Econ 705 : Preliminary Examination

August, 2011

Instructions:

(1) All the answers should be written legibly.

(2) In giving answers, try to mention conditions which justify the derivations.

(3) Even when you are not able to delineate precise conditions, try to provide a sketch of derivations and solutions as clearly as possible.

(1)(a-d) (50 Points) A researcher has a data set $\{Y_i, X_i\}_{i=1}^n$, where the observations are assumed to be related as

$$\begin{aligned} Y_i &= \beta_0 + P\{D_i^* = 1|X_i\}\beta_1 + u_i, \text{ and} \\ D_i^* &= 1\{\gamma_0 + \gamma_1 X_i \geq \varepsilon_i\}. \end{aligned}$$

Here $\{(\varepsilon_i, u_i, D_i^*)\}_{i=1}^n$ are *unobserved* random variables. We assume that $\{(Y_i, X_i, D_i^*, \varepsilon_i, u_i)\}_{i=1}^n$ are i.i.d. and that the conditional distribution of ε_i given X_i is $N(0,1)$ and that

$$Var(X_i) > 0 \text{ and } Var(P\{D_i^* = 1|X_i\}) > 0.$$

We also assume that the conditional distribution of u_i given X_i is $N(0, \sigma^2)$ for some parameter $\sigma^2 > 0$.

(a) (15 Points) Write down the log likelihood function of $\theta = (\beta_0, \beta_1, \gamma_0, \gamma_1, \sigma)$, and define the MLE.

(b) (15 Points) Show that the MLE is consistent (you may assume that the parameter θ is identified), and write down the asymptotic covariance matrix.

(c) (10 Points) Suppose that we are in the situation of (b) except that the conditional distribution of u_i given X_i has zero and variance σ^2 but is different from $N(0, \sigma^2)$. Does the MLE $(\hat{\beta}_0, \hat{\beta}_1)$ for (β_0, β_1) of (b) become inconsistent in general? Explain your answer.

(d) (10 Points) Suppose that we are in the situation of (b) except that the conditional distribution of ε_i given X_i has zero and variance 1 but is different from $N(0, 1)$. Does the MLE $(\hat{\beta}_0, \hat{\beta}_1)$ for (β_0, β_1) of (b) become inconsistent in general? Explain your answer.

(2)(a-d) (50 Points) A researcher has a data set $\{Y_i, X_i, Z_{1i}, Z_{2i}\}_{i=1}^n$, where the observations are assumed to be related as follows:

$$\begin{aligned} Y_i &= \beta_0 + \beta_1 X_i + u_i, \text{ and} \\ X_i &= \pi_0 + \pi_1 Z_{1i} + \pi_2 Z_{2i} + \varepsilon_i. \end{aligned}$$

Here $\{(\varepsilon_i, u_i)\}_{i=1}^n$ are unobserved random variables. We assume that $\{(Y_i, X_i, Z_{1i}, Z_{2i}, \varepsilon_i, u_i)\}_{i=1}^n$ are i.i.d. and that the conditional distribution of ε_i given (Z_{1i}, Z_{2i}) has mean zero and variance σ_ε^2. We assume that u_i and X_i are allowed to be correlated so that the regression model for Y_i suffers from endogeneity.

(a) (15 Points) Provide conditions under which β_0 and β_1 are identified and show how they are identified.

(b) (15 Points) Define a consistent estimator of (β_0, β_1) using (Z_{1i}, Z_{2i}) as a vector of instrumental variables, and show that the estimator is consistent.

(c) (10 Points) One would like to test the following null hypothesis:

$$\begin{aligned} H_0 &: \pi_1 + \pi_2 = 1 \text{ against} \\ H_1 &: \pi_1 + \pi_2 \neq 1. \end{aligned}$$

Propose a test statistic T_n and a critical value c such that $P\{T_n > c\} \to 0.05$ as $n \to \infty$ under H_0. (You need to explain how the convergence $P\{T_n > c\} \to 0.05$ follows.)

(d) (10 Points) Explain how you would test the IV exogeneity restrictions that $\mathbf{E}[Z_{1i}u_i] = \mathbf{E}[Z_{2i}u_i] = 0$.

End of the Exam