Search-Based Endogenous Illiquidity and the Macroeconomy*

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Abstract

We endogenize asset liquidity in a dynamic general equilibrium model with search frictions on asset markets. In the model, asset liquidity is tantamount to the ease of issuance and resaleability of private financial claims, which is driven by investors' participation on the search market. Limited funding ability of private claims creates a role for liquid assets, such as government bonds or fiat money, to ease funding constraints. We show that endogenizing liquidity is essential to generate the (positive) co-movement between asset liquidity and asset price. When the capacity of the asset market to channel funds to entrepreneurs deteriorates, investment drops while the hedging value of liquid assets increases. Our model thus demonstrates that shocks to the cost of financial intermediation can be an important source of flight to liquidity and business cycles.

Keywords: endogenous asset liquidity; financing constraints; general equilibrium

classification: E22; E44; G11

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1 Introduction

Illiquidity of privately issued financial assets arises from impediments to their issuance and subsequent transactions. Empirical evidence points to procyclical variation in the market liquidity of a wide range of financial assets.[1] The view that asset liquidity dries up during recessions has been further reinforced by the 2007-2009 financial crisis.[2]

Illiquid primary or secondary equity and debt markets reduce firms' ability to finance investment, which creates a role for liquid assets, such as fiat money or government bonds. These liquid assets provide insurance against funding constraints as they can be used for financing purposes at any time. For example, Ajello (2012) found that nonfinancial firms in the US fund 35% of fixed investment through financial markets, of which 76% through debt and equity issuance and 24% through portfolio liquidations; the rest 65% of fixed investment is funded by internal financing with liquid assets.

When funding constraints tighten in recessions, firms tend to rebalance their portfolios towards liquid assets - a phenomenon called "flight to liquidity". Variations in asset liquidity and the idea of liquidity hoarding as a hedging device against funding constraints goes back to Keynes (1936) and Tobin (1969). Nevertheless, the link between asset liquidity and aggregate fluctuations is often ignored in state-of-the-art dynamic general equilibrium models.

We propose a framework in which *endogenous* variation in asset liquidity interacts with macroeconomic conditions. We incorporate into an almost-standard real business cycle model a search market for privately issued financial assets together with fiat money that are not subject to search frictions. Search frictions give rise to asset illiquidity both on primary markets (issuance of new assets such as initial public offerings or borrowing for new startups) and secondary markets (liquidation of existing assets such as selling equipments or business units). Costly search captures the fact that it is costly and takes time to find buyers or sellers of financial assets. The search market structure thus can be interpreted as a stand-in for financial intermediation via either markets or banks, both of which involve a costly matching process between capital providers and seekers.

Asset illiquidity has both a physical and a price dimension: the physical dimension, to

[1] Studies by Huberman and Halka (2001), Chordia, Roll, and Subrahmanyam (2001), Chordia, Sarkar, and Subrahmanyam (2005) and Naes, Skjeltorp, and Odegaard (2011) assert that market liquidity is procyclical and highly correlated across asset classes such as bonds and stocks in the US.

[2] Dick-Nielsen, Feldhütter, and Lando (2012) identify a structural break in the market liquidity of corporate bonds at the onset of the sub-prime crisis. The liquidity component of spreads of all but AAA rated bonds increased and turnover rates declined, making refinancing more difficult. Commercial paper, which is largely traded on a search market with dealers as match-makers, experienced pronounced illiquidity reported by Anderson and Gascon (2009). In addition, money market mutual funds, the main investors in the commercial paper market, shifted to highly liquid and secure government securities. Finally, Gorton and Metrick (2012) show that the repo market has registered strongly increasing haircuts during the crisis.

which we refer to as saleability or market liquidity, is measured by the endogenous fraction of new or existing assets offered for sale that are successfully traded; the price dimension is captured by the sensitivity of the transaction price and unit intermediation costs to relative supply on the asset market. That is, there will be a spread between the effective purchasing price and the effective selling price. In this sense, money is fully liquid, since all of it can be sold immediately and there is no price spread in transaction.

The model shows how a drop in investor participation in the search market simultaneously reduces asset saleability, pushes down asset prices, and tightens funding constraints, which further generates "flight to liquidity" and dampens real economic activities. Our central contributions are to demonstrate that (i) circulation of privately issued financial claims and money depends on the search costs, and the shocks to the costs can be an important source of "flight to liquidity" and business cycles; (2) endogenizing asset liquidity is essential to generate positive comovement between asset saleability and asset prices.

Consider an economy where privately issued financial claims are backed by cash flow from physical capital, which is rented to final goods producers and owned by households. There is a continuum of households whose members are temporarily separated during periods. They face idiosyncratic investment risks. Some become workers, others entrepreneurs. Only the latter have access to investment opportunities for capital goods creation. All household members are endowed with a portfolio of liquid assets (money) and private claims, which we interpret as a catch-all for privately issued assets such as corporate bonds and equity.

To finance investment, entrepreneurs can use money accumulated before and/or operating profit; they can also issue new financial claims to their investment projects and/or liquidate their existing asset portfolio. Private claims (both new and old), unlike money, are only partially liquid. They are traded on a costly search market in which both buyers and sellers are matched by an intermediary for a fee. For simplicity, this fee is modelled as a dead-weight cost, which drives a wedge between the transaction price and the effective purchase and sale prices. Moreover, the intermediary determines the transaction price by maximizing the total surplus of a match, similar to the bargaining process in the labor search literature (Mortensen and Pissarides, 1994; Shimer, 2005).

Steady state equilibrium features different circulation of money and/or private claims. When search costs are small, private claims are liquid enough and money may not circulate; when search costs are too high, private claims will be dominated by money. Only when search costs are in a middle range can private claims and money co-exist. We focus on this region to capture assets with different liquidity in reality.

As the funding ability of private claims is limited by their partial liquidity and the intermediation costs incurred by buyers and sellers, entrepreneurs are financing constrained

and cannot fund the first-best level of investment. Money, on the other hand, is readily available for financing purposes at any time and there is no price spread. Since households value this hedging value against the illiquidity of private claims, money provides liquidity service and private claims demand a liquidity premium.

Then, we consider two types of *persistent* exogenous shocks: an aggregate productivity shock and a shock to the search costs, which we interpret as an "intermediation cost shock". The latter captures any generic disruption in the financial sector that increases the cost of providing intermediation services.

Negative aggregate productivity (TFP) shocks decrease the return to capital, make investment into capital goods less attractive, and hence crowd out investors from the search market. Negative intermediation cost shocks, on the other hand, make investment into liquid assets more attractive to hedge future investment. This reduces the incentive for investors to post costly buy orders on the search market.

In either case, the fall in demand on the asset market exceeds that of supply (under some regularity conditions), such that sellers have a lower chance of encountering a buyer. Hence, saleability of financial claims drops, which implies that entrepreneurs need to retain a larger equity stake in new investment projects. Their financing constraints tighten and the option of breaking off negotiations becomes less valuable. Entrepreneurs are thus willing to accept a lower price which further tightens financing constraints. In the aggregate, less resources are transferred to entrepreneurs. Real investment thus drops, and other economic activities slow down.

While both shocks generate procyclical asset (market) liquidity and prices, only intermediation cost shocks induce a persistent "flight to liquidity", measured by higher than usual liquidity premium. In the case of persistent negative TFP shocks, investors have a weak incentive to hedge against future investment. Note that negative TFP shocks reduce net worth and also tighten financing constraints such that money should be valued more. But less need for investment (because of lower current and future returns to capital) dominates and reduces money's hedging value for idiosyncratic investment risks. Adverse intermediation cost shocks, however, do not deteriorate the quality of investment itself either today or tomorrow. Investors thus strongly value the hedging service from money and rebalance towards it. Because of the portfolio rebalancing, asset price movements are more pronounced.

Finally, we check the model's implication of time series of macro variables, asset price, and liquidity premium. While both shocks can more or less capture the cyclical properties of macro variables in the data, only shocks to intermediation search costs can capture the countercyclical liquidity premium, and mildly procyclical but volatile asset price. Aggregate productivity shocks however generate strongly procyclical liquidity premium, and strongly

procyclical but mildly volatile asset price.

Note that market- and bank-based financial intermediation share the essential feature of matching savers and borrowers, such that our framework admits both interpretations of the intermediation process. On the one hand, the search and matching framework could echoes features of over-the-counter (OTC) markets, in which a large fraction of corporate bonds, asset-backed securities, and private equity is traded; the framework could also capture the costs in reallocating capital stock ownership across firms (see e.g., Eisfeldt and Rampini (2006)). Participation costs in these markets arise from information acquisition as well as brokerage and settlement services from dealers and market makers.[3] On the other hand, our framework can be seen as a reduced-form approach towards modeling the costly matching process between savers (investors) and the corporate sector through financial intermediaries.

To our knowledge, we are the first to incorporate endogenous asset liquidity in a dynamic macroeconomic model in a tractable way and to explore the feedback effects between asset liquidity and the real economy.[4] Kiyotaki and Moore (2012) (henceforth, KM) demonstrate how *exogenous* asset market liquidity interacts with aggregate fluctuations, in a model in which firms can only sell an *exogenous* fraction of private claims to finance new investment. However, as pointed out by Shi (2015), exogenous liquidity fluctuations lead to counterfactual asset price dynamics: A negative shock to asset saleability reduces the supply of financial assets, while demand remains relatively stable since the quality of investment projects is unaffected by liquidity shocks. The negative supply shock induces a persistent asset price boom that is at odds with the data. This counterfactual finding highlights the need to model asset liquidity endogenously, as we do in this paper.

Related Literature. Following KM and Shi (2015), we model liquidity differences between private claims and government-issued assets. Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) analyze such "unconventional policy" after an exogenous fall in liquidity in an extended KM model with a "zero lower bound".[5] In contrast, asset market liquidity is endogenous generated through the costly search market; the costly search also generates a spread between the purchasing price and the selling price of private claims.

The search literature provides a natural theory of endogenous liquidity as in Lagos and Rocheteau (2009) and has been applied to a wide range of markets such as OTC markets for asset-backed securities, corporate bonds, federal funds, private equity, housing amongst

[3] See, e.g., Dick-Nielsen, Feldhütter, and Lando (2012), Bao, Pan, and Wang (2011), Anderson and Gascon (2009).

[4] A recent study by Yang (2013) also considers endogenous asset liquidity. The difference is that we model liquid and illiquid assets together and the corresponding portfolio choice simultaneously.

[5] More generally, Kara and Sin (2013) show that market liquidity frictions induce a trade-off between output and inflation stabilization off the ZLB that can be attenuated by quantitative easing measures.

others (Duffie, Gârleanu, and Pedersen, 2005, 2007; Ashcraft and Duffie, 2007; Feldhutter, 2011; Wheaton, 1990; Ungerer, 2012). Rocheteau and Weill (2011) provides an extensive survey on search and liquidity. This literature shows that search frictions can explain substantial variation in a wide range of measures of asset market liquidity (e.g., bid-ask spreads and trading delays). Further, work by Den Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), and Petrosky-Nadeau and Wasmer (2013) has emphasized the role of search and matching frictions in credit markets and their impact on aggregate dynamics.[6]

Nevertheless, the joint behaviour of asset prices and asset saleability is generally not explored in a general equilibrium setting in the above two lines of research, such that mutual feedback effects are not considered.

An alternative approach to endogenizing liquidity uses information frictions, such as adverse selection models in Eisfeldt (2004) and Guerrieri and Shimer (2012). While endogenizing asset liquidity, these studies do not consider the feedback effects of fluctuations in liquidity on production and employment. A notable exception is Kurlat (2013), who extends KM with endogenous resaleability through adverse selection but neglecting the role of liquid assets. In Eisfeldt and Rampini (2009), firms need to accumulate liquid funds in order to finance investment. While the supply of liquid assets affects investment, secondary markets for asset sales are shut off as an alternative means of financing. In contrast to these contributions, we jointly model endogenous liquidity on primary and secondary markets, the role of liquid assets as the lubricant of investment financing, and feedback effects between asset liquidity and business cycles. In this sense, we compliment the studies of cyclical capital reallocation, such as in Eisfeldt and Rampini (2006) and Cui (2013a).

Our framework also differs along important dimensions from search-theoretic models of money, such as Lagos and Wright (2005) and Rocheteau and Wright (2005). In this literature, money has a transaction function in anonymous search markets. Recent extensions include privately created liquid assets such as claims to capital (Lagos and Rocheteau, 2008) or bank-deposits (Williamson, 2012) as media of exchange. Our framework rather emphasizes the role of financial assets - both public and private - as stores of value. That is, money and private claims are used for financing purposes. Moreover, our approach is able to generate endogenous variation in asset liquidity and the associated premia, because private claims are subject to search frictions themselves, rather than serving to overcome such frictions on other markets. These differences notwithstanding, a common tenet is that liquid assets play an important role in economic transactions by relaxing deep financial frictions.

[6]Further, Kurmann and Petrosky-Nadeau (2006) study search frictions associated with physical capital in a macroeconomic setting. As shown in Beaubrun-Diant and Tripier (2013), search frictions also help explain salient business cycle features of bank lending relationships.

By studying intermediation cost shocks which affect asset market liquidity, we complement the literature on financial shocks. Recent contributions by Jermann and Quadrini (2012), Christiano, Motto, and Rostagno (2014), and Jaccard (2013) identify financial shocks as an important source of business cycles. Our approach shows how such shocks may be endogenously amplified through the interlinkages between financial markets and the real economy.

2 The Environment

Time is discrete and infinite ($t = 0, 1, 2, ...$). The economy has three sectors: final goods producers, households, and financial intermediaries. Following Shi (2015), there is a continuum of households (with measure one) and each household has a continuum of members. Some members will be entrepreneurs, and some will be workers.[7] A period is divided into 4 sub-periods: the household's decision period, the production period, the investment period, and the consumption period.

The household's decision period. Aggregate shocks to productivity and liquidity are realized. All members *equally divide* the household's assets. The household holds (physical) capital stock, equity claims issued against capital stock by other households, and fully liquid assets (money). The household gives each member the instructions on the choices later contingent on whether the member will be an entrepreneur or a worker, especially how many purchasing quotes V_t and selling quotes U_t of equity claims.

The production period. each member receives a status draw: with a probability χ, she is an entrepreneur; otherwise, she is a worker. An entrepreneur has an investment project but no labor endowment, while a worker has a unit of labor endowment but no investment project. This χ is independent across members and over time. By the law of large numbers, each household thus consists of a fraction χ of entrepreneurs and a fraction $(1-\chi)$ of workers. Both groups are temporarily separated during each period and there is no insurance among them. Individual workers supply labor n_t to firms, earning a wage rate w_t; the total labor supply from the family is $N_t = (1 - \chi)n_t$.

The investment period. Entrepreneurs seek financing and undertake investment projects. Each project can transform 1 unit of consumption goods into 1 unit of capital stock. Asset markets are open in which individuals trade assets to finance new investments and carry out the portfolio choices instructed by their household.

Financial intermediaries facilitate asset transactions between sellers and buyers by im-

[7]The representative household with temporarily separated agents has been introduced in Lucas (1990) and applied to the KM framework in Shi (2015) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011).

plementing a costly matching process. Fully liquid assets, however, can be traded on a frictionless spot market. To abstract from government policies, we model liquid assets as non-interest bearing money in *fixed supply*.

The final sub-period. An entrepreneur consumes $c_t^i = \frac{C_t^i}{\chi}$ and a worker consumes $c_t^n = \frac{C_t^n}{1-\chi}$, where C_t^i and C_t^n are total consumption of entrepreneurs and workers respectively. Then, they return to households and pool all assets together. Note that superscript "i" stands for investment, and "n" stands for no investment.

2.1 A Representative Household

We specify the details for a representative household.

Preferences. The household objective is to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \left[\chi u(c_t^i) + (1-\chi) u(c_t^n) - (1-\chi) h(n_{t+s}) \right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor. $u(.)$ is a standard strictly increasing and concave utility function of consumption, and $h(.)$ captures the dis-utility derived from labour supply n_t. $\mathbb{E}_t$ is the expectation operator conditional on information at time t.

Balance Sheet. Physical capital (K_t), earning a return r_t, is owned by households and rented to final goods producers. There is a partially-liquid equity claim to the future return of every unit of capital, which household members can either retain or offer for sale to outside investors. We normalize equity by the capital stock, such that both depreciate at the same rate δ. Equity claims can be sold in a successful match at unit price q_t, which is determined by the zero-profit intermediary as explained in Section 2.2.

In addition, households can invest into nominal and fully liquid assets (money). Hence, at the onset of period t, households own a portfolio of liquid assets, equity claims on other households' return on capital, and own physical capital. These assets are financed by net worth plus equity claims issued to others (backed by some of their own physical capital). This financing structure gives rise to the following beginning-of-period balance sheet:

Table 1: Household's Balance Sheet

Assets		Liabilities	
liquid assets	B_t/P_t	equity issued	$q_t S_t^I$
other's equity	$q_t S_t^O$		
capital stock	$q_t K_t$	net worth	$q_t S_t + B_t/P_t$

All existing claims to capital need to be traded on the search market at price q_t for refinancing purposes. Similarly, for the fraction of the capital stock on which no outside equity claims have been written yet, it would need to be offered on the search market. It is, therefore, also valued at q_t. Therefore, besides liquid assets B_t, we only need to keep track of net equity, defined as

$$S_t = \text{equity claims on others' capital} \;+\; \text{unissued capital stock}$$

Assets Accumulations. Let S_t^j and B_t^j be net equity and money for j group members after distribution of the household, where j can be either "i" (entrepreneurs) group or "n" (workers) group. Because of the equal division $S_t^i = \chi S_t$ and $S_t^n = (1-\chi)S_t$. Similar division applies to liquid assets, $B_t^i = \chi B_t$ and $B_t^n = (1-\chi)B_t$.

The net equity evolves according to

$$S_{t+1}^j = (1-\delta)S_t^j + I_t^j - M_t^j, \tag{2}$$

where I_t^j is investment into capital goods, and M_t^j corresponds to quantity sales of equity claims.

2.1.1 Workers' flow-of-funds.

The household delegates equity purchases on the search market to workers, because they do not have investment opportunities ($I_t^n = 0$). Therefore, workers post asset purchasing quotes V_t to acquire new or old equity at a unit cost κ_t. On the search market, each posted position is filled with a probability $f_t \in [0,1]$, and an individual buyer expects to purchase an amount $M_t^n = -f_t V_t$. Notice that a worker's flow-of-funds constraint reads

$$C_t^n + \kappa_t V_t + q_t f_t V_t + \frac{B_{t+1}^n}{P_t} = w_t N_t + r_t S_t^n + \frac{B_t^n}{P_t}, \tag{3}$$

where labour income and the return on equity and money are used to finance consumption, search costs, and the new accumulation of equity claims and money. To simplify, we define the effective purchase price per unit of equity as

$$q_t^n \equiv q_t + \frac{\kappa_t^n}{f_t}, \tag{4}$$

where q captures the transaction price and $\frac{\kappa^n}{f}$ represents search costs per transaction (scaled by the probability of encountering a seller f). By using (2), $M_t^n = f_t v_t$, and the definition

of S_t^n and B_t^n, we write the flow-of-funds constraint (3) as

$$C_t^n + q_t^n S_{t+1}^n + \frac{B_{t+1}^n}{P_t} = w_t N_t + [r + q_t^n(1-\delta)](1-\chi) S_t + (1-\chi)\frac{B_t}{P_t}. \tag{5}$$

2.1.2 Entrepreneurs' flow-of-funds.

Entrepreneurs choose how many selling quotes U_t for sale at a unit cost κ_t in order to finance new investment ($I_t^i > 0$). These assets include existing equity claims on other households' capital stock and their own unissued capital stock (in total S_t^i), plus claims on new investment I_t^i. Then, the amount of private financial claims that are up for sale is bounded from above by an entrepreneur's existing equity holdings and the volume of new investment, $U_t \leq (1-\delta) S_t^i + I_t^i$. Offers are matched with a buyer with probability $\phi_t \in [0,1]$. Therefore, entrepreneurs expect to sell $M_t^i = \phi_t U_t$.

Notice that the returns on equity and money are used to finance consumption, search costs, and the accumulation of equity (with new investment taken into account) and money. The flow-of-funds constraint can thus be written as

$$C_t^i + I_t^i + \kappa_t U_t + \frac{B_{t+1}^i}{P_t} = r_t S_t^i + q_t \phi_t U_t + \frac{B_t^i}{P_t}, \tag{6}$$

We define the effective selling price of a unit of financial assets as

$$q_t^i \equiv q_t - \frac{\kappa_t^i}{\phi_t}. \tag{7}$$

When $\kappa_t > 0$, the effective selling price is below the transaction price. Hence, entrepreneurs not only face constraints regarding the quantity of equity that can be issued and resold, they also have to sell at a discount due to the intermediation cost $\frac{\kappa_t^i}{\phi_t}$ when liquidating financial claims.

Together with (2) and $M_t^i = \phi_t U_t$, the flow-of-funds constraint (6) becomes

$$C_t^i + I_t^i + q_t^i \left[S_{t+1}^i - I_t^i - (1-\delta) S_t^i\right] + \frac{B_{t+1}^i}{P_t} = r_t S_t^i + \frac{B_t^i}{P_t}. \tag{8}$$

Further, it is helpful to substitute out new investment by defining $0 < e_t < 1$, which denotes the fraction of total assets that entrepreneurs put on sale

$$U_t = e_t[(1-\delta) S_t^i + I_t^i].$$

Notice that the total number of sell quotes is bounded above by existing equity claims and

the first best investment. Further, we express $S^i_{t+1} = (1-\phi_t e_t)\left[(1-\delta)S^i_t + I^i_t\right]$ according to (2). Then, we can express $I^i_t = \frac{S^i_{t+1}-(1-\phi_t e_t)(1-\delta)S^i_t}{1-\phi_t e_t}$ and rewrite the flow-of-funds constraint (8) as

$$C^i_t + q^r_t S^i_{t+1} + \frac{B^i_{t+1}}{P_t} = r_t \chi S_t + \left[e_t\phi_t q^i_t + (1-e_t\phi_t)q^r_t\right](1-\delta)\chi S_t + \frac{\chi B_t}{P_t}, \tag{9}$$

$$\text{where} \quad q^r_t \equiv \frac{1-e_t\phi_t q^i_t}{1-e_t\phi_t}. \tag{10}$$

The left-hand side (LHS) of (9) captures entrepreneurs' spending on consumptions and accumulation of equity and money, while the right-hand side (RHS) represents entrepreneurial (total) *net-worth* including rental income from capital claims, the value of existing equity claims, and the real value of money. Note that a fraction $0 < e_t\phi_t < 1$ is saleable and, hence, valued at q^i_t, while a fraction $(1-e_t\phi_t)$ is retained and valued at q^r_t, which is the effective replacement cost of existing assets. To see this, notice that entrepreneurs can sell a fraction $e_t\phi_t$ of their financial assets at price q^i_t. For every unit of new investment, they will accordingly need to make a "down-payment" $(1-e_t\phi_t q^i_t)$ and retain a fraction $(1-e_t\phi_t)$ as inside equity. With this interpretation, if entrepreneurs replace existing assets by new assets issued against investment, q^r_t is indeed the effective replacement cost.Further, q_r captures the effect of search costs on equity accumulation: higher search costs decrease the effective sales price, which increases the down-payment that in turn depresses equity accumulation. Therefore, the entrepreneurs' ability to leverage will be lower if search costs are higher.

Notice that (9) involves gross investment. New investment can be backed out from $S^i_{t+1} = (1-e_t\phi_t)[(1-\delta)S^i_t + I^i_t]$ and (9). Formally, as all investment projects are carried out by entrepreneurs, aggregate investment I_t is

$$I_t = I^i_t = \frac{\left[\left(r_t + (1-\delta)e_t\phi_t q^i_t\right)S^i_t + \frac{B^i_t}{P_t}\right] - C^i_t}{1-e_t\phi_t q^i_t}. \tag{11}$$

which says that entrepreneurs' liquid net-worth net of consumption can be levered at $(1-e_t\phi_t q^i_t)^{-1}$ to invest in new capital stock.

2.1.3 The Household's Problem

Let $J_t(S_t, B_t)$ be the value of the representative household with net equity claims S_t and money B_t, given aggregate state variables taken as given by the household (the subscript t

of J indicates this).[8] Since at the end of t, workers and entrepreneurs reunite to share their stocks of equity and money, we have

$$S_{t+1} = S^i_{t+1} + S^n_{t+1}, \quad B_{t+1} = B^i_{t+1} + B^n_{t+1}. \tag{12}$$

Now, we know that (5), (9), and (12) are the three constraints that the household is facing, and we can formally write down the household problem

Problem 1:

$$J_t(S_t, B_t) = \max_{\{e_t, N_t, C^i_t, C^n_t, K_{t+1}, B_{t+1}\}} \{\chi u\left(\frac{C^i_t}{\chi}\right) + (1-\chi)\, u\left(\frac{C^n_t}{1-\chi}\right) - (1-\chi)h(\frac{N_t}{1-\chi}) + \beta \mathbb{E}_t\left[J_{t+1}(S_{t+1}, B_{t+1})\right]\}$$

subject to (5), (9), *and* (12).

2.2 Search and Matching

Search and Matching. Let $M(U_t,V_t)$ denote the flow of purchase-sell quotes matches. The matching function $M(U_t, V_t)$ captures the frictions in the market. The sources of the frictions are costs and time delays such as those due to the completion and processing of credit application, heterogeneities that are not modeled, and imperfect information flow. These matching is facilitated by financial intermediaries.

Note that we do not distinguish financial institutions (e.g., banks) and dealers in financial markets in our model. They are both captured by the financial sector with a costly matching technology, which intermediates the asset price.To the extent that intermediaries resemble banks, the matching fee could be interpreted as comprising screening and monitoring costs associated with successful matches. For a detailed discussion of these two types of agents and their impact on macroeconomic dynamics refer to De-Fiore and Uhlig (2011).

Following the labor search literature, M is concave and homogenous of degree 1 in (U, V) space with continuous derivatives. Let θ_t be the asset market tightness V_t/U_t, let $\phi_t = M(U_t, V_t)/U_t = M(1, \theta_t)$ denote the probability that 1 unit of assets can be sold (or asset liquidity), and $f_t = M(U_t,, V_t)/V_t = M(1/\theta_t, 1)$ be the probability that an asset purchasing quote that can be filled. Recall that ϕ_t also represents the fraction of financial assets that

[8] Once we proceed to the equilibrium definition, $\Gamma \equiv (K_t, B_t; A_t, \kappa_t)$ where K is the total capital stock, B is the total amount of money circulated, A is total factor productivity in final goods production. The exogenous stochastic processes for A and κ are specified in the numerical examples in Section 5.

can be sold *ex post* in a given period. Therefore, we refer to ϕ_t as asset saleability or asset (market) liquidity. Let

$$\lim_{\theta\to+\infty}\phi(\theta)=\lim_{\theta\to 0}f(\theta)=+\infty,\quad \lim_{\theta\to 0}\phi(\theta)=\lim_{\theta\to+\infty}f(\theta)=0.$$

A larger θ indicates that it is easier for the sellers to find potential buyers; also buyers have more difficulty in finding appropriate investment opportunities on the search market. The opposite is true, when θ goes to zero.

2.3 Asset Prices

The price of liquid assets is determined by the spot market clearing. In order to focus on the price of illiquid assets, we refer asset prices to the prices of equity claims, and refer the nominal prices to the inverse (consumption goods) prices of liquid assets. Next, we discuss how asset prices are determined through intermediaries. The details are in the Appendix.

Once a unit of offered assets is matched to a vacant asset position, intermediaries offer a price q to both parties. to maximize the total surplus by bargaining on behalf of each side of the trade. Notice that the amount of matched assets $m_{j,t}$ is predetermined at the point of bargaining. Therefore, buyers and sellers interact at the margin $m_{j,t}$, i.e., the match surplus for both buyers and sellers is the respective marginal value of an additional transaction.

Denote by J_t^n and J_t^i the transaction surplus of individual workers and entrepreneurs (from the point of view of the household) at time t. A buyer's surplus amounts to

$$J_t^n=-u'(\frac{C_t^n}{1-\chi})q_t+\beta\mathbb{E}_t\left[J_{S,t+1}(S_{t+1},B_{t+1})\right].$$

Intuitively, if the deal is successful, the buyer sacrifices q today but gains the household's value of one more unit of assets tomorrow.[9]

Similarly, the sellers' surplus is the marginal value to the household of an additional match for entrepreneurs

$$J_t^i=q_t-u'(\frac{C_t^i}{\chi})\left(\frac{1}{e_t\phi_t}\right)+\beta\left(\frac{1}{e_t\phi_t}-1\right)\mathbb{E}_t\left[J_{S,t+1}(S_{t+1},B_{t+1})\right],$$

which says that the seller gains $(q-e^{-1}\phi^{-1})$ today plus a continuation value from a successful match. The contemporary surplus reflects that entrepreneurs earn the bargaining price q ,

[9]Note that search market participation costs are already sunk at the bargaining stage. However, search costs are not ignored since households take them into account when determining optimal asset posting decisions by workers and entrepreneurs.

but spend $e^{-1}\phi^{-1}$ resources per additional match on new investment projects. The evolution of entrepreneurs' equity position can be expressed as the difference between offered and sold assets (i.e., $S^i_{t+1} = U_t - M_t = \left(e_t^{-1}\phi_t^{-1} - 1\right) M_t$). Entrepreneurs retain a fraction $(e^{-1}\phi^{-1} - 1)$ for each unit of successful matches as inside equity, which is brought back to the household. Therefore, the continuation value of a match consists of the marginal value of future assets to the household multiplied by $(e^{-1}\phi^{-1} - 1)$.

Note that all members within the groups of buyers and sellers are homogeneous, such that the type-specific valuations are identical in all matched pairs. We consider the case in which the transaction price q is determined by surplus division between buyers and sellers. That is, intermediaries set a price q to maximize

$$\max_q \{(J^i_t)^\omega (J^n_t)^{1-\omega}\} \tag{13}$$

where $\omega \in (0,1)$ is the fraction of the surplus that goes to sellers. This set-up is similar to bilateral (generalized) Nash bargaining between buyers and sellers over the match surplus. In the bilateral bargaining case, ω is the bargaining power of sellers. In this sense, our price setting is similar to the wage determining process in Ravn (2008) and Ebell (2011), where individual workers come to bargain on behalf of their respective households.

2.4 Equilibrium

To close the model, competitive firms rent aggregate capital stock K_t and hire aggregate labour N_t from households to produce output (general consumption goods) according to a standard Cobb-Douglas production function:

$$Y_t = A_t F\left(K_t, N_t\right),$$

where $F\left(K_t, N_t\right) = K_t^\alpha N_t^{1-\alpha}$, $\alpha \in (0,1)$, and A_t measures exogenous aggregate productivity. The profit-maximizing rental rate and wage rate are thus

$$r_t = A_t F_K(K_t, N_t), \quad w_t = A_t F_N(K_t, N_t). \tag{14}$$

Now, we are ready to define the recursive competitive equilibrium.

Definition 1:

The ***recursive competitive equilibrium*** *is a mapping $K_t \to K_{t+1}$, with associated consumption, investment, labour, and portfolio choices $\{C^i_t, C^n_t, N_t, e_t, I_t, S_{t+1}, B_{t+1}\}$, asset market tightness and asset liquidity $\{\theta_t, \phi_t, f_t\}$, and a collection of prices*

$\{P_t, q_t^i, q_t^n, q_t^r, w_t, r_t\}$, *given exogenous evolutions of aggregate productivity* A_t, *search costs* κ_t, *and positive fixed money supply* B, *such that*

1. *Given prices, the policy functions solve the representative household's problem (Problem 1). Aggregate investment is determined by in* (11);
2. *Final goods producers' optimality conditions in* (14) *hold;*
3. *Market clearing conditions hold, i.e.,*
   (a) *the capital market clears:* $K_{t+1} = (1-\delta) K_t + I_t$ *and* $S_t = K_t$;
   (b) *Asset liquidity* $\phi_t = M(1, \theta_t)$, *and the probability of filling purchasing quotes* $f_t = M(\theta_t^{-1}, 1)$;
   (c) q_t *solves* (13), *with the effective prices defined in* (4), (7) *and* (10);
   (d) *the market for liquid assets clears (money is in fixed supply):* $B_{t+1} = B_t = B$.

To verify that Walras' Law is satisfied, notice that the investment equation and the household budget constraint resemble the entrepreneurs' and workers' budget constraints (5) and (9). These two constraints imply the aggregate resource constraint

$$C_t + I_t + \kappa_t(V_t + U_t) = A_t K_t^{\alpha} N_t^{1-\alpha}. \tag{15}$$

For accounting purposes, aggregate investment is $I_t + \kappa_t(V_t + U_t)$. But real investment is I_t that will become capital stock at time $t+1$.

3 Equilibrium Characterization

The economy described in the previous section admits different types of equilibria, which can be distinguished by the activity of financial markets or intermediaries and the types of financial assets that circulate. One polar case is autarky, i.e., an equilibrium in which neither private claims nor money exist. In this equilibrium, both financial intermediation via the search market and the money market shut down and entrepreneurs finance investment fully with inside funding[10]

Although in practice some asset markets or parts of the banking system may temporarily shut down, on the macro level they do provide continuous support for resource reallocation.

[10]Such a complete breakdown of financial transactions may become self-fulfilling. For instance, when one party of the market does not participate, the other party would expect this inaction and stay out of the search market, reinforcing and further justifying the initial non-participation decision of their counterparts.

Therefore, we restrict our attention to the more realistic case of a non-autarky economy in which some types of financial claims exist. Once we focus on non-autarky economy, monetary economy can always exist if we let search market collapse. However, money is not backed by any real resources, while equity claims are backed by the rental return from capital stock. For this reason, we further restrict our attention to the equilibria in which private claims circulate together with money or private claims dominate money. In other words, private claims have priority to circulate; but if they become more costly and less liquid, money provides liquidity to make up the gap or even dominate private claims.

Then, the non-autarky case spans three types of equilibria. In the first, only public liquidity circulates. Intuitively, this is the case when search costs are prohibitively high for agents to participate in private asset markets. In the second, only private claims circulate. This would be the case if intermediation costs were sufficiently small, such that the return on private claims dominate the return on public liquidity. Finally, both private and public liquidity may coexist for intermediate levels of search costs.

For ease of exposition, we adopt a guess-and-verify strategy by first illustrating all equilibrium conditions under the assumption that both private claims and money co-exist. Then, we discuss under which parameter restrictions on intermediation search costs these equilibrium conditions will be met.

3.1 Households' Decisions

Suppose $\kappa > 0$ and the economy features both private and public liquidity. We will discuss the limiting case $\kappa = 0$ later. A necessary condition for private claims to exist is that the replacement cost $q_t^r \leq 1$. Otherwise, entrepreneurs only use internal funding for investment as the investment cost is always one unit of consumption goods. A necessary condition for public claims to exist is that the optimality condition of household holding money should be satisfied and we will derive the condition in the following.

Compared to workers, who value equity at price q_t^n, the price of equity is lower from the perspective of entrepreneurs as long as the search cost $\kappa_t > 0$. This is because when $\kappa_t > 0$, $q_t^n > q > q_t^i \geq 1 \geq q_t^r$. Therefore, the household will prompt entrepreneurs to spend whatever net worth they are not consuming on creating new equity. Entrepreneurs thus sell as many existing equity claims as possible and do not invest into money, i.e., $e_t = 1$ (or $U_t = (1-\delta)\chi S_t + I_t$) and $B_{t+1}^i = 0$.

In order to derive households' optimal decisions, we first consider the household-wide

budget constraint. We sum over the type-specific budget constraints (5) and (9) $\times \frac{q_t^n}{q_t^r}$:

$$\begin{aligned}\rho_t C_t^i + C_t^n + q_t^n S_{t+1} + \frac{B_{t+1}}{P_t} &= w_t N_t + [\chi\rho_t + (1-\chi)]\, r_t S_t \\ &+ [\chi\rho_t + (1-\chi) q_t^n]\,(1-\delta) S_t + [\chi\rho_t + (1-\chi)]\,\frac{R_t B_t}{P_t} \quad (16)\end{aligned}$$

where ρ_t is defined as the ratio between the effective purchasing price q_t^n and the effective replacement cost q_t^r , i.e.

$$\rho_t \equiv \frac{q_t^n}{q_t^r}. \quad (17)$$

The household then maximizes $J_t(S_t, B_t)$ subject to (16).

Labour choice. The first-order condition for labour from this optimization problem is

$$u'\left(\frac{C_t^n}{1-\chi}\right) w_t = \mu \quad (18)$$

which is a standard intra-period optimality condition. The marginal gain of extra consumption goods from earning wages should equal to the marginal dis-utility from working.

Risk sharing. The allocation of consumption goods between the two groups satisfies

$$u'\left(\frac{C_t^i}{\chi}\right) = \rho_t u'\left(\frac{C_t^n}{1-\chi}\right). \quad (19)$$

Notice that ρ_t is inversely related to risk-sharing among workers and entrepreneurs and measures the potential search market frictions.

When idiosyncratic risks can be fully insured like in a basic RBC model, entrepreneurs are not financially constrained and can implement the first-best investment schedule, such that the market price of equity equals its internal replacement cost. Therefore, $q_t = q_t^i = q_t^r = q_t^n = 1$. In such an unconstrained economy entrepreneurs do not need to restrain themselves and consume as much as workers. Therefore, full insurance implies $\rho_t = 1$.

In contrast, in an economy where idiosyncratic labor and investment risks are not insurable and the search market structure imposes further financing frictions, entrepreneurs cannot finance the first-best investment schedule. The market price of equity will remain above its replacement cost. In other words, we have $\rho_t > 1$ and $\frac{C_t^i}{\chi} < \frac{C_t^n}{1-\chi}$, i.e. entrepreneurs consume less than workers in order to expand investment. [11]

Portfolio choice. We now turn to the asset pricing formula for equity. The first-order

[11] Note that the absence of search costs, i.e. search cost $\kappa = 0$ is not a sufficient condition for full consumption risk insurance because labour income and wealth still differ across types, again leading to uninsurable investment risks. We will discuss this further.

condition for S_{t+1} is

$$q_t^n u'\left(\frac{C_t^n}{1-\chi}\right) = \beta \mathbb{E}_t J_{S,t+1}(S_{t+1}, B_{t+1}) \tag{20}$$

where J_S denotes the partial derivative of J w.r.t. S. With (19), the envelope condition implies that $J_{S,t}$ can be written as

$$J_{S,t} = u'\left(\frac{C_t^n}{1-\chi}\right)\left[\chi\rho_t\left(r_t+1-\delta\right)+\left(1-\chi\right)\left(r_t+(1-\delta)q_t^n\right)\right]. \tag{21}$$

Then, using (20) and (21), we obtain the asset pricing formula (Euler equation) for private claims

$$\mathbb{E}_t\left[\frac{\beta u'\left(C_{t+1}^n\right)}{u'\left(C_t^n\right)} r_{t+1}^e\right] = 1, \tag{22}$$

where the term $(1-\chi)$ cancels due to the homogeneity of the utility function. The second term in the expectations operator captures the internal return on equity from the perspective of the household:

$$r_{t+1}^e \equiv \chi\rho_{t+1} r_{t+1}^{ni} + (1-\chi) r_{t+1}^{nn}, \quad \text{where } r_{t+1}^{ni} \equiv \chi\rho_{t+1}\frac{r_{t+1}+(1-\delta)}{q_t^n}, \quad r_{t+1}^{nn} \equiv \frac{r_{t+1}+(1-\delta)q_{t+1}^n}{q_t^n}.$$

Intuitively, if a worker becomes an entrepreneur at time $t+1$, the return is r_{t+1}^{ni} because the household values each unit of next-period resources in the hands of entrepreneurs at ρ_{t+1}(note: an entrepreneur's marginal utility of consumption is ρ_{t+1} times that of a worker). On the other hand, if a worker does not change type at time $t+1$, the return to private claims is r_{t+1}^{nn}. The return from the point of view of the household is a weighted sum of r_{t+1}^{ni} and r_{t+1}^{nn}, with the weights representing the probability of agents to become entrepreneurs of workers next period.

Following similar steps, we can derive another asset pricing formula for money. The return from money is the inverse of inflation, with the latter being defined as

$$\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}.$$

The optimality condition for money holdings B_{t+1}^n is $u'\left(\frac{C_t^n}{1-\chi}\right)\frac{1}{P_t} = \beta\mathbb{E}_t\left[J_{B,t}\left(S_{t+1}, B_{t+1}\right)\right]$, where J_B denotes the partial derivative of J w.r.t. B. The return of money from the household's point of view is simply $\frac{(\chi\rho_{t+1}+1-\chi)}{\Pi_{t+1}}$, where the return accruing to a future entrepreneur

is again adjusted for ρ_{t+1}. Therefore, the asset pricing formula for money is

$$\mathbb{E}_t\left[\frac{\beta u'\left(C_{t+1}^n\right)}{u'\left(C_t^n\right)}\frac{\left(\chi\rho_{t+1}+1-\chi\right)}{\Pi_{t+1}}\right]=1. \tag{23}$$

Equations (18), (19), (22), and (23) summarize the household's choices.

3.2 Liquidity Premium

The above asset pricing formulae imply that private claims carry a liquidity premium, which compensates investors for impediments to transactions of these assets. For simplicity, we illustrate liquidity premium by focusing on the steady state values which will be denoted without time subscripts.

First, condition (23) implies that $\left(\chi\rho+1-\chi\right)\Pi^{-1}=\beta^{-1}$. If $\rho=1$, then $1/\Pi=\beta$, which is impossible with money in fixed supply and no government intervention. This is because $\Pi=1$ in steady state. Therefore, money will not be valued. If, however, money is valued, then $\Pi=1$ in steady state and

$$\rho=\rho^*=\chi^{-1}[\beta^{-1}-(1-\chi)]>1$$

As a result, the real interest rate on liquid assets Π^{-1} is lower than the rate of time preference β^{-1}.[12]

Further, as $\rho>1$, an individual entrepreneur is effectively financing constrained and consumes less than an individual worker as shown in (19). The reason for the binding financing constraints is that private claims are not fully liquid and cannot finance the first-best investment. By providing a liquidity service, government-issued assets (money in our model) mitigate financing constraints and are, therefore, valued. Conversely, privately issued assets demand a *liquidity premium* which amounts to the difference between the return from holding private claims and the return from holding money:

$$\Delta_t^{LP}=\mathbb{E}_t\left[\chi r_{t+1}^{ni}+(1-\chi)r_{t+1}^{nn}\right]-\mathbb{E}_t\left[\frac{1}{\Pi_{t+1}}\right]$$

As argued above, the liquidity premium is positive when entrepreneurs are financing constrained. That is, $\Delta_t^{LP}>0$ if two types of assets circulate and $\rho_t>1$ and. This fact is easiest to be seen in the steady state:

[12] Although we focus on fiat money, such that $P_t=P$ in the steady state (and $\Pi^{-1}=1<\beta^{-1}$), similar results obtain in an economy where the government issues interest-bearing securities.

Proposition 1:

Suppose that private claims and money exist in the steady state. Then, $r^{nn} > 1$ and money provides a liquidity service in the neighborhood around steady state. The steady state liquidity premium amounts to

$$\Delta^{LP} = \left(1 - \frac{1}{\rho}\right)(r^{nn} - 1)(1 - \chi) > 0$$

where the steady state value of ρ is $\rho = \chi^{-1}[\beta^{-1} - (1 - \chi)] > 1$.

Proof. See the Appendix B.1. □

Therefore, if $\kappa > 0$ and money does exist, we know that $\rho > 1$;[13] the liquidity premium is non-zero. Intuitively, the liquidity premium comes from two sources: one is that private claims are not fully resaleable as typically $\phi_t < 1$; the other is that, private claims have a spread Δ_t^s between the purchasing price and the selling price

$$\Delta_t^s \equiv q_t^n - q_t^i = \kappa\left(\frac{1}{\phi_t} + \frac{1}{f_t}\right)$$

As one can see from these aspects, the quantity liquidity aspect and price liquidity aspect are linked together. The linkage can be represented by the participation of buyers and sellers illustrated below.

3.3 Market Participation and the Circulation of Public and Private Liquidity

The asset price is set to maximize the total surplus of buyers and sellers. The sufficient and necessary first-order condition yields

$$\frac{\omega}{u'(\frac{C_t^i}{\chi})\left(q_t - \frac{1}{\phi_t}\right) + \left(\frac{1}{\phi_t} - 1\right)\beta\mathbb{E}_t\left[J_{S,t}\left(S_{t+1}, B_{t+1}\right)\right]} = \frac{1-\omega}{-u'(\frac{C_t^n}{1-\chi})q_t + \beta\mathbb{E}_t\left[J_{S,t}\left(S_{t+1}, B_{t+1}\right)\right]}. \tag{24}$$

[13] One might believe that the optimal policy is to reduce this liquidity premium by paying a nominal interest rate R, such that the real interest rate $\bar{R}$ equals time preferences rate β^{-1}. However, this is not necessarily true when government expenditures G is strictly positive and it directly enters the household's utility, as a benevolent government might still prefer a low real interest rate to finance the government expenditures (even when lump-sum taxes are available). The exploration of optimal monetary and fiscal policies is beyond the scope of our paper. But we invite the reader to a companion paper by Cui (2013b) for a reference.

By using the household's optimality condition for asset holdings (20) and the risk-sharing condition (19), we can derive an analytical solution for the asset price:

Proposition 2:

Suppose private claims exist. The bargaining solution simplifies to

$$\rho_t = \frac{\omega}{1-\omega}\theta_t. \tag{25}$$

Alternatively, (25) q_t *can be expressed as*

$$q_t = \frac{\rho_t\left(1+\frac{\kappa_t}{\omega}\right) - \frac{\kappa_t}{f_t}}{1+(\rho_t-1)\phi_t}. \tag{26}$$

Proof. See Appendix B.2. □

Proposition 2 is our main analytical result. This Proposition links the asset price with search costs and asset saleability. Importantly, in our model the Euler equation is not enough for determining asset price, as we need both asset saleability and asset price to be determined together. The bargaining solution (25) solves this issue by connecting asset saleability with asset price. This condition is in fact an "entry condition" similar to a free-entry condition in many previous papers in the asset search literature.

One can think of the "entry condition" in the following way. If the Euler equation determines the asset price, then the participation of buyers and sellers and search intensity need to be such that (25) is satisfied in order to link asset price and asset saleability. To illustrate further, the participation decisions can be seen from rewriting (25)

$$\frac{(1-\omega)q_t^n M_t}{\omega q_t^r M_t} = \theta_t = \frac{\kappa V_t}{\kappa U_t}$$

The left-hand side (LHS) is the ratio of buyers' valuation of asset transaction and sellers' valuation of asset transaction, weighted by their respective bargaining weights $(1-\omega)$ and ω; while the right-hand side (RHS) is the ratio of buyers' participation costs $(\kappa_t V_t)$ and sellers' participation costs $(\kappa_t U_t)$. Therefore, on the margin, the LHS needs to be the same as the RHS, such that the participation of buyers and sellers will deliver the gains that induce their participation. [14]

Remark: As a comparison, in a traditional asset pricing model, the Euler equation of

[14] Because we simplify the problem, we do not solve search intensity θ_t, purchasing quotes V, and selling quotes U directly. However, one can easily back out the search intensity from (25) or by reverse-engineering from S_{t+1}^i and S_{t+1}^n.

the investors will determine the asset price, given their consumption profiles. In a production economy, we need further conditions to determine consumption paths. But in either case, assets have full liquidity and $\phi_t = 1$.

Now, we can come back to the question when do private claims and money circulate. We focus on the neighborhood around steady state. There could exist two cut-off values of steady state κ that can characterize the existence of private claims and money (private and public liquidity).

On one hand, for private claims to exist, search costs cannot be too large. To see this, when financial markets are active, the replacement costs $q_t^r \leq 1$. Otherwise, entrepreneurs use only internal financing. Using the definition of q^r, we know that the selling price $q^i = q - \frac{\kappa}{\phi} \geq 1$. Notice that q is bounded as total resource from buyers is bounded; ϕ is also bounded by unity. Therefore, there must exist a threshold κ_2 leading to $q^i = 1$. A higher search cost $\kappa > \kappa_2$ reduces q^i to a level below unity, and private claims will not be issued by sellers.

On the other hand, for public liquidity (money) to exist, the search costs cannot be too small. Otherwise, private claims can provide enough liquidity compared to money with a low return. That is, even though it is costly to search the counterparties, there will be enough buyers for private claims to have a high return. Private claims provide enough liquidity and no one values money. In other words, money does not exist if financial frictions are not severe enough.

Proposition 3:

Private claims and money co-exist if and only if search costs κ satisfies

$$\kappa_1 \leq \kappa \leq \kappa_2 \tag{A1}$$

where κ_1 and κ_2 satisfy

$$\kappa_2 \equiv \frac{\rho^* - 1}{\gamma\rho^* + 1} M\left(1, \gamma\rho^*\right), \quad \gamma = \frac{1-\omega}{\omega}$$
$$\kappa_1 \equiv \max\{0, \tilde{\kappa}_1\}, \quad \tilde{\kappa}_1 = H(\chi, \beta, \delta, \alpha, \xi, \eta, \omega)$$

and H is some non-linear function specified in the Appendix. In addition, only private claims circulate when $\kappa \in [0, \kappa_1)$ and only money exists when $\kappa \in (\kappa_2, +\infty)$.

Proof. See the Appendix B.3. □

When $\kappa > \kappa_2$, entrepreneurs are still financially constrained, but the benefits of outside

financing cannot compensate the costs any more. The upper bound κ_2 in condition (A1) depends on the matching function, the bargaining weight ω, the discount factor β, and the probability of getting an investment opportunity χ. These parameters determine the value of trading a unit of equity claims and the willingness to participate. For example, more impatient participants (a lower β) will be further financing constrained (a higher ρ^*). Given that both $M(1, \gamma\rho^*)$ $\frac{\rho^*-1}{\gamma\rho^*+1}$ are increasing function of ρ^*, we know that they are willing to bear higher search costs. That is, the threshold κ_2 is higher with more impatient participants who will be further financing constrained.

The calculation of κ_1 and the verification of non-monetary equilibrium in $\kappa \in [0, \kappa_1)$ require the computation of steady state real value of liquidity L_t, which is

$$L_t = \frac{B_t}{P_{t-1}}.$$

Suppose money is valued and the asset pricing formula holds. Then, the steady state $\rho = \rho^* = \chi^{-1}\left[\beta^{-1} - (1-\chi)\right]$ is uniquely pinned down. Now, given exogenous parameters, we compute all the equilibrium conditions and check if indeed the equilibrium real value of liquidity $L(\kappa) \geq 0$. We can show that $\frac{dL}{d\kappa} > 0$ and therefore κ_1 is the point such that $L(\kappa) = 0$.

If $\kappa_1 > 0$, we know that money cannot exist when $\kappa \in [0, \kappa_1)$; If $\kappa_1 = 0$ (because $\tilde{\kappa}_1 \leq 0$), then money is always valued for any $\kappa > 0$. As we focus on equilibria in which private claims (backed by the real return from capital) have priority to circulate, only they exist when $\kappa \in [0, \kappa_1)$.

3.4 Search Costs, Asset Price, and Asset Liquidity

Now, we discuss the relationship among search costs, asset price, and asset liquidity.

When $\kappa \in (\kappa_2, +\infty)$, private claims will not circulate.

When $\kappa \in [\kappa_1, \kappa_2]$, money exists in the region, and $\rho = \frac{\beta^{-1}-(1-\chi)}{\chi}$ is uniquely pinned down by the asset pricing formula of money. Then, (25) implies that the search intensity $\theta = \omega\rho/(1-\omega)$. Since asset saleability ϕ and purchase probability f are only functions of θ, we know that asset price

$$q = \frac{\rho\left(1+\frac{\kappa}{\omega}\right) - \frac{\kappa}{f}}{1+(\rho-1)\phi} = \frac{\rho + \kappa\left(\frac{\rho}{\omega} - \frac{1}{f}\right)}{1+(\rho-1)\phi}$$

has a simple relationship with the search cost κ.

With the increase of search costs, on one hand, (25) implies that a higher κ pushes down

asset price since participation in the search market becomes more costly which drives buyers away from the market; on the other hand, an increase of κ in (25) pushes up asset price as it increases the costs of posting assets for sale. The net effect of higher intermediation costs on the equity price, therefore, depends on the parameters. However, the effective selling price q^i always decreases with κ and the spreads Δ^s always increase with κ.

Corollary 1:
Suppose the economy is in steady state and $\kappa \in [\kappa_1, \kappa_2]$. When only κ becomes larger, the selling price $q^i = q - \frac{\kappa}{\phi}$ always decreases, the spreads $q^n - q^i = \frac{\kappa}{f} + \frac{\kappa}{\phi}$ increases, while asset saleability ϕ is a constant. Asset price q decreases with κ if and only if $\phi < 1 - \omega$, or the following condition is satisfied

$$\xi \left(\rho^*\right)^{1-\eta} < (1-\omega)^{\eta} \omega^{1-\eta}, \tag{A2}$$

Proof. See the Appendix B.4. □

In other words, if the sensitivity of the demand changes to an increase κ is larger than the sensitivity of the supply changes, the bargained asset price will fall with an increased κ. To understand this, an increased of κ initially pushes up asset price as it needs to be compensated for a transaction to take place. However, demand will drop in response to the increase of κ. If the drop is large enough to overturn the initial impact of a large κ, asset price drops.

Notice that in steady state asset saleability ϕ does not move, as long as money is valued. Therefore, money serves as a lubricant in the economy. Because our convenient assumption of CRS technology in production and in matching, ρ is the same in the two regions $\kappa \in [\kappa_1, \kappa_2]$ or $\kappa \in (\kappa_2, +\infty)$. However, since financial markets exist in the region of $\kappa \in [\kappa_1, \kappa_2)$, more capital will be accumulated and wages will be higher even though the risk-sharing factor ρ is the same as in $\kappa \in [\kappa_2, +\infty)$,. Therefore, the consumption of both entrepreneurs and workers will be higher compared to the region of $(\kappa_2, +\infty)$.

In steady state, asset saleability ϕ does not change because ρ is uniquely determined by money's asset pricing formula, even though under (A2) q decreases with κ. In response to shocks, however, ϕ and q move together. For example, if there are aggregate productivity shocks, we can show that asset price q_t and asset saleability ϕ_t positively comove, provided that asset liquidity is not too large.

Corollary 2:

Suppose $\kappa \in [\kappa_1, \kappa_2]$. q_t can be expressed as a function of liquidity ϕ_t

$$q_t = \frac{\gamma\left(1+\frac{\kappa}{\omega}\right)\phi_t - \kappa}{\xi^{\frac{1}{1-\eta}}\phi_t^{\frac{\eta}{\eta-1}}\left[1+\left(\gamma(\xi^{-1}\phi_t)^{\frac{1}{1-\eta}}-1\right)\phi_t\right]}$$

Then, with only aggregate productivity shocks, *q_t correlates positively with asset saleability ϕ_t (i.e. $\frac{\partial q}{\partial \phi} > 0$) and negatively with the purchase rate f (i.e. $\frac{\partial q}{\partial f} < 0$), if*

$$\phi_t < \left[\frac{\eta}{1-\eta} + \left(1+\frac{\kappa}{\omega}\right)\right]\left[\frac{\eta}{1-\eta} + 2\gamma(\xi^{-1}\phi_t)^{\frac{1}{1-\eta}} - 1\right]^{-1}. \tag{A3}$$

When $\eta = 0.5$, the above sufficient condition simplifies to $\phi_t < \sqrt[3]{(1+\frac{\kappa_t}{2\omega})\gamma^{-1}\xi^2}$.

Proof. See Appendix B.2. □

Intuitively, the drop in saleability implies that a larger share of investment needs to be financed out of entrepreneurs' own funds. On one hand, this tightens the contemporaneous financing constraints of entrepreneurs. The threat point for entrepreneurs of breaking off negotiations over an additional asset sale and self-financing at the margin becomes less attractive. Entrepreneurs are thus more willing to accept a lower bargaining price. On the other hand, retaining a larger fraction of equity stakes also implies that entrepreneurs return more assets to the household, which relaxes the funding constraints of future generations of entrepreneurs. This effect supports the threat point, such that entrepreneurs ask for a higher transaction price in a successful match. Thus, a trade-off emerges between current and future funding constraints.

Proposition 2 shows that the contemporaneous effect dominates as long as the sales rate is small enough, because current financial constraints bind strongly. If financial frictions are sufficiently tight, entrepreneurs will have to accept a lower price when the demand side is less willing to participate. Our model can thus generate simultaneous decreases in asset saleability and the asset price through the simultaneous reaction of supply and demand; the demand is affected more and is reflected in the search intensity.

Remark: Proposition 2 is specific to shocks to aggregate productivity so that κ_t is fixed. We cannot obtain general results when κ_t is stochastic, as ϕ_t depends on κ_t and other macro variables. However, one can see that if κ's impact does not overturn the impact of ϕ_t on q_t, the positive comovement between asset price and asset liquidity remains. To show this, we turn to numerical simulations after model calibration.

Finally, when $\kappa \in [0, \kappa_1)$, only private claims exist and higher search costs act just like a traditional higher degree of capital-adjustment costs. The transaction price q and the

risk-sharing measure ρ increase with κ.

Corollary 3:
Suppose the economy is in steady state and $\kappa \in [0, \kappa_1)$, asset saleability ϕ, asset price q, and risk sharing factor ρ increase with κ.

Proof. See Appendix B.6. □

Intuitively, high search costs lead to less risk-sharing and therefore a higher ρ. Tobin's q (q^i for entrepreneurs and q^n for workers) needs to increase to reflect the higher degree of financing constraint. Due to the participation conditions, asset saleability ϕ needs to increase to encourage entry.

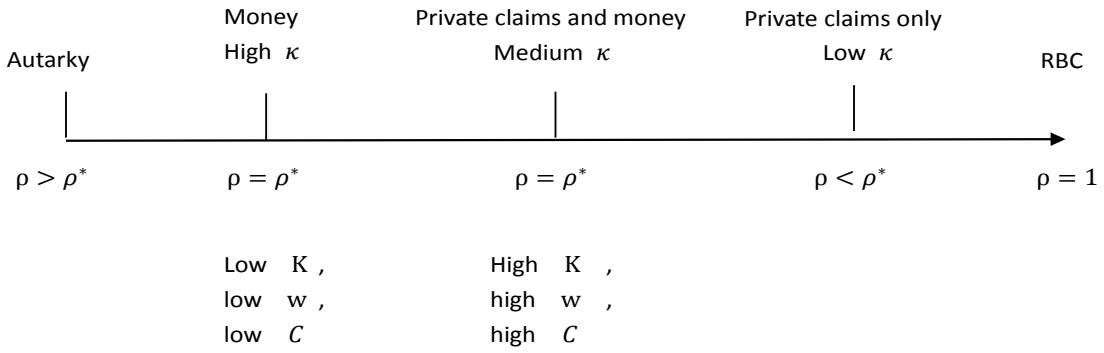
Remark: We can explicitly display a continuum of models on the horizontal axis with the division of ρ in Figure 1. A basic RBC model (see Appendix A.3 has full risk-sharing and $\rho = 1$. An autarky economy, without the existence of insurance markets, money, or private claims, features $\rho > \rho^*$. Between the autarky economy and the RBC economy, there could be a continuum of models, depending on how large the search costs are. When search costs $\kappa < \bar{\kappa}_1$ is small enough, only private claims exist and $1 < \rho < \rho^*$; when search costs $\kappa \geq \kappa_1$ is large enough, money will always be valued and $\rho = \rho^*$. However, there could be a region $\kappa \in [\kappa_1, \kappa_2)$ in which private claims and money co-exist. In this region, although $\rho = \rho^*$ is fixed, but the circulation of private claims facilitate the resources flow to entrepreneurs with investment opportunities; there will be a higher degree of capital accumulation, leading to a higher wage rate, and more consumption.

Remark: When $\kappa = 0$, asset price $q \neq 1$. It only implies that $q^n = q^i$. Entrepreneurs will still be financing constrained as there are uninsured labor income risks. Money may or may not be valued depending whether $\kappa_1 \geq 0$ or $\kappa_1 < 0$. If however we assume that labor income risks can be insured and $\kappa = 0$, we know that $q_t^i = q_t^n = q_t^r = \rho_t = 1$ (details in the Appendix A.4). In this case, if we denote total consumption $C_t = C_t^i + C_t^n$, the household's budget constraint becomes (16):

$$C_t + S_{t+1} = w_t N_t + r_t S_t + (1 - \delta) S_t,$$

which is very similar to the budget constraint in a basic RBC model (see the Appendix for details). In fact, $S_t = K_t$ in equilibrium we are essentially back to the RBC framework.

Figure 1: **A Continuum of Models** Models with different search costs and risk-sharing technology. ρ measures the difference of marginal utility of consumption between an entrepreneur and a worker. $\rho^* = \chi^{-1}[\beta^{-1} - (1-\chi)]$ is a pinned down by money's asset pricing formula. The RBC economy needs both $\kappa = 0$ and the risk-sharing of labor income. The autarky economy features the equilibrium in which search costs $\kappa \geq \kappa_2$ and no one values money. Steady state equilibrium with only private claims, only money, and the coexistence of the two depends on the search costs κ. The economy with both private claims and money features a higher degree of capital accumulation than the economy with only money. Therefore, the wage rate w is higher and consumption is higher.



4 Calibration

In the previous sections, we have developed a macro model with asset search. In this section, we calibrate the parameters using data on financial markets and aggregate economy. In the next section, we will use the calibrated model to measure the effect of aggregate productivity shocks and search cost shocks on liquidity premium, investment, consumption, and production.

4.1 Targets

Without loss of generality , we specify the matching function as

$$M(U,V) = \xi U^{\eta} V^{1-\eta} \tag{27}$$

where ξ is matching efficiency and $\eta \in [0,1]$ is the elasticity w.r.t. selling quotes. We choose a conventional CRRA utility function of consumption and a linear dis-utility of labor:[15]

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \quad h(n) = \mu n.$$

[15]This dis-utility function facilitates the steady state solution. The main results are robust to a more complicated specification. See the discussion in the calculation of steady state values in the Appendix.

We calibrate the steady state of the model to several long-run U.S. statistics. Parameters β, σ, and δ are chosen exogenously and are similar to a standard calibration. α and μ are set to target the investment-to-GDP ratio and working hours (Table 2). Note that GDP's counterpart in the data is the sum of real private consumption (note: C_t in the model) and real private investment (note: $I_t + \kappa(U_t + V_t)$ in the model). We use quarterly data 1971Q1 to 2014Q4 (accommodating asset price data we will use later for business cycle statistics) obtained from the FRED data set, and the investment-to-GDP ratio is roughly 20%.

Table 2: Steady state calibration

	Parameter	Baseline Value	Target/Source
Preferences and Production Technology			
Household discount factor	β	0.9850	Exogenous
Relative risk aversion	σ	2	Exogenous
Utility weight on leisure	μ	2.6904	Working time: 33%
Mass of entrepreneurs	χ	0.0540	Doms and Dunne (1998)
Depreciation rate of capital	δ	0.0250	Exogenous
Capital share of output	α	0.3750	Investment-to-GDP ratio: 20.0%
Search and Matching			
Supply sensitivity of matching	η	0.5000	Exogenous
Matching efficiency	ξ	0.2695	Saleability $\phi_u = 0.3000$
Search costs	$\bar{\kappa}_v$	0.0216	Tobins $q = 1.1500$
Bargaining weight of sellers	ω	0.5085	$B/(B + PqK) = 0.3015$

There are four search-market related parameters $\{\xi, \eta, \kappa, \omega\}$ and one parameter χ that is related to idiosyncratic investment risks. ξ and η are not independent due to the constant returns to scale matching technology on the search market. Without loss of generality, we set $\eta = 0.5$ and calibrate ξ.

We are then left with four independent parameters $\{\xi, \kappa, \omega, \chi\}$, which we calibrate jointly to match four targets. The parameter χ can be interpreted as the fraction of firms which adjust capital in a period. According to Doms and Dunne (1998), the annual fraction is 0.20 which translates to $\chi = 0.054$ at quarterly frequency (similar to Shi (2015)). Asset price q can be considered as Tobin's q in the data. Tobin's q ranges from 1.1 to 1.21 in the U.S. economy according to COMPUSTAT data. We target a value of $q = 1.15$.

Steady state saleability ϕ is calibrated at 0.30, which corresponds the ratio of funds raised in the market to fixed investment in the U.S. flow-of-funds data.[16] Finally, as $\kappa = 0$ is likely to generate an equilibrium without the existence of money or fully liquid public issued assets, we calibrate κ such that the liquidity-GDP ratio (real value of liquidity divided by GDP) is 30.1%. This number is roughly the ratio of total amount of money-equivalent assets

[16] Nezafat and Slavik (2010) use the US flow-of-funds data for non-financial firms to estimate the stochastic process of ϕ.

(including cash, checkable deposits, short-term Treasury bills) from the flow-of-funds data divided by GDP. With such calibrated κ, the search costs are only about 2% of total GDP.

Notice that with these parameters, assumptions (A1), (A2), and (A3) are satisfied and we should expect the asset price and asset saleability to positively co-move.

4.2 The Long-run Impacts of Search Costs

We examine the existence of public and private liquidity in steady state, by tracing κ over the positive domain. To do so, we first compute κ_1 and κ_2 by using Proposition 3:

$$\kappa_1 = 0.0054, \quad \kappa_2 = 0.0378$$

Therefore, when $\kappa \in [0, 0.0054)$, only private claims exist as they have enough liquidity values and dominate money; when $\kappa \in [0.0054, 0.0378]$, money will be valued, since the liquidity value of private claims drops; when $\kappa \in (0.0378, +\infty)$, only money circulates.

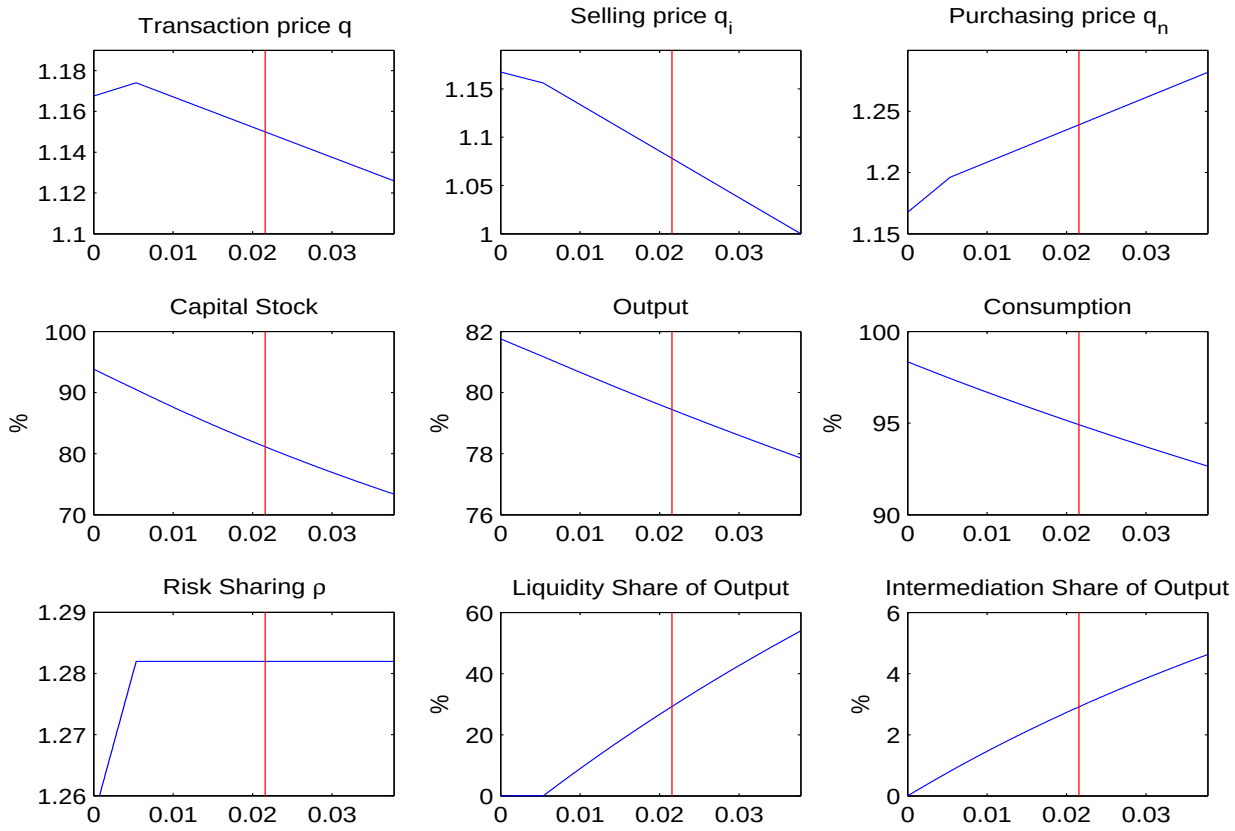
Now, we illustrate the impact of search costs on asset saleability, asset prices, and in particular the macroeconomy. To do so, we show how macro variables change when κ increase (Figure 2). For macro variables, we focus on consumption, investment, and total output; the quantities of these variables are normalized to 100 in the frictionless economy (i.e., a basic RBC model, see the details in the Appendix). Notice that $\kappa = 0$ is not equivalent to a basic RBC economy, as investment opportunities and labor income are still not fully insurable.

When we increase κ from 0 to κ_1, ρ rises from 1.26 to $\rho = \bar{\rho} = 1.282$. That is, risk-sharing become worse. When search costs increase, it is harder for search market to transfer funds. However, in this region, money is not valued as search market is liquid enough, and that is why liquidity share of output $L/Y = 0$. Asset prices q and q^n increase with search costs. Thus, search costs act like investment adjustment costs that consumes resources. In response, investment, consumption, and production drop with larger search costs.

Importantly, when the complete market is replaced by the financial market with search technology and with search costs $\kappa = 0$, capital accumulation is 7% lower than the first-best level which brings down marginal product of labor. Lower marginal product of labor reduces the demand for labor such that output is only about 82% of that in a basic RBC model. Consumption drops by 2% as result of less resource. When κ becomes larger, consumption, investment, and output are further reduced.

When $\kappa \in [\kappa_1, \kappa_2)$, money is valued and there is co-existence of money and private claims. The higher the search costs, the larger need to hold liquid assets. The liquidity share of output (L/Y) thus increases monotonically with search costs from 0% to 54%. In

Figure 2: **Comparative Statics**. Consumption, investment, and output are expressed as percentage of the quantities in a frictionless model (i.e., a basic RBC model). Liquidity share of output is L/Y and intermediation share of output is $\kappa(U+V)/Y$. The red vertical line cuts the horizontal axes through the calibrated κ.



contrast to the region of $[0, \kappa_1)$, q decreases as search costs increases, reflecting the theoretical results discussed in Corollary A2. q_i decreases to 1 when $\kappa = \kappa_2$ and entrepreneurs decide not to participate in the financial market.

As κ increases, consumption, investment, output are further depressed. When κ increase from κ_1 to κ_2, investment, output, and consumption drop by about 20%, 8%, and 6%, respectively. At the same time, the total search costs only increase to 4.8% of total resources produced. We thus see the amplification effects from less developed financial markets, modeled as increased of search costs.

In sum, when search costs become larger, money is valued more. Low return from money and higher costs from trading private claims imply that physical capital stock will be underaccumulated, leading to lower consumption and production.

5 Equilibrium Responses to Shocks

This section uses numerical tools to illustrate system dynamics after exogenous shocks. We illustrate the dynamics when standard aggregate productivity shocks hit, and when financial markets temporarily experience troubles modeled as temporary changes of search costs. The second shock can also be thought of as productivity shocks that mainly change the production of resources transfers. Unlike the standard aggregate productivity shocks that affect the production of final consumption goods, the search costs shocks affect the reduction of the final goods in order to make valuable investment.

5.1 Aggregate Productivity Shocks and Search Cost Shocks

We consider a standard AR(1) process for aggregate productivity, i.e.,

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A$$

with $0 < \rho_A < 1$ and i.i.d. $\epsilon_t^A \sim N(0, \sigma_A^2)$. We further introduce a shock to the cost of financial intermediation, which in our asset search framework corresponds to a change in the participation costs. We let

$$\log(1 + \kappa_t) = \rho_\kappa \log(1 + \kappa_{t-1}) + (1 - \rho_\kappa) \log(1 + \kappa) + \epsilon_t^\kappa$$

with $0 < \rho_\kappa < 1$ and i.i.d. $\epsilon_t^\kappa \sim N(0, \sigma_\kappa^2)$. Rather than affecting the production frontier of the economy, this shock simply impairs the capacity of the financial sector to intermediate funds between workers and entrepreneurs. Both in a market and a banking context, such an

increase in intermediation costs may, for example, be triggered by rising uncertainty about counterparty risk. Or in a banking network structural, temporary failures of critical banks in the network may result in significant rise of intermediation costs. In contrast to productivity shocks, intermediation cost shocks do not directly affect production, such that they unfold their effects primarily through the endogenous responses of asset saleability and prices which further have an impact on production. In order to have a fair comparison, the persistence and the size of shocks target the volatility (0.02) and 1st order correlation (0.91) of GDP's cyclical components. By using only productivity shocks, we have

$$\rho_A = 0.90, \sigma_A = 0.008.$$

By using only shocks to intermediation costs, the exercise gives

$$\rho_\kappa = 0.82, \sigma_\kappa = 0.012.$$

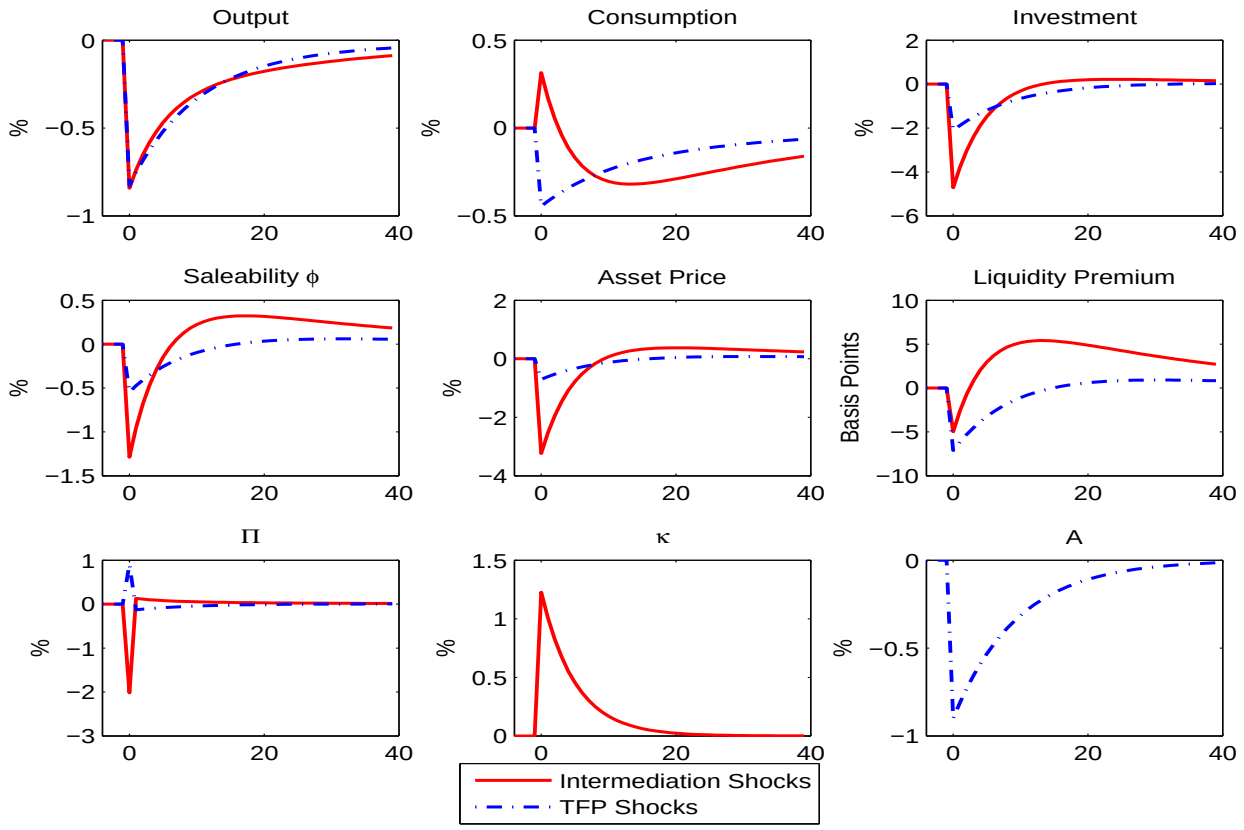
We use these parameters to show two numerical simulations in the following. By design, these two shocks will generate very similar aggregate output dynamics. The focus will be the different paths of other variables.

Negative aggregate productivity shocks. Suppose an adverse productivity shock hits the economy at time 0 (see the dynamics of A_t in Figure 3). This shock depresses the rental rate of capital and its value to the household. Search for investment into entrepreneurs is less attractive and the amount of purchase orders from workers drops. The demand-driven fall is reflected in the sharp drop in asset saleability ϕ. This endogenous decline of asset liquidity amplifies the initial shock in two ways: (1) it reduces the quantity of assets that entrepreneurs are able to sell; (2) the asset price, i.e. private assets' resale value falls - though only modestly - in line with our analytical result in Proposition 2. Both effects constrain entrepreneurs and thus tighten their financing constraints. As a result, investment falls; consumption also falls because of fewer resources.

In principle, money's liquidity service becomes more valuable to households when private claims' liquidity declines. However, in the case of a persistent TFP shock, lower expected returns to capital make *future investment less attractive.* This effect works against the incentive to hedge against asset illiquidity for future investment. The first effect has a positive impact on liquidity premium, while the second effect has a negative impact.

Which effect dominates depends on the calibration and is thus an empirical question. In our calibration, the decline in profitability of investment projects is sufficient for liquidity premium to drop. This fall in demand for liquid assets is reflected in the decrease of their price $1/P$, which leads to a surge in inflation $\pi = P/P_{-1}$ on impact. To the extent that total

Figure 3: Impulse responses after a standard deviation shock to aggregate productivity or intermediation search costs at time 0. The units of liquidity premium are annualized changes in basis points. Units of other variables are percentage changes from their steady state levels.



factor productivity reverts back to the steady state while asset liquidity is still subdued, hedging becomes more attractive which explains the relatively fast recovery of the liquidity premium.

Remark: In our framework, the liquidity (resaleability) of financial assets is endogenously generated through the features of the search market. In the absence of search frictions, these liquidity effects would not occur after adverse shocks. In a RBC world, negative TFP shocks primarily affect the demand for capital goods and thus reduce the optimal level of investment. In addition to this effect, entrepreneurs are financing constrained in our model, which strongly amplifies the response of investment as argued above.

Intermediation shocks. Suppose a shock hits the economy at time 0 (see the dynamics κ in Figure 3). The output dynamics in this scenario are by construction similar to those of the productivity shock.

Note that higher search costs bind resources. Both the substitution and income effects induce households to adjust their portfolios. Realizing that search market participation is

more costly now and later, households seek to reduce their exposure to private financial claims. On the supply side, financing-constrained entrepreneurs would still like to sell as many assets as possible in order to take full advantage of profitable investment opportunities. Thus, asset demand on the search market shrinks relative to supply, which reduces the likelihood for sellers to be matched with buyers, such that asset saleability falls.

Since the sharp drop in asset liquidity tightens entrepreneurs' financing constraints substantially, the threat point of abandoning the bargaining process with a potential buyer worsens. Entrepreneurs as sellers are willing to accept a lower price. The bargaining price thus falls strongly and amplifies the initial shock by depressing entrepreneurs' net worth further. This effect is mirrored in a significant decline of investment activity, the impact response of which is about six times stronger than that of output.

As saving via the financial market becomes more expensive, workers reduce their labor supply and consume slightly more after the initial shock. Entrepreneurs consume much less because of significant financing constrained. As a result, aggregate consumption increases slightly initially, while output falls almost three times immediately because of fewer labor supply. However, lower investment into the capital stock soon reduces the marginal product of labor and the wage rate. As labour income falls, consumption persistently drops below the steady state.

While the intermediation cost shock depresses the demand for and liquidity of private assets, it substantially increases the hedging value of money and money becomes more valuable. To see this, note that future investment remains profitable since the productivity of capital is not affected by the shock. To take advantage of future investment opportunities, households seek to hedge against the persistent illiquidity of private claims by expanding their liquidity holdings. Therefore, the liquidity premium rises, although it falls slightly at the beginning due to lower norminal price level, another sign of "flight to liquidity". With the accumulation of liquidity, buyers later have more resources to buy private claims which improves the market liquidity. That is why asset price and asset saleability overshoot above the steady state levels after about 3 years.

5.2 Business Cycle Statistics

The equilibrium dynamics suggest two key results. (1) In order to reconcile declining asset saleability with falling asset prices, liquidity must be an endogenous phenomenon. In other words, it must be a consequence, rather than a cause of economic disturbances. (2) Both standard productivity and intermediation cost shocks affect the hedging value of liquid assets. However, only the latter unambiguously implies a negative co-movement between

the liquidity premium and aggregate output. Having described the transition dynamics, we further compare with data the model's predictions for the cyclical behaviors of macro and finance variables.

For asset price, we use the Wilshire 5000 price full cap index from 1971Q1-2014Q4, as it covers possibly the largest number of stocks that are traded. 1971Q1 is the first quarter that this index is available. For asset liquidity, it is difficult to measure real and time costs in transaction as they depend on many factors such as the size of a trade, the timing, trading venue, etc. Further, the information needed to calculate transaction costs is often unavailable. Thus, in previous studies, a number of measures are used to evaluate asset liquidity, such as bid-ask spreads and trading volume. But many of these measures do not have a long enough time series for macro models. Or they may have their own issues of measuring liquidity. For example, a large trading volume implies more active markets but may also be correlated with volatility, which can reduce market liquidity.

Following Naes, Skjeltorp, and Odegaard (2011), we choose a simple and popular proxy of illiquidity measure for private claims and money-like government issued assets: the Amihud (2002) measure. This measure can be easily obtained from quarterly, monthly, or even daily data:

$$ILR_{i,T} = \frac{\sum_{t=1}^{T} \frac{|R_{i,t}|}{VOL_{i,t}}}{D_T}$$

where D_T is the number of trading days within a time window T, $|R_{i,t}|$ is the absolute return on day t for an asset i, and $VOL_{i,t}$ is the trading volume (in units of currency) on date t. $ILR_{i,T}$ measures asset illiquidity, as a high $ILR_{i,T}$ implies low liquidity (there is a high price impact of trades). Therefore, the Amihud measure captures how the price moves for each volume unit of trades. From this measure, we know that liquid assets will tend to have a low Amihud measure. They carry a liquidity premium such that the return will be low; the saleability of liquid assets are high and the volume will tend to be large.

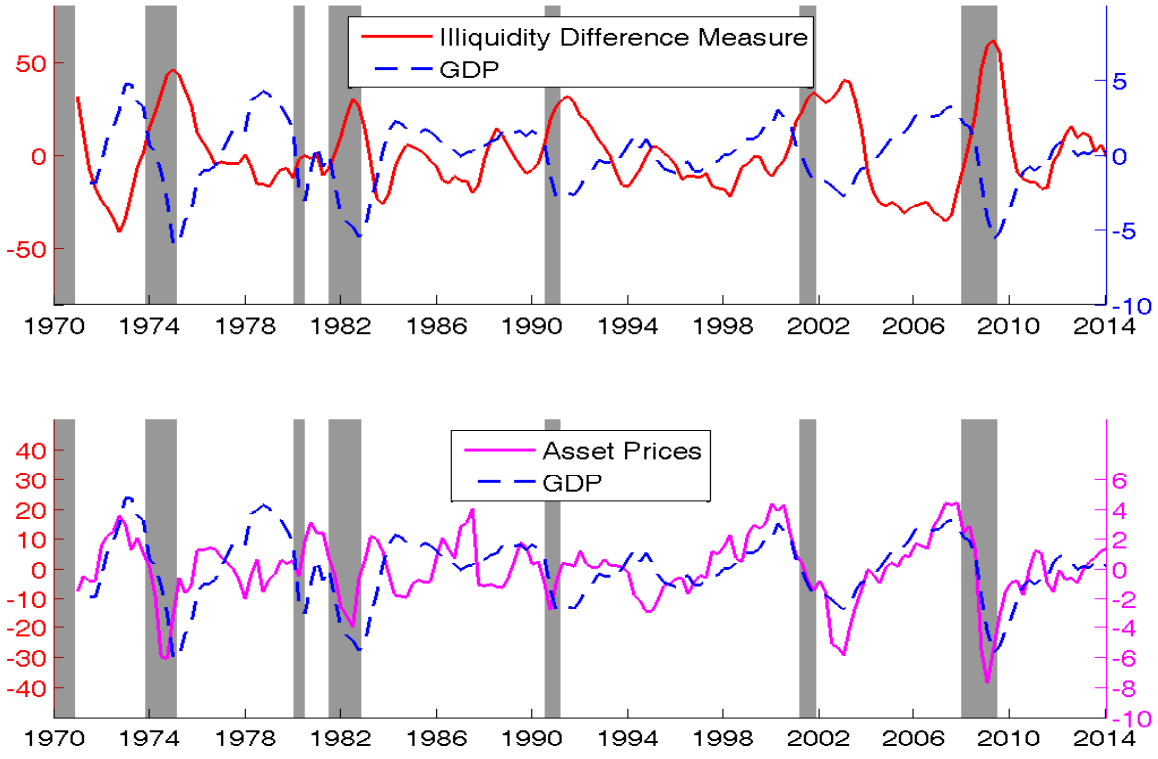
Our goal is to obtain a measure that can show liquidity difference between private claims and money-like government issued assets. We then calculate two illiquidity measures for these two types of assets ILR_T^P and ILR_T^M, and call the difference

$$ILR_T^D = ILR_T^P - ILR_T^M$$

as the *illiquidity difference measure*. Therefore, the liquidity premium from our model should exhibit similar patterns from the illiquidity difference measure.

To calculate the illiquidity measure, we use data on stock prices, returns, and trading volume. We obtain monthly sample data from CRSP (the Center for Research in Security

Figure 4: **Cyclical components of illiquidity difference measure, asset prices, and GDP.** All series are cyclical components of HP filtered original series times 100. The shaded areas are NBER dated recessions.



Prices) for the period 1971 to 2014 and then average it to be quarterly. Furthermore, we obtain an aggregate ILR_T^P by equally weighting cross-sectional $ILR_{i,T}$ measures. Note that we restrict attention to stocks listed at the NYSE to keep the sample as homogenous as possible. In addition, we use the similar strategy to compute the ILR_T^M from the data of 3-month Treasury bills.

We use the HP filter (with a smoothing coefficient of 1600) to de-trend all time series. Asset prices tend to fall in recessions, which tend to be associated with portfolio rebalancing towards liquid assets (Figure 4). Not surprisingly, the illiquidity difference measure correlates negatively with GDP (-0.67), while asset prices correlate positively with GDP (0.51).

Some key business cycle statistics of the model in comparison to the data are reported in Table 3, where only aggregate productivity shocks are considered. Our main targets are consumption, investment, asset prices, and liquidity premium. Again, since we do not have a direct observation of liquidity premium (or it is very hard to construct), the illiquidity difference measure ILR_t^D is used to check against model implied liquidity premium. Therefore,

Table 3: Business cycle statistics with only aggregate productivity shocks

	Relative volatility $\frac{\sigma_x}{\sigma_y}$		Correlation $\rho(x,y)$		1st auto-correlation	
Variable x	Data	Model	Data	Model	Data	Model
Output	0.02	0.02	1.00	1.00	0.91	0.91
Consumption	0.58	0.66	0.93	0.98	0.89	0.94
Investment	3.21	2.26	0.95	0.98	0.86	0.89
Liquidity premium	9.31	6.62	-0.67	0.78	0.92	0.87
Asset Price	4.88	0.70	0.51	0.82	0.81	0.87

Table 4: Business cycle statistics with only intermediation shocks

	Relative volatility $\frac{\sigma_x}{\sigma_y}$		Correlation $\rho(x,y)$		1st auto-correlation	
Variable x	Data	Model	Data	Model	Data	Model
Output	0.02	0.02	1.00	1.00	0.91	0.91
Consumption	0.58	0.82	0.93	0.48	0.89	0.97
Investment	3.21	3.86	0.95	0.78	0.86	0.80
Liquidity premium	9.31	13.03	-0.67	-0.50	0.92	0.97
Asset Price	4.88	2.64	0.51	0.58	0.81	0.79

even though the volatility of liquidity premium and the illiquidity measure might not be the same, the correlation between these two should be close to 1.

Similar to a basic RBC model, consumption and investment volatility, the correlation of macroeconomic variables with GDP, and first-order autocorrelations are roughly in line with the data. However, the liquidity premium and the asset price move too little in the model. Besides, the model-implied positive correlation (0.78) between the liquidity premium and GDP falls short of the data (-0.67). All these statistics confirm the result obtained in the impulse responses exercises.

As a comparison, Table 4 shows the relevant statistics when there are only intermediation shocks. Unlike the economy with productivity shocks only, the volatility of the liquidity premium and the asset price are much higher. However, liquidity premium fluctuates more than the data, perhaps because ILR^D is an imperfect measure for asset illiquidity; the volatility of asset price is still lower than the data, perhaps reflecting that asset price in practice has bubble components that we do not model. Compared to Table 3, the volatility of investment is closer to the data, while consumption becomes more volatile.

Importantly, the model with shocks to intermediation search costs successfully generates countercyclical movements (correlation -0.50) in the liquidity premium, mimicking the liquidity hoarding typically observed in recessions. That is, recessions might be caused by different reasons and the dynamics of liquidity premium could be a good indicator. In addi-

tion, although asset price is still procyclical, the correlation (0.58) is closer to the data (0.51) and is much smaller than the correlation (0.82) generated from the model with only aggregate productivity shocks. The reason behind is that asset price q seems to have overshooting above steady state after search cost shocks (Figure 3).

5.3 A Discussion

So far, we have illustrated the endogenous asset illiquidity channel and how shocks to aggregate productivity and search costs affect the channel. We discuss three related issues that the readers may have.

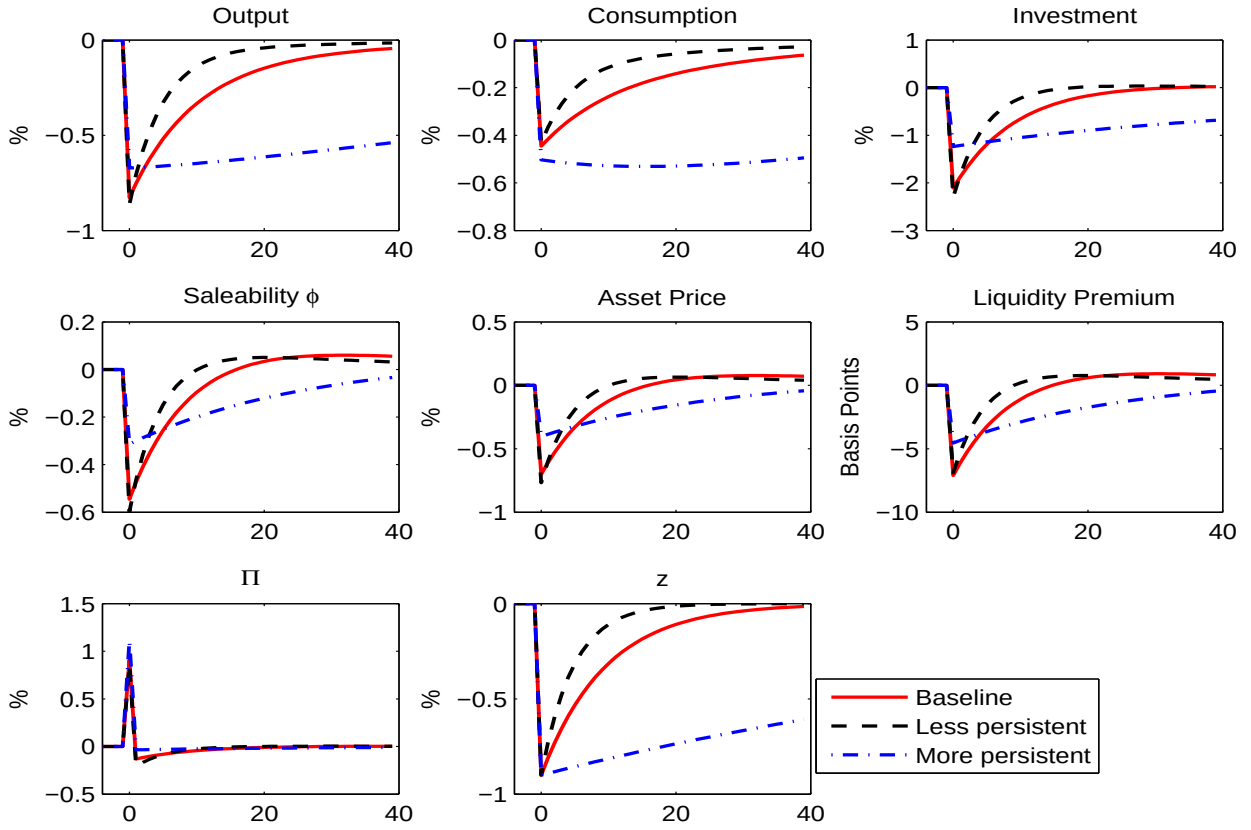
First, aggregate productivity shocks in fact have different effects on hedging value of money. From previous exercises, we learn that if aggregate productivity is persistently below the steady state level, the need for investment is depressed persistently and thus the willingness to hedge idiosyncratic investment risks drops. Nevertheless, low productivity implies lower net worth such that entrepreneurs are more financing constrained. This effect should raise the hedging motive and the willingness to hold money.

The first effect seems to dominate the second effect as in the baseline experiment displayed in Figure 3. But to highlight the second effect, we set ρ_A to a higher (+10%) or a lower (-10%) value and compare the differences with the baseline experiment. After changing the persistence of shocks, the equilibrium dynamics are similar to the baseline simulation (Figures 5). But different degrees of persistence alternate the magnitude and the speed of the adjustment of macro and finance variables.

We can see that the second effect is displayed. When negative aggregate *productivity* shocks are perceived to be more persistent in the future, the hedging value of liquid assets will be depressed for longer. It takes longer for the liquidity premium to revert back to the steady state. However, in the first few periods after shocks, the drop of liquidity premium is less than that in the baseline, even though later the drop becomes larger. Because of persistently lower net worth, this difference shows that agents find them to be financially constrained longer which raises the hedging value of money. Therefore, the real value of money helps entrepreneurs finance investment and helps workers purchase private claims, which prevents output, investment, saleability, and asset price dropping to the levels in the baseline.

Second, the endogenous asset liquidity is crucial for having the positive co-movement between asset price and asset saleability (or market liquidity). Kiyotaki and Moore (2012) and Shi (2015) consider an exogenous and persistent reduction of ϕ. Since entrepreneurs are financing constrained by ϕ, this variation depresses the supply of assets on financial markets.

Figure 5: Impulse responses after a standard deviation shock to aggregate productivity at time 0. The units of liquidity premium are annualized changes in basis points. Units of other variables are percentage changes from their steady state levels.



Demand of private claims, on the hand, is rarely affected as rental rate from capital stock does not change although a persistent lower ϕ reduces slightly the demand. Therefore, adverse liquidity shocks generate asset price boom which is far from the practice. Another way of seeing this is that a higher degree of financing constraint implies a higher Tobin's q.

Our endogenous liquidity framework thus demonstrates that financial shocks need to strongly affect the demand side in order to overturn this anomaly in the reaction of asset prices. The higher search costs directly reduce the search intensity from the demand side. Moreover, demand side shocks are more severe on impact, the more persistent they are. This is because buyers who perceive financial markets to be illiquid for an extended period, anticipate that holding additional equity claims may constrain their own funding ability in the future and thus become even less inclined to buy them.[17]

[17]The above discussion leads to a final check of the endogenous liquidity mechanism. An alternative way of thinking financial disturbance is to shock the matching function itself. More specifically, we shock the matching efficiency ξ in order to check whether an efficiency problem generated from the financial sector could lead to drop of asset price and liquidity. This line of reasoning is very similar to the productivity

In reality, when the financial sector has troubles, the transfer of resources for investment purposes becomes more costly. Banks hesitate to fund investment ideas because information flow slows down; it may take longer time and may be more costly for investors to screen new projects and for acquirers to purchase old business units. All these reflected why demand for financial assets such as in IPOs and mergers and acquisitions could be substantially reduced.

Third, the shocks to search costs can be thought of as investment-specific technological shocks. While Shi (2015) suggests that aggregate productivity shocks are necessary to overturn the asset price anomaly generated by exogenous liquidity shocks, our simulations show that pure financial shocks are sufficient, provided that liquidity is modelled endogenously. The financial shocks considered here are different from productivity shocks, since they affect investment via financing constraints rather than directly reducing the production frontier of the economy.

To see this, recall the goods market clearing condition (15)

$$C_t + I_t + \kappa_t (V_t + U_t) = Y_t.$$

Aggregate productivity shocks directly affect the right-hand side and then affect consumption and investment on the LHS, while intermediation cost shocks directly affect the investment-related costs $\kappa_t (V_t + U_t)$ on the LHS and then affect labor supply and output on the RHS. Such cost shocks may thus be interpreted as a particular form of investment-specific technology shocks (see, e.g., Greenwood, Hercowitz, and Krusell (1997), Fisher (2006) and Primiceri, Justiniano, and Tambalotti (2010)), whose impact is amplified by their effect on endogenous market participation.

However, the shocks to search costs share some similarity with aggregate productivity shocks if one interpret the costs as the efficiency of producing financial services. In fact, if we regard $Y_t - \kappa_t(V_t + U_t)$ as total output, then an increase of search costs consumes more resources when transferring resources for investment, and thus leading to a lower level of output. With this interpretation, our theory suggests that it might be worthwhile to distinguish financial services and other types of output in national accounts and in future research, in order to estimate the costs (or the efficiency) of resources transfer.

shocks in a standard RBC exercise. But these shocks affect the financial sector itself (instead of to the goods producer sector). An adverse efficiency shock, for example because of excess-borrowing and later contagious bank run, makes the financial sector functioning less as before. Nevertheless, the answer is negative. When matching technology is worse, the dominant force is still the supply of capital in which asset price will increase. For the sake of space, the detail simulation and comparison is available upon request.

6 Conclusion

We endogenize asset liquidity in a macroeconomic model with search frictions. Endogenous fluctuation of asset liquidity may be triggered by shocks that affect asset demand and supply on the search market either directly (intermediation cost shocks), or indirectly (productivity shocks). By tightening entrepreneurs' financing constraints, they feed into investment, consumption and output. Our model is able to capture both a physical and a price dimension of asset liquidity. In particular, we show that asset prices can co-move with asset saleability. The *endogenous* nature of asset liquidity is key to match this positive correlation, as *exogenous* liquidity shocks would act as negative supply shocks on the asset market and lead to asset price booms in recessions.

We also show that the liquidity service provided by intrinsically worthless government-issued assets, such as money, is higher when financing constraints bind tightly. As a result, shocks to the cost of financial intermediation increase the hedging value of liquid assets, enabling our model to replicate the "flight to liquidity" or countercyclical liquidity premium observed in U.S. data.

Our search framework can be interpreted as a model of market-based financial intermediations. It can also be seen as a short-cut to modelling bank-based financial intermediation: Financial intermediaries help channel funds from investors to suitable creditors in need of outside funding, which resembles a matching process. Adding further texture by explicitly accounting for intermediaries' balance sheets would open interesting interactions between liquidity cycles and financial sector leverage and maturity transformation.

Regarding government interventions, our framework suggests that, as in KM, open market operations in the form of asset purchase programs can have real effects by easing liquidity frictions. However, government demand may crowd out private demand due to congestion externalities in an endogenous liquidity framework. Therefore, future research could focus on the optimal design of conventional and unconventional monetary as well as fiscal policy measures in the presence of illiquid asset markets.

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Appendix

A Equilibrium Conditions

A.1 Recursive Competitive Equilibrium

Using the fact that total consumption $C_t = C_t^n + C_t^i$ and (19) (which implies that $C_t^i = \frac{\chi}{1-\chi}\rho_t^{-1/\sigma} C_t^n$), we know $C_t^n = \rho_t^n C$ and $C_t^i = \rho_t^i C$, where

$$\rho_t^n \equiv \frac{1-\chi}{1-\chi+(u')^{-1}(\rho_t)\chi}, \quad \rho_t^i \equiv \frac{(u')^{-1}(\rho_t)\chi}{1-\chi+(u')^{-1}(\rho_t)\chi}.$$

where $(u')^{-1}$ is the inverse function of marginal utility of consumption. When we use the CRRA utility $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, $\rho_t^n \equiv \frac{1-\chi}{1-\chi+\rho_t^{-1/\sigma}\chi}$, $\rho_t^i \equiv \frac{\chi\rho_t^{-1/\sigma}}{1-\chi+\rho_t^{-1/\sigma}\chi}$. We further define the real liquidity as $L_t = \frac{B_t}{P_{t-1}}$. Given the aggregate state variables (K_t, A_t, κ_t), we solve the equilibrium system

$$\left(K_{t+1}, L_{t+1}, C_t, I_t, N_t, \rho_t, \rho_t^i, \rho_t^n, \phi_t, f_t, q_t, q_t^i, q_t^n, r_t, w_t, \Pi_t\right)$$

together with the exogenous laws of motion of (A_t,κ_t). To solve for these 16 endogenous variables, we use the following equilibrium conditions:

1. The representative household's optimality conditions:

$$u'\left(\frac{\rho_t^n C_t}{1-\chi}\right) w_t = h'(\frac{N_t}{1-\chi}), \quad \rho_t^n \equiv \frac{1-\chi}{1-\chi+(u')^{-1}(\rho_t)\chi}, \quad \rho_t^i \equiv \frac{(u')^{-1}(\rho_t)\chi}{1-\chi+(u')^{-1}(\rho_t)\chi}.$$

$$1 = \beta\mathbb{E}_t\left[\frac{u'(\rho_{t+1}^n C_{t+1})}{u'(\rho_t^n C_t)}\left[\chi\rho_{t+1}+1-\chi\right]\frac{1}{\Pi_{t+1}}\right] \tag{28}$$

$$1 = \beta\mathbb{E}_t\left[\frac{u'(\rho_{t+1}^n C_{t+1})}{u'(\rho_t^n C_t)}\frac{\left(\chi\rho_{t+1}+1-\chi\right)r_{t+1}+(1-\delta)\left(\chi\rho_{t+1}+(1-\chi)\,q_{t+1}^n\right)}{q_t^n}\right] \tag{29}$$

$$I_t = \frac{\left[\left(r_t+(1-\delta)\phi_t q_t^i\right)\chi S_t + \chi L_t\right] - \rho_t^i C_t}{1-\phi_t q_t^i} \tag{30}$$

2. Final goods producers:

$$r_t = \alpha A_t\left(\frac{K_t}{N_t}\right)^{\alpha-1}, \quad w_t = (1-\alpha)A_t\left(\frac{K_t}{N_t}\right)^{\alpha} \tag{31}$$

3. Market clearing:

   (a) The household's budget constraint:

$$\begin{aligned}\left(\rho_t\rho_t^i+\rho_t^n\right)C_t + L_{t+1} + q_t^n K_{t+1} &= w_t N_t + \left[\chi\rho_t+(1-\chi)\right]\frac{L_t}{\Pi_t} + \left[\chi\rho_t+(1-\chi)\right]r_t K_t \\ &+ \left[\chi\rho_t+(1-\chi)q_t^n\right](1-\delta)K_t \end{aligned} \tag{32}$$

   (b) Capital accumulation: $K_{t+1} = (1-\delta)K_t + I_t$

   (c) Given the matching function (note: $\gamma \equiv \frac{\omega}{1-\omega}$,)

$$\phi_t = M(1, \frac{\rho_t}{\gamma}), \quad f_t = M(\frac{\gamma}{\rho_t}, 1)$$

When we specify $M(U_t, V_t) = \xi U_t^{\eta} V_t^{1-\eta}$, we have $\phi_t = \xi \left(\gamma^{-1}\rho_t\right)^{1-\eta}$, $\quad f_t = \xi^{\frac{1}{1-\eta}} \phi_t^{\frac{\eta}{\eta-1}}$

(d) Asset Prices

$$q_t = \frac{\rho_t \left(1 + \frac{\kappa_t}{\omega}\right) - \frac{\kappa_t}{f_t}}{1 + (\rho_t - 1)\,\phi_t}, \quad q_t^i = q - \frac{\kappa_t}{\phi_t}, \quad q_t^n = q + \frac{\kappa_t}{f_t}$$

(e) Liquid assets in fixed supply (note: $L_t = \frac{B_t}{P_{t-1}}$, $\Pi_t = \frac{P_t}{P_{t-1}}$): $L_{t+1} = \frac{L_t}{\Pi_t}$.

A.2 The Steady State

The following illustrates one particular approach of computing the steady state values of 16 variables when both private claims and money are valued. For ease of exposition, we directly use $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ and $h(n) = \mu n$. In fact, no numerical solver is necessary because of the design of the model. Again, we use the variable itself without the time subscript to denote the steady state.

First, notice that market clearing for liquid assets implies that $\Pi = 1$. Next, we use (28) to obtain

$$\rho = \chi^{-1}\left[\beta^{-1} - (1 - \chi)\right] \tag{33}$$

and therefore

$$\rho^n \equiv \frac{1-\chi}{1 - \chi + \rho^{-1/\sigma}\chi}, \quad \rho^i \equiv \frac{\chi \rho^{-1/\sigma}}{1 - \chi + \rho^{-1/\sigma}\chi} \tag{34}$$

With ρ, we know that

$$\phi = M(1, \frac{\rho}{\gamma}), \quad f_t = M(\frac{\gamma}{\rho}, 1) \tag{35}$$

Again, when we specify $M(U_t, V_t) = \xi U_t^{\eta} V_t^{1-\eta}$ we have $\phi = \xi \left(\gamma^{-1}\rho\right)^{1-\eta}$ and $f = \xi^{\frac{1}{1-\eta}} \phi^{\frac{\eta}{\eta-1}}$. Next, we can compute asset prices

$$q = \frac{\rho \left(1 + \frac{\kappa}{\omega}\right) - \frac{\kappa}{f}}{1 + (\rho - 1)\,\phi}, \quad q^i = q - \frac{\kappa}{\phi}, \quad q^n = q + \frac{\kappa}{f} \tag{36}$$

From (29) and (31), we have

$$r = \frac{\frac{q^n}{\beta} - (1 - \delta)\left(\chi\rho + (1 - \chi)\, q^n\right)}{\chi\rho + 1 - \chi} \tag{37}$$

$$w = (1 - \alpha)\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}, \quad C = \left(\frac{w}{\mu}\right)^{1/\sigma} \frac{1-\chi}{\rho^n} \tag{38}$$

Now, we need to solve real liquidity value L and capital stock K. One can simplify (30) and (32) to be

$$\rho^i C + dK = \chi L$$

$$(\rho\rho^i + \rho^n)C = \left[\frac{(1-\alpha)r}{\alpha} + q^n\left(\beta^{-1} - 1\right)\right] K + \chi(\rho - 1)L$$

where we use $I = \delta K$ and $d = \delta(1 - \phi q^i) - \chi(r + (1 - \delta)\phi q^i)$. Then, we can solve real liquidity and capital stock as

$$L = \frac{\left(\rho\rho^i + \rho^n\right) d + \rho^i \left[\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1} - 1)\right]}{\chi\left[\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1} - 1) + (\rho - 1)d\right]} C \tag{39}$$

$$K = \frac{1}{\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1} - 1) + (\rho - 1)d} C \tag{40}$$

Finally, we express labor supply N and (physical) investment as a function of K

$$N = \left(\frac{r}{\alpha}\right)^{\frac{1}{1-\alpha}} K, \quad I = \delta K \tag{41}$$

Finally, the similar steps still go through with other types of utility function $u(.)$ and $h(.)$. A different $u(.)$ only changes the computation of ρ^i and ρ^n. A different $h(.)$ (especially a non-linear $h(.)$) modifies (38) to

$$C = (u')^{-1}\left(\frac{h'\left(\frac{N}{1-\chi}\right)}{w}\right)\frac{1-\chi}{\rho^n}$$

Then, we need to guess a value of N and check whether the guess is correct by following (39), (40), and (41). For these reasons, the following proofs do not rely on a particular choice of $u(.)$ and $h(.)$ either.

A.3 A Corresponding basic RBC Model

We briefly describe the corresponding basic RBC model relative to our model with uninsurable investment and labor income risks. As is well known, one can solve the planner's solution to a RBC model. The planner maximizes

$$V_t(K_t) = \max_{C_t, N_t, K_{t+1}} \{\frac{C_t^{1-\sigma}-1}{1-\sigma} - \mu N_t + \beta \mathbb{E}_t\left[V_{t+1}(K_{t+1})\right]$$
$$s.t. \quad C_t + K_{t+1} = A_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta)K_t \tag{42}$$

where $\mathbb{E}_t$ is conditioning on aggregate productivity shocks A_t, as only A_t will be the aggregate shocks. The optimality conditions of labor supply and capital accumulation are

$$(C_t)^{-\sigma}(1-\alpha)A_t\left(K_t/N_t\right)^{\alpha} = \mu, \tag{43}$$

$$1 = \beta \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left[\alpha A_t\left(K_t/N_t\right)^{\alpha-1} + 1 - \delta\right]\right]. \tag{44}$$

Therefore, equations (42), (43), and (44) solve (K_{t+1}, C_t, N_t) given the state variables (K_t, A_t). Calculating the steady state values is relatively straightforward. We substitute out capital-labor ratio K/N in (43) and (44), and obtain

$$\frac{K}{N} = \left(\frac{\mu C^{\sigma}}{1-\alpha}\right)^{\frac{1}{\alpha}}, \quad C = \left(\frac{1-\alpha}{\mu}\right)^{\frac{1}{\sigma}}\left[\frac{\beta^{-1}-(1-\delta)}{\alpha}\right]^{\frac{\alpha}{(\alpha-1)\sigma}}.$$

Therefore, from the social resources constraint (42) and by using the capital labor-ratio, we obtain capital and labor as

$$K = \left[\frac{\beta^{-1}-(1-\delta)}{\alpha} - \delta\right]C, \quad N = \left[\frac{\beta^{-1}-(1-\delta)}{\alpha} - \delta\right]C/\left(\frac{\mu C^{\sigma}}{1-\alpha}\right)^{\frac{1}{\alpha}}.$$

A.4 Alternative Modeling: Consumption Sharing

Suppose we further assume that the household members share consumption after trading, we will not have ρ_t to measure the degree of consumption risk-sharing. A buyer's surplus amounts to

$$J_t^n = -u'(C_t)q_t + \beta \mathbb{E}_t\left[J_{S,t+1}(S_{t+1}, B_{t+1})\right].$$

The sellers' surplus is the marginal value to the household of an additional match for entrepreneurs

$$J_t^i = u'(C_t)\left(q_t - \frac{1}{e_t\phi_t}\right) + \beta\left(\frac{1}{e_t\phi_t} - 1\right)\mathbb{E}_t\left[J_{S,t+1}(S_{t+1}, B_{t+1})\right].$$

Then, intermediaries maximize joint surplus $(J_t^i)^{\omega}(J_t^n)^{1-\omega}$ by picking a specific price q_t, which gives rise to (by following similar derivation in the proof of Proposition 2)

$$q_t = 1 + \left[\frac{2\omega-1}{1-\omega}\phi_t - 1\right]\frac{\kappa}{f_t}$$

Importantly, when $\kappa \to 0$, we see that $q_t \to 1$. This result implies that when household members share consumption goods together, $\kappa \to 0$ implies that the economy looks as if it is a basic RBC economy.

B Proofs

B.1 Proposition 1

We first rewrite two asset pricing formulae in steady state

$$\rho\chi r^{ni} + (1-\chi) r^{nn} = \beta^{-1}, \quad \rho\chi \frac{1}{\Pi} + (1-\chi)\frac{1}{\Pi} = \beta^{-1} \tag{45}$$

Since $\Pi = 1$, one knows that $\rho = \frac{\beta^{-1}-(1-\chi)}{\chi}$. We keep writing $1/\Pi$ to denote the return from money. Notice that $r^{nn} = \frac{r+(1-\delta)q^n}{q^n} > \frac{r+(1-\delta)}{q^n} = r^{ni}$. Then, $r^{nn} > 1$; otherwise, the two asset pricing formulae in (45) cannot simultaneous hold. Further, rearranging (45), we have

$$\chi r^{ni} = \frac{\beta^{-1} - (1-\chi) r^{nn}}{\rho}, \quad \chi \frac{1}{\Pi} = \frac{\beta^{-1} - (1-\chi)\frac{1}{\Pi}}{\rho}$$

and the liquidity premium Δ^{LP} can be expressed as $\Delta^{LP} = \chi r^{ni} + (1-\chi) r^{nn} - \frac{1}{\Pi} = \left(1 - \frac{1}{\rho}\right)(r^{nn} - \frac{1}{\Pi})(1-\chi)$. Since $\rho > 1$ and $r^{nn} > 1$, we know that $\Delta^{LP} > 0$. □

B.2 Proposition 2

We first simplify the bargaining solution to (24)

$$\frac{\omega}{\rho\left(q_t - \frac{1}{\phi_t}\right) + \frac{1-\phi_t}{\phi_t} q_t^n} = \frac{1-\omega}{q_t^n - q},$$

by using the first-order condition (20) and the risk-sharing condition (19) from the household. Then

$$\omega \frac{\kappa_t}{f_t} = (1-\omega)\left[\rho_t\left(q_t - \frac{1}{\phi_t}\right) + \frac{1-\phi_t q_t^i}{\phi_t}\frac{(1-\phi_t)\, q_t^n}{1-\phi_t q_t^i}\right].$$

Using the definition $\rho_t \equiv \frac{q_t^n}{q_t^r} = \frac{(1-\phi_t)q_t^n}{1-\phi_t q_t^n}$, the above identity is simplified to (25)

$$\omega \frac{\kappa_t}{f_t} = (1-\omega)\,\rho_t\left(q_t - q_t^i\right) \iff \rho_t = \frac{\omega}{1-\omega}\frac{\phi_t}{f_t}.$$

Now, one can further express q_t in terms of ϕ_t. Using the definition $\rho_t \equiv \frac{q_t^n}{q_t^r} = \frac{(1-\phi_t)q_t^n}{1-\phi_t q_t^i} = \frac{(1-\phi_t)\left(q_t + \frac{\kappa_t^n}{f_t}\right)}{1-\phi_t q_t + \kappa_t^i}$, we can express q as

$$q_t = \frac{\rho_t\,(1+\kappa_t) - (1-\phi_t)\frac{\kappa_t}{f_t}}{1 + (\rho_t - 1)\,\phi_t} = \frac{\rho_t\left(1 + \frac{\kappa_t}{\omega}\right) - \frac{\kappa_t}{f_t}}{1 + (\rho_t - 1)\,\phi_t},$$

where the second equality uses (25) again. □

B.3 Proposition 3

We use the guess-and-verify strategy. Suppose the private claims and money can exist. Then, all the equilibrium conditions from (33)-(41) are satisfied.

We search for the threshold κ_2 that yields $q^i \geq 1$ when $\kappa \leq \kappa_2$. Using the asset price derived in Lemma 2, $q_t = \frac{\rho_t(1+\frac{\kappa_t}{\omega})-\frac{\kappa_t}{\phi}}{1+(\rho_t-1)\phi}$, the selling price $q_t^i = q_t - \frac{\kappa_t}{\phi}$ becomes $q_t^i = \frac{\rho_t(1+\frac{\kappa_t}{\omega})-\frac{\kappa_t}{f_t}-\frac{\kappa_t}{\phi_t}-(\rho_t-1)\kappa_t}{1+(\rho_t-1)\phi_t}$. Therefore, $q_t^i \geq 1$ is equivalent to

$$\rho_t(1+\frac{1-\omega}{\omega}\kappa_t - \phi_t) + \kappa_t + \phi_t \geq 1 + \frac{\kappa_t}{f_t} + \frac{\kappa_t}{\phi_t}.$$

Using the bargaining solution (25) $\rho_t = \frac{\omega}{1-\omega}\frac{\phi_t}{f_t}$, one can simplify the above inequality to

$$(1-\phi_t)(\rho_t - 1 - \frac{\kappa_t}{f_t} - \frac{\kappa_t}{\phi_t}) \geq 0 \iff \rho_t - 1 \geq \frac{\kappa_t}{f_t} + \frac{\kappa_t}{\phi_t} = \frac{\kappa_t\,(\theta+1)}{M(1,\theta_t)}$$

where we use the fact that $\phi_t \in [0,1]$ together with the definition of f_t and ϕ_t. By using the relationship $\rho_t = \frac{\omega}{1-\omega}\theta_t$ we can simplify the above condition to

$$\kappa_t \leq \frac{\rho_t - 1}{\gamma\rho_t + 1} M\left(1, \gamma\rho_t\right)$$

Finally, we know that ρ is bounded above by $\rho^* = [\beta^{-1} - (1-\chi)]/\chi$ (again, ρ^* is pinned down from the asset pricing formula of money in steady state). Given that $M(1,\gamma\rho)$ and $\frac{\rho-1}{\gamma\rho+1}$ are increasing functions of ρ, we know that the threshold $\kappa_2 = \frac{\rho^*-1}{\gamma\rho^*+1} M\left(1, \gamma\rho^*\right)$.

Next, we will calculate the threshold κ_1 as a function H of other parameters. Suppose both private claims and money circulate. Then, the asset pricing formula of money holds. Therefore, $\rho = \chi^{-1}\left[\beta^{-1} - (1-\chi)\right]$ and (34)-(41) hold. Importantly, ϕ and f are not functions of κ. Further,

$$q^i = q - \frac{\kappa}{\phi} = \frac{\rho + \kappa\left[\frac{\rho}{\omega} - \frac{1}{f} - \frac{1}{\phi} - (\rho-1)\right]}{1+(\rho-1)\phi} = \frac{\rho + \kappa\,(\phi-1)\,(\frac{1}{\phi}+\frac{1}{f})}{1+(\rho-1)\phi}$$

$$q^n = q + \frac{\kappa}{f} = \frac{\rho + \kappa\left[\frac{\rho}{\omega} + \frac{(\rho-1)\phi}{f}\right]}{1+(\rho-1)\phi}$$

and we can express

$$q^i = c_{i1} + \kappa c_{i2}, \quad q^n = c_{n1} + \kappa c_{n2},$$

where

$$c_{i1} = \frac{\rho}{1+(\rho-1)\phi}, \quad c_{i2} = -\frac{(1-\phi)\,(\frac{1}{\phi}+\frac{1}{f})}{\phi\,[1+(\rho-1)\phi]}, \quad c_{n1} = \frac{\rho}{1+(\rho-1)\phi}, \quad c_{n2} = \frac{\frac{\rho}{\omega} + \frac{(\rho-1)\phi}{f}}{1+(\rho-1)\phi},$$

By inspecting the coefficients, we know that except $c_{i2} < 0$, others are strictly positive. For similar reasons, we can express r from (37)

$$r = c_{r1} + \kappa c_{r2}$$

where

$$c_{r1} = \frac{\left[1-\beta(1-\delta)(1-\chi)\right]\rho}{1+(\rho-1)\phi} - \beta(1-\delta)\chi\rho, \quad c_{r2} = \frac{\left[1-\beta(1-\delta)(1-\chi)\right](\frac{\rho}{\omega} + \frac{(\rho-1)\phi}{f})}{1+(\rho-1)\phi} > 0$$

Money exists iff $L \geq 0$. Since $K > 0$, from (39) and (40), we know that the denominator of the two equation has to be positive. Then, $L \geq 0$ iff $\left(\rho\rho^i + \rho^n\right) d + \rho^i\left[\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1}-1)\right] \geq 0$, or equivalently

$$\delta(1-\phi q^i) + \frac{\rho^i\left[\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1}-1)\right]}{(\rho\rho^i + \rho^n)} \geq \chi(r + (1-\delta)\phi q^i)$$

If we plug in the expression of q^i, q^n, and r, we have

$$\left[\frac{\rho^i}{(\rho\rho^i+\rho^n)}\frac{(1-\alpha)}{\alpha}-\chi\right]r+\frac{\rho^i\left(\beta^{-1}-1\right)q^n}{(\rho\rho^i+\rho^n)}-\phi\left[\delta+\chi(1-\delta)\right]q^i+\delta\geq 0$$

Let $\zeta_r=\left[\frac{\rho^i}{(\rho\rho^i+\rho^n)}\frac{(1-\alpha)}{\alpha}-\chi\right]$, $\zeta_n=\frac{\rho^i}{(\rho\rho^i+\rho^n)}\left(\beta^{-1}-1\right)$, and $\zeta_i=\phi\left[\delta+\chi(1-\delta)\right]$, then

$$\kappa\left(\zeta_r c_{r2}+\zeta_n c_{n2}-\zeta_i c_{i2}\right)\geq-\delta-\zeta_r c_{r1}-\zeta_n c_{n1}+\zeta_i c_{i1}$$

Notice that $\zeta_r=\chi\left[\frac{1}{\chi\rho+(1-\chi)/\rho^\sigma}\frac{1-\alpha}{\alpha}-1\right]>\chi\left[\beta^{-1}\frac{1-\alpha}{\alpha}-1\right]>0$, $\zeta_n>0$, and $\zeta_i>0$, then $\zeta_r c_{r2}+\zeta_n c_{n2}-\zeta_i c_{i2}>0$. This implies that we can express

$$\kappa\geq\frac{-\delta-\zeta_r c_{r1}-\zeta_n c_{n1}+\zeta_i c_{i1}}{\zeta_r c_{r2}+\zeta_n c_{n2}-\zeta_i c_{i2}}$$

so that the threshold $\tilde{\kappa}_1=H(\chi,\beta,\delta,\alpha,\xi,\eta,\omega)=\frac{\zeta_i c_{i1}-\delta-\zeta_n c_{n1}-\zeta_r c_{r1}}{\zeta_r c_{r2}+\zeta_n c_{n2}-\zeta_i c_{i2}}$.

Finally, with some tedious algebra, one can show that $\partial L/\partial\kappa>0$ by using the expression (39) together with $\rho=\chi^{-1}\left[\beta^{-1}-(1-\chi)\right]$ and all other equilibrium conditions. That is, $L(\kappa)$ is an increasing function of κ. Therefore, when private claims and money co-exist, liquidity value increases with search costs κ. Liquidity value L only cross zero once at $\kappa=\tilde{\kappa}_1$.

B.4 Corollary 1

When $\kappa\in[\kappa_1,\kappa_2]$, $\rho=\rho^*$ and $\theta=\frac{1-\omega}{\omega}\rho$. Then, ρ, $\phi(\theta)$, and $f(\theta)$ are functions of parameters that are independent of search costs κ. Then, the spread $q^n-q^i=\kappa\left(\frac{1}{\phi}+\frac{1}{f}\right)$ increases with search costs.

Further, we know that $q^i=q-\frac{\kappa}{\phi}=\frac{\rho+\kappa\left[\frac{\rho}{\omega}-\frac{1}{f}-\frac{1}{\phi}-(\rho-1)\right]}{1+(\rho-1)\phi}$. q^i is a decreasing function of κ because $\frac{\rho}{\omega}-\frac{1}{f}-\frac{1}{\phi}-(\rho-1)<0$. To see this, $\frac{\rho}{\omega}-\frac{1}{f}-\frac{1}{\phi}-(\rho-1)<0$ is equivalent to

$$\frac{\rho(1-\omega)}{\omega}-\frac{1}{f}<\frac{1-\phi}{\phi}\iff\frac{1}{f}(\phi-1)<\frac{1-\phi}{\phi}$$

where we use the relationship $\rho=\frac{\omega}{1-\omega}\frac{\phi}{f}$. The inequality is then trivially satisfied.

Finally, $\frac{\partial q}{\partial\kappa}>0$ is equivalent to

$$\frac{\gamma}{\omega}\phi-1<0\to\phi<1-\omega.$$

where we use the definition for γ. Since in the steady state $\rho=\rho^*=\chi^{-1}\left[\beta^{-1}-(1-\chi)\right]$ and $\phi=\xi\left(\gamma^{-1}\rho\right)^{1-\eta}$, the above inequality is equivalent to

$$\xi\left(\rho^*\right)^{1-\eta}<(1-\omega)^\eta\omega^{1-\eta}g^{1-\eta}.\quad\square$$

B.5 Corollary 2

Using $f_t=\xi\left(\gamma_t^{-1}\rho_t\right)^{-\eta}$ and $\phi_t=\xi\left(\gamma_t^{-1}\rho_t\right)^{1-\eta}$, we can rewrite q_t as a function of liquidity ϕ_t

$$\begin{aligned}q&=\frac{\rho_t^{1-\eta}\left(1+\frac{\kappa_t}{\omega}\right)-\kappa_t\gamma_t^{-\eta}\xi^{-1}}{\rho_t^{-\eta}\left[1+(\rho_t-1)\,\xi\left(\gamma_t^{-1}\rho_t\right)^{1-\eta}\right]}=\frac{\gamma^{1-\eta}\xi^{-1}\phi_t\left(1+\frac{\kappa_t}{\omega}\right)-\kappa_t\gamma_t^{-\eta}\xi^{-1}}{\gamma^{-\eta}(\xi^{-1}\phi_t)^{\frac{\eta}{\eta-1}}\left[1+\left(\gamma_t(\xi^{-1}\phi_t)^{\frac{1}{1-\eta}}-1\right)\phi_t\right]}\\&=\frac{\gamma_t\phi_t\left(1+\frac{\kappa_t}{\omega}\right)-\kappa_t}{\xi^{\frac{1}{1-\eta}}\phi_t^{\frac{\eta}{\eta-1}}\left[1+\left(\gamma_t(\xi^{-1}\phi_t)^{\frac{1}{1-\eta}}-1\right)\phi_t\right]}\end{aligned}$$

By differentiating the asset price from (26) with respect to ϕ_u, we obtain

$$\begin{aligned}&\frac{\partial q}{\partial \phi_t}\left[\xi^{\frac{1}{1-\eta}}\phi_t^{\frac{\eta}{\eta-1}}\left[1+(\rho_t-1)\,\phi_t\right]\right]\\&=\gamma_t\left(1+\frac{\kappa_t}{\omega}\right)-q_t\frac{\partial}{\partial \phi_t}\left[\xi^{\frac{1}{1-\eta}}\phi_t^{\frac{\eta}{\eta-1}}\left[1+\left(\gamma_t(\xi^{-1}\phi_t)^{\frac{1}{1-\eta}}-1\right)\phi_t\right]\right],\end{aligned}\tag{46}$$

where $\frac{\partial}{\partial \phi_t}\left[\xi^{\frac{1}{1-\eta}}\phi_t^{\frac{\eta}{\eta-1}}\left[1+\left(\gamma_t(\xi^{-1}\phi_t)^{\frac{1}{1-\eta}}-1\right)\phi_t\right]\right]=\rho_t^{-1}\gamma_t\left[\phi_t\left(2\rho_t-\frac{1-2\eta}{1-\eta}\right)-\frac{\eta}{1-\eta}\right]$.Note that $2\rho-\frac{1-2\eta}{1-\eta}=\frac{\eta}{1-\eta}+2\rho-1$, then a sufficient and necessary condition for $\frac{\partial q}{\partial \phi_t}>0$ is for the RHS of (46) to be non-negative. This is the case, whenever

$$\phi_t \quad < \quad \left[\frac{\eta}{1-\eta}+\left(1+\frac{\kappa_t}{\omega}\right)\frac{\rho_t}{q_t}\right]\left[\frac{\eta}{1-\eta}+2\rho_t-1\right]^{-1}.$$

This condition requires ϕ_t to be small enough for the asset price and asset liquidity to correlate positively. Replacing ρ and notice that $\rho/q\geq 1$, we know that a sufficient condition is

$$\phi_t \quad < \quad \left[\frac{\eta}{1-\eta}+\left(1+\frac{\kappa_t}{\omega}\right)\right]\left[\frac{\eta}{1-\eta}+2\gamma_t(\xi^{-1}\phi_t)^2-1\right]^{-1}.$$

When $\eta=0.5$, $\frac{\eta}{\eta-1}=1$ and the sufficient condition becomes $\phi_t<\sqrt[3]{\frac{\xi^2(1+\frac{\kappa_t^i}{2\omega})}{\gamma_t}}$.

Finally, note that $\frac{\partial q_t}{\partial \phi_t}>0$ implies $\frac{\partial q_t}{\partial f_t}<0$, because $\frac{\partial q_t}{\partial \phi_t}=\frac{\partial q_t}{\partial \phi_t}\frac{\partial \phi_t}{\partial f_t}$ and $\frac{\partial \phi_t}{\partial f_t}=\frac{\eta-1}{\eta}\xi^{\frac{1}{\eta}}f_t^{-\frac{1}{\eta}}<0$. That is, the same parameter restriction that ensures $\frac{\partial q_t}{\partial \phi_t}>0$ also ensures $\frac{\partial q_t}{\partial f_t}<0$. □

B.6 Corollary 3

When $\kappa\in[0,\kappa_1)$, we know that money does not circulate. Therefore, the asset pricing formula for money will not be satisfied in order to pin down ρ. Instead, $L=0$, and from (39) we know that ρ solves the following equation

$$\left(\rho\rho^i+\rho^n\right)d+\rho^i\left[\frac{(1-\alpha)r}{\alpha}+q^n(\beta^{-1}-1)\right]=0$$

where all equilibrium variables can be expressed as a function of ρ if we use equations (34)-(41). The implicity function theorem implies that $\partial\rho/\partial\kappa>0$ (where the derivation is tedious and available upon requests). Then, ϕ is also an increasing function of κ. Using the relationship (36) again, one knows that q and q^n are all increasing function of κ. Note that to illustrate κ's impact on ρ , ϕ, q, and q^n, we provide a numerical example after we calibrated the model. □