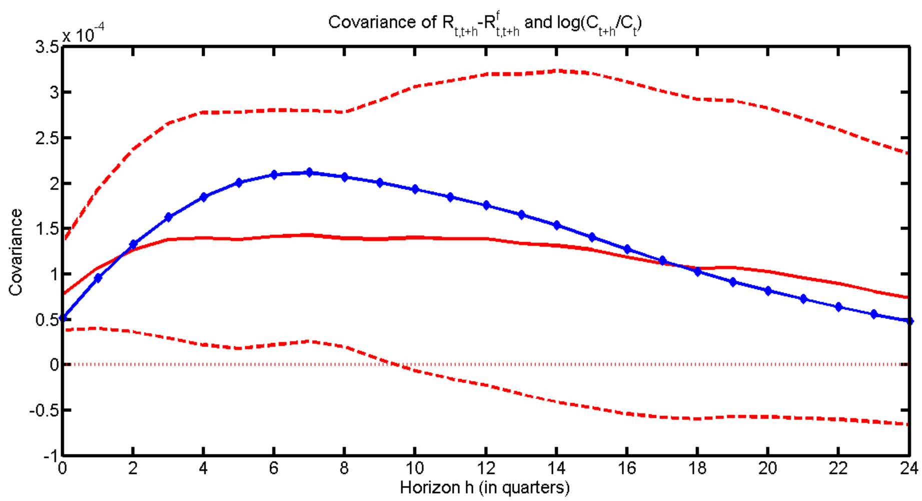


(b) Scale $j = 4, 5$

Figure 7: Covariance of $\log(\frac{C_{t+h+1}}{C_{t+h}})$ and $R^{ei}_{t,t+1}$ divided by h (model with solid diamonds, $j = 5$ and $j = 4, 5$).



(a) Scale $j = 5$



(b) Scale $j = 4, 5$

Figure 8: Covariance of $\log(\frac{C_{t+h}}{C_{t+1}})$ and $R^{ei}_{t,t+h}$ divided by h. Model with solid diamonds.

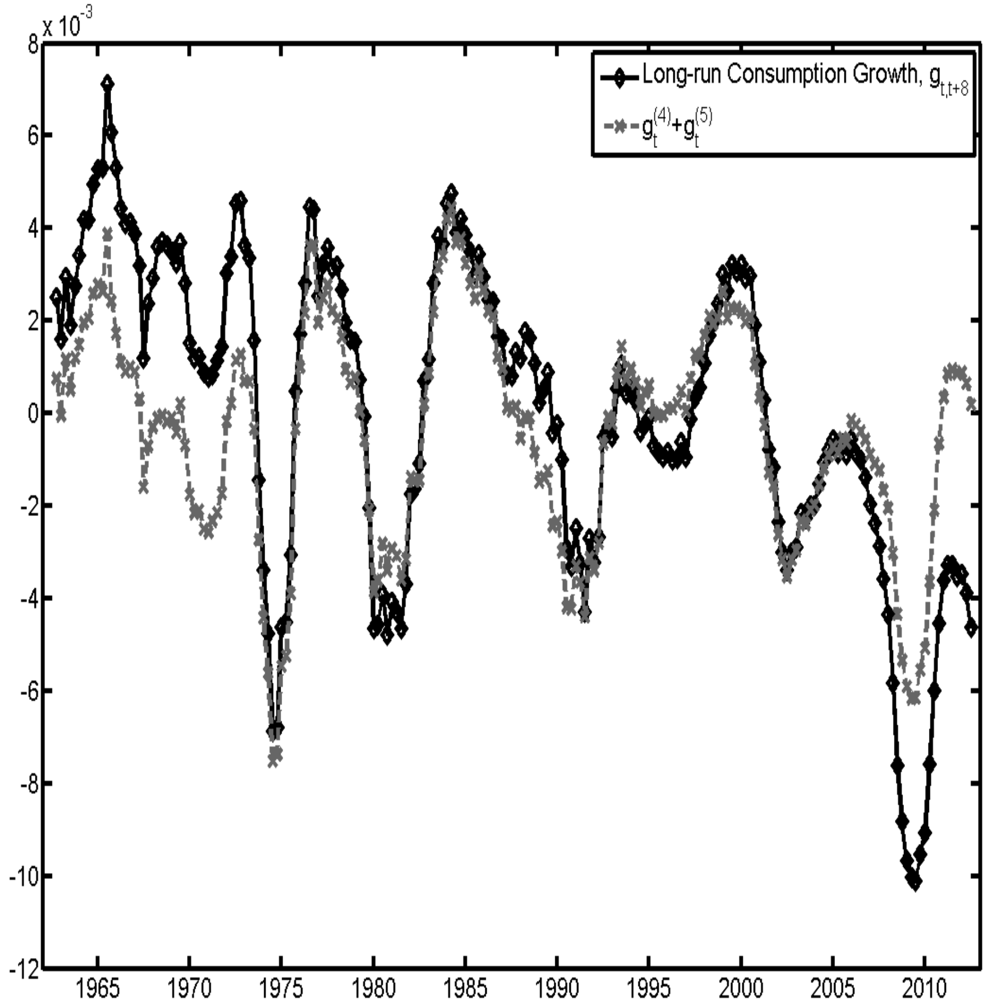


Figure 9: Comparison between consumption growth aggregated over 2 years, $g_{t,t+8}$, and the componens $g_t^{(4)}$ and $g_t^{(5)}$ capturing cycles between 2 and 4 years, and between 4 and 8 years, respectively.

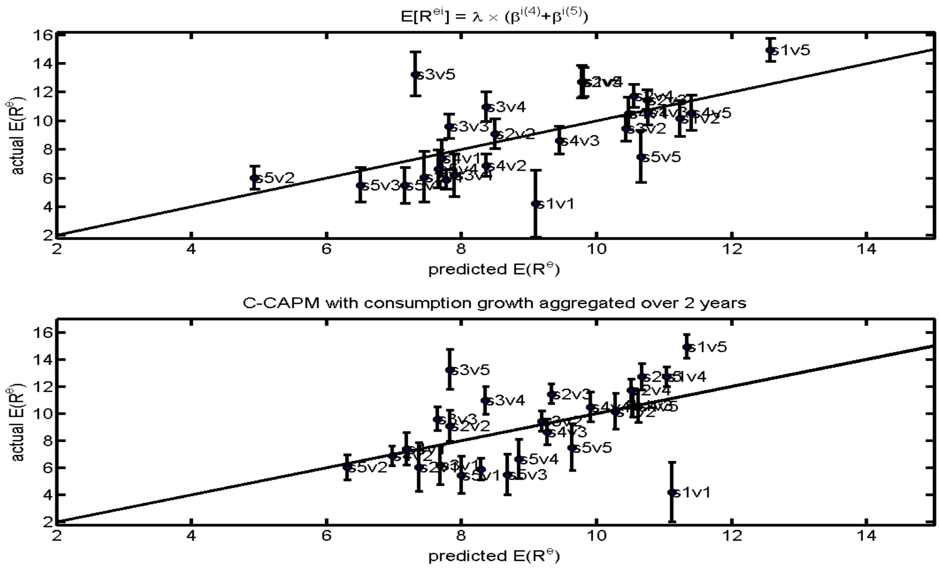


Figure 10: **Cross-Sectional Fit**. The figure plots fitted versus average actual excess returns (% per year) of the scale-based CCAPM ($j = 4, 5$) and the long-run consumption-capm model (Panel B) for the 25 Fama-French portfolios.

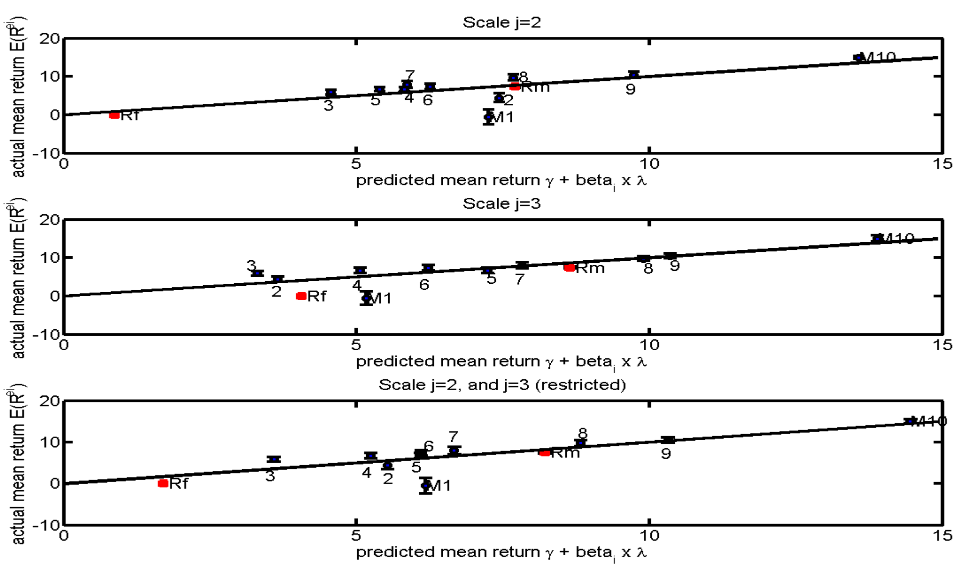


Figure 11: **Cross-Sectional Fit**. The figure plots fitted versus average actual excess returns (% per year) of consumption-capm model across scales for the 10 momentum sorted portfolios.

Panel A: β_1 - second-pass regression

Constant	λ_1	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	-2.663	7.26	5.61	68.406	25	0.000	
(-)	(0.737)						
8.778	-0.161	2.79	2.43	62.358	24	0.000	0.01
(3.312)	(0.388)						

Panel B: β_2 - second-pass regression

Constant	λ_2	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	0.880	3.86	3.18	71.430	25	0.000	
(-)	(0.304)						
9.584	-0.061	2.80	2.44	63.929	24	0.000	0.01
(2.578)	(0.303)						

Panel C: β_3 - second-pass regression

Constant	λ_3	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	2.210	3.62	3.10	71.118	25	0.000	
(-)	(0.710)						
9.292	-0.073	2.81	2.42	63.959	24	0.000	0.00
(4.276)	(0.486)						

Panel D: β_4 - second-pass regression

Constant	λ_4	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	1.168	3.02	2.48	60.075	25	0.000	
(-)	(0.387)						
6.692	0.308	2.78	2.37	56.915	24	0.000	0.02
(2.443)	(2.443)						

Panel E: β_5 - second-pass regression

Constant	λ_5	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	1.292	2.45	1.98	70.780	25	0.000	
(-)	(0.409)						
2.512	0.945	2.39	1.96	63.909	24	0.000	0.28
(3.741)	(0.342)						

Panel D: β_6 - second-pass regression

Constant	λ_6	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	-1.047	6.12	5.30	67.460	25	0.000	
(-)	(0.368)						
8.302	-0.138	2.74	2.30	61.359	24	0.000	0.05
(2.499)	(0.134)						

Table 1: **One factor model - 25 Portfolios Formed on Size and Book-to-Market.** Second-pass regressions with a constant. The Table reports: the estimates of the prices of risk on the consumption component and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the R^2 of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year.

Panel A: β_1 and β_5 - second-pass regression

Constant	λ_1	λ_5	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	-0.307	1.232	2.39	2.00	67.246	24	0.000	
(-)	(0.310)	(0.435)						
1.991	-0.241	0.970	2.35	1.97	62.235	23	0.000	0.30
(4.318)	(0.398)	(0.355)						

Panel B: β_2 and β_5 - second-pass regression

Constant	λ_2	λ_5	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	0.166	1.080	2.36	1.92	70.497	24	0.000	
(-)	(0.267)	(0.258)						
0.651	0.139	1.026	2.36	1.92	63.881	23	0.000	0.29
(3.513)	(0.320)	(0.360)						

Panel C: β_3 and β_5 - second-pass regression

Constant	λ_3	λ_5	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	-0.134	1.365	2.45	1.96	70.579	24	0.000	
(-)	(0.401)	(0.421)						
3.955	-0.556	1.044	2.33	1.83	63.883	23	0.000	0.31
(3.063)	(0.338)	(0.326)						

Panel D: β_4 and β_5 - second-pass regression

Constant	λ_4	λ_5	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	0.403	0.879	2.23	1.80	57.346	24	0.000	
(-)	(0.347)	(0.240)						
-5.812	0.848	1.226	2.14	1.64	55.777	23	0.000	0.42
(3.686)	(0.390)	(0.370)						

Panel E: β_5 and β_6 - second-pass regression

Constant	λ_5	λ_6	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	1.166	-0.180	2.30	1.83	67.131	24	0.000	
(-)	(0.325)	(0.157)						
1.492	0.974	-0.159	2.28	1.81	61.354	23	0.000	0.34
(3.362)	(0.345)	(0.135)						

Table 2: **Fifth Component is fixed - 25 Portfolios Formed on Size and Book-to-Market.** Second-pass regressions with a constant. The Table reports: the estimates of the prices of risk on the consumption component and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the R^2 of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year.

Panel A: Unrestricted C-CAPM, $E[R^{ei}_{t,t+1}] = \lambda_4 \beta^{i(4)} + \lambda_5 \beta^{i(5)}$ **- second-pass regression**

Constant	λ_4	λ_5	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	0.403	0.879	2.23	1.80	57.346	24	0.000	
(-)	(0.347)	(0.240)						
-5.812	0.848	1.226	2.14	1.64	55.777	23	0.000	0.42
(3.686)	(0.390)	(0.370)						

Panel B: Restricted C-CAPM, $E[R^{ei}_{t,t+1}] = \lambda_{restr}(\beta^{i(4)} + \beta^{i(5)})$ **- second-pass regression**

Constant	λ_{restr}	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	0.631	2.31	1.87	59.677	25	0.000	
(-)	(0.204)						
-3.605	0.881	2.19	1.71	58.017	24	0.000	0.39
(3.137)	(0.317)						

Table 3: **H0: $\boldsymbol{\lambda}_4 = \boldsymbol{\lambda}_5$ 25 Portfolios Formed on Size and Book-to-Market** Second-pass regressions without and with a constant. The Table reports: the estimates of the prices of risk on the consumption component j and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the R^2 of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year. In Panel B, we restrict the two price of risk to be the same.

Panel A: Component $j = 1$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	-3.131	7.05	5.04	25.594	10	0.004	
(-)	(0.816)						
9.932	0.476	0.98	0.76	21.733	9	0.010	0.31
(2.788)	(0.393)						

Panel B: Component $j = 2$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	0.919	0.63	0.48	20.491	10	0.025	
(-)	(0.246)						
2.853	0.642	0.41	0.34	19.649	9	0.020	0.88
(2.923)	(0.362)						

Panel C: Component $j = 3$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	2.220	1.66	1.40	32.073	10	0.000	
(-)	(0.590)						
11.975	-0.657	1.15	0.89	25.453	9	0.003	0.05
(4.178)	(0.552)						

Panel D: Component $j = 4$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	1.231	1.20	0.95	30.241	10	0.001	
(-)	(0.328)						
5.088	0.557	1.13	0.87	25.712	9	0.002	0.09
(2.447)	(0.316)						

Panel E: Component $j = 5$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	1.405	1.08	0.97	24.307	10	0.007	
(-)	(0.376)						
3.799	0.833	0.99	0.81	22.911	9	0.006	0.29
(3.673)	(0.649)						

Table 4: **Portfolios Sorted by SIZE** Second-pass regressions without and with a constant. The Table reports: the estimates of the prices of risk on the consumption component j and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the R^2 of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year.

Panel A: Component $j = 1$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\lVert\alpha\rVert$	χ^2-stat	DoF	p-value	R^2
0	-3.926	3.36	2.73	29.146	10	0.001	
(-)	(0.941)						
8.648	-0.057	1.61	1.30	15.391	9	0.081	0.00
(2.163)	(0.279)						

Panel B: Component $j = 2$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\lVert\alpha\rVert$	χ^2-stat	DoF	p-value	R^2
0	1.081	2.87	2.51	27.450	10	0.002	
(-)	(0.264)						
10.772	-0.264	1.54	1.25	17.606	9	0.04	0.08
(2.455)	(0.213)						

Panel C: Component $j = 3$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\lVert\alpha\rVert$	χ^2-stat	DoF	p-value	R^2
0	1.946	1.77	1.28	27.454	10	0.002	
(-)	(0.460)						
5.018	0.856	1.36	1.13	17.677	9	0.039	0.28
(2.390)	(0.364)						

Panel D: Component $j = 4$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\lVert\alpha\rVert$	χ^2-stat	DoF	p-value	R^2
0	1.282	2.11	1.81	26.099	10	0.000	
(-)	(0.308)						
7.233	0.231	1.58	1.27	17.772	9	0.038	0.04
(2.246)	(0.255)						

Panel E: Component $j = 5$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\lVert\alpha\rVert$	χ^2-stat	DoF	p-value	R^2
0	1.233	1.46	1.31	21.608	10	0.017	
(-)	(0.288)						
4.021	0.686	1.08	0.95	17.077	9	0.048	0.55
(2.730)	(0.286)						

Table 5: **Portfolios Sorted by Book-to-Market** Second-pass regressions without and with a constant. The Table reports: the estimates of the prices of risk on the consumption component j and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the R^2 of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year.

Panel A: Component $j = 1$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	-2.398	5.79	4.22	62.204	10	0.004	
(-)	(0.831)						
12.537	2.293	2.91	2.51	47.953	9	0.000	0.37
(2.411)	(0.498)						

Panel B: Component $j = 2$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	0.952	2.82	2.05	43.234	10	0.000	
(-)	(0.258)						
0.863	0.852	2.80	2.00	37.100	9	0.000	0.42
(2.521)	(0.204)						

Panel C: Component $j = 3$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	1.727	3.43	2.56	56.987	10	0.000	
(-)	(0.374)						
4.062	1.037	2.06	1.37	48.456	9	0.000	0.68
(2.440)	(0.246)						

Panel D: Component $j = 4$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	0.761	5.25	4.34	55.097	10	0.000	
(-)	(0.263)						
14.736	-0.923	2.86	1.89	46.673	9	0.000	0.39
(2.413)	(0.287)						

Panel E: Component $j = 5$ - second-pass regression

Constant	λ_j	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	1.015	5.18	3.97	62.164	10	0.000	
(-)	(0.349)						
19.674	-2.004	2.01	1.64	41.874	9	0.000	0.70
(3.101)	(0.510)						

Table 6: **Portfolios Sorted by Momentum** Second-pass regressions without and with a constant. The Table reports: the estimates of the prices of risk on the consumption component j and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the R^2 of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year.

Panel A: Unrestricted C-CAPM, $E[R^{ei}_{t,t+1}] = \lambda_2\beta^{i(2)} + \lambda_3\beta^{i(3)}$ - second-pass regression

Constant	λ_2	λ_3	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	0.689	0.628	2.44	1.79	43.229	9	0.000	
(-)	(0.454)	(0.423)						
4.607	-0.105	1.118	2.06	1.45	36.682	8	0.000	0.69
(1.897)	(0.288)	(0.378)						

Panel B: Restricted C-CAPM, $E[R^{ei}_{t,t+1}] = \lambda_{restr}(\beta^{i(2)} + \beta^{i(3)})$ - second-pass regression

Constant	λ_{restr}	$\sqrt{\alpha^2}$	$\|\alpha\|$	χ^2-stat	DoF	p-value	R^2
0	0.649	2.44	1.80	47.275	10	0.000	
(-)	(0.163)						
1.702	0.523	2.32	1.55	41.939	9	0.000	0.60
(2.592)	(0.121)						

Table 7: **H0: $\boldsymbol{\lambda}_2 = \boldsymbol{\lambda}_3$ - Portfolios Sorted by Momentum** Second-pass regressions without and with a constant. The Table reports: the estimates of the prices of risk on the consumption component j and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the R^2 of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year. Panel B restricts the two price of risk to be the same.