Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

August 2010

Instructions

This exam has 5 questions and a total of 100 points.

You have two hours to complete it.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Good luck!

1. (20 points) A consumer's preferences on consumption bundles $x \in \mathbb{R}_+^K$ admits a strictly quasiconcave utility representation $u(x)$. Let $p \in \mathbb{R}_{++}^K$ denote the price vector and $w \in \mathbb{R}_+$ denote the income. Show that if $u(x)$ is homogeneous of degree one, then the Marshallian demand function $\mathbf{x}(p, w)$ and the indirect utility function $v(p, w)$ take the following forms:

$$\begin{aligned} \mathbf{x}(p, w) &= w\widetilde{\mathbf{x}}(p), \\ v(p, w) &= w\widetilde{v}(p). \end{aligned}$$

2. (20 points) Consider an economy with two households and two commodities. Assume household 1 has utility function

$$u^1(x_1, x_2) = \min(x_1, \frac{3}{4}x_2),$$

and household 2 has utility function

$$u^2(x_1, x_2) = \min(x_1, \frac{1}{4}x_2).$$

Assume initial endowments are $e^1 = (\alpha, 1)$ and $e^2 = (1 - \alpha, 1)$.

(a) Compute the equilibrium price correspondence as a function of α for all $\alpha \in (0, 1)$.

(b) Argue that this correspondence is upper hemicontinuous, or show that it is not. Here, "argue" means to provide a sketch of the proof, omitting tedious details but pointing out what you omit.

3. (20 points) Suppose Fred is considering purchasing flood insurance for his house. If Fred does not buy flood insurance, his wealth will be $w > 0$ if there is no flood and $w - L$ if there is a flood, where $L < w$. The probability of a flood is $\pi \in (0, 1)$. The cost of a policy that pays K if a flood occurs is cK, where $c < 1$.

Assume Fred can choose any $K \in [0, L]$, and he chooses K so as to maximize his expected utility. Assume his von Neumann-Morgenstern utility function u is a differentiable, strictly increasing, and strictly concave function of his final wealth.

(a) Find the first order condition that characterizes Fred's choice of K, assuming it is an interior solution. State conditions on the parameters that imply his choice of K is interior.

(b) For what value(s) of c will Fred purchase full insurance? Does the answer depend on the form of the utility function? Why or why not?

(c) Drop the assumptions that u is differentiable and concave – assume only that u is strictly increasing and a utility maximizing choice exists. Show that the K that Fred chooses is weakly increasing in the probability π of a flood occurring.

4. (15 pts) Consider an economy with 3 consumers, $k \geq 1$ public goods, $x = (x_1, \dots x_k) \in \mathbb{R}^k_+$, and one private good y. Consumer i's utility from a bundle (x, y_i) is $u^i(x, y_i)$, where u^i is a continuous and strictly increasing function. Her endowment of private good is $\bar{y}_i > 0$, and she has no endowment of public goods. The government produces public good. If it produces the vector x of public goods, each consumer pays a cost $t \cdot x$ for them, where $t \in \mathbb{R}^k_{++}$.

   (a) (10 pts) Assuming $k = 1$, make further assumptions, as few and weak as possible, that will guarantee a public goods bundle (Condorcet winner) exists that cannot be beaten in a majority vote by any other bundle x. Indicate your reasoning (i.e., sketch a proof).

   (b) (5 pts) Same as (a), but for $k = 2$.

5. (25 pts) Consider a two-person exchange setting with two private goods, x and y. The consumption sets are $\mathbb{R}^2_+$. Consumer i's endowments are $\bar{x}_i$ and $\bar{y}_i$, both positive. Consumer i's utility function is $u^i(x_i, y_i, \theta_i)$, where $\theta = (\theta_1, \theta_2)$ is a random variable distributed on a nondegenerate square, $\Theta = [\underline{\theta}_1, \bar{\theta}_1] \times [\underline{\theta}_2, \bar{\theta}_2]$, according to a distribution function $F(\theta_1, \theta_2)$. The realization of θ_i is privately known to consumer i (it is her "type"), but to no one else. Each u^i is continuous and strictly increasing in its first two arguments. Furthermore, for each i there exists θ_i, θ'_i, (x_i, y_i), and $(\hat{x}_i, \hat{y}_i)$ such that

$$u^i(x_i, y_i, \theta_i) > u^i(\hat{x}_i, \hat{y}_i, \theta_i) \text{ and } u^i(x_i, y_i, \theta'_i) < u^i(\hat{x}_i, \hat{y}_i, \theta'_i).$$

   (a) (5 pts) Define a revelation mechanism for this setting, and state what it means for it to be dominant strategy incentive compatible.

   (b) (5 pts) We seek a dominant strategy incentive compatible mechanism that for any θ, yields an allocation satisfying two criteria. First, it should be feasible, except perhaps for violating the nonnegativity constraints, $y_i \geq 0$. Second, if it is feasible, then it should be efficient except that not all of good y is consumed, i.e., there is no way of making a Pareto improvement (ex post) without consuming more of good y.

   Make as few as possible, and as weak as possible, additional assumptions that will imply the existence of such a mechanism.

   (c) (15 pts) Using the assumptions you made in (b), define the desired mechanism, prove that it is dominant strategy incentive compatible, and show that it satisfies the two desired criteria.