

1 The Stochastic Neoclassical Growth Model with Heterogeneous Households

Consider the stochastic neoclassical growth model. The resource constraint in the economy is given by

$$C_t + K_{t+1} = z_t K_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t.$$

where upper case letters denote aggregate variables. The technology shock z_t follows a finite Markov chain with transition matrix $\pi(z_{t+1}|z_t)$ and associated invariant distribution Π . Denote by $z^t = (z_0, z_1, \dots, z_t)$ the history of these shocks. The initial capital stock in the economy is given by K_0 .

There are two groups $i = 1, 2$ of households of equal measure (normalized to 1). Each group has a period utility function defined over consumption $c_t^i \geq 0$ and labor $l_t^i \geq 0$

$$u^i(c_t^i, l_t^i)$$

and discounts the future at common time discount factor β . A household of type i has labor productivity $\theta^i > 0$ so when this household supplies l_t^i hours of work, the effective labor supply of that household is given by

$$\theta^i l_t^i.$$

Here θ^i is a fixed parameter, for both i , and we assume

$$\theta^1 + \theta^2 = 1$$

Each household of group i owns initial capital stock $k_0^i = \theta^i K_0$, and thus

$$k_0^1 + k_0^2 = K_0.$$

1. Let

$$\pi = \begin{pmatrix} \rho_1 & 1 - \rho_1 & 0 & 0 \\ 1 - \rho_1 & \rho_1 & 0 & 0 \\ 0 & 0 & \rho_2 & 1 - \rho_2 \\ 0 & 0 & 1 - \rho_2 & \rho_2 \end{pmatrix}$$

where ρ_1, ρ_2 are parameters with $\rho_j \in (0, 1)$ for $j = 1, 2$. Find the invariant distribution(s) associated with π .

2. Suppose the social planner attaches Pareto weights μ^1, μ^2 to the two types of households. State the social planner problem recursively and clearly identifying state variables and control variables.
3. Define a recursive competitive equilibrium.
4. Use the recursive formulation of the household problem and the marginal pricing condition to derive the intratemporal and intertemporal optimality conditions for each household type.
5. Now suppose that

$$u^i(c_t^i, l_t^i) = \frac{(c_t^i)^{1-\sigma} - 1}{1-\sigma} - \frac{(l_t^i)^{1+\nu}}{1+\nu}$$

for both i . Use your results from the previous question to give sufficient conditions on the parameters (β, σ, ν) such that aggregate allocations and prices in this economy with heterogeneous households coincide with those obtained in a representative agent economy where the representative household has labor productivity $\theta = 1$ and a utility function of the form

$$u(c_t, l_t) = \frac{(c_t)^{1-\sigma} - 1}{1-\sigma} - \frac{(l_t)^{1+\nu}}{1+\nu}$$

6. Suppose we introduce a one period, risk free bond into the economy with heterogenous households. Under the conditions derived in the previous question
 - (a) Would there be trade of this bond in equilibrium? You have to justify your answer.
 - (b) Give the price of this risk free bond, as a function of consumption allocations. Note that I do not ask you to solve for equilibrium consumption allocations. Also note that you can answer this question even without having answered the previous question.