Dynamic Panics: Theory and Application to the Eurozone*

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Job Market Paper‡

Abstract

This paper documents a new type of dynamic lender coordination problem in sovereign debt markets that I call a *dynamic panic*. During a dynamic panic, expectations of future negative investor sentiments reduce the willingness of the sovereign to repay in the future and thus translate to negative investor sentiments today. I demonstrate existence of these crises for short-term debt even when there is no underlying multiplicity and show that they imply a high degree of fragility on standard Markov-perfect equilibria. When the debt is of longer maturity I show that such panics resemble the recent Eurozone crisis. I explore policy implications and find that interest rate ceilings are an ineffective policy tool but that liquidity provision by the ECB may be welfare-improving. Motivated by this result, I perform a simple DSGE estimation exercise to determine investors' ex ante forecast of such panics and the welfare consequences of liquidity provision. Using Bayesian methods and Spanish CDS spreads, I find that investors' forecast of such a crisis ex-ante was once every 7.37 years, which is in close accordance with the realized frequency of 7.5 years. I also find that liquidity provision by the ECB is likely welfare-improving.

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1. Introduction

Ever since the seminal contribution of Eaton and Gersovitz (1981), there has been a sort of a 'folk theorem' in the sovereign debt and default literature that market sentiments can have real effects. The intuition is as follows: If investors expect the sovereign to default, they will charge him a higher interest rate; this in turn makes it more costly to repay and so he defaults more often. However, if lenders do not expect default, they will charge a low spread which in turn makes borrowing less costly and reduces default probability. This intuition was formalized in some form in the models of Calvo (1988) and Cole and Kehoe (1996).

However, the recent quantitative literature in this field lacks these multiplicity dynamics. Models such as Aguiar and Gopinath (2006), Arellano (2008), and Chatterjee and Eyigungor (2012) tend to produce unique equilibria. Authors in this vein have even used this result to motivate application of method of moments in quantitative analysis. Further, in recent work Auclert and Rognlie (2014) have demonstrated theoretically that the baseline equilibrium used in this quantitative literature is in fact unique.

While this determinacy result is appealing from a tractability and econometric perspective, the lack of multiplicity dynamics is problematic on an intuitive level given that many empirical crises do exhibit such confidence-driven dynamics e.g. Mexico's 1994 Tequila Crisis or the recent crisis in the Periphery Eurozone countries. This paper reconciles this apparent inconsistency by showing that such confidence-driven crises can in fact arise in the canonical sovereign debt framework despite the underlying equilibrium uniqueness. Further, equilibria exhibiting such crises can be computed and simulated using the same tools employed by this literature.

In particular, I outline a new type of dynamic lender coordination problem, which I term a *dynamic panic*. During a dynamic panic, lenders today anticipate that lenders tomorrow will charge a very high spread on debt issued by the sovereign. This will in turn reduce the value of the repayment tomorrow to the sovereign, who will default more often as a consequence. Lenders today panic as a result and demand higher spreads, which increases the contemporaneous frequency of default. Thus, we can have a sequence of lenders who are all panicking about the behavior of future lenders.

This intuition can be observed graphically in Figure 1. When the confidence light is green, lenders in the future offer the sovereign a high price on his debt, which induces the sovereign to repay more often. Given a persistent process for confidence, represented by the thickness of the arrows, lenders today will react to this lower default frequency and offer a higher debt price today. The opposite holds true if the light is red and investors are panicking.

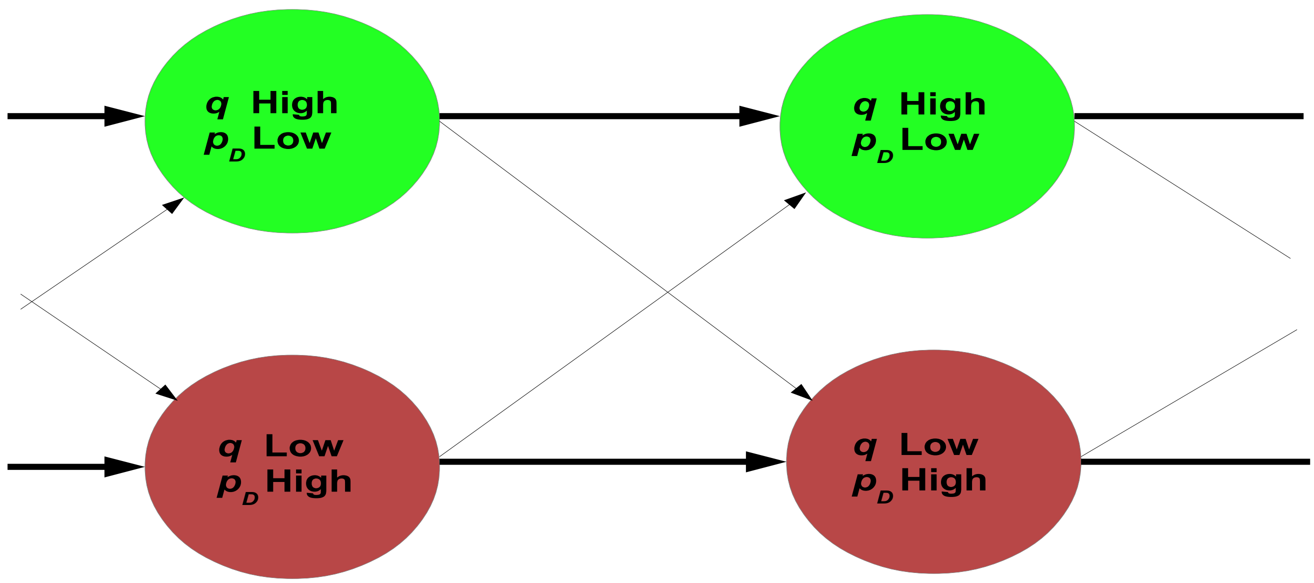

Figure 1: Dynamic Panics: A Simple Illustration

All that is needed to induce such dynamics is some persistent, non-fundamental object, which I call confidence. A persistent notion of confidence gives lenders *across time* a way to communicate with each other and thus to coordinate on malignant spread dynamics.[1] These dynamics are sustained in turn by optimal sovereign default behavior.

In this paper, I find conditions under which such crises exist and characterize their basic properties. I show that when non-fundamental confidence has effects, it does act as a panic i.e. it uniformly shifts

[1]There is empirical evidence to suggest that confidence or beliefs are persistent in nature even when fundamentals are accounted for. See, for example, Barsky and Sims (2009). One could reconcile this by assuming that it is costly for investors to acquire information and thus prior beliefs are updated slowly. For the purposes of my model, I will assume this fact.

the demand schedule for debt. These shifts are rationally anticipated and priced. Further, I show that the existence of these crises implies a fragility on the typical Markov-perfect equilibrium that is computed from fundamentals. In particular, I show that in addition to the equilibrium that is typically found in computation, there is always at least one other distinct set of policy and pricing functions that are only ϵ away from satisfying the equilibrium conditions. ϵ can easily be smaller than numerical convergence criteria, and so caution should be used when interpreting quantitative models in this vein.

I also show how these crises can differ based on the maturity structure of the debt. In particular, dynamic panics with longer maturity debt can generate persistent waves of borrowing: High borrowing at high spreads and low borrowing at low spreads. These are peculiar features of the Eurozone crisis that have attracted the attention of the recent literature (Lorenzoni and Werning (2013), Corsetti and Dedola (2013), and Conesa and Kehoe (2012)). My model generates such crises as an equilibrium outcome.

The intuition is as follows: When a confidence shock hits the economy, lenders anticipate that the sovereign will borrow aggressively *tomorrow*, diluting the value of the long-term debt they purchase.[2] As a result, they demand to be compensated with higher spreads today. The sovereign, upon seeing these spontaneous high spreads today, will choose to borrow into them rather than reduce consumption or default, since both of these remain very costly given the non-fundamental nature of the spread shock. This justifies the lenders' fear of excessive borrowing provided the confidence regime is persistent.

Dynamic panics also have new implications for policy, which I explore. I find that in this environment, an interest rate ceiling would be ineffective. Such a policy has been proposed for Eurozone countries by Lorenzoni and Werning (2013) and Corsetti and Dedola (2013). The justification these authors provide for interest rate ceilings is that there are two ways a sovereign can raise the same amount of revenue: A small amount of debt at low spreads, which are low because the probability of default is low, or a large amount of debt at high spreads, which are high because the probability of default is high. These authors argue that distressed Eurozone countries were 'stuck' in the latter situation and that a simple, credible cap on the market rate would be enough to rule out the sub-optimal equilibrium. However, during a dynamic panic, the sovereign is always borrowing on the left side of this 'Laffer curve', even during a crisis. An interest rate ceiling is thus isomorphic to revenue cap on debt issuance and will only reduce government

[2] See Hatchondo and Martinez (2009) or Chatterjee and Eyigungor (2012) for an in depth discussion of debt dilution in long-term debt

consumption and increase the likelihood of default.

Even though an interest rate ceiling is ineffective, I do find that liquidity provision[3] i.e. the credible pledge by the central bank to purchase debt at potentially sub-market rates, is an effective policy tool if the goal is to remove confidence fluctuations. However, its welfare consequences are ambiguous, since some moral-hazard based default will remain after implementation.

Motivated by the applicability of dynamic panics to the recent crisis in Peripheral Europe, I explore quantitatively the model's empirical and policy implications with an empirical exercise. In particular I perform a structural estimation on an off-the-shelf DSGE model modified to include persistent, time-varying default probabilities to answer two key questions: First, what was the ex-ante likelihood of a transition into a dynamic panic? And second, was liquidity provision by the ECB welfare-improving?

My structural estimation on Spanish data suggests that these crises may in fact occur frequently. The probability of switching confidence regimes in any given quarter is estimated to be around 3.39%, which is roughly once every 7.37 years. This figure is quite robust and is computed from spread data before the crisis. It does not rely whatsoever on the relative frequency of these events in the data. Given that the crisis took place roughly 7.5 years after the inception of the monetary union,[4] this figure is in close accordance with realized events and tells us that if anything, investors anticipated such crises more often than they occurred.

This result helps to solve the puzzle of low sovereign debt spreads throughout the early 2000's as well. Lane (2012) articulates this puzzle as follows: "(T)he low spreads on sovereign debt...indicated that markets did not expect substantial default risk and certainly not a fiscal crisis of the scale that could engulf the euro system as a whole." On the contrary, I argue such low spreads were completely compatible with a rational long-term dynamic panic. This is because during normal times investors do not fear default, but the possibility that the economy will enter a regime in which default is more likely. Such a regime will dilute the value of long-term debt without obliterating it. Therefore, one can have very low spreads prior to a long-term dynamic panic even if during such a crisis default becomes a highly probable event.

[3]When I use the term, 'liquidity provision' in the paper, I mean it in the same context as De Grauwe (2011) and Corsetti and Dedola (2013) i.e. it is a pledge by the central bank to act as the lender of last resort to the sovereign government and thus to potentially purchase government debt at submarket rates.

[4]'Inception' is a vague term here, since the launch of the Euro took several years following its initial circulation in 1997. The external validity measure of 7.5 years assumes the initial date is the full launch, which occurred in 2002Q1, and that the crisis occurred in 2009Q3.

To answer the second question, the empirical exercise suggests that so long as liquidity provision does not induce moral-hazard default more than once every 9.7 years that it will indeed improve welfare. This is substantially less than historical default trends outlined by Reinhart and Rogoff (2010), and so liquidity provision is likely welfare-improving.

This paper makes several methodological and theoretical contributions. First, it outlines a new dynamic lender coordination problem, which is the susceptibility of the reputation-lending environment to dynamic, persistent sunspots; second, it provides a new general existence theorem for non-fundamental dynamics that does not rely on the existence of underlying multiplicity of equilibria; third, it generates 'gambling for redemption' or borrowing into high spreads under fewer assumptions than Conesa and Kehoe (2012); fourth, it suggests that interest rate ceilings, which rely on Laffer curve multiplicity, would be an ineffective policy in combating such crises; fifth, it develops a new type of Markov-switching DSGE model and implements a new solution method to understand quantitatively episodes of crises and default; and sixth, it develops several widely applicable techniques that can be used to reduce the computational burden of approximating Markov-switching DSGE solutions.

The rest of the paper is organized as follows. In Section 2, I review the relevant literature. In Section 3, I describe the canonical model employed in the quantitative sovereign debt literature and in Section 4 I characterize how this model reacts to non-fundamental fluctuations. Section 5 applies the results to the Eurozone in an empirical exercise by estimating an off-the-shelf DSGE model modified to include time-varying default probabilities. It also outlines in detail the solution technique. Section 6 concludes.

2. Literature Review

As of yet, the academic literature has had little time to keep pace with developments that took place in the Eurozone over the past 6 years or so. However, several noteworthy pieces have emerged that have tried to deal seriously with the peculiar circumstances surrounding the sovereign debt crisis in the Eurozone. These papers have been both empirical and structural. On the empirical side, recent work has taken aim at demonstrating the confidence-driven nature of this crises by documenting an unusually weak correlation between economic fundamentals and CDS spreads. Some prominent examples include De Grauwe and Ji (2013) and Aizenman et al. (2013).

On the structural side, much emphasis has been placed on the unusual phenomenon of borrowing into high spreads and its concomitant drastic effect on debt-to-GDP ratios. Conesa and Kehoe (2012) have termed this phenomenon 'gambling for redemption,' and have argued that being mired in a deep recession is a necessary condition for such behavior. In their framework, countries start with high initial debt levels and are shocked into a deep recession. However, there is some probability that the country will recover. The government can under certain conditions 'gamble' on this probability, hoping that a recovery occurs before default does. In a companion paper, Conesa and Kehoe (2014), they show that when the government is gambling for redemption, third-party bailouts at penalty interest rates would prove ineffective, since such policies raise the cost of repayment and would only induce default at current debt levels.

Another structural model designed to understand this unusual borrowing into high spreads is Lorenzoni and Werning (2013), who argue for a 'Laffer-curve' type multiplicity in the spirit of Calvo (1988). They argue that revenue is non-monotonic in debt issuance, since the government can raise the same amount of revenue by issuing little debt at low spreads or a large quantity of debt at high spreads. In their model, they assume that the government can only commit to a level of debt *revenue* and not to a level of debt *issuance*. They thus argue that what occurred in the Eurozone was that a large group of countries got 'caught' on the right side of the Laffer curve, which made servicing debt more expensive and thus increased default probabilities. They term such a crisis a 'slow-moving' crisis. They also argue that when the debt is long-term, expectations regarding not only immediate equilibrium behavior, but also equilibrium behavior in the future can drive fluctuations in prices today and provide a simple continuous-time example of such dynamics.

Corsetti and Dedola (2013) also argue for a Laffer-curve type multiplicity but emphasize instead not the slow-moving nature of the crisis, but the inability of the central bank's 'printing press' to alleviate these crises. They emphasize that the central bank can only help avert such crises by acting as the lender of last resort for the government and being willing to step in and provide liquidity at potentially sub-market rates.

Broner et al. (2014) build their theory around private investment, claiming that a self-fulfilling panic arose in which investors feared default and so demanded higher spreads; domestic banks then began to chase these high-yield sovereign bonds instead of investing in productive capital; this drop in investment precipitated a recession that lowered tax revenues and increased the incentive to default, thus justifying the inital panic by the sovereign's lenders.

My work differs from these works along several dimensions, the most pronounced of which is the capacity to rationalize the *coexistence* of crisis and non-crisis periods in a rational expectations framework. For instance, in both Conesa and Kehoe (2012) and Lorenzoni and Werning (2013), the analysis begins conditional on either a deep recession or a sudden expectations shock, both of which must have been unforeseen. In my paradigm, lenders rationally anticipate and price such crises and thus I can use CDS spread data *before* the crisis to determine the likelihood of a such a crisis, which is certainly a relevant parameter if we are interested in preventing future crises.

Other work that has tried to understand the Eurozone crisis and policy implications include Bocola (2014) and Bi and Traum (2012), who use likelihood-based estimation techniques and spread data to determine default frequencies and policy implications. This paper is in this vein to the extent that it performs a similar exercise to these.

Prior to the Eurozone crisis, a substantial literature had evolved outlining the dynamics of sovereign debt and default episodes. Noteworthy papers in this tradition include Eaton and Gersovitz (1981),Bulow and Rogoff (1988), and Aguiar and Gopinath (2006). There is a nice summary of this tradition in Aguiar and Amador (2013a). This literature has also developed a branch that explicitly considers debt of longer maturities, of which prominent examples include Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012), Arellano and Ramanarayanan (2012), Broner et al. (2013), and Aguiar and Amador (2013b). A common thread in this tradition is the lack of multiplicity or self-fulfilling dynamics. This paper brings such non-fundamental dynamics back into this strand.

Also prior to the Eurozone crisis, there was a reasonably consistent and well-developed way of understanding a majority of international financial crises, known as the 'sudden stop' mechanism. This phenomenon was coined by Dornbusch et al. (1995) and subsequently described on by Calvo (1998), Mendoza and Arellano (2002), and Neumeyer and Perri (2005). Rodrik and Velasco (1999) and Summers (2000) have noted that such international financial crises typically exhibit bank-run dynamics and consequently may rely heavily on market sentiment. As I described in the introduction, though the Eurozone in many respects did not exhibit the sudden stop characteristics of its emerging-markets predecessors.

On the theoretical frontier, this paper also contributes the literature on sunspots, since that is the tool I choose to model non-fundamental confidence. This literature started with the works of Azariadis (1981) and Cass and Shell (1983). Farmer and Guo (1994) and Farmer and Benhabib (1994) extended these models with extrinsic uncertainty to business cycle frameworks, placing emphasis on the need for strategic complementarities to dissolve the typical determinacy results. Shell (2008) provides a nice summary of the prerequisite conditions for the existence of sunspot activity and Hoelle (2014) discusses in depth the relationship between sunspot activity and multiplicity of equilibria when markets are incomplete. My work contributes to this literature by providing a general existence theorem for sunspot dynamics in a reputation-lending environment that is distinct from randomization over multiplicity of equilibria.

The last relevant strand of literature that this work advances is that of Markov-switching DSGE models. This literature, which started with Hamilton (1989), has made the case that parameter instability in the form of regime-switching is often key to understanding macroeconomic time-series. Since that contribution several authors have demonstrated the applicability of this class of models to understanding the Great Moderation (Kim and Nelson (1999)), the 'Great Inflation' of the 1970's (Bianchi (2013)), and even to some extent sovereign default episodes (Bi and Traum (2012) or Bocola (2014). Due to its wide-ranging applicability, numerous solution techniques have been prescribed, most focusing around solving the Linear Rational Expectations (LRE) model once the full general equilibrium model has been linearized. Examples of this includeFarmer et al. (2009) and Davig and Leeper (2007). One recent work, Foerster et al. (2013), solves these Markov-switching models from the full initial model instead of the implied LRE model. I make heavy use of their framework in my solution of the model.

In particular, I provide a set of tools to broaden the applicability of the method of Foerster et al. (2013); I also demonstrate how their method could be applied to models of sovereign default, provide that default behavior is taken as exogenous.

3. Model

3.1. Environment

In this section I construct a small open economy model in the tradition of Eaton and Gersovitz (1981) and Arellano (2008) and show how the model reacts to non-fundamental confidence.

There are three stochastic processes: A fundamental endowment shock that can take values in the set $\mathcal{Y}$, a continuous, fundamental default preference shock that can take values in the set $[\underline{m}, \bar{m}]$, and a non-fundamental confidence shock that can take values in the set Ξ. Both the endowment and the confidence shocks are assumed to be persistent and $\mathcal{Y}$ and Ξ are assumed to be discrete sets. The continuous preference shock is assumed to be iid over time.

3.2. Sovereign Borrower

The sovereign borrower makes all decisions for the country. In particular, it chooses a level of consumption, c, how much to borrow from abroad, $b' \in \mathcal{B}$, and whether or not to default. It is assumed that debt is long-term as in Chatterjee and Eyigungor (2012) i.e. debt matures stochastically at a rate λ and pays a coupon κ for each period that it does not mature.

I will focus on Markov-Perfect Equilibria and so I can write the sovereign's problem recursively. Taking as given the demand schedule for its debt from foreign investors, $q(y, b')$, the government solves the following Bellman, which is conditional on repayment this period:

$$V(y, \xi, b) = \max_{c \geq 0, b' \in \mathcal{B}} u(c) + \beta \mathcal{V}(y, \xi, b') \tag{1}$$

$$\text{s.t. } c \leq y - [\lambda + (1 - \lambda)\kappa]b + q(y, \xi, b')[b' - (1 - \lambda)b]$$

Default is chosen before the sovereign borrows in each period, implying that

$$\mathcal{V}(y, \xi, b') = E_{(\tilde{y}, \tilde{\xi}, \tilde{m})|(y,\xi)}[\max\{V(\tilde{y}, \tilde{\xi}, b') + \tilde{m}, X(\tilde{y})\}]$$

where $\tilde{m}$ is our continuous, iid preference shock over default. I assume that when the country defaults, it is excluded from credit markets forever and that it suffers some additive output loss, $\phi(y)$ in each period. Thus, we can express the value of default as

$$X(y) = u(y - \phi(y)) + \beta E_{\tilde{y}|y}[X(\tilde{y})]$$

Notice that because there is no re-entry, the value of default is an exogenously specified function independent of any particular equilibrium.

3.3. Foreign Investors

There is a unit mass of competitive foreign investors. These foreign investors are assumed to be risk-neutral and deep-pocketed. Their outside option yields a risk-free rate, R. Thus, their optimality condition is given period by period by a simple no-arbitrage condition that also pins down the price of debt just as in Chatterjee and Eyigungor (2012). They take the behavior of the sovereign, which implies a price of debt defined recursively by:

$$q(y,\xi,b') = \frac{1}{R} E_{(\tilde{y},\tilde{\xi},\tilde{m})|(y,\xi)} \Big[1\{V(\tilde{y},\tilde{\xi},b') + \tilde{m} \geq X(\tilde{y})\} \times \big[\lambda + (1-\lambda)(\kappa + q(\tilde{y},\tilde{\xi},a(\tilde{y},\tilde{\xi},b')))\big] \Big] \tag{2}$$

where $a(\tilde{y},\tilde{\xi},b')$ is the anticipated level of borrowing that will be undertaken tomorrow in the state $(\tilde{y},\tilde{\xi},b')$.

3.4. Equilibrium Definition

A **Markov-Perfect Equilibrium** is a set of functions $V(y,\xi,b)$, $a(y,\xi,b)$, and $q(y,\xi,b')$ such that

1. $V(y,\xi,b)$ and $a(y,\xi,b)$ satisfy Recursion 1 when given $q(y,\xi,b')$
2. $q(y,\xi,b')$ solves Recursion 2 given $V(y,\xi,b)$ and $a(y,\xi,b)$

A **Confidence-Waves Equilibrium** is a Markov-Perfect Equilibrium in which $H(y,\xi_1,b) \neq H(y,\xi_0,b)$ for some equilibrium object H at some state $(y,b) \in \mathcal{Y} \times \mathcal{B}$ for some $\xi_1 \neq \xi_0$, since in such an equilibrium non-fundamental confidence has real effects. A drop from ξ_1 to ξ_0 when $\xi_1 > \xi_0$ will be called a **Dynamic Panic**.

4. Theoretical Results

In what follows I characterize the theoretical properties of Confidence-Waves Equilibria. This section will be divided into two subsections: In the first, I assume that the debt is short-term to outline the basic properties of dynamic panics. In the second, I relax the short-maturity assumption and show how debt of longer maturities reacts to dynamic panics. In this latter section, I make an explicit connection the recent crisis in the Eurozone and outline policy implications as well.

4.1. Short-Term Results

The crux of the results to follow come from the existence theorem. However, I first highlight some relevant results from Arellano (2008) and Auclert and Rognlie (2014) that generalize to my environment:

Proposition 4.1 (Fundamental Uniqueness). *Any Markov-Perfect Equilibrium that does not depend on ξ is unique.*

Proof See Auclert and Rognlie (2014).

This proposition, which I take as motivational, dictates that any Markov-Perfect equilibrium that is independent of confidence fluctuations is unique.[5] This is the result that the computational literature has implicitly relied on in some capacity when performing quantitative analysis and it rules out the possibility for self-fulfilling crisis dynamics.

The next result is my own and demonstrates that despite this uniqueness result we can in fact have real effects from non-fundamental activity.

Theorem 4.2 (Existence). *Suppose that*

1. $\min \mathcal{B} > 0$ *and sufficiently high*
2. *The range of* $\tilde{m}$ *is sufficiently wide*
3. $u(c)$ *is sufficiently steep*
4. ξ *follows a sufficiently persistent process*

Then a Confidence-Waves Equilibrium exists.

A graphical example of this result can be found in Figure 2:[6] There is a unique fundamental equilibrium demand schedule given by the blue dashed line, while a Confidence-Waves Equilibrium oscillates between the green line and the red line. The dynamics of the non-fundamental equilibrium play out as a panics, with the high confidence schedule being quite close to the fundamental equilibrium and the low confidence schedule reflecting a sudden, negative sentiments shock. This effect is strongest for a low endowment point, since marginal utilities are steepest at this point, which accords with the theorem.

Theorem 4.2 is an important result for two reasons. First, it provides a general existence theorem for equilibria with non-fundamental dynamics and is thus a contribution in itself; second, it gives us an idea of how the confidence waves equilibria work on a mechanical level.

[5] Although existence is typically taken for granted by the literature, it is not difficult to show. For the sake of brevity, I forgo the proof.

[6] To calibrate these examples, I simply use the parameterization of Chatterjee and Eyigungor (2012) but I shorten the maturity of the debt, remove the continuous endowment shock, and add a large but non-binding subsistence level of consumption to induce steeper flow utility functions.

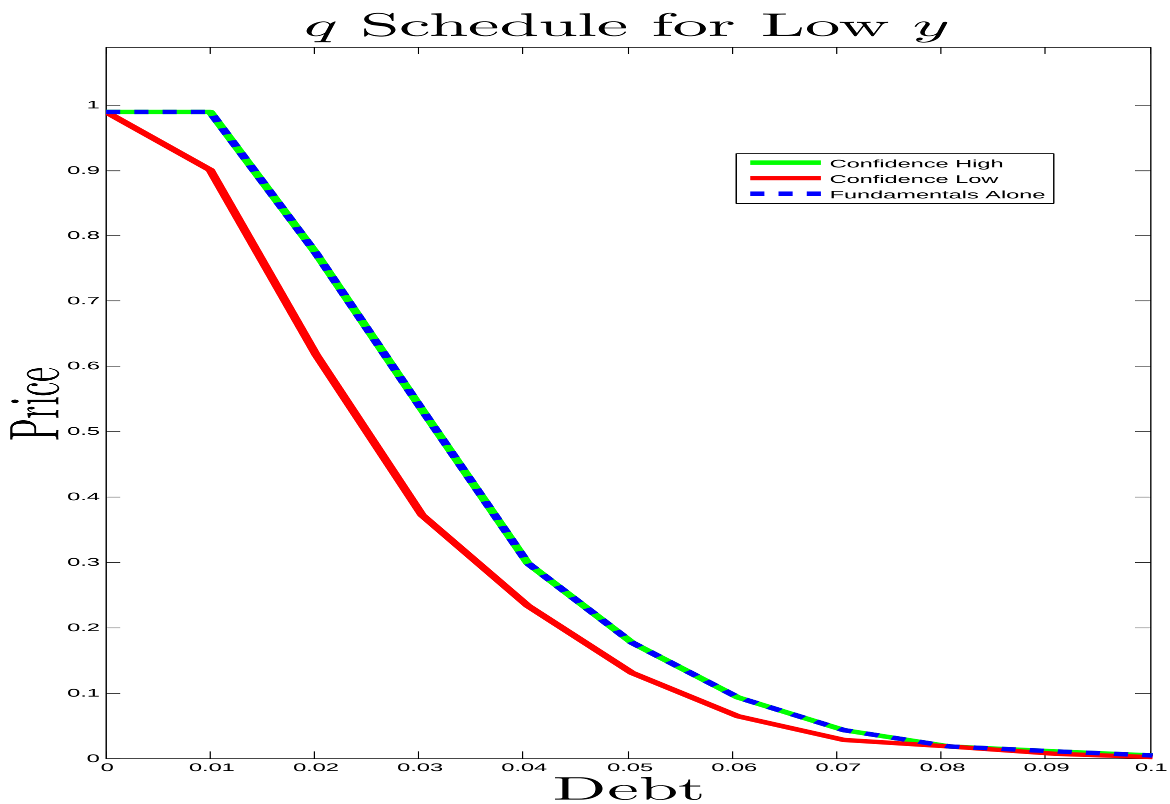

Figure 2: Confidence-Waves Equilibrium: Demand Schedule

A general existence result for non-fundamental or sunspot dynamics is difficult to demonstrate because most existence claims rely on fixed point theorems. Most fixed-point theorems in turn rely on some notion of completeness or compactness. However, the space in which sunspots are active is *neither* a complete lattice nor a compact set, since it requires that the policy function be *strictly monotone* in the sunspot, and the set of strictly increasing functions is neither a compact set nor a complete lattice. Theorem 4.2 does in fact rely on Brouwer's Fixed-Point Theorem, but to ensure that the theorem is valid I construct a compact subset of policy functions on which the sunspot is active. I then show that the functional equation operates on this compact subset.

However, the existence theorem still tells us a great deal about how confidence-waves equilibria *can work*. To see why, let us briefly discuss why the theorem holds. This existence theorem holds *precisely because* the Contraction Mapping Theorem fails: Whenever the continuation value is perturbed, this changes the propensity to default and thus the equilibrium price today. Thus, whenever the continuation value changes *so does the flow utility.* It is this interconnectedness which makes the functional equation not a contraction and it is this same interconnectedness that I use in the theorem to find a compact subset of functions that are strictly increasing in non-fundamental confidence. In particular, the discount rate of the agent normally grinds away any non-fundamental fluctuations in the infinite horizon, since $\beta < 1$ and $u(c)$ is not directly impacted by changes to V. However, in this case, when β diminishes non-fundamental fluctuations, they can be re-inflated through the flow utility function, since $u(c)$ now depends explicitly on the continuation value through the equilibrium price.

The first two conditions of the theorem simply ensure that if confidence affects value functions in the future that this will be reflected in the consumption possibilities set today. They are necessary theoretically can be ignored computationally.

The second two conditions of the theorem ensure that for a given change in the consumption possibilities set induced by the price change the sovereign's contemporaneous value changes by a sufficient amount. It highlights the fact that there is a hidden cardinality in these sovereign default models: The flow utility function can be steep or flat *relative* to the risk-neutral lenders' no-arbitrage condition. Steepness is required for Confidence-Waves Equilibria to exist. These latter two conditions tend to be necessary when finding Confidence-Waves Equilibria computationally.

We can derive several corollaries from this substantial result, which I now present and discuss in turn.

Corollary 4.3 (Panic Dynamics). *In any Confidence-Waves Equilibrium whose existence is guaranteed by Theorem 4.2, we must have that if $x_1 > x_0$, then $q(y, x_1, b') > q(y, x_0, b')$ for all $(y, b) \in Y \times B$*

Corollary 4.3 tells us how confidence will impact the equilibrium dynamics. In particular, it tells us that shifts in confidence will imply monotone shifts in the pricing function. Thus, we are correct in calling them dynamic panics. The pricing schedules will never cross, though in computational examples they may coincide at certain points.

Corollary 4.4 (Rationally Priced). *Fix the value and policy functions of the sovereign. If $\bar{x}$ is the maximum of Ξ, then $q(y, \bar{\xi}, b')$ is increasing in the degree of persistence. Likewise, if $\underline{\xi}$ is the minimum of Ξ, then $q(y, \underline{\xi}, b')$ is decreasing in the degree of persistence.*

This result simply tells us that dynamic panics are rational phenomena and will be priced accordingly. The price of debt during periods of high confidence will be lower if the probability of a panic increases, since it is more likely that the economy will transition into a regime with higher default probabilities. This result will be invoked heavily during the empirical exercise, in which I use spread data to identify the likelihood of such panics.

This last result is in many ways the most surprising. It highlights the potential danger in the Markov-Perfect structure typically assumed in computation.

Corollary 4.5 (Markov Fragility). *Whenever the conditions of Theorem 4.2 hold, then $\forall \epsilon > 0$ there exists at least one set of policy and pricing functions $\{\hat{V}, \hat{a}, \hat{q}\} : \mathcal{Y} \times \Xi \times \mathcal{B} \rightarrow \mathcal{R}$ such that*

1. $\{\hat{V}, \hat{a}, \hat{q}\}$ *is distinct from the unique fundamental Markov-Perfect Equilibrium*
2. $\{\hat{V}, \hat{a}, \hat{q}\}$ *is no further than ϵ away from satisfying the equilibrium conditions*

Corollary 4.5 follows from the fact that in any Confidence-Waves Equilibrium guaranteed by Theorem 4.2 there will be a distance $\theta > 0$ between the value functions at different confidence levels and that θ is increasing in the degree of persistence of the non-fundamental process ξ. Thus, we can increase the persistence of ξ to such a degree that the probability of leaving is arbitrarily small and thus at least one confidence regime becomes arbitrarily close to an equilibrium. Even if one confidence-regime approaches the fundamental equilibrium, the other must be a distance $\theta > 0$ from it. However, we know from uniqueness that we will never arrive at another equilibrium.

Corollary 4.5 is illustrated graphically in Figure 3. We can see that as we decrease η, which is the probability that ξ switches values, the confidence-waves demand schedules become further and further apart. The demand schedule for $\eta = 1e-10$ even passes numerical convergence criterion for an equilibrium, which suggests some caution in interpreting equilibria computed under the assumption of Markov-perfection.

4.2. Long-Term Results and the Eurozone Crisis

Having now outlined the basic properties of Confidence-Waves Equilibria and dynamic panics, I now explore what happens when we increase the maturity of the debt. In particular, I will show that long-term dynamic panics exhibit several features peculiar to the Eurozone crisis, including excessive persistence and

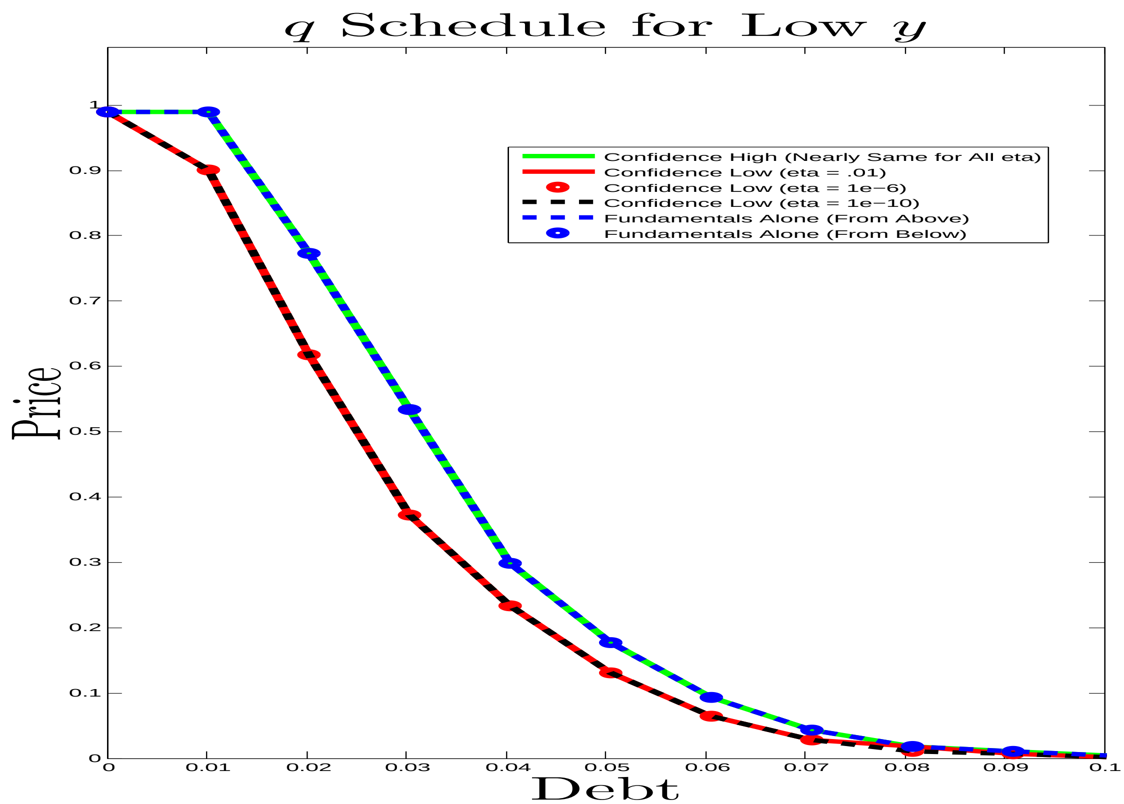

Figure 3: Confidence-Waves Equilibria for Various η (Probability of ξ Switching)

borrowing into high spreads. Because of this, I explore several policy implications of long-term dynamic panics and discuss briefly their applicability to the Eurozone crisis

I begin by defining a new term:

Definition The confidence-waves equilibrium is **Default Relevant** if the value of the confidence matters for the default decision of the sovereign.

The discreteness of the state space is important for the notion of default-relevance, since any active sunspot in a continuous state-space would always be default-relevant.

Theorem 4.6. *With short-term debt, a Markov-Perfect Equilibrium is a Confidence-Waves Equilibrium if and only if that Confidence-Waves Equilibrium is default relevant.*

Proof See Theoretical Appendix.

Theorem 4.6 tells us that if the sunspot has any real effects, it must at some point make the difference between the sovereign defaulting and repaying. This proposition disappears when we extend the maturity of the debt. It will still be the case that default relevance is sufficient to have an active sunspot, but it will no longer be necessary, since the sunspot can affect the future price of the debt if it affects borrowing behavior.

So how can borrowing alone drive non-fundamental activity? Figure 4 demonstrates. In particular, notice that with longer term debt lenders care not only about whether the sovereign defaults tomorrow, but about the future price of the debt. Thus, if lenders in period t anticipate lenders in $t+1$ to panic, it need not be the case that default probabilities actually rise in $t+1$. All that needs to happen is the default probabilities in $t+2$ rise, since this is sufficient to drive down the price in $t+1$, causing lenders in t to panic. This can happen through excessive borrowing.

To see this more clearly, suppose that lenders panic in period t. Then,

1. There is an increased expected default probability in $t+2$
2. This reduces the expected price in $t+1$
3. Price reduction in $t+1$ induces aggressive sovereign borrowing since this shock was completely non-fundamental i.e. consumption reduction and default are still costly options
4. Lower expected price in $t+1$ also implies low price in t

Although the causal chain of a long-term dynamic panic is given above, in equilibrium we will simply have two different regimes: One in which spreads, default probabilities, and borrowing are all low and another in which all of these objects are high. This can be seen in the numerical example provided in Figures 5 and 6.[7] This example shows how shifts in confidence can affect only borrowing and pricing behavior. In particular, we get two distinct pricing and borrowing regimes: One in which low borrowing occurs at low spreads and one in which high borrowing occurs at high spreads.

[7] To calibrate these examples, I simply use the parameterization of Chatterjee and Eyigungor (2012) and add a large but non-binding subsistence level of consumption to induce steeper flow utility functions.

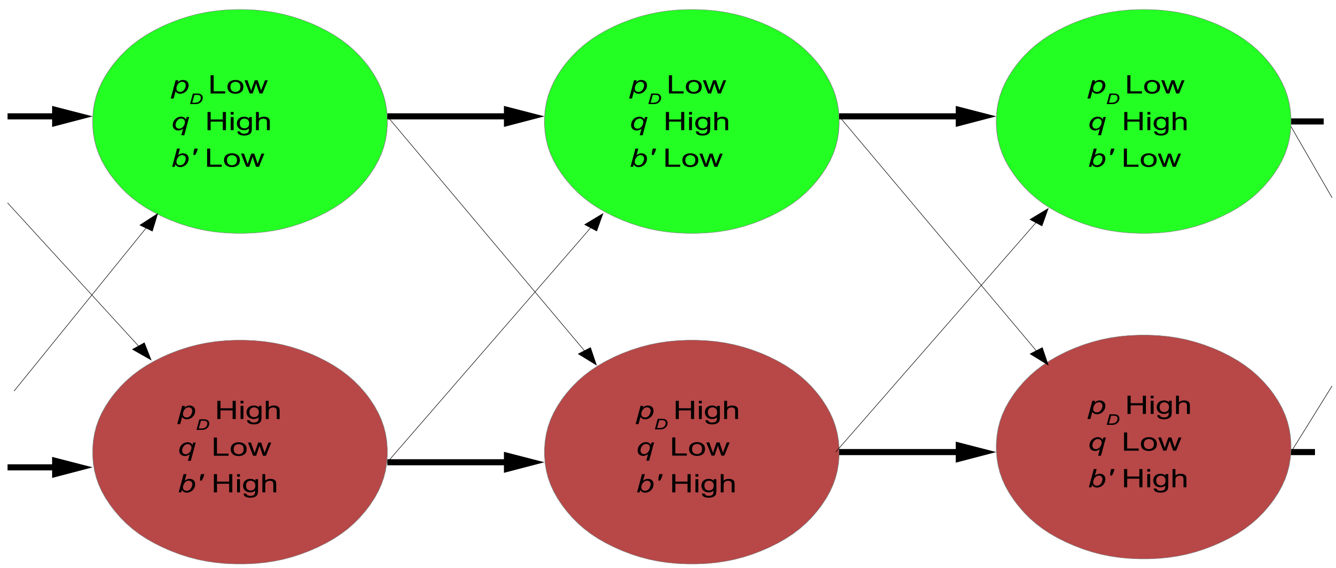

Figure 4: Long-Term Dynamic Panics: An Illustration

Further, we can see from a simulation path that a regime change is associated with higher levels of borrowing. Figure 6 compares the spreads and debt-to-GDP paths of two sample economies which face the same endowment shocks. The first economy does not respond to non-fundamental activity and the second economy is in a confidence-waves equilibrium, experiencing a dynamic panic. One can see that when the confidence falls, its effect is to simultaneously increase both the spread and the debt position of the sovereign. While the debt position does not increase substantially, the spreads experience a massive spike when confidence falls: From roughly 6% to nearly 12%. They stay consistently higher as the panic persists.

There are numerous provable characteristics of these long-term dynamics panics that I will now outline.

Proposition 4.7. *If the debt price falls uniformly when confidence drops, it must be the case that borrowing increases in some states of the world relative to high confidence.*

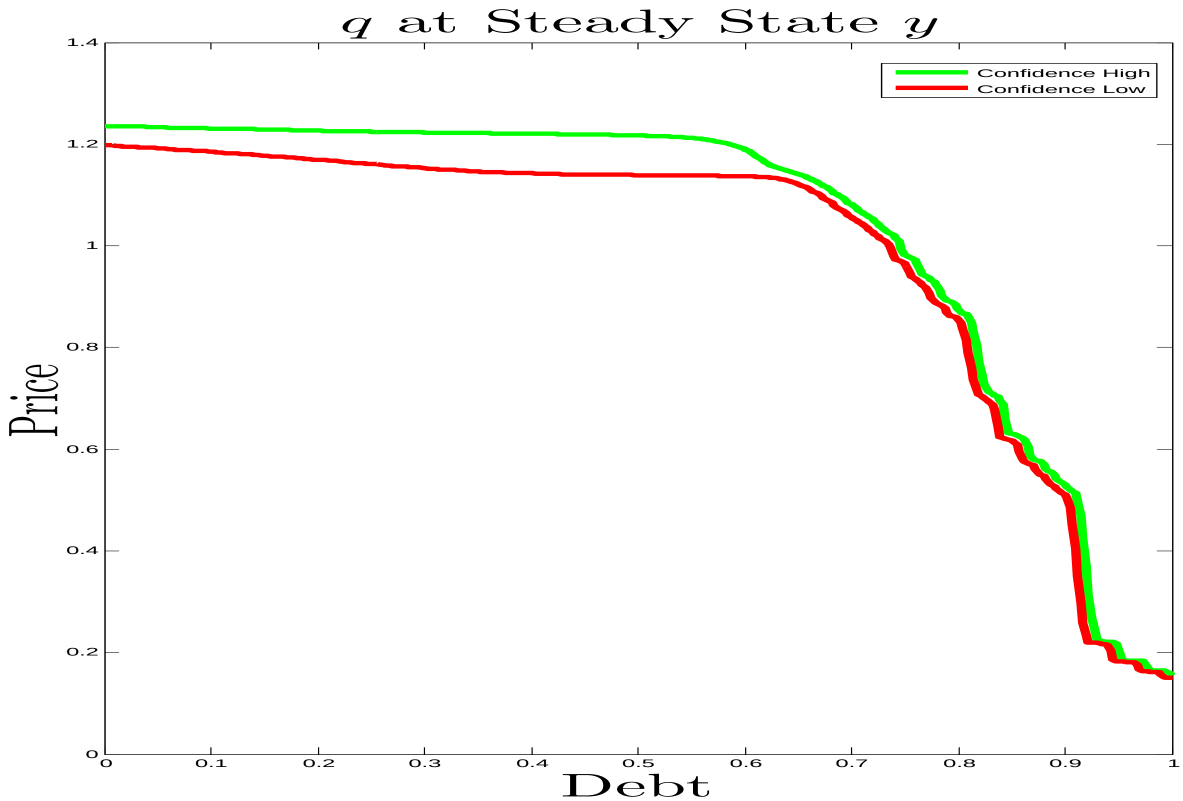

Figure 5: Demand Functions in a Long-Term Dynamic Panic (Not Default Relevant)

Proof To see why borrowing must necessarily increase in some states of the world, suppose that it did not. Suppose that, faced with a confidence shock that lowered debt prices, the sovereign delevered in every state relative to its behavior in that state with high confidence. Fixing this behavior, we turn to the pricing function, which is a conditional contraction on q. Note that when confidence shifts down, the sovereign delevers in every state. But this would necessarily *increase* the debt price relative to its high confidence counterpart, since the continuation value of the debt is increasing in B', the new debt taken on tomorrow,[8] and confidence is persistent. But this contradicts the fact that q drops with confidence. Therefore, in some states of the world, the sovereign must react to the investor panic with higher borrowing. ∎

Proposition 4.8. *Suppose that we are in a region of the state space in which the sovereign increases its borrowing when confidence drops. Then the probability of default increases even if the equilibrium is not default-relevant.*

[8]For an argument for this, see Chatterjee and Eyigungor (2012) who provide a proof of this. Their results generalize to my environment.

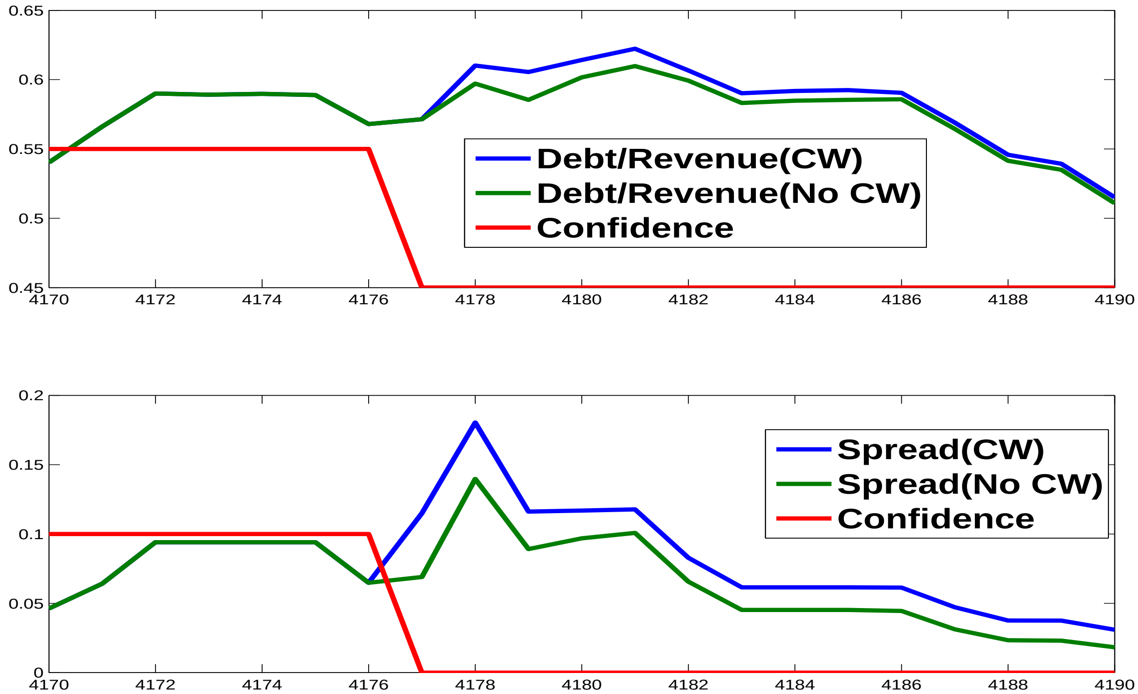

Figure 6: Simulation of a Long-Term Dynamic Panic (Not Default Relevant)

Proof This proof is simple. The value of repayment for the sovereign is increasing in b while the value of default is flat in b. Therefore, the default policy is weakly decreasing in b. Therefore, reducing b i.e. borrowing more *increases* the probability of default in the next period. This occurs even if confidence never directly impacts the default decision. $\blacksquare$

This proposition most aptly describes how deteriorating confidence has affected the Eurozone. In most countries it has not caused a panic-driven default. However, it has increased default risk *because* it has caused sovereign governments to borrow excessively in the face of this confidence.

I now explore two new policy implications in this environment. The first is the efficacy of a rate ceiling. Many authors, including Corsetti and Dedola (2013) and Lorenzoni and Werning (2013) have argued that an interest rate ceiling could have been an effective tool in combating malignant market sentiments. The reason is the following: A graph of revenue versus debt at any debt-auction ought to be parabolic since low levels of debt with have high prices and thus raise revenue but high levels of debt will have lower prices due to increased default probabilities and thus actually lower revenue. Some examples of such Laffer-curves can be seen in Figure 7.

These authors follow Calvo (1988) in asserting that the confidence crisis experienced by the Eurozone was a result of the sovereign winding up on the right-hand side of this Laffer-curve i.e. raising the same amount of revenue but with higher debt and worse prices. In the presence of such a crisis, an interest rate ceiling can be an effective tool since it forces the investors to coordinate on the good equilibrium on the left-hand side of the Laffer curve.

This policy implication is lost when the crisis at hand is a confidence-waves crisis and not a Calvo-style crisis, as is made clear by the following proposition.

Proposition 4.9. *During a dynamic panic, a binding interest rate ceiling is equivalent to a revenue cap on debt issuance. Thus a binding, temporary interest rate ceiling will increase the probability of default.*

Proof First, note that whether the economy is in a crisis or not the sovereign optimally borrows on the left-hand side of the 'Laffer curve', which plots revenue against debt issuance. This is because the sovereign can commit to not only to revenue raised at debt auctions, but also to the amount of debt issued. Thus, given to issuance options yielding the same revenue, the sovereign will always choose the one with less debt, since the value function is decreasing the level of debt.

During a dynamic panic, the entire Laffer curve shifts but the sovereign continues to remain on the left-hand side of it. Therefore, if we implement a binding interest-rate ceiling, it will necessarily lower the quantity of debt that can be issued. This is because the demand curve for debt is downsloping[9], so a price floor (rate ceiling) translates directly to a ceiling on debt issuance. A ceiling on debt issuance also places a ceiling on the revenue that can be raised, since the sovereign is located on an upsloping portion of the Laffer curve. Since the ceiling is temporary, tomorrow the sovereign can expect to resume with the equilibrium dynamics.

Denote the value of the sovereign who faces the original equilibrium demand functions with an interest rate ceiling as $\hat{V}(y, \xi, b')$. Note that since $\hat{V}(y, \xi, b')$ is the objective function of the same maximiza-

[9]See Chatterjee and Eyigungor (2012) for a proof of this.

tion as $V(y,\xi,b')$ but with an additional constraint, specifically one on revenue, we will necessarily have $\hat{V}(y,\xi,b') \leq V(y,\xi,b')$. Therefore, the probability of default has risen. $\blacksquare$

There is an intuitive graphical exposition of Proposition 4.9 in Figure 7. The black line represents the debt cap imposed by the rate ceiling. Consider the level of revenue raised by the horizontal dashed line. The typical Calvo-style multiplicity dictates that the sovereign is on the far-right intersection with the blue curve, and thus a rate ceiling such as this forces investors to coordinate back on the good equilibrium on the left side of the curve.

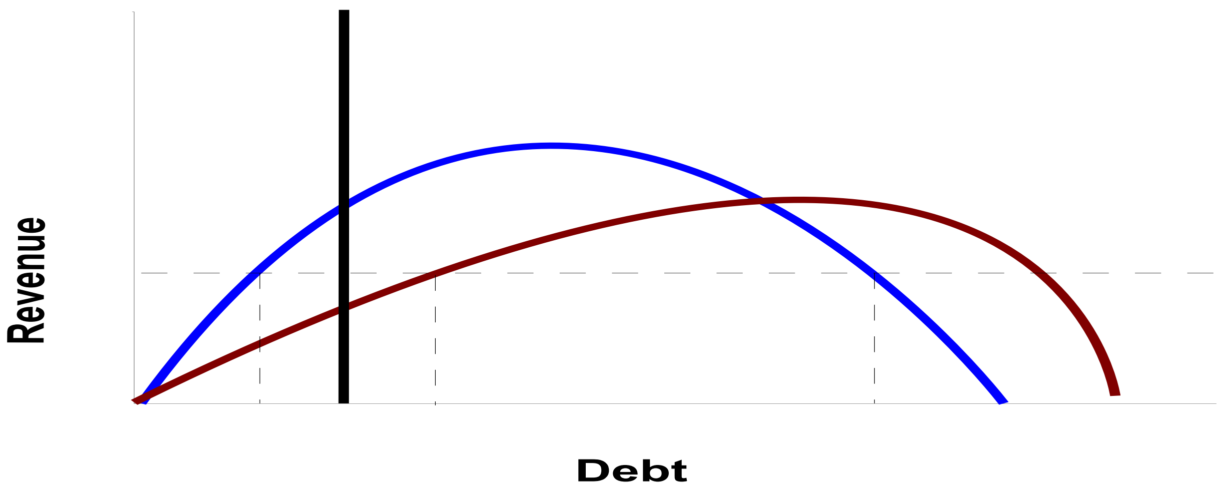

Figure 7: Laffer-Curves: Non-crisis (Blue) and Crisis (Red)

However, during a dynamic panic, we are not on the right side of the blue curve; we are in fact on the left side of a new red curve that implies less revenue raised for any given level of debt. The debt cap imposed by the black line then simply implies a revenue cap. Given this revenue cap, consumption must drop in the case of repayment and so repayment becomes less attractive and default frequencies rise.

It is important to note that an interest rate ceiling is different than the arguably successful measures that the ECB took to avert the crisis with the establishment of the European Financial Stability Facility (EFSF) or the Outright Monetary Transactions (OMT) bond-buying program. As noted by Corsetti and Dedola (2013), these programs were note rate ceilings but guarantees that the ECB would purchase government debt at sub-market interest rates. I follow De Grauwe (2011) in calling such a policy *liquidity provision*. The next proposition demonstrates that in fact such a policy may be effective.

Proposition 4.10. *Suppose that there is a continuous, additive iid endowment shock s of very small variance.*[10] *Then liquidity provision can eliminate the impact of confidence fluctuations without the need to actually purchase any assets. The resulting economy will still suffer from a weakly positive probability of default.*

Proof See Theoretical Appendix.

Proposition 4.10 tells us that the ECB can in fact judiciously apply provision of liquidity to eliminate the impact of malignant market sentiments, as they effectively did with the OMT Program and the EFSF. Such a policy does not actually require a purchasing of the debt, so long as the implied demand schedule is consistent with an equilibrium not subject to confidence fluctuations.

What is less clear from this proposition alone are the welfare implications of such a policy. It is not clear that the ECB would want to remove confidence fluctuations in the first place. After all, when the economy is not experiencing a long-term panic the sovereign can get great prices on his debt issuance. Would they want to trade the occasional long-term panic for a more constant and obvious moral hazard problem that would drive up borrowing costs at all times? The answer is ultimately quantitative, and depends on the severity of the underlying moral hazard probability in an equilibrium without confidence fluctuations. The next section explores this possibility.

5. Empirical Exercise

In the previous sections, I outlined the theoretical properties of confidence-waves equilibria. Now, I seek to understand the recent Eurozone crisis in the context of these dynamics. After my theoretical analysis of Confidence-Waves Equilibria and dynamic panics, I am left primarily with two lingering questions with regard to the Eurozone: What was the anticipated frequency of a long-term dynamic panic? And was

[10] The only reason to include this small shock is to guarantee existence in the model without confidence fluctuations. See Chatterjee and Eyigungor (2012) for more details.

the provision of liquidity through programs such as the EFSF and OMT actually welfare-improving? To answer these questions, I will employ an off-the-shelf DSGE model and add persistent, time-varying default probabilities. I will show that a shift in these default probabilities induces the same reaction from model observables as a fully endogenous dynamic panic.

I am more interested in obtaining accurate estimates for only a few relevant parameters. Therefore I prefer a structural estimation on an otherwise standard model to, for instance, simulated method of moments on the theoretical model for several reasons. First, the accuracy of the estimates can be greatly increased by adding several ingredients tangential to the theory but important to model fit, such as active households with endogenous labor supply, investment adjustment costs, and several exogenous shocks; in fact, Smets and Wouters (2007) show that Bayesian estimation of DSGE models, appropriately constructed, can outperform unrestricted VAR specifications in out-of-sample performance. For this same reason, welfare conclusions drawn for the sake of policy implications are more plausible when taken from this more standard model than the heavily stylized theoretical model. Second, such models can be solved much faster and thus are more easily estimated. And third, it can be shown at great difficulty that all of the theoretical results continue to go through when the government and households make decisions separately, provided the households value government spending separably and the government maximizes only this portion of household utility.

My identification of the stochastic structure of the confidence regimes is motivated by Corollary 4.4, which tells us that the probabilities of transitions are priced into the spreads. Thus, I can use spread data to estimate the probabilities of regime switching. Since I am interested in only a few novel parameters of an otherwise standard model, I will employ full-information likelihood-based techniques to derive my estimates. Several authors interested in full-information estimation have taken the approach of estimating models with sovereign default by specifying an exogenous rule for fiscal policy and default, instead of actually allowing for endogenous default choice, as I have done in the theoretical section. Some noteworthy examples include Bi and Traum (2012) and Bocola (2014). With an exogenous fiscal rule, the equilibrium is determinate and the likelihood function is well-defined.

However, just because the equilibrium is determinate does not mean that the model exhibits steady-state dynamics. These authors address this problem by applying the particle filter method of Fernandez-

Villaverde and Rubio-Ramirez (2007), which uses simulated 'particles' to approximate the likelihood of a given parameterization from the full non-linear specification. Rather than taking this approach, I modify the model such that it can be linearized and adopt the method of Foerster et al. (2013). The innovation in my approach is in treating a default and its concomitant costs as parameters and then perturbing those parameters to get an accurate solution.

With the perturbation method described in Foerster et al. (2013), I can capture all of the essential dynamics of confidence-waves equilibria while taking confidence and default as exogenous regime shifts. This approach will result in determinacy and substantially faster computation time. This simple model will produce spreads that explicitly price not only the probability of a sovereign default, but also of a dynamic panic.

In employing the technique of Foerster et al. (2013), I make a handful of technical contributions to speed up the computation as well. In particular, they show in their paper that the primary bottleneck to solving a DSGE model with parameter instability involves solving a quadratic system. I demonstrate that, for a large class of models, many of the unknowns in this quadratic system can be determined before explicitly solving the system. In particular, I argue that the response of investment to changes in the capital stock is the only object that needs to be solved for in the quadratic system. Once this rule is known, the rest of the equilibrium decision rules can be derived from a simple linear system. This exponentially reduces computational time while retaining the nice property that one still derives all possible first-order approximations.

5.1. Specification

In this section I present a standard DSGE model such that it retains the intuition of dynamic panics while simultaneously being suited to estimation methods. First, I have a unit mass of standard, neoclassical growth households with endogenous labor supply and preferences as in Greenwood et al. (1988). They have a constant degree of relative risk aversion, σ, and a Frisch elasticity of labor supply, χ. These households save in capital and can only trade in domestic markets.

There is a unit mass of competitive final goods firms with a Cobb-Douglas technology that experience an aggregate productivity shock, z_t. There is also a competitive investment goods sector that produces subject to convex adjustment costs $\Phi\left(\frac{i_t}{k_{t-1}}\right)$.

The government's budget constraint remains the same, but it will follow a simple fiscal rule instead of maximizing household utility. In particular, government expenditures will follow an exogenous AR(1) process in its log:

$$\log(g_t) = (1-\rho_g)\log(g^\star) + \rho_g \log(g_{t-1}) + \sigma_g \epsilon_t \tag{3}$$

To generate non-trivial borrowing behavior, I also specify the tax policy rule. What the government does not raise in domestic taxes it borrows from abroad in defaultable debt. In the event of default, it is assumed that the government sets $\tau_t = g_t$ i.e. it must use current taxes to finance all expenditures. When it is not in default, it sets taxes in response to its current debt level and the exogenous interest rate it faces from the outside investors. In particular, I assume that the government chooses a lump-sum tax policy of the following form:

$$\tau_t - \tau^\star = \gamma_b\left(b_{t-1} - \frac{\hat{\gamma}_b}{\gamma_b} b^\star\right) + \gamma_R(R_t - R^\star) + \gamma_g(g_t - g^\star) \tag{4}$$

$\tau^\star$, $b^\star$, $R^\star$, and $g^\star$ are target levels that will, in equilibrium, reflect the steady state values. The term $\hat{\gamma}_b$ simply adjusts the fiscal rule for the maturity of the debt and possibility of default. It is given by

$$\hat{\gamma}_b = \gamma_b - \lambda - (1-\lambda)\kappa + \lambda Q_{ss}$$

where Q_{ss} is the steady state price of foreign government debt. This rule is similar to those in Schmitt-Grohé and Uribe (2007) and Leeper (1991), but with the addition of explicit consideration of the foreign interest rate that will induce a downward sloping demand curve for foreign assets in response to exogenous fluctuations in the interest rate.

The parameter γ_R governs the response of the government to exogenous changes in the interest rate that it faces. When $\gamma_R > 0$, then the government raises taxes and thus lowers its debt issuance in response to interest rate shocks, generating a downsloping demand curve for foreign assets as a function of the exogenous interest rate.

γ_g determines the extent to which the government responds to shocks in spending with taxes and γ_b governs how aggressively the government responds its debt: If γ_b is high, then high debt levels are quickly adjusted; if γ_b is low, then large debt levels will linger longer. Taken together, the magnitude of $(\gamma_b, \gamma_R, \gamma_g)$

can be interpreted as a measure of fiscal discipline, since they determine the extent to which adverse shocks are funded by painful domestic taxes relative to defaultable foreign debt.

The price of debt, Q_t, continues to reflect the probability of default, but to generate more plausible debt price dynamics I allow for stochastic re-entry after default and for haircuts. In particular, I assume that in each period of default a fraction $1 - \hat{\delta}$ of the face value of the bond is destroyed. This implies a pricing recursion as follows:

$$Q_t = \frac{1}{R_t} E\left[[1 - d_{t+1}][\lambda + (1 - \lambda)(\kappa + Q_{t+1})] + d_{t+1}\hat{\delta}Q_{t+1} \right]$$

I allow for the outside option, R_t to fluctuate over time. Notice that this recursion is valid when the sovereign is in default as well.

Lastly, I assume that long-term dynamic panics enter the model exogenously as a regime shift. However, I invoke their properties from the theoretical model. In particular, we know

1. A long-term dynamic panic is associated with a change in both sovereign behavior and foreign investors' demand for debt
2. During a long-term dynamic panic, the government borrows more than it otherwise would
3. During a long-term dynamic panic, the probability of default rises.

to generate plausible long-term dynamic panic, I take this last characteristic to be truly exogenous i.e. I assume that default occurs stochastically but that it has a greater likelihood during a confidence-waves crisis.

This simple assumption gives me the other relevant properties of a confidence-waves. First, it will generate higher spreads since investors demand compensation for the higher possibility of default, and thus both parties will change their behavior during a crisis. Second, it will generate increased borrowing on the part of the government, since it must fill the same primary deficit with lower-priced debt.

Thus, a long-term dynamic panic will be associated with higher spreads, higher external borrowing, and greater default probabilities. We will also see a slump in investment and a concomitant contraction in output and consumption. This happens for two complementary reasons: First, expected productivity falls during such a panic, since default productivity costs are more likely in the future; second, consumption may actually be higher during a default, since foreign debt obligations are repudiated. Both of these effects

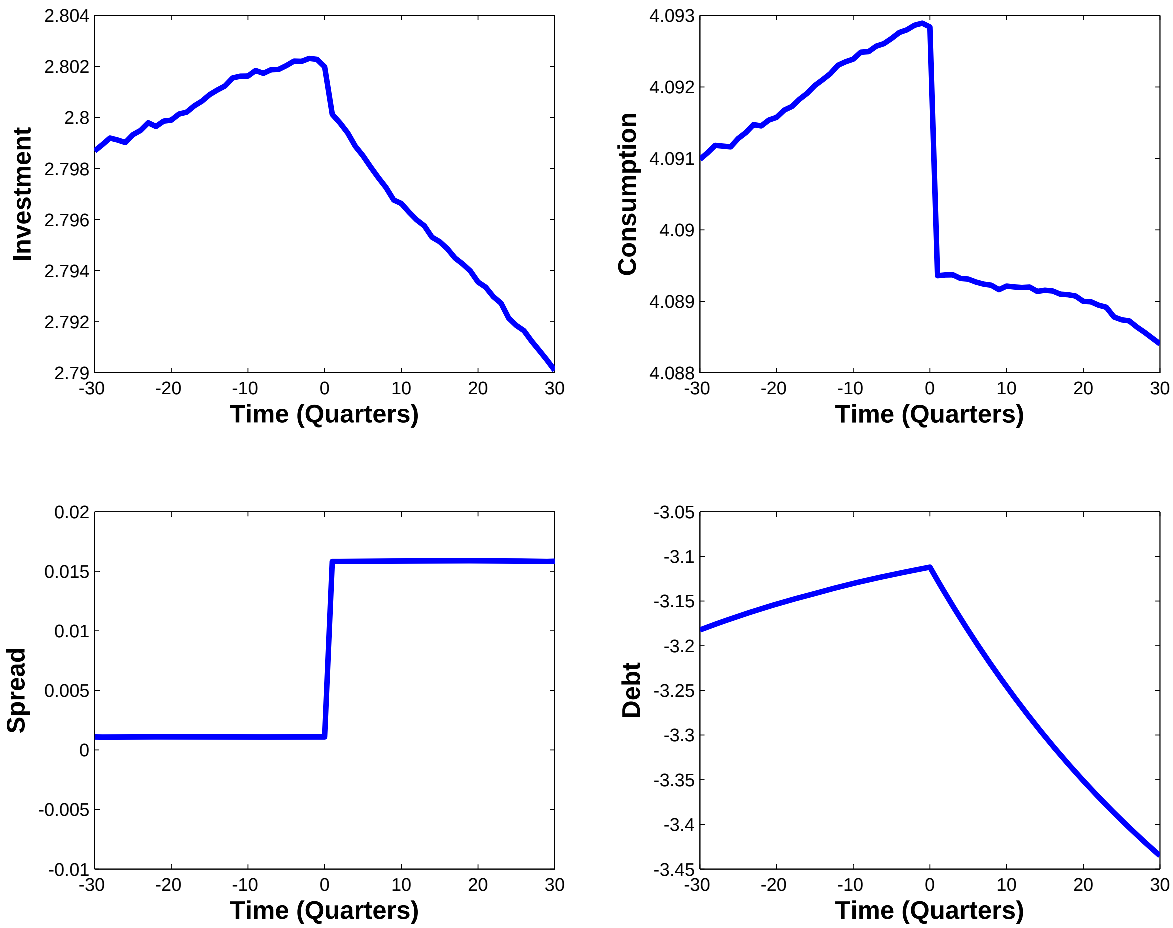

Figure 8: Mean Trajectories: Long-Term Dynamic Panic at t = 0

create a strong disincentive to save that is reflected in low investment. All of these trends can be seen in the simulation averages of the modified model in Figure 8.[11]

Lastly, for the purposes of the estimation, I assume that foreign interest rates and labor productivity follow AR(1) processes as well and that productivity drops during a default.

$$\log(z_t) = (1-\rho_z)\log(z^\star(s_t)) + \rho_z \log(z_{t-1}) + \sigma_z \epsilon_{z,t}$$

$$\log(R_t) = (1-\rho_R)\log(R^\star) + \rho_R \log(R_{t-1}) + \sigma_R \epsilon_{R,t}$$

[11] The trajectories in this figure are computed using the estimated parameters I later derive.

where s_t denotes the current regime, of which default is a possibility.

Note that a lender shock will increase the spread today even though the risk-free rate is differenced out. This is because the lender shocks are persistent i.e. $\rho \in (0,1)$. While higher rates today are differenced out of the spread, higher anticipated future rates are not. Rather, they bring down the price of debt tomorrow and thus drive down the price of debt today.

5.2. Parameter Instability

I consider an equilibrium in which four key parameters are subject to switching: $(z^\star, rr, d, p_D)$, where rr is the recovery rate on bonds in default. I denote these parameters by the vector $\theta(s_t)$, where $s_t \in \{1,2,3\}$ i.e. there are three distinct regimes. The parameters take the following values in the three different regimes:

$$\begin{pmatrix} z^\star(s_t) \\ rr(s_t) \\ d(s_t) \\ p_D(s_t) \end{pmatrix} \in \{ \begin{pmatrix} \mu \\ \delta_u \\ 0 \\ p_H \end{pmatrix}, \begin{pmatrix} \mu \\ \delta_u \\ 0 \\ p_L \end{pmatrix}, \begin{pmatrix} \mu_D \\ \hat{\delta} \\ 1 \\ 1-\pi_{RE} \end{pmatrix} \} \tag{5}$$

where $\mu_d < \mu$. $\hat{\delta}$ governs the recovery rate of bonds in default; δ_u is never observed on the equilibrium path and so can be judiciously chosen to ensure a well-defined steady state. The change from $s_t = 1$ to $s_t = 2$ will behave as the completely endogenous confidence-switching regimes described in the general theoretical model. Estimating the transition between these regimes and the implications for policy of this switch is the primary goal of this exercise, since a default has yet to occur in the Spanish data. In particular, the transition matrix $P = (p_{s',s})_{s',s=1,2,3}$ will be given by:

$$P = \begin{bmatrix} 1-\eta-p_H & \eta & p_H \\ \eta & 1-\eta-p_L & p_L \\ \pi_{RE} & 0 & 1-\pi_{RE} \end{bmatrix} \tag{6}$$

It is assumed that $p_L > p_H \geq 0$ i.e. default is more likely in the low-confidence regime. I will also assume that $\eta < .5$ i.e. the regimes are persistent. Notice that I allow for stochastic re-entry with probability ξ in the event of a default and that it is assumed, as is the case in the general model, that the sovereign re-enters credit markets with high confidence.

5.3. Model Solution

The equilibrium conditions can be written in the following form:

$$E_t[f(y_{t+1}, y_t, x_t, x_{t-1}, \chi\epsilon_{t+1}, \epsilon_t, \theta_{t+1}, \theta_t)] = \mathbf{0}_{n_x+n_y} \tag{7}$$

where $y_t = (c_t, i_t, Q_t)$ are the primary control variables, $x_t = (k_t, b_t, R_t, z_t, g_t)$ are the endogenous state variables, and $\theta_t = (p_D(s_t), z^\star(s_t), d(s_t))$ are those parameters that are subject to regime switching, and s_t denotes the current regime. $\bar{\chi}$ is the perturbation parameter. It is convenient to write the equilibrium conditions in this way because I can then apply the method of Foerster et al. (2013). Note that the function f looks as follows:

$$\begin{bmatrix} \textbf{(1)} \;\; \left[c_t - \kappa_1\left(z_t^{1-\alpha}k_{t-1}^{\alpha}\right)^{\frac{1+\chi}{1+\alpha\chi}}\right]^{-\sigma} - \beta\left[c_{t+1} - \kappa_1\left(\left[z^\star(s_{t+1})^{1-\rho_z} z_t^{\rho_z} e^{\sigma_z\bar{\chi}\epsilon_{z,t+1}}\right]^{1-\alpha} k_t^{\alpha}\right)^{\frac{1+\chi}{1+\alpha\chi}}\right]^{-\sigma} \times \\ \left(\kappa_2\left[\left[z^\star(s_{t+1})^{1-\rho_z} z_t^{\rho_z} e^{\sigma_z\bar{\chi}\epsilon_{z,t+1}}\right]^{1-\alpha} k_t^{\frac{\alpha-1}{1+\chi}}\right]^{\frac{1+\chi}{1+\alpha\chi}} + (1-\delta)\left(1 + \Phi\left(\frac{i_{t+1}}{k_t}\right) + \Phi'\left(\frac{i_{t+1}}{k_t}\right)\frac{i_{t+1}}{k_t}\right)\right) \\ \textbf{(2)} \;\; c_t + \left[1 + \Phi\left(\frac{i_t}{k_{t-1}}\right)\right] i_t + g_t - [1-d(s_t)][\lambda + (1-\lambda)(\kappa + Q_t)]b_{t-1} + Q_t b_t - \kappa_0\left[z_t^{1-\alpha}k_{t-1}^{\alpha}\right]^{\frac{1+\chi}{1+\alpha\chi}} \\ \textbf{(3)} \;\; -Q_t b_t + d_t Q_t rr(s_t) b_{t-1} + \\ [1-d(s_t)]\left[(\lambda + (1-\lambda)(\kappa + Q_t) - \gamma_b)b_{t-1} + \gamma_R R_t + (\gamma_g - 1)g_t + \left(\tau^\star + \hat{\gamma}_b b^\star - \gamma_R R^\star - \gamma_g g^\star\right)\right] \\ \textbf{(4)} \;\; k_t - (1-\delta)k_{t-1} - i_t \\ \textbf{(5)} \;\; \log(R_t) - (1-\rho_r)\log(R^\star) - \rho_R\log(R_{t-1}) - \sigma_R\epsilon_{R,t} \\ \textbf{(6)} \;\; \log(g_t) - (1-\rho_g)\log(g^\star) - \rho_g\log(g_{t-1}) - \sigma_g\epsilon_{g,t} \\ \textbf{(7)} \;\; \log(z_t) - (1-\rho_z)\log(z^\star(s_t)) - \rho_z\log(z_{t-1}) - \sigma_z\epsilon_{z,t} \\ \textbf{(8)} \;\; Q_t - \frac{1}{R_t}\left[[1-d(s_{t+1})][\lambda + (1-\lambda)(\kappa + Q_{t+1})] + d(s_{t+1}) rr(s_t) Q_{t+1}\right] \end{bmatrix} \tag{8}$$

I seek a solution to this model of the following form:

$$y_t = g(x_{t-1}, \epsilon_t, \bar{\chi}, s_t), \;\; y_{t+1} = g(x_t, \bar{\chi}\epsilon_t, \chi, s_{t+1}), \;\; x_t = h(x_{t-1}, \epsilon_t, \bar{\chi}, s_t) \tag{9}$$

An exact solution to this model is computationally burdensome and, given the model's design, unnecessary. Instead, I will find a linear approximation to the model around a non-stochastic steady state. I will search for a set of matrices $\{g_{ss}(s_t), h_{ss}(s_t)\}_{s_t=1,n_s}$, where $g_{ss}(s_t)$ has dimension $n_y \times (n_x + n_\epsilon + 1)$ and $h_{ss}(s_t)$ has dimension $n_x \times (n_x + n_\epsilon + 1)$. When in a regime s_t, $g_{ss}(s_t)$ will map deviations in $(x_{t-1}, \epsilon_t, \bar{\chi})$ from their non-stochastic steady state into deviations of y_t from its non-stochastic steady state. Thus, if $\hat{z}_t$ is the steady-state deviation of an equilibrium object, z_t, then $\hat{y}_t = g_{ss}(s_t)[\hat{x}_{t-1}', \epsilon_t, \bar{\chi}]'$ and $\hat{x}_t = h_{ss}(s_t)[\hat{x}_{t-1}', \epsilon_t, \bar{\chi}]'$

when in a regime s_t.[12]

In order to perturb this model, I must have a well-defined notion of a steady state that is independent of the Markov-switching regimes. To do so, I follow Foerster et al. and perturb the parameters $(d(s_t), z^\star(s_t))$ as follows:

$$\begin{pmatrix} z^\star(\chi, s_t) \\ rr(\chi, s_t) \\ d(\chi, s_t) \end{pmatrix} = \begin{pmatrix} \bar{z} \\ \bar{rr} \\ \bar{d} \end{pmatrix} + \bar{\chi} \begin{pmatrix} \hat{z}(s_t) \\ \hat{rr}(s_t) \\ \hat{d}(s_t) \end{pmatrix} \tag{10}$$

where $\hat{z}(s_t) = z^\star(s_t) - \bar{z}$, $\hat{rr}(s_t) = rr(s_t) - \bar{rr}$, and $\hat{d}(s_t) = d(s_t) - \bar{d}$ and $(\bar{z}, \bar{rr}, \bar{d})$ are taken to be the ergodic mean of $(z^\star(s_t), rr(s_t), d(s_t))$. I construct the steady state of the dynamic system in terms of the $(\bar{z}, \bar{rr}, \bar{d})$, and thus the steady state is independent of the current regime. I calibrate $\hat{\delta}$ and choose $\delta_u(\hat{\delta})$ to ensure that $\bar{rr} = 1$, which guarantees that the following expression holds:

$$f(\bar{y}, \bar{y}, \bar{x}, \bar{x}, 0, 0, \bar{\theta}, \bar{\theta}) = \mathbf{0}_{(n_x+n_y)\times 1}$$

i.e. the equilibrium conditions equate to zero at the non-stochastic steady state.

In order to solve this system, I must take a series of derivatives of Equation 8 with respect to all endogenous objects and the Markov-switching parameters and evaluate them at the steady state. Foerster et al. (2013) demonstrate that a first-order approximation to the solutions g and f can then be obtained in two steps. The first step entails solving the following quadratic system for $\{D_{1,n_x} g_{ss}(s_t), D_{1,n_x} h_{ss}(s_t)\}_{s_t=1}^{n_s}$, which are the first n_x columns of the approximated policy rules and laws of motion, respectively, for each state. The relevant quadratic system is given below:

$$A(s_t) \begin{bmatrix} \mathbf{I}_{n_x} \\ D_{1,n_x} g_{ss}(1) \\ \ldots \\ D_{1,n_x} g_{ss}(n_s) \end{bmatrix} D_{1,n_x} h_{ss}(s_t) = B(s_t) \begin{bmatrix} \mathbf{I}_{n_x} \\ D_{1,n_x} g_{ss}(s_t) \end{bmatrix} \tag{11}$$

for all s_t. Where $A(s_t)$ is an $(n_x + n_y) \times (n_x + n_s n_y)$ matrix and $B(s_t)$ is an $(n_x + n_y) \times (n_x + n_y)$ matrix. Both are functions of the derivatives of Equation 8 and their full specification can be found in Foerster et

[12] The non-stochastic steady state of ϵ_t and $\bar{\chi}$ are 0. $\bar{\chi}$ is 1 in the perturbation solution.

al. (2013).

Once a solution to Equation 11 has been found, the remaining elements of the matrices h_{ss} and g_{ss} can be found by solving a simple linear system that is provided in the computational appendix. If there are multiple mean-square stable approximations, I denote the one that provides the *higher* likelihood to be the true one.[13]

I validate the accuracy of the first-order approximation by checking the unconditional Euler Equation errors, as suggested by Foerster et al. (2013). I find that at the posterior mean, $\log_{10}(EE\ Error) = -3.6078$.[14] To put this figure in perspective, note that when this object is -3 (-4), there is a \$1 error for every \$1,000 (\$10,000) of consumption determined by the Euler Equation.

5.3.1. Reducing the Dimensionality

Equation 11 is the matrix representation of a quadratic system with $n_s n_x(n_x + n_y)$ equations and the same number of unknowns. Foerster et al. (2013) suggest the use of Grobner bases to solve for all possible solutions to this system. While this method is satisfyingly exhaustive, its full implementation can be burdensome, as computational time is for most algorithms is exponential or even doubly exponential in the number of potential solutions. In their expository examples, the number of unknowns i.e. $n_s n_x(n_x+n_y)$ is never more than 8.

The solution I seek has a substantially larger dimensionality. In particular, $n_s n_x(n_x + n_y) = 3 \times 5 \times 8 = 120$. Given an exponential rate, even one iteration could take hundreds of thousands of years to compute, which is clearly impractical. However, there is much that we know about how the equilibrium operates that can be imposed on the solution before we even begin that allow me to reduce the dimensionality of th system. In particular, I develop a new method called the **Capital-Motion Algorithm** that reduces the quadratic system to 3 equations in 3 unknowns, which can be solved in hundredths of a second. The Capital-Motion Algorithm can be implemented in a wide class of models beyond the one at hand, and thus render this solution method more applicable in general.

The **Capital-Motion Algorithm** proceeds in 4 steps:

1. Fix the coefficients governing exogenous laws of motion.

[13] Note that a multiplicity of first-order approximations *does not imply* a multiplicity of equilibria. The equilibrium of the model is demonstrably determinant.

[14] This error is much smaller than comparable models described by Foerster et al. (2013) that also have some form of adjustment costs.

2. Use the resource constraint to express consumption in terms of investment.
3. Solve a smaller quadratic system to determine the derivative $i_k(s_t)$ in each state i.e. how investment responds to a shock to capital.
4. For each solution $i_k(s_t)$, solve a linear system to determine all other model derivatives.

We can demonstrate a very useful property of the Capital-Motion Algorithm as it pertains to our case:

Theorem 5.1. *The Capital-Motion Algorithm reduces the dimensionality of the quadratic system governing the model solution from 120 equations/unknowns to 3 equations/unknowns and still delivers all solutions to the original system.*

Proof See Computational Appendix.

Theorem 5.1 is tailored to the model at hand for simplicity, but it can be generalized to a wider class of models: Essentially any model for which the crux of the intertemporal dynamics is the Euler equation. It relies on the fact that the key unknowns in the quadratic system are the coefficients governing the future capital choice in each state with respect to shocks to current capital, of which there are n_s i.e. one for each state. The decision of the agent to consume or to save today will reflect his propensity to consume or save tomorrow *in response to the same shock*; hence, when the rule is linear the relevant system is quadratic. Given the information imposed on the system so far, knowledge of saving behavior tomorrow in response to a capital shock in each of the different states is sufficient for determining saving behavior today in response to the same shock, and thus we can solve for these objects.

Once the saving response to a capital shock has been determined, the other equilibrium objects can be solved for linearly, since none of the others require solving an intertemporal problem, which is the source of the problem's quadratic nature.[15] In other words, a shock to any other object in the model will only be affected by future objects insofar as it has adjusted investment *today*.

Theorem 9.4 reduces the dimensionality of the problem drastically, to the point where there are only n_s coefficients that must be solved for, which is 3 in this model. Upon reaching this point, I can easily solve

[15] A general form of Proposition 9.4 would entail solving for the number of endogenous equilibrium objects that jointly affect the intertemporal decision. For instance, if we were to introduce cyclical fiscal policy into the model, then we would need to solve a quadratic system of $2n_s$ equations and unknowns, since the dynamics of investment and debt be interdependent.

the model for all possible solutions in fractions of a second and proceed with the rest of the solution as described in the previous section.

5.4. Estimation Procedure

5.4.1. Data

I use three quarterly time series data from Spain from 2001 until 2012: GNP, 5-year CDS spreads, and the public current account as a fraction of GNP. I take gross external government debt and GNP from the ECB Statistical Data Warehouse and I take spread data on 5-year debt from the MARKIT database. The first two objects require detrending, for which I use a Hodrick-Prescott filter (Hodrick and Prescott (1997)) to remain agnostic. The spread data requires no de-trending, though I reduce the frequency from daily to quarterly by means of an average. The filtered data can be found in Figure 11 in the Computational Appendix.[16]

I use Spanish data because De Grauwe (2011) and others have noted that Spain's crisis seems most confidence-driven i.e. spreads tend to co-move the least with its fundamentals relative to other countries in the Eurozone periphery, although they all exhibit these trends. Since it is this shift in confidence that I seek to estimate, I find Spanish data to be the most appropriate tool available.

5.4.2. Identification

The shocks are all identified because they have orthogonal impacts on the observable variables: An interest rate shock today will impact positively debt and spreads but leave contemporaneous output untouched; a government policy shock will have a negative impact on debt but leave spreads and output untouched; and a productivity shock will increase contemporaneous output without impacting either the debt level or the spreads.

There are two potential sources of identification of the probability of regime switching: First, there is the length of time that the economy spends in one period versus another; and second, there is the fact that the probability of a regime switch is priced into the spreads. Given the relatively short span of the data, I follow the second approach for identification.

To identify this probability from spread data, I make a couple of identifying assumptions. First, I assume that the probability of regime switching is symmetric. This condition, although perhaps not necessary, is

[16] In this figure, I mark the start date of the crisis as 36 quarters, which corresponds to 2009Q4.

the condition under which I can ensure existence in the theoretical model. Second, I assume that there is no probability of default in normal times i.e. $p_H = 0$. Thus, any positive spread that we see in normal times reflects the possibility of a regime shift into a panic state in which a default is possible. The divergence in the spreads in the panic state plus the symmetry assumption will allow me to jointly pin down the probability of default and the probability of a confidence shift. Note that the probability of a default in a panic is not uniquely determined by the spread, however. Private sector expectations, through investment and consumption, help provide additional identification regarding this probability.

Last, I denote the start of the crisis i.e. the regime shift as occurring in 2009Q4, which roughly dates the start of the sovereign debt crisis by most accounts. Since there is little dispute regarding the start date of the crisis, I prefer this route to estimating the start date, an approach taken by, for instance, Bianchi (2013).

5.4.3. Calibration

I calibrate many of the model's key parameters and estimate only 5: η, p_L, γ_b, γ_g, and γ_r. The parameterization is given in Table 1. A few require discussion.

First, I choose z_L based on Mendoza and Yue (2012) i.e. 5% less than normal times. I choose the mean risk-free rate to be 0.01. To match the volatility, I choose to set the volatility of this shock to the estimated volatility of a T-bill to an emerging market from Neumeyer and Perri (2005). As for the defaultable debt, I assume a 4% coupon, which is in line with the average for Spanish long-term debt, and choose λ to match the average maturity of Spanish debt, which was 6.5 years at the time of the crisis.

The specification of the government spending and TFP shocks is taken from Arias et al. (2007) who calibrate these parameters in a simple RBC model. Though they calibrate to the US, several authors have noted that the Spanish economy before the crisis was not terribly different from the US in its cyclical properties (see Puch and Licandro (1997)).

I calibrate π_{RE} such that the average default lasts 2 years, which is roughly the length of Grecian exclusion from international credit markets following its default in 2012. The results do not substantially change even when this parameter varies widely. I also calibrate the haircut following default to match that of Greece on average. In particular, note that the face value of a bond in default is given by

$$\begin{aligned}\tilde{b_t} &= \sum_{\tau=1}^{\infty} \pi_{RE}(1-\pi_{RE})^{\tau-1}\left(\frac{\hat{\delta}}{R^\star}\right)^{\tau} b_t \\ \rightarrow \frac{\tilde{b_t}}{b_t} &= \sum_{\tau=1}^{\infty} \pi_{RE}(1-\pi_{RE})^{\tau-1}\left(\frac{\hat{\delta}}{R^\star}\right)^{\tau} \qquad (12)\end{aligned}$$

I equate this expression to the average recovery rate of the face value of Greek debt, which was 29.5%, to determine the value of $\hat{\delta}$.

I calibrate impatience,[17] adjustment costs, the Frisch elasticity of labor supply, labor disutility, capital share of income, intertemporal elasticity of substitution, and capital depreciation to standard values in the RBC literature.

I take the fraction of average foreign debt to GNP as well as government spending to GNP direction from Spanish data.

5.4.4. Estimation Procedure

The remaining 5 parameters, which are those of most interest, are estimated using a Bayesian approach. I specify the priors in Table 2. I take a fairly agnostic stance with regard to the prior distributions, assuming that they all fall in the range $[0, 1]$[18] and that the government does not respond to adverse shocks 1 to 1 with taxes i.e. it smooths such shocks with debt. Changing these priors change the results only negligibly.

Their distribution is attained via a Random-Walk Metropolis Algorithm as outlined in Schorfheide and An (2007). To derive the likelihood of a particular parameterization, I solve the model to a first-order approximation using the algorithm described previously. I then place this approximation into state-space form and apply the Kalman filter, taking the series on output, the public current account, and spreads to be my observables.[19]

After specifying the prior, I follow the RWM outlined in Schorfheide and An (2007) to obtain a modal estimate and simulate draws from the posterior distribution. Although the entire distribution is of interest for constructing credible sets and understanding how the data operate, I am most interested in the point estimates given by the mode, since they will provide my estimates of probability of regime switches.

[17]This parameter is on the high end of standard discount rates. This simply helps to ensure that household cares enough about the future to disinvest when a default is likely. A low depreciation rate serves this same purpose.

[18]The domain restriction imposed by the priors helps deliver comparable estimates across different models.

[19]I initialize the Kalman filter mean at the non-stochastic steady state and the variance according to the non-crisis regime.

Parameter	Value	Rationale
σ	2	RBC Standard
α	0.33	RBC Standard
β	0.999	RBC Standard
δ	0.005	RBC Standard
z_H	1.0	RBC Standard
Φ''	0.455	RBC Standard
χ	0.4	RBC Standard
ψ	1.5	RBC Standard
$R^\star$	1.01	RBC Standard
z_L	0.95	Mendoza and Yue (2012)
$b^\star$	-0.19653 $\times y_{ss}$	Spanish Data (ECB SWD)
$g^\star$	0.375289 $\times y_{ss}$	Spanish Data (ECB SWD)
ρ_z	0.95	Arias et al. (2007)
σ_z	0.0031	Arias et al. (2007)
σ_g	0.0077	Arias et al. (2007)
ρg	0.98	Arias et al. (2007)
σ_r	0.0063	Neumeyer and Perri (2005)
ρ_r	0.81	Neumeyer and Perri (2005)
π_{RE}	0.125	Match Grecian Exclusion
λ	0.0385	Match Average Maturity
κ	0.04	Match Average Coupon
$\hat{\delta}$	0.7861	Equation 12
p_H	0.0	Calibrated (Identification)

Table 1: Calibrated Parameters

Parameter	Distribution	Parameter 1: a	Parameter 2: b	Implied Mode
γ_b	Beta	3.0	3.0	0.5
γ_g	Beta	3.0	3.0	0.5
γ_R	Beta	3.0	3.0	0.5
η	Beta	1.010	3.0	0.01
p_L	Beta	1.002	3.0	0.10

Table 2: Prior Distributions

Parameter	Median (CW)	Mean (CW)	Credible Set (CW)	Median (CW)	Mean (No CW)	Credible Set (No CW)
γ_b	0.0584	0.0616	[0.0267, 0.1109]	0.0646	0.0704	[0.0280, 0.1330]
γ_g	0.5095	0.5064	[0.4037, 0.5987]	0.4869	0.4803	[0.3579, 0.5813]
γ_r	0.6441	0.6327	[0.3402, 0.8816]	0.6319	0.6156	[0.3050, 0.8626]
η	0.0339	0.0472	[0.0080, 0.1394]	N/A	N/A	N/A
p_L	0.2204	0.2710	[0.0744, 0.6336]	0.0297	0.0304	[0.0072, 0.0556]

Table 3: Posterior Statistics and 90% Credible Sets

5.5. Results

In this section I compare the results of the model with confidence waves and without in terms of model fit and parameter estimates. For the model without confidence waves, I impose that $p_L = p_H$ and that $\eta = 0.0$. The estimates and 90% credible sets for the both models are given in Table 3 below. The mean and the median are similar in all regards, but for the purposes of inference I will take the posterior median to be my primary estimate. The posterior distributions can be found in Figures 15 and 16 in the Computational Appendix.

First, notice that the estimate of the probability of regime-switching is actually not that unlikely: The data suggest a median estimate of 3.39%, which corresponds to roughly 7.37 years. Recall that this figure is identified from spread data prior to the crisis and spread levels during the crisis. It does not use the relative frequency of the regimes. If we take 2002 to be the initial date at which investors fully anticipated monetary union for the foreseeable future, this figure seems quite reasonable: Investors expected such a crisis to occur on average every 7.37 years and it took about 7.5 years for one to occur. The closeness of this non-targeted moment speaks strongly to the validity of the model.

The mean estimate of η is actually slightly higher than the median estimate: 4.72%. If we took this to be our estimate, it would imply that investors anticipated a crisis once every 5.3 years. This suggests that, if anything, investors anticipated such crises to occur more frequently than they actually did, not less frequently. Even though the credible set reaches all the way to 13.94%, Figure 15 shows that most of the mass is concentrated around the mean and median and that there is a long, thin tail on the right-hand side.

Though I do not report it in the table, the posterior odds ratio of the model with confidence-waves relative to the model without them is 94.39 given an even prior across the models, which is greater than one. This suggests that the data favor the model with confidence waves.

Notice further that the estimates regarding the fiscal rule tell us something interesting as well. In particular, for government spending and foreign shocks, they are higher in the model with confidence waves.[20] These coefficients are a model-based measure of fiscal discipline or responsibility, since they govern the strength with which the sovereign responds to it debt and foreign shocks with painful domestic tax adjustments. The model without confidence waves suggests a certain degree of fiscal 'irresponsibility' that is not necessarily there when one accounts for the large shock experienced by the sovereign during a confidence-waves crisis. These results suggest that the need for austerity packages targeted at fiscal discipline may not be as large as would be suggested by a cursory look at the data.

The reason for this in the context of the model is that, when there are no dynamic panics, the model interprets the high spreads as coming solely from lender shocks. These shocks will increase debt levels, but in the data, this debt hangs around and in fact grows. There are three ways the model can accommodate this: First, the government cannot respond too strongly to this faster accumulation of debt with a lower γ_b; second, it can generate higher levels of government spending (which is not observed) to induce the higher debt levels so long as those spending shocks are not counteracted i.e. lower γ_g; and third, it can not respond as strongly to those interest rate shocks with taxes i.e. lower γ_R. However, in the model with confidence-waves, *not all of interest rate shocks are lender shocks.* Part of them are intrinsically caused by the equilibrium actions of the sovereign, as is reflected by the higher default probability. Because of this, the sovereign can maintain several higher measures of fiscal discipline while simultaneously experiencing a panic.

Last, let us turn to the estimated probability of default. Notice first that it is quite high in the model with confidence waves, with default expected to occur with probability 22.04% in any given quarter. This is in accordance with Proposition 4.8. However, this default probability is not terribly well-identified, as can be seen by the broad span of the credible set. This is simply because the transition probabilities are pinned down by the mean of the spreads in each regime, and there are only 9 quarters in the data used to

[20] It is true that the estimated 'responsibility' parameter governing the response to past debt levels is higher in the model without confidence-waves, but the largest and most active shocks in this model are the government spending shocks. This can be seen in Figure 13 in the Computational Appendix. Thus, the parameter governing the government's response to this shock is the most telling measure of fiscal virtue and it is this one that is most profoundly smaller in the model without confidence waves.

pin down the default transition. The credible set for the confidence-wave estimate is smaller because there are 36 quarters over which to average, implying greater accuracy in the face of unrelated lender shocks.

Finally, notice that estimated probability of default is higher in the model with dynamic panics. This result is the least surprising, since the dynamic panics model has the ability to fit two different default probabilities while the other model is required to fit only one. The model without such panics thus places the uniform default probability somewhere between the crisis and non-crisis estimates.

5.6. Policy Experiment: Liquidity Provision

Given a set of estimated parameters, we can now begin to think seriously about the model's implications for policy. The key policy question tends to revolve around the provision of liquidity by the European Central Bank i.e. should the ECB act as the lender of last resort in sovereign debt markets for the constituent countries of the Eurozone? We know that it did with the Emergency Lending Facility and the Outright Monetary Transactions bond-buying program, but was this optimal? The theoretical model of dynamic panics is ambiguous on this point, and so we need the data to provide the answer.

So-called fundamentalists, such as Issing (2011) have argued no, citing both the problem of moral hazard and cross-subsidization from fiscally responsible countries to fiscally irresponsible ones. Others, such as De Grauwe (2011), take a multiple-equilibrium view, arguing that panic in financial markets caused self-fulfilling increases in the likelihood of default, since such panic raises the cost of debt repayment.

We can analyze the provision of liquidity in the context of this model with a simple exercise. Let us assume that, contrary to the government, the ECB cares about utility from both private and public consumption. Suppose further that if the ECB begins providing liquidity, two things happen: First, confidence-waves are eliminated, since the ECB is not affected by market sentiments;[21] and second, the country defaults more often because of moral hazard i.e. such provision encourages fiscal irresponsibility since member countries can rely on the ECB to purchase the debt and fill revenue gaps if times turn bad.

We can approximate the value function of the household in each regime, compute its certainty-equivalent consumption,[22] and then ask ourselves whether provision of liquidity could improve on this in each regime.

[21] It is not difficult to demonstrate that an equilibrium always exists in the absence of confidence fluctuations. Chatterjee and Eyigungor (2012) provide a proof of this when the state space is discrete. An equilibrium of this type would come into action if confidence-waves were eliminated.

[22] Given the GHH preferences, I take certainty-equivalent consumption to be the constant stream of consumption the household must receive if it provides $l = 0$ forever to be indifferent with its situation in the recursive equilibrium.

We certainly do not know by exactly how much the moral hazard problem would increase the default probability, but we can ask ourselves *how bad* it would have to be for liquidity provision to become sub-optimal. The estimated model suggests that liquidity provision can improve welfare so long as the moral-hazard induced likelihood of default does not exceed 2.59% in any given quarter.

5.6.1. Procedure

In computing the welfare statistics, I need to ensure that my approximation of the value function is accurate for the task at hand. This is not an easy task, given that the regime-specific 'steady states'[23] may be far away from the ergodic distribution of model objects.

I overcome this problem via simulation. In particular, I approximate the unconditional value of being in one regime or another as follows:

1. Simulate the model for a long time e.g. $N_{inner} = 10,000,000$ quarters
2. Compute the average household value conditional on being in a given regime for each regime.
3. Invert this value to obtain the certainty equivalent consumption implied by the simulation.
4. Repeat Steps 1-3 many times e.g. $N_{outer} = 1000$
5. Take as an estimate of the certainty-equivalent consumption in each regime its average over the N_{outer} simulations.

Given the stationarity of the model, this estimate will converge to the true certainty-equivalent consumption as $N_{inner} \to \infty$. The use of an outer loop N_{outer} helps to speed up the process by providing independent estimates instead of relying on ergodicity. In practice, this generates estimates for CEC which vary from each other by less than $1e-4$, which I take to be the desired tolerance when comparing the efficacy of policy.

To determine the threshold default frequency under liquidity provision, I repeat the above procedure several times in the context of an interval bisection, since welfare will be decreasing in the likelihood of default.[24]

[23] These should not be confused with the model's non-stochastic steady state, which is regime-independent.
[24] Though I have no guarantee that welfare is decreasing in the default probability, in every numerical application it is.

5.6.2. Optimal Policy

Figure 9 shows the difference in certainty-equivalent consumption between the model with liquidity provision and the baseline model for different degrees of the moral hazard problem. The threshold default frequency will equate the certainty-equivalent consumption under the liquidity policy to that in the baseline model. The graph shows this to be 2.59%, which translates to once every 9.65 years.

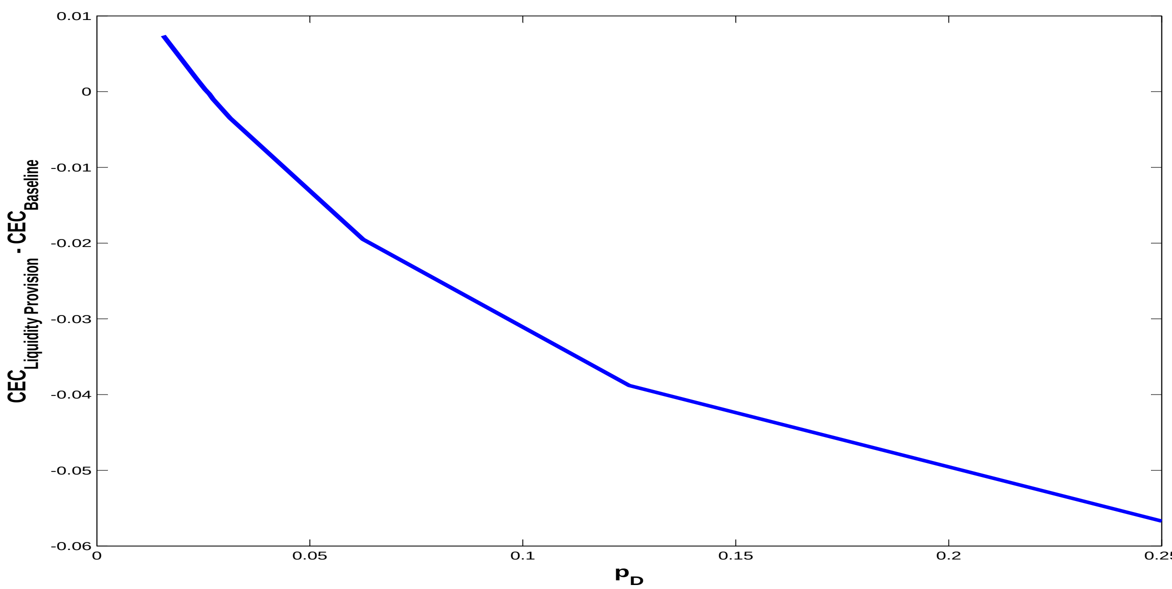

Figure 9: Welfare Difference: Liquidity Provision and Baseline Model

This figure is the same for crisis times and non-crisis times, so there is no time-inconsistency. The reason for this is because of the endogenous labor supply. When the economy enters a crisis, consumption and investment fall as taxes rise to service mounting debt burdens; however, labor supply also falls. The net effect of these two on welfare is essentially zero, though they have large implications for other model objects.

The default frequency in the absence of confidence-waves is lower than the frequency of confidence-waves in the baseline model. This does not speak to any particular welfare implications of the model, but simply reflects the fact that with confidence-waves it takes at least two periods to default instead of one. To match in welfare terms this default structure, defaults must occur less frequently if they are not preceded by a confidence-waves crisis.

So can we expect liquidity provision to be welfare-improving? The model puts no rigor on the frequency of such default as it pertains to the Eurozone, but we can compare it to the historical experience of other countries. For instance, Reinhart and Rogoff (2010) show that the average external default rates for Brazil and Greece are once every 20 and 31.3 years, respectively over the past two centuries. Since the welfare threshold for our estimated Spanish data is once every 9.65 years, the provision of liquidity by the central bank in the Eurozone is quite likely to be welfare-improving for its member countries.

It is important to note that the exercise here does not explicitly incorporate the welfare of all member of a monetary union, but only those in distress. There is a fear, express by Issing (2011) and others, regarding the cross-subsidization of member countries implied by liquidity provision facilities. However, as history has shown and as the model predicts, it is sufficient for the ECB to declare credibly that it is willing to purchase the debt of distressed member countries. They need not actually do it in many cases.[25]

6. Conclusion

In this paper, I documented the existence of a new type of international financial crises, which I call dynamic panics. I demonstrated their existence in the standard quantitative sovereign debt model as well as characterized their basic properties. In particular, I showed that they appear as true panics in the sense of monotone price shifts, and that their existence implies that the uniqueness result associated with Markov-perfection on fundamentals alone is fragile. I also show that such panics can affect both long-term and short-term debt, but if they affect short-term debt it must at some point act through the default channel. However, with long-term debt we can have crises driven solely by borrowing behavior, which I argue occurred in the Eurozone periphery. I further demonstrated that in this environment interest rate ceilings are ineffective since the sovereign is always on the left side of its Laffer curve, even during a crisis.

I then performed an empirical exercise on a standard DSGE model with time-varying default probabilities Spanish data to estimate the relevant parameters. I outlined a new algorithm to speed up the computation of this class of models that can be generalized to increase their applicability. The estimation of this model told us that the median estimate of the probability of a confidence-wave crisis, as determined

[25]Some distressed countries, such as Greece and Ireland did receive rescue packages from the ECB's Emergency Lending Facility. However, it is not clear that at the time these amounted to liquidity provision, since the program was assumed to be of a temporary nature.

by spreads, is 3.39%, which is validated externally by the actual frequency of such crises relative to the inception of the Euro. Further, allowing the possibility of these crises improves the fit of the structural model according to the posterior odds ratio.

I also showed that empirical measures of fiscal responsibility are markedly higher once confidence-waves are accounted for, and thus the need for austerity packages may not be as large as would be suggested by a cursory glance at the data. Further, I demonstrated that the provision of liquidity by the ECB is optimal provided it does not induce moral-hazard based default more than once every 9.65 years on average.

This paper lays the groundwork for much potential future research. For instance, I have only just begun to outline the theoretical properties of these confidence-waves and have only been able to prove their existence for short-term debt, though computational examples with long-term debt can be found. An existence theorem for the case of long-term debt would likely be quite enlightening. Also, on the theoretical frontier, this paper explored some of the practical implications of functional operators that are not contractions. The tools presented in these proofs may be generalizable to other classes of asset pricing models as well.

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8. Appendix A: Theoretical Proofs

8.1. Proof of Theorem 4.2

For the sake of intuition, I will highlight the specific requirements outlined in the theorem as the proof proceeds. The proof proceeds in six steps.

8.1.1. Step 1: m Shock Large Enough Rule out Degenerate Equilibria

Here, we ensure that the default preference shock is large enough to rule out degenerate equilibria. First, assume that

$$\bar{m} \geq X(y) - \big(u(y+b) + \beta \bar{V}(y)\big) \tag{13}$$

$$\underline{m} < X(y) - \left(u(y + b - \frac{1}{R}\bar{b}) + \beta E[X(\tilde{y})]\right) \tag{14}$$

for every (y, b) in the state space. We define $\bar{V}$ to be the value the sovereign receives from receiving the risk-free rate with full commitment forever. It will act as an upper bound on the set of equilibrium value functions.

These two conditions tell us two things:

- Default must occur in some states of the world and not in others
- For every level of debt and output, there exists a sufficiently bad m-shock to induce default and a sufficiently good m-shock to induce repayment

The assumption of a large range of m need not be restrictive, since we have put no condition at this point on the probability structure of m. For instance, most mass could be concentrated near zero with very large, thin tails.

8.1.2. Step 2: Difference the Valuation of Repayment and Default: Call it M

Define a new object, $M(y, m, \xi, b)$, which is defined by differencing the values of repayment and default.

$$M(y, m, \xi, b) = m + u(y + b - q(y, \xi, b^\star)b^\star) - u(y - \phi(y)) + \beta E[\max\{M(\tilde{y}, \tilde{m}, \tilde{\xi}, b^\star), 0\}] \tag{15}$$

where $b^\star \in B$ is the optimal choice of debt conditional on repayment. Notice that the sovereign repays if and only if $M(y, m, \xi, b) \geq 0$. Define the right-hand side of this functional operator to be T.

8.1.3. Step 3: Demonstrate that $(TM)_m = 1$ in any Equilibrium

This is fairly easy to see. Since m is iid it has no impact on the pricing function since it has no lasting effects. Further, since it is solely a preference shock over default, which occurs before the auction (Eaton-Gersovitz timing), then its realization will not affect the choice of $b^\star$ during the auction and so we will have $M_m = 1$ by a simple derivative.

8.2. Step 4: Define a Closed Subset in Which M is Strictly Increasing in ξ

Let $\mathcal{M}(\theta)$ denote the subset of functions $M : Y \times [\underline{m}, \bar{m}] \times \Xi \times B \to \mathcal{R}$ for which $M_m = 1$ and the following holds:

$$M(y, m, \xi_1, b) - M(y, m, \xi_0, b) \geq \theta > 0 \tag{16}$$

for some $\xi_1 > \xi_0$ and some $\theta > 0$. Define $\xi^\star(y, m, b)$ to be the lowest ξ such that $M(y, m, \xi^\star(y, m, b), b) \geq 0$. $\xi^\star$ may not in fact exist, in which case the government always defaults regardless of the realization of ξ.

8.2.1. Step 5: Show that $\exists \theta > 0$ such that the Equilibrium Operator maps $M(\theta) \to M(\theta)$

Suppose that we had $M \in \mathcal{M}(\theta)$ for some $\theta > 0$. We will solve for θ explicitly later. Take the difference of $\hat{M} = TM$ between ξ_1 and ξ_0 that is less than ξ_1 when the rest of the state is the same. By the principle of optimality, I can evaluate $TM(y, m, \xi_1)$ at $b_0^\star$ to derive

$$
\begin{aligned}
\hat{M}(y,m,\xi_1,b) - \hat{M}(y,m,\xi_0,b) \geq\ & u(y+b-q(y,\xi_1,b_0^\star)b_0^\star) - u(y+b-q(y,\xi_0,b_0^\star)b_0^\star) \\
& + \beta E_{\tilde{y},\tilde{m}}\left[\sum_{\xi \geq \xi^\star(\tilde{y},\tilde{m},b_0^\star)} M(\tilde{y},\tilde{m},\tilde{\xi},b_0^\star)p(\tilde{\xi}|\xi_1)\right] - \\
& \beta E_{\tilde{y},\tilde{m}}\left[\sum_{\xi \geq \xi^\star(\tilde{y},\tilde{m},b_0^\star)} M(\tilde{y},\tilde{m},\tilde{\xi},b_0^\star)p(\tilde{\xi}|\xi_0)\right] \qquad (17)
\end{aligned}
$$

Notice that if $p(\cdot|\xi_1)\ fosd\ p(\cdot|\xi_0)$, then we will have that the last two terms are weakly positive because M is nondecreasing in ξ since it is in $\mathcal{M}(\theta)$. We can then write the pricing function explicitly in terms of M to derive

$$
\begin{aligned}
\hat{M}(y,m,\xi_1,b) - \hat{M}(y,m,\xi_0,b) \geq u\left(y+b-\frac{1}{R}E_{\tilde{y},\tilde{m}}\left[\sum_{\xi \geq \xi^\star(\tilde{y},\tilde{m},b_0^\star)} 1\{M(\tilde{y},\tilde{m},\tilde{\xi},b_0^\star)p(\tilde{\xi}|\xi_1)\}\right]b_0^\star\right) - \\
u\left(y+b-\frac{1}{R}E_{\tilde{y},\tilde{m}}\left[\sum_{\xi \geq \xi^\star(\tilde{y},\tilde{m},b_0^\star)} 1\{M(\tilde{y},\tilde{m},\tilde{\xi},b_0^\star)p(\tilde{\xi}|\xi_0)\}\right]b_0^\star\right)
\end{aligned}
$$

We know that $1\{M(y,m,\xi,b)\}$ is a nondecreasing function of ξ, and so we know that this expression must be ≥ 0. However, we wish to claim more than that. We wish to bound this expression away from zero by some $\theta > 0$. We can do this without too much difficulty given the assumptions we've made so far. To do so, note the following:

For every y, there are threshold values of m for which we repay or default regardless of the level of confidences or debt, $[m_L(y), m_H(y)]$. Since we know that M is continuous and increasing in m, it must be the case that $\exists m^\star(y,\xi,b) \in [m_L(y), m_H(y)]$ such that

$$M(y, m^\star(y,\xi_1,b),\xi_1,b) = 0$$

We know further that because $M \in \mathcal{M}(\theta)$ that

$$M(y, m^\star(y,\xi_1,b),\xi_0,b) < 0$$

We therefore know that there is a positive mass of m realizations such that the default decision is impacted by the level of confidence. In particular, since $M_m = 1$ everywhere, we know that the set of m for which confidence matters for default is at least $(m^\star, m^\star+\theta)$, which has a positive mass.

Now, suppose that in addition to first-order stochastic dominance, we have an even stronger notion of persistence, which is that there exists some $\bar{\pi} > 0$ such that $F(\tilde{\xi}|\xi_0) - F(\tilde{\xi}|\xi_1) \geq \bar{\pi}\ \forall \tilde{\xi} \neq \bar{\xi}$. Suppose that there the cardinality of Ξ is $N_\xi \geq 2$ In this case, we can see that there will be a boundable difference in the pricing functions given as follows:

$$q(y,\xi_1,b') - q(y,\xi_0,b') \geq \frac{1}{R}\sum_{\tilde{y}\in Y}\left[F_m\left(m^\star(\tilde{y},\xi_1,b')+\theta\right) - F_m\left(m^\star(\tilde{y},\xi_1,b')\right)\right]p(\tilde{y}|y)\frac{(N_\xi-1)}{N_\xi}\bar{\pi} > 0$$

Call the RHS $\kappa(m^\star,\theta)$. Then we can express the difference in $\hat{M}$ as

$$\begin{aligned}\hat{M}(y,m,\xi_1,b) - \hat{M}(y,m,\xi_0,b) \geq\ & u\left(y+b-\left[q(y,\xi_0,b_0^\star)+\kappa(m^\star,\theta)\right]b_0^\star\right) - \\ & u\left(y+b-q(y,\xi_0,b_0^\star)b_0^\star\right)\end{aligned}$$

To finish this step, we need only find a $\theta^\star$ such that for every point in the state space the RHS is $\geq \theta^\star$ i.e. $\forall (y,m,b) \in Y\times[\underline{m},\bar{m}]\times B$, the following holds:

$$u\left(y+b-\left[q(y,\xi_0,b_0^\star)+\kappa(m^\star,\theta^\star)\right]b_0^\star\right) - u\left(y+b-q(y,\xi_0,b_0^\star)b_0^\star\right) \geq \theta^\star \tag{18}$$

It is fairly clear that Equation 18 will hold provided the flow utilities are fairly steep. How steep? I provide an example. Suppose that $u(c) = \log(c-\bar{c})$ and that m is distributed uniformly. Then this condition becomes

$$\log\left(y+b-\left[q(y,\xi_0,b_0^\star)+\frac{N_\xi-1}{N_\xi}\bar{\pi}\frac{\theta^\star}{[\bar{m}-\underline{m}]}\right]b_0^\star-\bar{c}\right) - \log\left(y+b-q(y,\xi_0,b_0^\star)b_0^\star-\bar{c}\right) \geq \theta^\star$$

To find this bound, let us find the minimum possible distance between these two:

$$\log\left(\bar{y}+\underline{b}-\frac{\bar{b}}{R}-\bar{c}+\frac{N_\xi-1}{N_\xi}\bar{\pi}\frac{\theta^\star}{[\bar{m}-\underline{m}]}\underline{b}\right) - \log\left(\bar{y}+\underline{b}-\frac{\bar{b}}{R}-\bar{c}\right) \geq \theta^\star$$

This can be rearranged as follows:

$$\frac{\bar{y}+\underline{b}-\frac{\bar{b}}{R}-\bar{c}-\frac{N_\xi-1}{N_\xi}\bar{\pi}\frac{\theta^\star}{[\bar{m}-\underline{m}]}\underline{b}}{\bar{y}+\underline{b}-\frac{\bar{b}}{R}-\bar{c}} \geq \exp(\theta^\star)$$

Notice that we are now dealing with the following expression:

$$\frac{a+b\theta}{a} \geq \exp(\theta) \tag{19}$$

for some constants a and b. This expression will hold true for a range of small, positive θ, provided $b/a > 1$. One way to do this is to ramp up $\underline{b}$ while ensuring that $\underline{b}$ and $\bar{b}$ are fairly close together i.e. $\underline{b}-\frac{\bar{b}}{R}$ remains constant. Another is to insert a high subsistence level of consumption, $\bar{c}$, accompanied by a small variance in endowments, $\bar{y}-\underline{y}$ to ensure positive consumption. Both of these can be done judiciously without changing the thresholds on m and thus without violating the assumptions on the necessary range of m. A combination of these two approaches will work as well: The first increases the relevance of the pricing function for the value function this period i.e. increases b and the second ensures that the flow payoffs are sufficiently steep over the consumption possibilities set i.e. reduces a.

We can see the maximum possible $\theta^\star$ graphically in Figure 10. The green line is a graph of $\frac{a+b\theta}{a}$ and the blue line is a graph of $\exp(\theta)$. Provided $b/a > 1$, we will have a small region for which the green line is larger. The point $\theta^\star$ is the upper bound of this region.

8.2.2. Step 6: The Right Fixed Point Theorem

We now know the conditions under which $T : \mathcal{M}(\theta) \rightarrow \mathcal{M}(\theta)$. Suppose that these conditions are met. We now consider the operator $\hat{T}$, which operates on pricing functions and is defined as follows:

$$\hat{T}q(y,\xi,b') = \frac{1}{R}E\left[1\{V(\tilde{y},\tilde{m},\tilde{\xi},b';q) \geq X(\tilde{y})\}\right]$$

where V is the fixed point value function conditional on a pricing function q. We know that this exists and is unique because when q is taken as given the sovereign's Bellman equation becomes a contraction.

Define $\mathcal{Q}(\theta)$ to be the space of positive, real-valued functions on $Y \times \Xi \times B$ bounded above by $\frac{1}{R}$ and

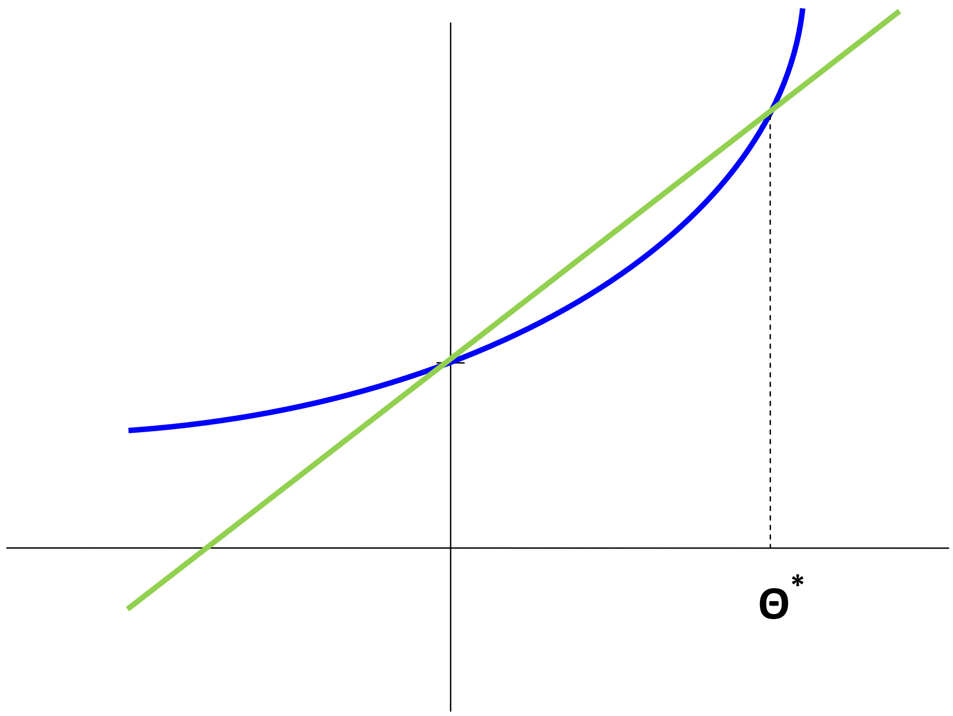

Figure 10: Graphical Illustration of $\theta^{\star}$: Log utility and m-Uniform

for which when $\xi_1 > \xi_0$ the following holds for all $(y, b) \in Y \times B$:

$$q(y, \xi_1, b) - q(y, \xi_0, b) \geq \frac{N_\xi - 1}{N_\xi} \bar{\pi} \frac{\theta}{[\bar{m} - \underline{m}]} \tag{20}$$

By the arguments established in Step 6, we know that if $q \in \mathcal{Q}(\theta)$ and the conditions in Step 6 are met, then the value functions of the sovereign at the fixed point, V, will be at least θ away from each other. This will imply that $\hat{T}q \in \mathcal{Q}(\theta)$.

Therefore, since $Y \times \Xi \times B$ is a discrete and finite set, we have that $\mathcal{Q}(\theta)$ is a compact set of real-valued functions for which q is strictly increasing the ξ. Since m is continuously distributed and the flow utility function of the sovereign is continuous[26], we will have further that $\hat{T}$ is a continuous map. Thus, Brouwer's fixed point theorem holds and a $q^\star$ exists such that $\hat{T}q^\star = q^\star$.

By the construction of $\hat{T}$, we will also be guaranteed an $M^\star \in \mathcal{M}(\theta)$ such that $TM^\star = M^\star$[27], and thus an equilibrium in which confidence has real effects exists. ∎

8.3. Proof of Theorem 4.6

Let the default decision be given by $d(y, \xi, b)$. When the equilibrium is not default relevant, then for any (y, b), we will have $d(y, \xi_1, b) = d(y, \xi_0, b) = d(y, b)$. Thus, we will have

$$q(y, \xi_1, b') = \frac{1}{R} E[1 - d(\tilde{y}, \tilde{\xi}, b')] = q(y, \xi_0, b')$$
$$\to q(y, \xi_1, b') = q(y, \xi_0, b') = q(y, b')$$

and so the sunspot does not affect the price. To see the converse, note that the government's value function only depends on the sunspot insofar as it affects the price:

$$V(y, \xi, b) = \max_{b' \in \mathcal{B}} u(y - b + q(y, \xi, b')b') + \beta E[\max\{V(\tilde{y}, \tilde{\xi}, b'), X(\tilde{y})\}]$$

Suppose that $q(y, \xi, b') = q(y, b')$ and that q is fixed. Now define $\hat{V}$ as follows,

$$\hat{V}(y, b) = \max_{b' \in \mathcal{B}} u(y - b + q(y, b')b') + \beta E[\max\{\hat{V}(\tilde{y}, b'), X(\tilde{y})\}]$$

Since $X(\cdot)$ is exogenously given under the assumption of no re-entry, the contraction mapping theorem tells us that a unique V exists that satisfies the first functional equation, and a unique $\hat{V}$ exists that satisfies the second functional equation. Since the flow payoffs in the two functional equations are identical, $\hat{V}$ must satisfy the first functional equation as well. By uniqueness, this implies that $V(y, \xi_1, b) = \hat{V}(y, b) = V(y, \xi_0, b)$, and thus the sunspot has no effect. Since the sunspot has no effect in V, it has no effect on the default decision. ∎

8.4. Proof of Proposition 4.10

Here, I assume that there is some small, additive iid endowment shock $\tilde{s}$, such that the model becomes isomorphic to the model of Chatterjee and Eyigungor (2012). Thus, the existence result they provide

[26] See Chatterjee and Eyigungor (2012) for a more in-depth discussion of this.
[27] Note that we get this even though M is not discrete and thus perhaps not in a compact set.

for long-term debt *without* confidence fluctuations still holds. Denote the equilibrium price of debt to be $q(y, b')$.

The European Central Bank can pledge liquidity by guaranteeing to purchase debt at a schedule $q(y, b')$ for the foreseeable future. If it does so, it will induce the sovereign to adopt the policy rules from the equilibrium free of confidence shifts. When this happens, investors will lend to the sovereign at the price $q(y, b')$, since it is in fact an equilibrium price, and the ECB never actually has to purchase the debt. ∎

9. Appendix B: Computational Appendix

9.1. MSDSGE Model Solution

We seek a solution of the form:

$$y_t = g(x_{t-1}, \epsilon_t, \chi, s_t), \; y_{t+1} = g(x_t, \chi\epsilon_t, \chi, s_{t+1}), \; x_t = h(x_{t-1}, \epsilon_t, \chi, s_t) \tag{21}$$

Foerster et al. (2013) demonstrate that a first-order approximation to the solutions g and f can be obtained in two steps. The first step entails solving the following quadratic system for $\{D_{1,n_x} g_{ss}(s_t), D_{1,n_x} h_{ss}(s_t)\}_{s_t=1}^{n_s}$:

$$A(s_t) \begin{bmatrix} \mathbf{I}_{n_x} \\ D_{1,n_x} g_{ss}(1) \\ \ldots \\ D_{1,n_x} g_{ss}(n_s) \end{bmatrix} D_{1,n_x} h_{ss}(s_t) = B(s_t) \begin{bmatrix} \mathbf{I}_{n_x} \\ D_{1,n_x} g_{ss}(s_t) \end{bmatrix} \tag{22}$$

for all s_t. Where $A(s_t)$ is an $(n_x + n_y) \times (n_x + n_s n_y)$ matrix given by

$$A(s_t) = \left[\sum_{s'=1}^{n_s} p_{s_t,s'} D_{2n_y+1,2n_y+n_x} f_{ss}(s', s_t), \; p_{s_t,1} D_{1,n_y} f_{ss}(1, s_t) \;,\; \ldots \;,\; p_{s_t,n_s} D_{1,n_y} f_{ss}(n_s, s_t) \right] \tag{23}$$

And $B(s_t)$ is an $(n_x + n_y) \times (n_x + n_y)$ matrix given by:

$$B(s_t) = -\sum_{s'=1}^{n_s} p_{s_t,s'} \left[D_{2n_y+n_x+1,2(n_y+n_x)} f_{ss}(s', s_t), \; D_{n_y+1,2n_y} f_{ss}(s', s_t) \right] \tag{24}$$

The second step uses the result from the first step and the involves solving two linear systems, the first being

$$\begin{bmatrix} D_{n_x+1,n_x+n_\epsilon} g_{ss}(1) \\ \dots \\ D_{n_x+1,n_x+n_\epsilon} g_{ss}(n_s) \\ D_{n_x+1,n_x+n_\epsilon} h_{ss}(1) \\ \dots \\ D_{n_x+1,n_x+n_\epsilon} h_{ss}(n_s) \end{bmatrix} = \left[\Theta_\epsilon, \Phi_\epsilon\right]^{-1} \Psi_\epsilon \tag{25}$$

and the second being

$$\begin{bmatrix} D_{n_x+n_\epsilon+1,n_x+n_\epsilon+1} g_{ss}(1) \\ \dots \\ D_{n_x+n_\epsilon+1,n_x+n_\epsilon+1} g_{ss}(n_s) \\ D_{n_x+n_\epsilon+1,n_x+n_\epsilon+1} h_{ss}(1) \\ \dots \\ D_{n_x+n_\epsilon+1,n_x+n_\epsilon+1} h_{ss}(n_s) \end{bmatrix} = \left[\Theta_\chi, \Phi_\chi\right]^{-1} \Psi_\chi \tag{26}$$

The matrices $\{\Theta_\epsilon, \Phi_\epsilon, \Psi_\epsilon, \Theta_\chi, \Phi_\chi, \Psi_\chi\}$ can be constructed from the solutions to the quadratic system and selected derivatives of the function f. See Foerster et al. (2013) for more details.

The objects from the model that I match to the data are as follows:

- Output: y_t
- Public Current Account: $-\frac{b_t-(1-\lambda)b_{t-1}}{y_t}$
- Spread: $\frac{\lambda+(1-\lambda)(\kappa+Q_t)}{Q_t} - R_t$

Note that I compute the spread in the same way as Chatterjee and Eyigungor (2012), by assuming that the price tomorrow is expected to be the same as the price today.

9.2. Proof of Theorem 5.1

For the purposes of the following results, I denote the ith row and the jth column of $h_{ss}(k)$ to be h_{ij}^k. The same notation applies to the coefficients g_{ij}^k. I consider all rows of these matrices but *only* the first n_x columns, since this is all that is required in the quadratic system. We can reduce the dimensionality of the quadratic problem over a series of three propositions.

Proposition 9.1. *The coefficients governing the stochastic process are predetermined. If row i is an exogenous stochastic process, then we will have that $h^k_{ii} = \rho(i)$ for all states k, where $\rho(i)$ is the degree of persistence of the process in row i. All other elements in row i must be zero*

Proof This follows mechanically since exogenous stochastic processes are, by definition, not affected by any of the other equilibrium objects. Further, an AR(1) process itself is already linear in its past values, with a coefficient equal to the persistence, so any valid approximation must reflect this. ∎

This simple and intuitive step reduces the dimensionality of the problem by $n_s n_x n_{exo}$, where n_{exo} is the number of exogenous stochastic processes. In our case, the dimensionality of the problem drops by 45, which is a substantial improvement but nowhere near large enough yet. We can continue the reduction, though, with another result, which follows from exogeneity of the fiscal rule:

Proposition 9.2. *The coefficients governing the evolution of government debt process are predetermined.*

Proof This follows from the exogeneity of the fiscal rule. The impact of interest rates, productivity, and policy shocks as well as the impact of past debt can be determined via the relevant derivatives of this fiscal rule. ∎

This step reduces the dimensionality by $n_x n_s = 15$. With the exogenous processes predetermined, I now turn to the consumption-saving decision, which is at the crux of the model solution:

Proposition 9.3. *Given coefficients on investment movements, the coefficients on capital and consumption movements are uniquely determined. Coefficients on capital movements will be the same as the coefficients on investment movements with the exception of capital itself, which requires an additional $1 - \delta$. If row i' corresponds to consumption and row i corresponds to capital, then $g^k_{i'j} = f^k_j - h^k_{ij}$. f^k_j will either be a known constant or some linear combination of other unknowns of the matrices h^k and g^k.*

Proof This result comes from the fact that the problem faced by the household is the marginal allocation of additional income to capital. Thus, given a positive shock to the budget set of the household, if we know how much of that additional income was allocated to capital, then we simply subtract that amount from the size of the shock to determine how much was allocated to consumption. Therefore, the constants f_j^k are simply the impact on the budget constraint in state k from a unit shock to the state variable in column j. $\blacksquare$

In my case, the set f_j^k are known constants. This proposition, upon implementation, reduces the dimensionality by $2 \times n_s n_x$, which in our case reduces the dimensionality by 30. This proposition is useful because it is extremely applicable. The consumption-saving decision is the cornerstone of modern macroeconomics and nearly every recursive problem entails this decision at some level. Thus, this technique could have near universal applicability for those seeking to implement the method of Foerster et al. (2013) to derive an approximation to a given equilibrium.

We can reduce the dimensionality once more before we actually solve for the approximation with the following proposition:

Proposition 9.4. *Suppose that the consumption coefficients have been removed from the system using the technique described in Proposition 9.3. If i is the row containing the investment coefficient, a solution to a quadratic system of size k entailing only $\{h_{ii}^k\}_{k=1,n_s}$ is both necessary and sufficient to solve the entire quadratic system.*

To see this result, note that after the requirements thus far that have been imposed will imply a quadratic system of 15 equations in 15 unknowns: These unknowns are the linear response of investment to the 5 states variables in each of the 3 states. This system can be stated as follows after some tedious algebra (or, more easily, by use of symbolic engine) for a set of parameter-determined constants $\{c_{i,j}\}$:

$$0 = c_{1,0} + c_{1,1}i_{k,1}^2 + c_{1,2}i_{k,1}i_{k,2} + c_{1,3}i_{k,1}i_{k,3} + \sum_{i=1}^{3} c_{1,i+3}i_{k,i}$$

$$0 = c_{2,0} + c_{2,1}i_{k,2}^2 + c_{2,2}i_{k,1}i_{k,2} + c_{2,3}i_{k,2}i_{k,3} + \sum_{i=1}^{3} c_{2,i+3}i_{k,i}$$

$$0 = c_{3,0} + c_{3,1}i_{k,3}^2 + c_{3,2}i_{k,1}i_{k,3} + c_{3,3}i_{k,2}i_{k,3} + \sum_{i=1}^{3} c_{3,i+3}i_{k,i}$$

$$0 = c_{4,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{4,(i-1)*3+j}i_{b,i}i_{k,j} + \sum_{i=1}^{3} c_{4,i+9}i_{b,i} + \sum_{i=1}^{3} c_{4,i+12}i_{k,i}$$

$$0 = c_{5,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{5,(i-1)*3+j}i_{b,i}i_{k,j} + \sum_{i=1}^{3} c_{5,i+9}i_{b,i} + \sum_{i=1}^{3} c_{5,i+12}i_{k,i}$$

$$0 = c_{6,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{6,(i-1)*3+j}i_{b,i}i_{k,j} + \sum_{i=1}^{3} c_{6,i+9}i_{b,i} + \sum_{i=1}^{3} c_{6,i+12}i_{k,i}$$

$$0 = c_{7,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{7,(i-1)*3+j}i_{R,i}i_{k,j} + \sum_{i=1}^{3} c_{7,i+9}i_{b,i} + \sum_{i=1}^{3} c_{7,i+12}i_{R,i}$$

$$0 = c_{8,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{8,(i-1)*3+j}i_{R,i}i_{k,j} + \sum_{i=1}^{3} c_{8,i+9}i_{b,i} + \sum_{i=1}^{3} c_{8,i+12}i_{R,i}$$

$$0 = c_{9,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{9,(i-1)*3+j}i_{R,i}i_{k,j} + \sum_{i=1}^{3} c_{9,i+9}i_{b,i} + \sum_{i=1}^{3} c_{9,i+12}i_{R,i}$$

$$0 = c_{10,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{10,(i-1)*3+j}i_{g,i}i_{k,j} + \sum_{i=1}^{3} c_{10,i+9}i_{b,i} + \sum_{i=1}^{3} c_{10,i+12}i_{g,i}$$

$$0 = c_{11,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{11,(i-1)*3+j}i_{g,i}i_{k,j} + \sum_{i=1}^{3} c_{11,i+9}i_{b,i} + \sum_{i=1}^{3} c_{11,i+12}i_{g,i}$$

$$0 = c_{12,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{12,(i-1)*3+j} i_{g,i} i_{k,j} + \sum_{i=1}^{3} c_{12,i+9} i_{b,i} + \sum_{i=1}^{3} c_{12,i+12} i_{g,i}$$

$$0 = c_{13,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{13,(i-1)*3+j} i_{z,i} i_{k,j} + \sum_{i=1}^{3} c_{13,i+9} i_{z,i}$$

$$0 = c_{14,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{14,(i-1)*3+j} i_{z,i} i_{k,j} + \sum_{i=1}^{3} c_{14,i+9} i_{z,i}$$

$$0 = c_{15,0} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{15,(i-1)*3+j} i_{z,i} i_{k,j} + \sum_{i=1}^{3} c_{15,i+9} i_{z,i}$$

Let us denote a solution to this system by $I^n = \{i^n_{j,i}\}_{j\in\{k,b,R,g,z\},\ i=1,3}$, where n indexes the solution from a possible set of many solutions. We can glean from this system that the first the equations are in fact an isolated quadratic system i.e. they contain 3 equations in 3 unknowns: $i_{k,1}, i_{k,2}, i_{k,3}$. None of the other coefficients enter into this system. Further, conditional on having a solution to this system, the remaining 12 equations are linear in their 12 unknowns.

Thus, we can find a solution to this large system as follows:

1. Solve the subsystem given by the first three equations for all solutions, $\{i^n_{k,1}, i^n_{k,2}, i^n_{k,3}\}_{n=1,N}$, where there are N determinate solutions.
2. For each solution, $n \leq N$, fix $\{i^n_{k,1}, i^n_{k,2}, i^n_{k,3}\}$ as constants and solve Equations 4-15 as a linear system. This will, of course, yield either no solution, a unique solution, or a continuum of solutions for each n.

Let $\hat{N} \leq N$ be number of total solutions for which the linear system for which Step 2 yields a unique solution.[28] If at n the linear system had no solution, then $\{i^n_{k,1}, i^n_{k,2}, i^n_{k,3}\}$ could not have formed the basis of a solution in the first place, and if at n the linear system had a continuum of solutions, then the approximation (not the equilibrium) would be indeterminate and thus of no use.

Now, I argue that this procedure yields all determinate approximations to the entire system and only those determinate approximations. The latter claim is easy to understand: By construction, any solution constructed with this procedure must be a determinate equilibrium.

[28] In practice, both N and $\hat{N}$ almost invariably equal 8.

The former is also fairly trivial: Suppose that there was another solution, $I^{\hat{n}}$, to the entire system that was not found via this procedure. Then we could isolate the terms $\{i^{\hat{n}}_{k,1}, i^{\hat{n}}_{k,2}, i^{\hat{n}}_{k,3}\}$ from this solution and apply them to the first three equations. Because the procedure did not find them, this subsystem will not be satisfied. But this contradicts the fact that $\hat{n}$ was indeed a solution to the system, since all conditions do not hold with equality. Thus, our procedure must find *all* valid solutions and *only* valid solutions. ∎

9.3. Kalman Filter: Observables

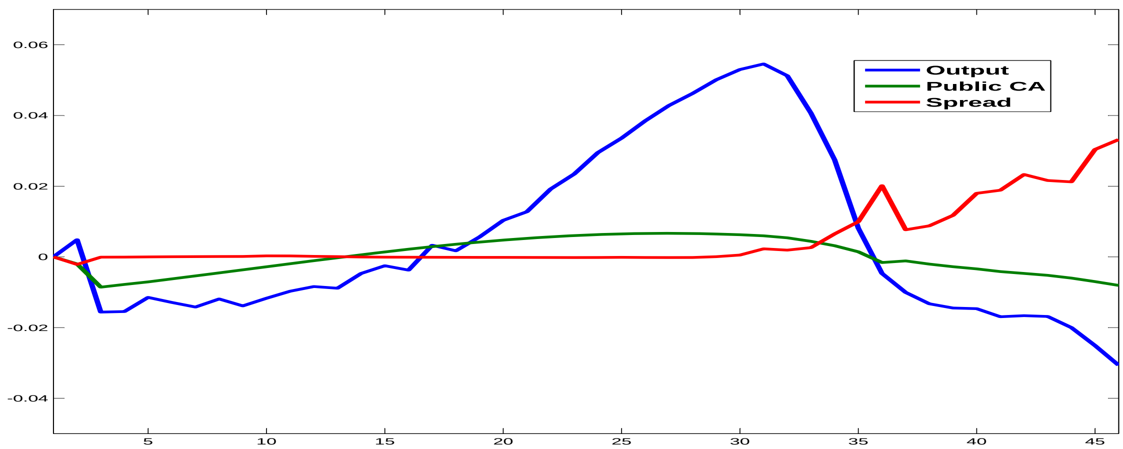

Figure 11: Model-Implied Observables: Data

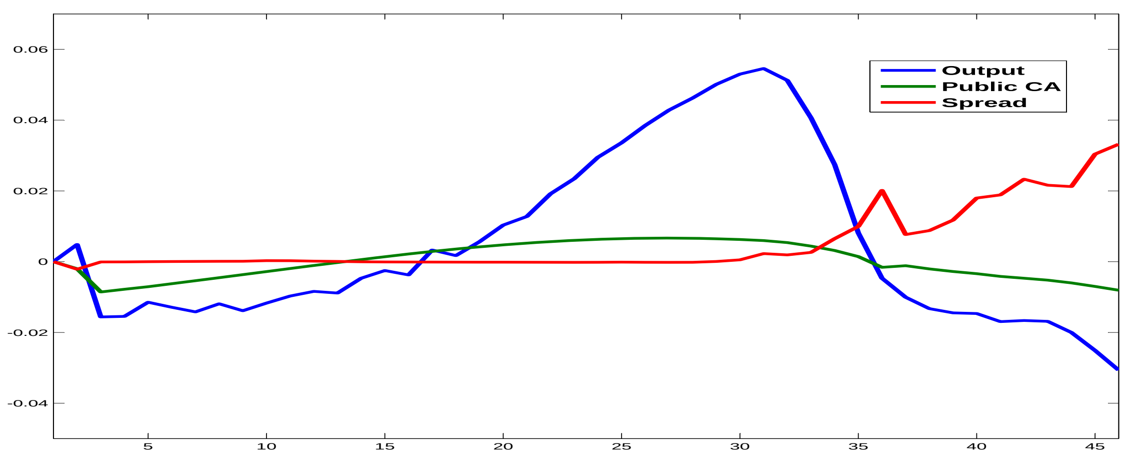

Figure 12: Model-Implied Observables: Baseline Kalman Filter Predictions

9.4. Kalman Filter: Model Components

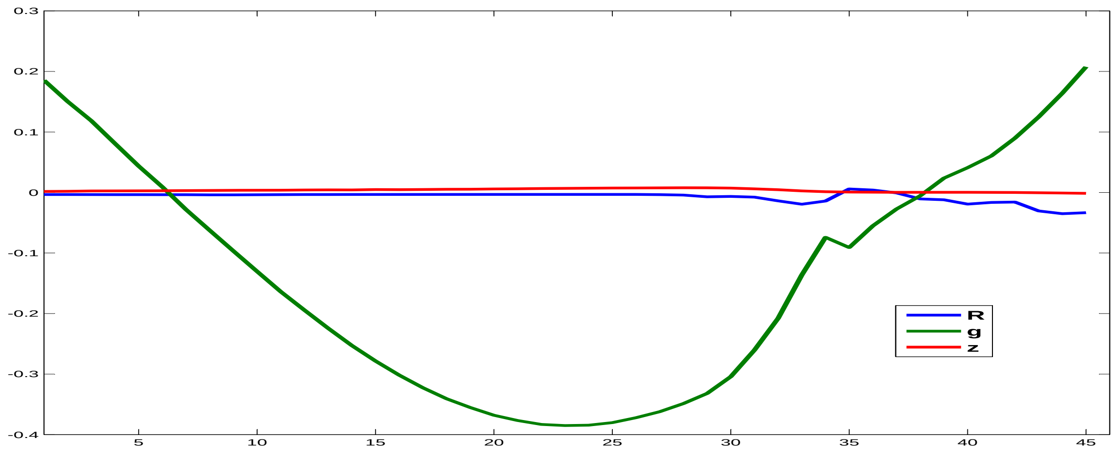

Figure 13: Exogenous Shocks: Deviations from Steady State

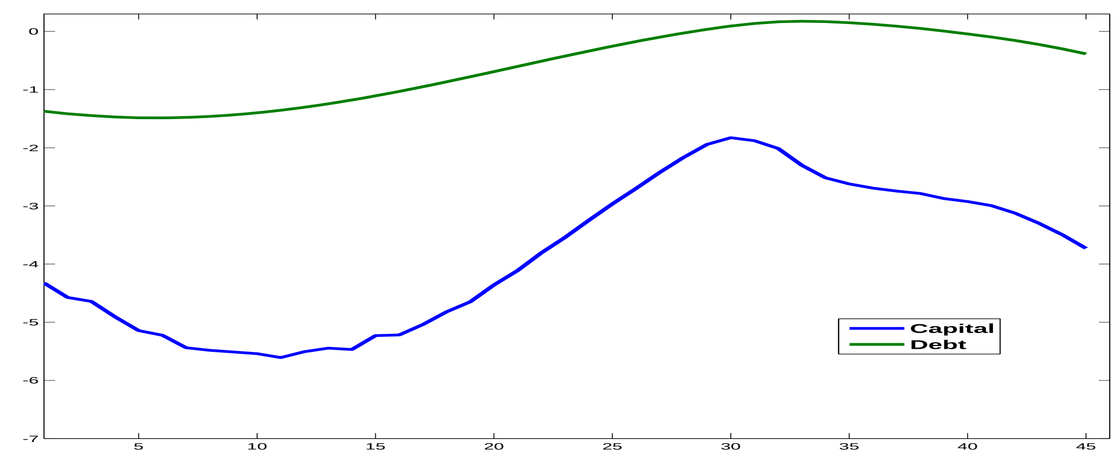

Figure 14: Endogenous State Variables: Deviations from Steady State

9.5. Posterior Distributions

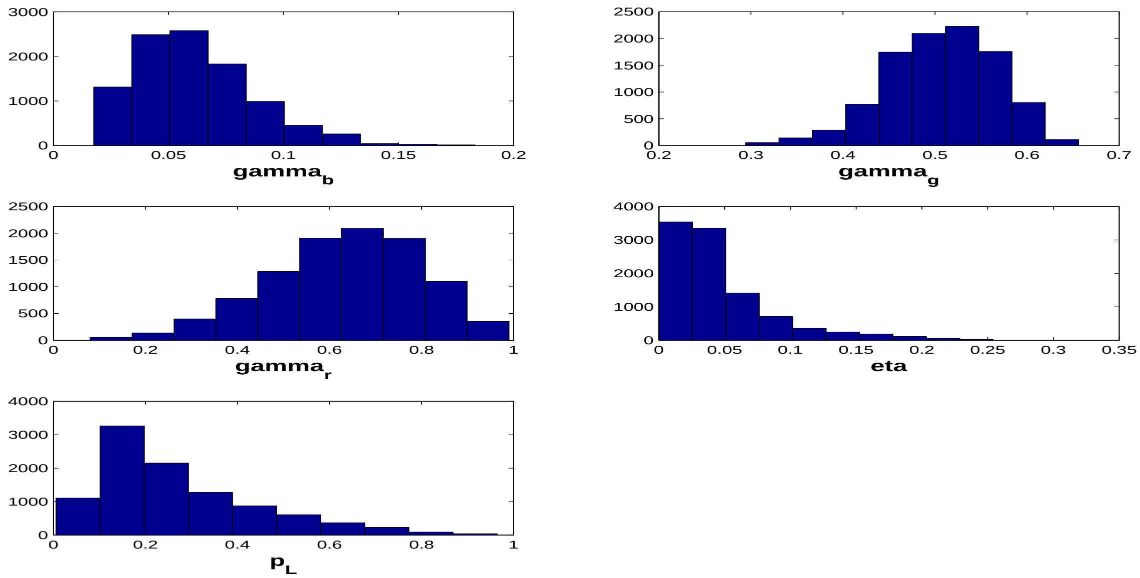

Figure 15: Posterior Distributions of Baseline Model

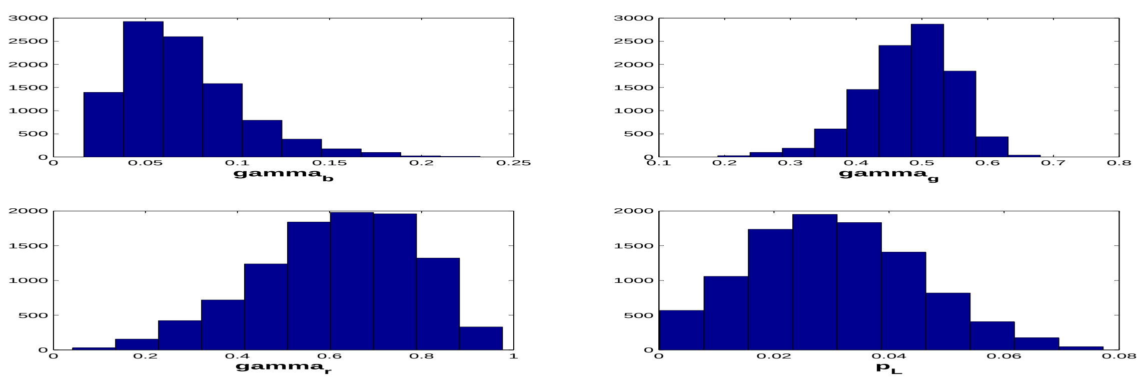

Figure 16: Posterior Distributions of Model Without Dynamic Panics