706 Prelim
August 2014
Good luck!

Macroeconomic propagation mechanisms convert serially-uncorrelated "impulses," or shocks (e.g., technology shocks), into serially-correlated aggregates (e.g., real consumption growth). With that as background, suppose for now that you know that U.S. real consumption growth, x_t, follows a covariance stationary $AR(2)$ process with weak white noise innovations.

1. Provide a detailed characterization of x_t via its Wold decomposition. Is it a complete characterization? Are the innovations associated with its Wold representation uncorrelated? Independent? Gaussian?

2. What is the unconditional innovation variance of x_t? Must it be finite? What is the conditional innovation variance of x_t? Is it necessarily smaller than the unconditional variance?

   Now suppose that you know that x_t follows a covariance stationary $AR(2)$ process with conditionally-Gaussian $ARCH(6)$ innovations.

3. Write down the full conditionally-Gaussian $AR(2) - ARCH(6)$ process for x_t. What must be true of the AR and $ARCH$ parameters to ensure covariance stationarity? How would you modify the process to allow the response of volatility to depend on the signs of innovations? Write down the modified process. Why/when might such a modification be useful?

   Now suppose that you know only that x_t follows a covariance stationary process, but you decide to *fit* a conditionally-Gaussian $AR(2) - ARCH(6)$ *model*.

4. How would you the estimate the model by Gaussian pseudo-MLE, and what are the properties of the resulting estimates?

5. How would you diagnose the specification adequacy (as regards conditional mean dynamics, conditional variance dynamics, and conditional density) of your fitted $AR(2) - ARCH(6)$ model?