

Microeconomic Theory I  
Preliminary Examination  
University of Pennsylvania

August 15, 2011

**Instructions**

This exam has 5 questions and a total of 100 points.

You have two hours to complete it.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise. Write clearly so that you might get partial credit.

Good luck!

1. (20 pts) Suppose  $u$  is a continuous utility function representing a continuous and locally non-satiated preference relation on  $\mathbb{R}_+^n$ . Let  $p \in \mathbb{R}_{++}^n$  be a price vector. Let  $e(p, u)$  and  $h(p, u)$  be the expenditure function and the Hicksian demand correspondence, respectively. Prove the following properties:

- (a) (5 pts) Show that  $e(p, u)$  is homogenous of degree one in  $p$  : for any  $p, u$ , and a scalar  $\lambda \in \mathbb{R}_{++}$ ,  $e(\lambda p, u) = \lambda e(p, u)$ .
- (b) (5 pts) Show that  $e(p, u)$  is nondecreasing in  $p$  and strictly increasing in  $u$ .
- (c) (5 pts) Show that  $e(p, u)$  is concave in  $p$ .
- (d) (5 pts) Suppose  $p, p' \in \mathbb{R}_{++}^n$  and let  $x \in h(p, u)$  and  $x' \in h(p', u)$ . Then

$$(p' - p) \cdot (x' - x) \leq 0.$$

2. (25 pts) There are two commodities, leisure and consumption. Three workers wish to insure themselves against the possibility of being laid off and getting no wage. They know that exactly one of them will be laid off and unemployed. Each worker earns 10 when employed and 0 when unemployed. Each worker consumes exactly what they earn in the period plus benefits from or payments for insurance. Each worker has 3 units of leisure if unemployed and 1 unit of leisure if employed. The workers set up Arrow-Debreu markets to insure each other against unemployment. The utility function of each worker has the form  $\mathbb{E}u(l(s), c(s))$  where  $l(s)$  is consumption of leisure,  $c(s)$  is consumption of income,  $s$  is the state of the world, and  $\mathbb{E}$  is the expected value operator.

For each of the following utility functions, find an Arrow-Debreu equilibrium, and state whether workers consume more or less when they are unemployed as compared to when they are employed. (Hint: make use of the symmetry of the problem.)

- (a)  $u(l, c) = l^{1/3}c^{2/3}$ .
  - (b)  $u(l, c) = \frac{1}{3} \ln l + \frac{2}{3} \ln c$ .
  - (c) As stated, the aggregate leisure time given up by the two employed workers is 4 units and their combined earnings were 20. Suppose that there was the possibility that instead of a worker being laid off, jobs could be shared so that each worker gave up  $4/3$  units of leisure time and earned  $20/3$ . Explain why at the ex ante stage before it is known who will be laid off workers would prefer this to the Arrow-Debreu equilibrium for any concave utility function  $u(l, c)$ .
  - (d) Suppose in the Arrow-Debreu equilibria you calculated for parts (a) and (b) worker 1 was the worker who was unemployed. Would this unemployed worker be better off than in the work sharing arrangement in part c?
3. (15 pts) In class we used a separating hyperplane theorem to prove one of the fundamental welfare theorems.
- (a) State precisely the separating hyperplane theorem.
  - (b) State carefully the welfare theorem that the separating hyperplane theorem is used to prove.
  - (c) Describe carefully how the separating hyperplane theorem is used in this proof.

4. (20 pts) There are two states,  $\omega = a, b$ . Let  $p = \Pr(\omega = a)$ . There are two consumers and one good. Their state-contingent endowments are  $(x_{1a}, x_{1b}) = (1, 0)$  and  $(x_{2a}, x_{2b}) = (0, 1)$ . The Bernoulli utility function of consumer  $i$  is  $u_i(x) = x^{\theta_i}$ , defined on  $\mathbb{R}_+$ . Assume each  $\theta_i \in (0, 1)$ .
- (a) (5 pts) Find all efficient state-contingent allocations  $(x_1, x_2)$ .
- (b) (15 pts) Now suppose that for each  $i$ , the parameter  $\theta_i$  is the privately known type of consumer  $i$ . It is equally likely to be  $\theta'_i$  or  $\theta''_i$ , where  $0 < \theta'_i < \theta''_i < 1$ .  
The consumers can commit to a mechanism that will determine the state-contingent allocation as a function of the messages they send. Characterize the set of dominant strategy mechanisms that achieve, for each type vector, a corresponding efficient allocation.
5. (20 pts) Consider a trading environment in which a consumer of type  $v \in [\underline{v}, \bar{v}]$  obtains (Bernoulli) utility  $qv - p$  if she obtains  $q$  units of a divisible good and pays a total of  $p$  dollars for them. Assume  $0 < \underline{v} < \bar{v}$ .
- (a) (12 pts) Let  $q : [\underline{v}, \bar{v}] \rightarrow \mathbb{R}_+$  be an arbitrary function specifying an amount of the good for each type of consumer. Prove from first principles that  $q$  is part of an incentive compatible revelation mechanism if and only if it is nondecreasing.
- (b) (8 pts) Now let  $p : [\underline{v}, \bar{v}] \rightarrow \mathbb{R}$  be a continuously differentiable function specifying a payment for each type of consumer. Find a condition on  $p$  that is necessary and sufficient for it to be part of an incentive compatible revelation mechanism, and describe your reasoning.