Economics 702, Dirk Krueger

1 A Pure Exchange Economy with Household Heterogeneity

Consider a stochastic pure exchange economy where the current state of the economy is described by $s_t \in S = \{s_1, \ldots, s_M\}$. Event histories are denoted by s^t and the initial node s_0 is fixed. Probabilities of event histories are given by $\pi_t(s^t)$. There are 2 different types of households with equal mass normalized to 1. Households potentially differ in their endowment stream $\{e_t^i(s^t)\}$, their initial asset position a_0^i and their time discount factors $\beta_i \in (0,1)$. Preferences for each household over consumption allocations $c^i = \{c_t^i(s^t)\}$ are given by

$$u^i(c^i) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} (\beta_i)^t \, \pi_t(s^t) U(c_t^i(s^t)).$$

where $U(.)$ is strictly increasing and strictly concave.

1. Suppose that $S = \{s_1, s_2\}$ and that $\pi_t(s^t)$ is Markov with transition matrix
$$\pi(s'|s) = \begin{pmatrix} \rho & 1-\rho \\ 1-\kappa & \kappa \end{pmatrix}$$
where $\rho, \kappa \in [0,1]$ are parameters. For which parameter combinations (ρ, κ) is the associated invariant distribution Π
   (a) Unique?
   (b) Satisfies $\Pi = (0.8, 0.2)$?

2. Suppose that households can trade a full set of Arrow securities. Define a sequential markets equilibrium, for arbitrary $\{a_0^i\}_{i \in 1,2}$ with $\sum_i a_0^i = 0$.

3. Define a recursive competitive equilibrium.

4. Suppose that the aggregate endowment satisfies, for $t \geq 0$
$$e_t(s^t) = 2(1+g)^t$$

with $g > 0$, and that $\beta_1 = \beta_2$ as well as

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}.$$

The individual endowments satisfy $e_t^i(s^t) > 0$ for all i, t, s^t, but no further assumptions are given, of course apart from

$$\sum_{i=1}^{2} e_t^i(s^t) = e_t(s^t) = 2(1+g)^t.$$

Characterize as fully as possible the sequential market equilibrium consumption allocations and the prices of Arrow securities.

5. Compute the risk-free interest rate in this economy.

6. Now suppose that households cannot borrow, that is, impose $a_{t+1}^i(s^{t+1}) \geq 0$ and also assume that $1 > \beta_1 > \beta_2$ and $a_0^1 = a_0^2 = 0$ and

$$e_t^i(s^t) = (1+g)^t$$

for both $i = 1, 2$. Repeat questions 4. and 5.