Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

June 2010

Instructions

This exam has 5 questions and a total of 160 points.

You have two hours to complete it.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Good luck!

1. (30 pts) A consumer's preference relation is strictly convex and has a quasilinear utility function representation, $U(\mathbf{x}, m) = u(\mathbf{x}) + m$, on the consumption set $\mathbb{R}^K_+ \times (-\infty, +\infty)$. Assume the price of commodity $K + 1$ is 1, so the budget constraint is $\mathbf{p} \cdot \mathbf{x} + m \leq y$, where y is income and $\mathbf{p} \in \mathbb{R}^K_{++}$ is the price vector for the first K commodities.

   (a) (15 pts) Denote the consumer's Marshallian demand functions as $\mathbf{x}(\mathbf{p}, y)$ and $m(\mathbf{p}, y + z)$. Prove that for any $z \in \mathbb{R}$,

   $$\mathbf{x}(\mathbf{p}, y) = \mathbf{x}(\mathbf{p}, y + z) \quad \text{and} \quad m(\mathbf{p}, y + z) = m(\mathbf{p}, y) + z.$$

   (b) (15 pts) Show that the indirect utility function has the form

   $$v(\mathbf{p}, y) = \phi(\mathbf{p}) + y.$$

2. (35 pts) Three hunters will hunt for deer tomorrow in a game park with exactly two deer. Each hunter will catch at most one deer (by park regulations), and both deer will be caught. There are thus three states of the world tomorrow: state $s = 1, 2, 3$ represents the event that each hunter **except** hunter s catches a deer. Letting ω^i denote hunter i's initial endowment of contingent deer meat, we have

   $$\omega^1 = (0, 1, 1), \ \omega^2 = (1, 0, 1), \ \omega^3 = (1, 1, 0).$$

   Today (date $t = 0$) they arrange for how the meat from any deer caught tomorrow (date $t = 1$) will be shared. The utility function of hunter i is

   $$U^i(x_i) = \Sigma_{s=1}^3 \pi_s^i u^i(x_s^i),$$

   where x_s^i is his consumption of deer meat in state s, and π_s^i is his belief probability that state s will occur. Assume u^i is continuous, strictly concave, and strictly increasing.

   (a) (10 pts) Suppose the hunters agree that the state probabilities are $(1/2, 1/4, 1/4)$. (Hunter 1 is believed to be twice as likely to not catch a deer as is either of the other two.) Show that at any interior Pareto efficient allocation, hunter 1 will consume the same amount of deer meat regardless of who catch the deer. (You can assume for this part that each u_i is differentiable.)

   For the remaining parts, assume instead that each hunter is so self-confident that he believes he will surely catch a deer: $\pi_i^i = 0$ for each i. But he is not as confident about the others: $\pi_s^i > 0$ for $s \neq i$.

   (b) (10 pts) Prove that if (x^{1*}, x^{2*}, x^{3*}) is a Pareto efficient allocation, then $x_i^{i*} = 0$ for each i.

   (c) (15 pts) Prove that competitive equilibria exist, letting p_s denote the price at date 0 for contingent deer meat in state s at date 1. (You cannot simply quote the existence result proved in class, since here the preferences are **not** strongly monotone because each $\pi_i^i = 0$.)

3. (30 pts) Walrasian equilibrium with production.

   (a) (10 pts) State precisely the definition of a Walrasian equilibrium for an economy with production.

   (b) (10 pts) Under standard assumptions on preferences and interior endowments, what conditions on the production technology are sufficient for a Walrasian equilibrium with production to exist? (Little if any credit will be given for trivial conditions such as "the production set is empty".)

   (c) (10 pts) Give an example in which one of the conditions on the technology you gave in (b) is not satisfied, and a Walrasian equilibrium does not exist.

4. (35 pts) There are 2 potential buyers of an object, which is initially owned by a potential seller. The value of buyer i for the object is v_i, which he does not know. However, he privately observes a signal θ_i about it. The signals are independently and uniformly distributed. The value of buyer i is

$$v_i = \theta_1 + \theta_2.$$

   The seller's value for the object is a constant, $v_0 \in [0, 2]$. If agent i receives the object for a price p, his utility is $v_i - p$. If he pays p and does not receive the object, his utility is $-p$.

   (a) (5 pts) Describe an (ex post) efficient allocation.

   (b) (15 pts) Does there exist a Bayesian incentive compatible revelation mechanism that achieves an efficient allocation? Prove your answer.

   (c) (15 pts) Find a symmetric equilibrium of the second price auction that has a reserve price (lowest acceptable bid) r, where $r \in [0, 2]$.

5. (30 pts) (30 pts) Consider the following principal-agent model. The agent chooses effort $e \in \{e_L, e_H\}$, where $0 \leq e_L < e_H \leq 1$. His effort choice is unobserved by the principal. It determines the density function of profit as

$$f(\pi|e) = ef_1(\pi) + (1 - e)f_0(\pi)$$

   on a nondegenerate interval $[\underline{\pi}, \bar{\pi}]$, upon which both f_1 and f_0 are positive and continuous. The profit realization is observed by all and contractible. The agent's utility when he chooses e and is paid w is $v(w) - g(e)$. Assume

$$v' > 0,\ v'' < 0,\ g(e_L) < g(e_H).$$

   The principal's utility is $\pi - w$ if the realized profit is π and she pays the agent w. The parties agree ex ante to a contract $w(\pi)$ that pays the agent a wage as a function of realized profit.

   (a) (10 pts) Find an additional assumption under which the first best can be achieved, i.e., under which a contract $w : [\underline{\pi}, \bar{\pi}] \to \mathbb{R}$ exists such that a first-best efficient allocation is achieved when the agent takes his best effort given this contract. State your reasoning.

   (b) (20 pts) Suppose $w^{SB}(\pi)$ is a contract that induces e_H, and the expectation of $w^{SB}(\tilde{\pi})$ conditional on e_H is less than that of any other contract that induces e_H and gives the agent the same expected utility. Find an assumption that implies w^{SB} is strictly increasing. State your reasoning.