

Dynamic Legislative Bargaining with Veto Power*

Salvatore Nunnari

University of California, San Diego

Department of Political Science

sunnari@ucsd.edu

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Abstract

Many assemblies grant one or more of their members the right to block decisions even when a proposal has secured the necessary majority—a veto right. In this paper, I analyze the consequences of veto power in an infinitely repeated divide-the-dollar bargaining game with an endogenous status quo policy. The division of the dollar among legislators is unchanged until the committee agrees on a new allocation, which becomes the new status quo. In each period, a committee member is randomly recognized to propose a new division of the dollar. If a majority that includes the veto player prefers this proposal, it is implemented; otherwise, the dollar is divided according to the previous period's allocation. I show that a Markov equilibrium of this dynamic game exists, and that, irrespective of the discount factor of legislators, their recognition probabilities, and the initial division of the dollar, policy eventually gets arbitrarily close to full appropriation of the dollar by the veto player. Finally, I analyze some measures to reduce the excessive power of the veto player: reducing his or her recognition probability, expanding the committee by increasing the number of veto players, and changing the identity of the veto player in each bargaining round.

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1 Introduction

A large number of important voting bodies grant one or several of their members the right to block decisions even when a proposal has secured the necessary majority—a *veto right*. One prominent example is the United Nations Security Council, where a motion is approved only with the affirmative vote of nine members, including the concurring vote of the five permanent members (China, France, Great Britain, Russia, and the U.S.). Another important example is the U.S. President’s ability to veto congressional decisions. Additionally, in assemblies with asymmetric voting weights and complex voting procedures, veto power may arise implicitly: this is the case of the U.S. in the International Monetary Fund and the World Bank governance bodies (Leech and Leech 2004).¹

The existence of veto power raises two concerns. First, the ability of an agent to veto policies increases the possibility of legislative stalemate, or “gridlock”. Second, although the formal veto right only grants the power to block undesirable decisions, it could *de facto* allow veto members to impose their ideal decision on the rest of the committee (Russell 1958, Woods 2000, Blum 2005).²

In this paper, I investigate the consequences of veto power in a dynamic bargaining setting where the location of the current *status quo policy* is determined by the policy implemented in the previous period. This is an important feature of many policy domains—for instance personal income tax rates or entitlement programs—where legislation remains in effect until the legislature passes a new law. In each of an infinite number of periods, one of three

¹Many other institutions grant veto power to some of their decision makers. For example, some corporate boards of directors grant minority shareholders a “golden share”, which confers the privilege to veto any decision. This share is often held by members of the founding family, or governments in order to maintain some control over privatized companies and was widely used in the European privatization wave of the late 90s and early 2000s. For instance, the British government had a golden share in BAA, the UK airports authority; the Spanish government had a golden share in Telefónica; and the German government had a golden share in Volkswagen.

²These concerns were expressed by the delegates of the smaller countries when the founders of the United Nations met in San Francisco in June 1945 (Russell 1958, Bailey 1969), and they have been a crucial point of contention in the ongoing discussion over how to reform the UN Security Council to improve its credibility and reflect the new world order (Fassbender 1998, Weiss 2003, Bourantonis 2005, Blum 2005). A similar debate has recently arisen regarding the IMF’s and WB’s voting weights determination (Woods 2000, Rapkin and Strand 2006).

legislators, one of whom is a veto player, is randomly recognized to make a proposal on the allocation of a fixed endowment. The proposed allocation is implemented if it receives at least two affirmative votes, including the vote of the veto player. Otherwise, the status quo policy prevails and the resource is allocated as it was in the previous period. In this sense, the status quo policy evolves endogenously.

In this simple setting, I answer two basic questions: To what extent is the veto player able to leverage his veto power into outcomes more favorable to himself? As this leverage is found to be substantial, I then turn to a second question: What are the effects of institutional measures meant to reduce the power of the veto player?

In particular, I fully characterize a Markov Perfect Equilibrium (MPE) and prove it exists for any discount factor, any initial divisions of the resources, and any recognition probabilities.³ In this MPE, the veto player is eventually able to move the status quo policy arbitrarily close to his ideal point. That is, the veto player is eventually able to fully appropriate all resources, irrespective of the discount factor, the recognition probabilities, and the initial division of the resources.

When agents are impatient, this result comes from the fact that non-veto legislators support any proposal that gives them at least as much as the status quo. Thus, it takes at most two proposals by the veto player to converge to full appropriation of the dollar. When legislators are patient—that is, when they care, even minimally, about the effect of the current policy on future outcomes—the ability of the veto player to change the policy to his advantage remains, but is reduced. This occurs because, when other committee members receive a proposal that increases the veto player’s share, they take into account the associated reduction in their future bargaining power and demand a premium to support it. However, unless legislators are perfectly patient, this premium is always smaller than the share of resources not already allocated to the veto player, and the policy displays a *ratchet effect*:

³The only general existence result for dynamic bargaining games applies to settings with stochastic shocks to preferences and the status quo (Duggan and Kalandrakis 2010). As these features are not present in my model, proving existence is a necessary step of the analysis.

with the possible exception of the first period, the share to the veto player will only stay constant (if he is not proposing) or increase (if he is proposing).

The speed of convergence to the veto player's ideal outcome is decreasing in the discount factor of the committee members, as the premium demanded by non-veto legislators increases in their patience. In contrast with the impatient case, when agents are patient, this premium is always positive and, thus, it takes an infinite number of bargaining periods to converge to full appropriation of the dollar by the veto player.

This result suggests that the ability to oppose any decision is indeed a powerful right and guarantees a strong leverage on long run outcomes. Therefore, I analyze potential mechanisms to weaken veto power and find that extreme outcomes are difficult to avoid in the long run. First, I investigate the effect of reducing the agenda setting power of the veto player. As long as the veto player has a positive probability of recognition, he will be able to extract all resources. However, the speed of convergence to this outcome decreases as this probability decreases. Second, adding an additional veto player does not increase the ability of non-veto players to retain a share of the resources in the long run, but it reduces the ability of each veto player to accrue all the resources: whenever a veto player proposes, he has to share what he extracts from the other legislators with the other veto player. Finally, when the veto right is randomly re-assigned in every period—rather than permanently held by one legislator—the long term outcome is still extreme: policy eventually converges to an absorbing set where all resources go to either the proposer or the veto player.

This paper contributes primarily to the theoretical literature on the consequences of veto power in legislatures. A large number of studies build on models of legislative bargaining à la Baron and Ferejohn (1989) to examine the role of veto power in policy making. Most of these papers model specific environments and focus on the case of the U.S. Presidential veto. These works analyze conditions under which an executive may exercise veto power (Matthews 1989, Groseclose and McCarty 2001, Cameron 2000), evaluate the effect of presidential veto on spending (Primo 2006) or the distribution of pork barrel policies (McCarty

2000*b*), disentangle the effect of veto and proposal power (McCarty 2000*a*), and provide a rationale for the emergence of veto points inside Congress (Diermeier and Myerson 1999). More closely related to this paper, Winter (1996) shows that the share of resources to veto players is decreasing in the cost of delaying an agreement, so that the share of resources to non-veto players declines to zero as the cost of delay becomes negligible, that is, as legislators become infinitely patient. Banks and Duggan (2000) derive a similar result in a more general model of collective decision making. A common limitation of this literature, and the main point of departure with my paper, is the focus on static settings: the legislative interaction ceases once the legislature has reached a decision, and policy cannot be modified after its initial introduction. In these frameworks, any conclusion on the effect of veto power on policy outcomes depends heavily on the specific assumptions on the status quo policy (Krehbiel 1998, Tsebelis 2002). In this paper, the status quo policy is not exogenously specified but is rather the product of policy makers' past decisions.

In this sense, the present study belongs to a strand of recent literature on legislative policy making with an endogenous status quo and farsighted players (Baron 1996, Kalandrakis 2004, Bernheim, Rangel and Rayo 2006).⁴ However, with the exception of Duggan, Kalandrakis and Manjunath (2008), who model the specific institutional details of the American presidential veto and limit their analysis to numerical computations, this literature does not explore the consequences of veto power. The most related work to mine is Kalandrakis (2004) who analyzes a similar environment without a veto player. This institutional variation generates stark differences in strategies and long run outcomes, as the voting behavior of patient players mimics the behavior of impatient ones, and policy quickly converges to an absorbing set where the proposer extracts all resources in all periods.⁵ In my setting, the discount factor affects voting strategies and—even if the irreducible absorbing set has the

⁴See also Epple and Riordan (1987), Ingberman (1985), Bowen and Zahran (2009), Dziuda and Loeper (2010), Cho (2005), Gomes and Jehiel (2005), Lagunoff (2008, 2009), Barseghyan, Battaglini and Coate (2010), Battaglini and Coate (2007, 2008), Baron and Herron (2003), Baron, Diermeier and Fong (2011), Diermeier and Fong (2011), Duggan and Kalandrakis (2010), Kalandrakis (2009), Penn (2009).

⁵Convergence to this absorbing set is not deterministic, as it depends on the identity of the proposer recognized in each period, but it happens in finite time, in a maximum expected time of 2.5 periods.

veto player extracting all resources—convergence to absorption typically takes an infinite number of periods, during which the veto player shares resources with one non-veto player.

The paper is organized as follows. Section 2 gives a detailed presentation of the legislative setup and introduces the equilibrium notion. Section 3 outlines the equilibrium analysis and gives the main results. Section 4 investigates the consequences of measures to reduce the power of the veto player. In Section 5 I justify the equilibrium I have selected from among the multiplicity of equilibria that exist. Section 6 concludes with a discussion of the limitations of my approach and of the future directions for the study of veto power in dynamic frameworks.

2 Model and Equilibrium Notion

2.1 Model

Three agents repeatedly bargain over a legislative outcome \mathbf{x}^t for each $t = 1, 2, \dots$. One of the three agents is endowed with the power to veto any proposed outcome in every period. I denote the veto player with the subscript v and the two non-veto players with the subscript $j = \{1, 2\}$. The possible outcomes in each period are all possible divisions of a fixed resource (a dollar) among the three players, that is \mathbf{x}^t is a triple $\mathbf{x}^t = (x_v^t, x_1^t, x_2^t)$ with $x_i^t \geq 0$ for all $i = v, 1, 2$ and $\sum_{i \in \{v, 1, 2\}} x_i = 1$. Thus, the legislative outcome \mathbf{x}^t is an element of the unit simplex in \mathbb{R}_+^3 , denoted by Δ . Figure 1 represents the set of possible legislative outcomes, $\mathbf{x} \in \Delta$, in \mathbb{R}^2 . The vertical dimension represents the share to (non-veto) player 1, while the horizontal dimension represents the share to (non-veto) player 2. The remainder is the share that goes to the veto player. Thus, the origin is the point where the veto player gets the entire dollar.

The Bargaining Protocol. At the beginning of each period, one agent is randomly selected to propose a new policy, $\mathbf{z} \in \Delta$. Each agent has the same probability of being

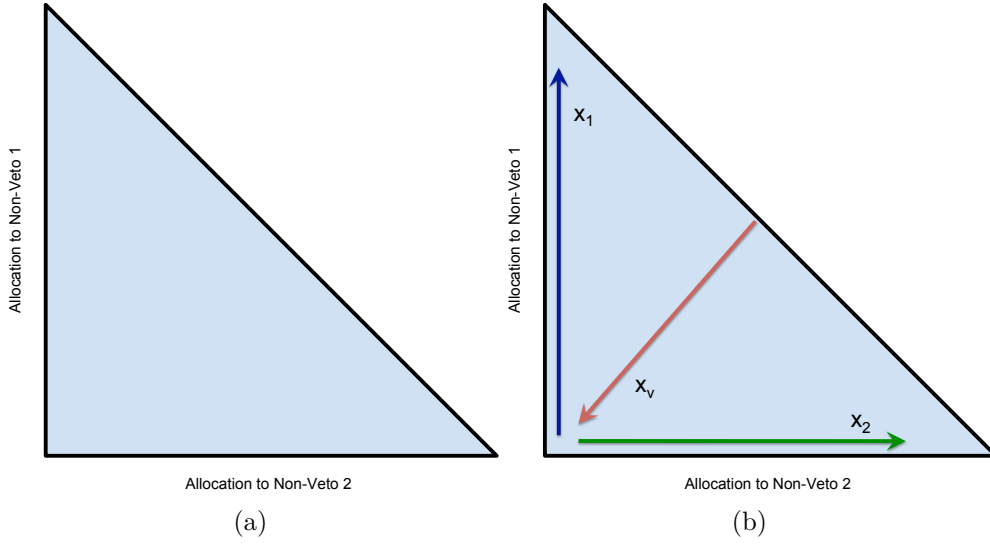


Figure 1: The set of possible legislative outcomes in each period, Δ

recognized as policy proposer, that is $\frac{1}{3}$.⁶ This new proposal is voted up or down, without amendments, by the committee. A proposal passes if it gets the support of the veto player and at least one other committee member. If a proposal passes, $\mathbf{x}^t = \mathbf{z}$ is the implemented policy at t . If a proposal is rejected, the policy implemented is the same as it was in the previous period, $\mathbf{x}^t = \mathbf{x}^{t-1}$. Thus, the previous period's decision, \mathbf{x}^{t-1} , serves as the *status quo policy* in period, t . The initial status quo \mathbf{x}^0 is exogenously specified.

Stage Utilities. Agent i derives stage utility $u_i : \Delta \rightarrow \mathbb{R}$, from the implemented policy \mathbf{x}^t . I assume players' utilities depend only on their share of the dollar, and that payoffs are linear, so that $u_i(\mathbf{x}) = x_i$. Players discount the future with a common factor $\delta \in [0, 1)$, and their payoff in the game is given by the discounted sum of stage payoffs.

2.2 Equilibrium

Strategies. In general, strategies are functions that map histories, that is, vectors that records all proposals as well as all voting decisions that precede an action, to the space of proposals Δ and voting decisions {yes; no}. In what follows, though, I restrict analysis

⁶I will relax this assumption in Section 4.1.

to cases when players condition their behavior only on a summary of the history of the game that accounts for payoff-relevant effects of past behavior (Maskin and Tirole 2001). Specifically, define the state in period t as the previous period's decision \mathbf{x}^{t-1} , and denote the state by $\mathbf{s} \in S$, so that we have $\mathbf{s} = \mathbf{x}^{t-1}$ and $S = \Delta$. I restrict attention to Markov strategies such that agents behave identically ex ante, that is, prior to any mixing, in different periods with state \mathbf{s} , even if that state arises from different histories.

In general, a *mixed Markov proposal strategy* for legislator i is a function $\mu_i : S \rightarrow \mathcal{P}(\Delta)$, where $\mathcal{P}(\Delta)$ denotes the set of Borel probability measures over Δ . For the purposes of this analysis, it is sufficient to assume that for every state \mathbf{s} , μ_i has finite support. Thus, the notation $\mu_i[\mathbf{z}|\mathbf{s}]$ represents the probability that legislator i makes the proposal \mathbf{z} when recognized, conditional on the state being \mathbf{s} . A *Markov voting strategy* is an *acceptance correspondence* $A_i : S \rightarrow \Delta$, where $A_i(\mathbf{s})$ represents the allocations for which i votes *yes* when the state is \mathbf{s} . Then, a *Markov strategy* is a mapping $\sigma_i : S \rightarrow \mathcal{P}(\Delta) \times 2^\Delta$, where for each \mathbf{s} , $\sigma_i(\mathbf{s}) = (\mu_i[\cdot|\mathbf{s}], A_i(\mathbf{s}))$.

Continuation Values and Expected Utilities. In this dynamic game, the *expected utility* of agent i from the allocation implemented in period t does not only depend on his stage utility, but also on the discounted expected flow of future stage utilities, given a set of strategies. In order to define properly the *continuation value* of each status quo, I will first introduce the concepts of the *win set* and *transition probabilities*.

For a given set of voting strategies, define the win set of state $\mathbf{s} \in \Delta$, $W(\mathbf{s})$, as the set of all proposals that beat \mathbf{s} by the voting rule described above. In this setting, $W(\mathbf{s})$ is the collection of all proposals \mathbf{x} to which the veto player and at least one non-veto player vote yes. This differs from a simple majority rule, where the win set would be the collection of all proposals \mathbf{x} to which at least two agents, irrespective of their identity, vote yes.

Then, for a triple of Markov strategies $\boldsymbol{\sigma} = (\sigma_v, \sigma_1, \sigma_2)$, we can write the transition

probability to decision \mathbf{x} when the state is \mathbf{s} , $Q[\mathbf{x}|\mathbf{s}]$ as follows:

$$Q[\mathbf{x}|\mathbf{s}] \equiv I_{W(\mathbf{s})}(\mathbf{x}) \sum_{i=\{v,1,2\}} \frac{1}{3} \mu_i[\mathbf{x}|\mathbf{s}] + I_{\{\mathbf{s}\}}(\mathbf{x}) \sum_{i=\{v,1,2\}} \frac{1}{3} \sum_{\mathbf{y}: \mu_i[\mathbf{y}|\mathbf{s}]>0} I_{\Delta \setminus W(\mathbf{s})}(\mathbf{y}) \mu_i[\mathbf{y}|\mathbf{s}] \quad (1)$$

where $I_{W(\mathbf{s})}(\mathbf{x})$ is the indicator function that takes value of 1 when \mathbf{x} is a proposal that beats the status quo and 0 otherwise, $I_{\{\mathbf{s}\}}(\mathbf{x})$ is the indicator function that takes value of 1 when $\mathbf{x} = \mathbf{s}$ and 0 otherwise, and $I_{\Delta \setminus W(\mathbf{s})}(\mathbf{y})$ is the indicator function that takes value of 1 when \mathbf{y} is a proposal that does not beat the status quo and 0 otherwise. The first part of (1) reflects the probability of transition to allocations that are proposed by one of the three players and are approved, which is the probability of transition $Q[\mathbf{x}|\mathbf{s}]$ if $\mathbf{x} \neq \mathbf{s}$. The probability of staying in the same state, $Q[\mathbf{x}|\mathbf{s}]$, is given by the probability that $\mathbf{x} = \mathbf{s}$ is proposed (the first term), plus the probability that a proposal $\mathbf{x} \neq \mathbf{s}$ is proposed and rejected by the floor (the second term).

Equipped with this notation, I now define the *continuation value*, $v_i(\mathbf{s})$, of legislator i when the state is \mathbf{s} :

$$v_i(\mathbf{s}) = \sum_{\mathbf{x}: Q[\mathbf{x}|\mathbf{s}]>0} [u_i(\mathbf{x}) + \delta v_i(\mathbf{x})] Q[\mathbf{x}|\mathbf{s}] \quad (2)$$

Using (2), we can finally write the expected utility of legislator i , $U_i(\mathbf{s})$, as a function of the allocation implemented in period t , \mathbf{x}^t :

$$U_i(\mathbf{x}^t) = x_i^t + \delta v_i(\mathbf{x}^t) \quad (3)$$

Given that non-veto legislators are otherwise identical, I focus on Markov proposal and voting strategies that are *symmetric* with respect to the two non-veto legislators. A Markov equilibrium is symmetric if it has the following property: for any state $\mathbf{s} \in \Delta$, define \mathbf{s}^{12} by switching s_1 and s_2 , that is $\mathbf{s}^{12} = [s_v, s_2, s_1]$. Then an equilibrium is symmetric if $\sigma_1(\mathbf{s}) = \sigma_2(\mathbf{s}^{12})$ for any $\mathbf{s} \in \Delta$.

Equilibrium Notion. We can finally define the equilibrium solution concept as a variant of Markov perfect Nash equilibrium with a standard refinement on voting strategies:

Definition 1 *A symmetric Markov perfect Nash equilibrium in stage-undominated voting strategies (MPE) is a pair of Markov strategy profiles (symmetric for the two non-veto players), $\sigma^* = \{\sigma_v^*, \sigma_1^*, \sigma_2^*\}$, where $\sigma_v^* = (\mu_v[\cdot|\mathbf{s}], A_v^*(\mathbf{s}))$, and $\sigma_j^* = \{(\mu_j[\cdot|\mathbf{s}], A_j^*(\mathbf{s}))\}_{j=1}^2$, such that for all $i = v, 1, 2$ and all $\mathbf{s} \in \Delta$:*

$$\mathbf{y} \in A_i^*(\mathbf{s}) \iff U_i(\mathbf{y}) \geq U_i(\mathbf{s}) \quad (4)$$

$$\mu_i^*[\mathbf{z}|\mathbf{s}] > 0 \Rightarrow \mathbf{z} \in \arg \max\{U_i(\mathbf{x})|\mathbf{x} \in W(\mathbf{s})\} \quad (5)$$

An equilibrium, as specified in (4), requires that legislators vote yes if and only if their expected utility from the status quo is not larger than the expected utility from the proposal. Such stage undominated voting strategies rule out uninteresting equilibria where voting decisions constitute best responses solely due to the fact that legislators vote unanimously, and thus a single vote cannot change the outcome. The fact that proposers optimize over all feasible proposals, that is over all proposals that would be approved by a winning coalition composed of the veto player and at least one other legislator, is ensured by (5).

3 Equilibrium Analysis

Proving existence of a symmetric MPE of this dynamic game, and characterizing it, constitutes a challenging problem due to the cardinality of the state space. Thus, I propose natural conditions on strategies, and show that these conditions define an equilibrium. The first condition is that equilibrium proposals involve *minimal winning coalitions* (Riker 1962), such that at most one of the two non-veto players receives a positive fraction of the dollar in each period. Second, the proposer proposes the *acceptable* allocation—that is, an allocation in the win set of the status quo, $\mathbf{x} \in W(\mathbf{s})$ —that maximizes his current share of the dollar.

Finally, I prove that these strategies, and the associated continuation values, are part of a symmetric MPE that satisfies conditions (4) and (5).

The remainder of this section describes the dynamics of this equilibrium and explores the mechanisms behind the results. To build intuition, I start from the case where legislators are impatient, $\delta = 0$, and only care about their current allocation, and then move to the general case with patient legislators, $\delta \in (0, 1)$. In both cases, the equilibrium exhibits the two features mentioned above, and legislators' patience changes only the set of allocations they prefer to the status quo.

3.1 Impatient Legislators

When legislators are impatient, they value only current allocations. Then, the expected utility agent i derives from an allocation $\mathbf{x}^t \in \Delta$ is:

$$U_i(\mathbf{x}^t) = x_i^t + \delta v_i(\mathbf{x}^t) = x_i^t$$

Therefore, regardless of the other agents' proposal and voting strategies, it is optimal for legislator i to accept any proposal that allocates to him at least as much as the status quo, and to reject everything else.

$$A_i(\mathbf{s}|\delta = 0) = \{\mathbf{x} \in \Delta | x_i \geq s_i\}$$

Confronted with these acceptance sets, the proposer will propose the allocation that gives him the highest share of the dollar among all those that are supported by the veto player, and at least one non-veto player.

Figures 2(a) and 2(b) show the acceptance set of the veto player, and the optimal proposal strategy of, respectively, non-veto player 1 and non-veto player 2, when the status quo policy is \mathbf{s}^0 . Each non-veto proposer simply makes the veto player indifferent between the status quo

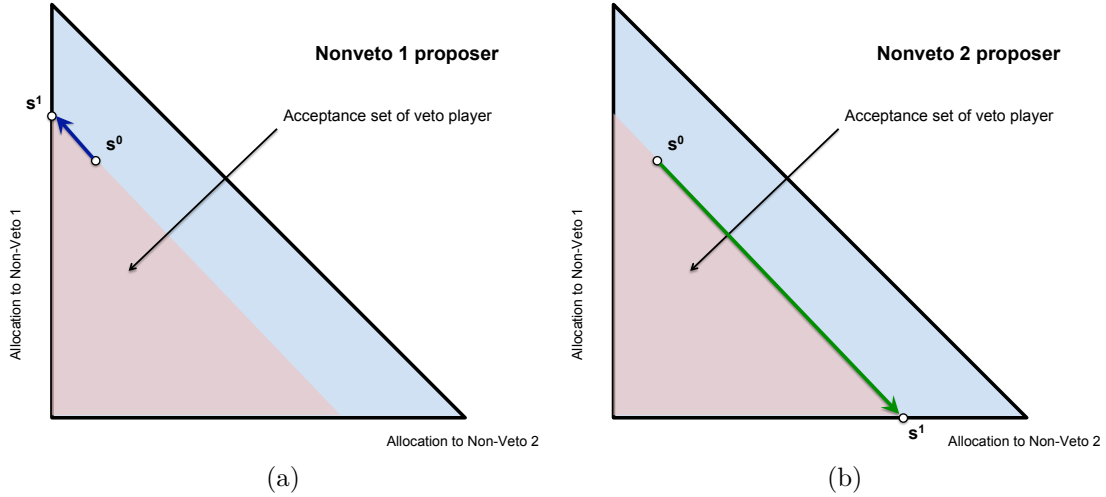


Figure 2: Non-veto players' equilibrium proposal strategies for state s^0 and $\delta = 0$.

and his proposal⁷ and assigns the remainder to himself, disenfranchising the other non-veto player, whose no vote cannot stop passage.

On the other hand, when the veto player is the proposer, he needs to secure a yes vote from one non-veto player to change the policy. He will, thus, build a coalition with the *poorer* non-veto player—the non-veto player who receives the least in the status quo—giving him as much as he is granted by the status quo. An impatient non-veto player will accept this proposal. This equilibrium proposal strategy is depicted in the left-hand panel of Figure 3. When he is not the proposer, the veto player will oppose any reduction to his allocation. Moreover, whenever he proposes, he will be able to increase his share by exactly the amount held by the non-veto player who receives the most in the status quo—the *richer* non-veto player. Given these simple strategies, the equilibrium of the game with $\delta = 0$ has two important features. First, the allocation to the veto player displays a *ratchet effect*: it can only stay constant or increase. Second, the veto player is able to steer the status quo policy to his ideal point in at most two proposals, as he can pass any $\mathbf{x} \in \Delta$ when the poorer non-veto player has zero. Thus, as the veto player can oppose all subsequent changes, he will get the whole dollar in all subsequent periods. The right-hand panel of Figure 3 depicts

⁷The veto player's indifference curve is defined by the diagonal with slope -1.

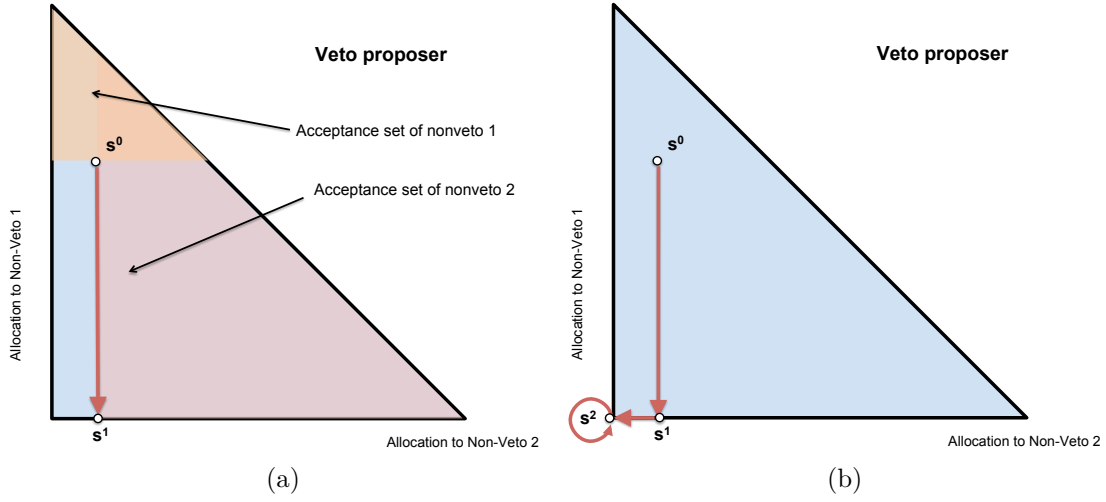


Figure 3: Veto player's equilibrium proposal strategy for state s^0 and $\delta = 0$.

these transitions when the initial status quo is s^0 and the veto player is randomly assigned to be the proposer in the first two periods.

3.2 Patient Legislators

In the more general case, where legislators care about future outcomes, similar results hold. In particular, equilibrium proposals still involve minimal winning coalitions, and the proposer still picks the acceptable allocation that maximizes his current share. However, the acceptance sets of all legislators are now different, and the set of allocations each agent (weakly) prefers to the status quo policy changes with the discount factor, as legislators take into account the impact of the current allocation on future rounds. Not surprisingly, this has important consequences for the dynamics of the game. In the remainder of this section, I first analyze the case when the proposer is the veto player, and then the case when the proposer is a non-veto player.

To help with the exposition, partition the space of possible divisions of the dollar into two subsets, $\bar{\Delta}$, and $\Delta \setminus \bar{\Delta}$. Define $\bar{\Delta} \subset \Delta$ as the set of states $\mathbf{x} \in \Delta$ in which at least one

non-veto legislator gets zero:

$$\bar{\Delta} = \{\mathbf{x} \in \Delta \mid x_i = 0 \text{ for some } i = \{1, 2\}\}$$

Note that, if all proposals on the equilibrium path entail minimal winning coalitions, then $\bar{\Delta}$ is an absorbing set, and it is reached in at most one period from any initial status quo allocation. Moreover, define the *demand* of legislator i as the minimum amount he requires to accept a proposal $\mathbf{x} \in \bar{\Delta}$.

Definition 2 *For a symmetric MPE, non-veto legislator j 's demand when the state is \mathbf{s} is the minimum amount $d_j(\mathbf{s}) \in [0, 1]$ such that for a proposal $\mathbf{x} \in \bar{\Delta}$ with $x_j = d_j(\mathbf{s})$, $x_v = 1 - d_j(\mathbf{s})$, we have $U_j(\mathbf{x}) \geq U_j(\mathbf{s})$. Similarly, veto legislator v 's demand when the state is \mathbf{s} is the minimum amount $d_v(\mathbf{s}) \in [0, 1]$ such that for a proposal $\mathbf{x} \in \bar{\Delta}$ with $x_v = d_v(\mathbf{s})$, $x_j = 1 - d_v(\mathbf{s})$, for $j = 1, 2$, we have $U_v(\mathbf{x}) \geq U_v(\mathbf{s})$.*

3.2.1 Non-Veto Proposer

When a non-veto player is proposing, he needs to secure the vote of the veto player in order to change the current status quo. As a consequence, a proposal that results in a minimal winning coalition assigns a positive share only to the proposer and, if necessary, to the veto player. If the non-veto proposer wants to maximize his current share of the dollar, he will propose the veto player's demand to the veto player, and assign the remainder of the dollar to himself. Therefore, to characterize the equilibrium proposal strategies of a non-veto player, we need to identify the acceptance set of the veto player.

A patient veto player is not indifferent between all states in which he receives the same allocation, and might be better off with allocations that reduce his current share when these decrease his future coalition building costs. This occurs because the future status quo policy affects the future leverage the veto player has when he is the proposer. In this event, he needs to secure the vote of just one non-veto player, and he will, thus, build a coalition with

the non-veto player who demands the least, and extract the remainder. As shown below, the demand of each non-veto player is a positive function of what he gets if the policy is unchanged and, therefore, a veto player's coalition building costs when the status quo is \mathbf{s} are a positive function of $\min\{s_1, s_2\}$.

Thus, a veto player prefers an allocation \mathbf{s}' where he gets s'_v and $\min\{s'_1, s'_2\} = s'_{nv}$ to an alternative allocation \mathbf{s}'' with $s''_v = s'_v$ but $\min\{s''_1, s''_2\} = s''_{nv} > s'_{nv}$. If the veto player is recognized in the following period, he will be able to increase his share more in the state \mathbf{s}' than in \mathbf{s}'' .

Figure 4 depicts the acceptance set of a patient veto player for two different values of $\delta > 0$. While an impatient veto player never supports an allocation that reduces his share, a patient one is willing to move from an interior allocation where he gets a higher share, to an allocation towards the edges of the simplex where both he and one non-veto player have a smaller share. In the Appendix, I characterize the amount the veto player demands to accept a proposal that brings the status quo into $\bar{\Delta}$ —where one non-veto player gets nothing—as:

$$d_v = \max\left\{s_v - \frac{\delta}{3-2\delta}s_{nv}, 0\right\} \quad (6)$$

where s_{nv} is the allocation of the poorer non-veto player in the status quo. The reduction accepted by the veto player increases with his discount factor δ and the share to the poorer non-veto player s_{nv} . An impatient veto player does not accept any new division of the dollar that gives him less than the status quo. The same is true for a patient veto player when the status quo is in $\bar{\Delta}$ and, thus, $s_{nv} = 0$. Note also that the reduction a veto player is willing to accept could be more than what he has in the status quo, in which case his demand is bounded below by 0.

Having identified the acceptance set of the veto player, the non-veto proposer will thus propose the point in the acceptance set of the veto player that maximizes the proposer's stage utility. These are depicted in the right-most panel of Figure 4. A non-veto proposer

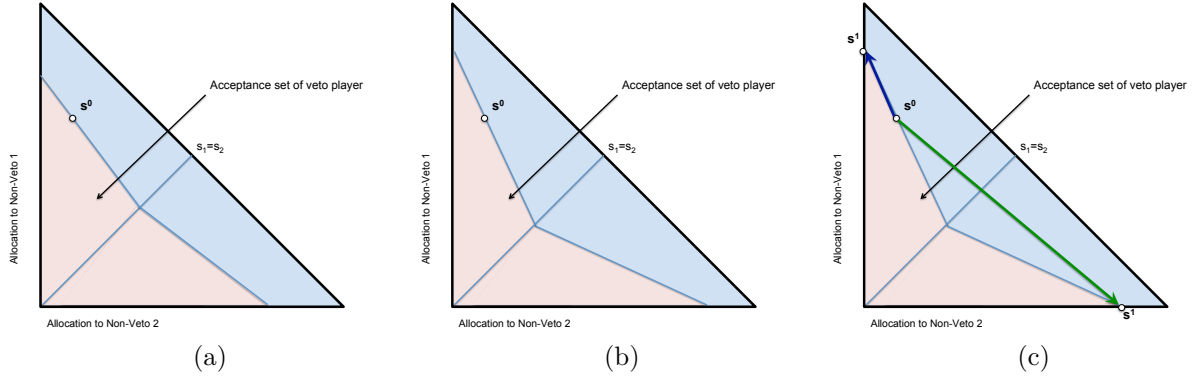


Figure 4: Veto's acceptance set and non-veto's proposal strategies for state \mathbf{s}^0 : (a) $A_v(\mathbf{s}^0)$ when $\delta = \delta_1 > 0$; (b) $A_v(\mathbf{s}^0)$ $\delta = \delta_2 > \delta_1$, (c) equilibrium proposal of non-veto 1 (blue arrow) and non-veto 2 (green arrow).

will completely expropriate the other non-veto player, give the veto player his demand, and allocate the remainder to himself. When the state is in $\bar{\Delta}$, the non-veto proposer can only get $1 - s_v$, but when the state is in $\Delta \setminus \bar{\Delta}$ he can extract an higher amount, namely $1 - d_v$.

3.2.2 Veto Proposer

A similar analysis holds for the veto proposer. As mentioned above, when the veto player desires to pass a proposal with a minimal winning coalition, he is not bound to include any specific legislator. Thus, he selects the legislator who accepts the highest increase to the veto player's share—that is, the legislator with the lowest demand—as his coalition partner. With impatient legislators, this is the poorer non-veto player, who accepts any allocation that assigns him a share greater than or equal to his share in the status quo, regardless of the distribution of the dollar among the other players. However, a patient non-veto player evaluates the impact of the current proposal on his future bargaining power.

The bargaining power of a non-veto player decreases with the share held by the veto player in the status quo, s_v . A patient non-veto player values both what he has and the allocation to the veto player, and prefers an allocation $\mathbf{s}' \in \bar{\Delta}$ where he gets $s'_j = 0$ and the veto gets s'_v to an alternative allocation $\mathbf{s}'' \in \bar{\Delta}$ with $s''_j = s'_j$ but $s''_v > s'_v$. The difference between these allocations arises when he is recognized in $t + 1$, as he will gain the support of

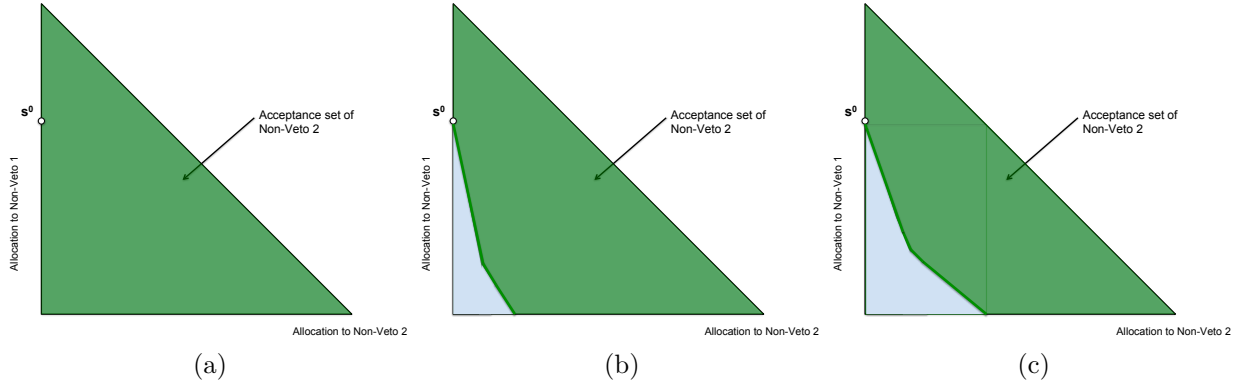


Figure 5: Non-veto 2's acceptance sets for state \mathbf{s}^0 where $s_1 > s_2$: (a) $A_2(\mathbf{s}^0)$ when $\delta = 0$, (b) $A_2(\mathbf{s}^0)$ when $\delta = \delta_1 > 0$; (c) $A_2(\mathbf{s}^0)$ $\delta = \delta_2 > \delta_1$.

the veto player only for proposals that give him no more than $1 - s_v$. Figure 5 depicts the acceptance set of the poorer non-veto player for a state $\mathbf{s}^0 \in \overline{\Delta}$ and three increasing values of the discount factor.

The veto player's coalition partner now demands a *premium* to vote in favor of an allocation that increases the veto player's share. In other words, the veto player has to compensate his coalition partner with a short term gain in stage utility for the long term loss in future bargaining power. The Appendix shows that the demand of the poorer non-veto player for states $\mathbf{s} \in \overline{\Delta}$ is:

$$d_{nv} = \frac{\delta}{3-2\delta} \overline{s_{nv}} \quad (7)$$

where $\overline{s_{nv}}$ is the allocation to the richer non-veto player in the status quo. Some properties of d_{nv} are worth noting. First, d_{nv} is smaller than $\overline{s_{nv}}$ for any $\delta \in [0, 1)$. This means that, whenever agents are not perfectly patient ($\delta = 1$), the veto proposer can increase his share, as he can assign himself $1 - d_{nv} > s_v = 1 - \overline{s_{nv}}$. Second, the premium paid by the veto player to his coalition partner is monotonically increasing, and convex, in δ and linearly increasing in $\overline{s_{nv}}$: d_{nv} converges to $\overline{s_{nv}}$ as δ converges to 1, and d_{nv} converges to 0 as δ converges to 0. The fact that the premium is always positive is crucial for the long term dynamics of the game. In particular, this implies that the ratchet effect described above still functions,

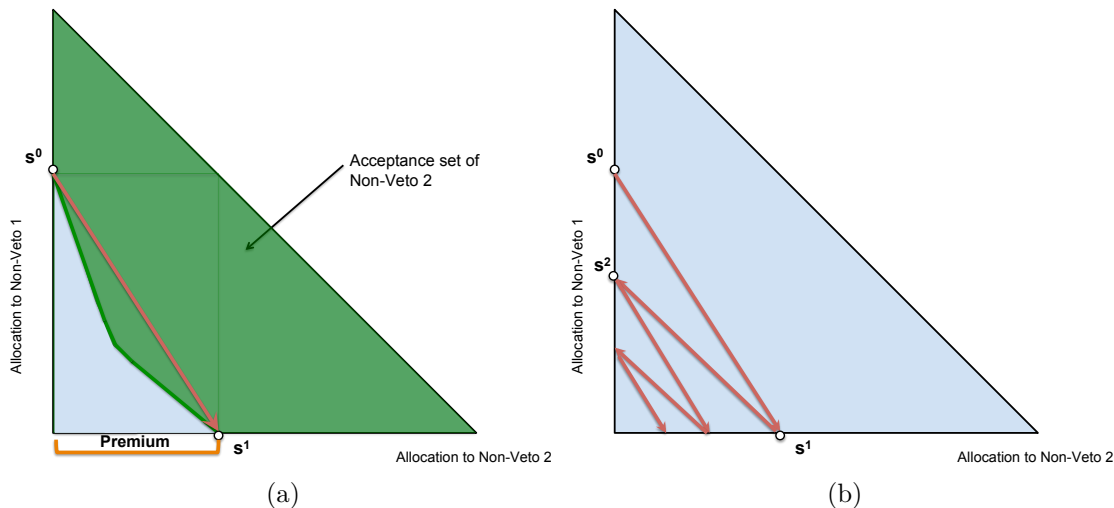


Figure 6: Veto's equilibrium proposal strategy for state s^0 and $\delta > 0$.

albeit at a slower rate such that the convergence to the veto player's ideal point happens asymptotically. Figure 6(b) shows how the state would evolve when the veto player always proposes.

One additional equilibrium difference between patient and impatient legislators is that the veto player mixes between coalition partners for some states in the interior of the simplex when the allocations to the two non-veto players are close. This is necessary to guarantee that the proposer's choice of a partner is a best response to what they demand. If the veto player always picked the poorer non-veto player as coalition partner, the poorer player would become the most expensive coalition partner. To see why, note that the demand of a legislator depends both on the current allocation and on the continuation value of the status quo policy. Under pure proposal strategies, the richer non-veto player is sure to be excluded from any future coalition and, when his allocation in the status quo is not much different than the allocation of the poorer non-veto player, this lower continuation value makes him less demanding. In this case, it would not be optimal for the veto player to always propose to the poorer non-veto player.⁸

⁸Mixed proposal strategies are a common feature of stationary subgame perfect equilibria in models of legislative bargaining à la Baron and Ferejohn (1989), and in Markov perfect equilibria of dynamic legislative bargaining models, for the same reason discussed above. See, for example, Banks and Duggan (2000, 2006), Kalandrakis (2004, 2009), and the discussion in Duggan (2011).

3.3 Results

Proposition 1 provides a summary of the discussion above:

Proposition 1. *For any $\delta \in [0, 1)$ and any initial division of the dollar, $\mathbf{s}^0 \in \Delta$, there exists a symmetric MPE that induces a Markov process over outcomes such that:*

- *For any state $\mathbf{s} \in \Delta \setminus \bar{\Delta}$ there is probability 1 of transition to $\bar{\Delta}$.*
- *$\bar{\Delta}$ is an absorbing set.*
- *All proposals give a positive allocation at most to a minimal winning coalition.*
- *For some $\mathbf{s} \in \Delta \setminus \bar{\Delta}$, the veto proposer mixes between possible coalition partners that have positive and nearly equal allocation under the status quo. For the remaining $\mathbf{s} \in \Delta$, the veto proposer proposes d_{nv} to the poorer non-veto player.*
- *For all $\mathbf{s} \in \Delta$, the non-veto proposers proposes d_v to the veto player.*
- *For all $\mathbf{s} \in \bar{\Delta}$, $d_v = s_v$ and $d_{nv} \geq s_{nv}$.*

Figure 7 explores the states for which mixing occurs in equilibrium. In regions C and D of Figure 7 the veto player mixes between coalition partners. These regions evolve from left to right as the discount factor grows. In regions B and C of Figure 7 the veto player is willing to accept nothing. Note that mixing occurs when the non-veto players have nearly equal allocations and that the set of status quo policies where mixing occurs grows with δ . This happens because the weight players put in the probability of inclusion in future coalitions diminishes with δ . For $\delta = 0$ coalition building costs are solely determined by status quo allocations, and, thus, there are pure strategy proposals. Regions B and C shrink as δ decreases as well: the lower the discount factor, the lower the benefit the veto player receives from reducing future coalition building costs. For $\delta = 0$, the veto player never accepts anything less than what the status quo grants him, s_v . For status quo allocations

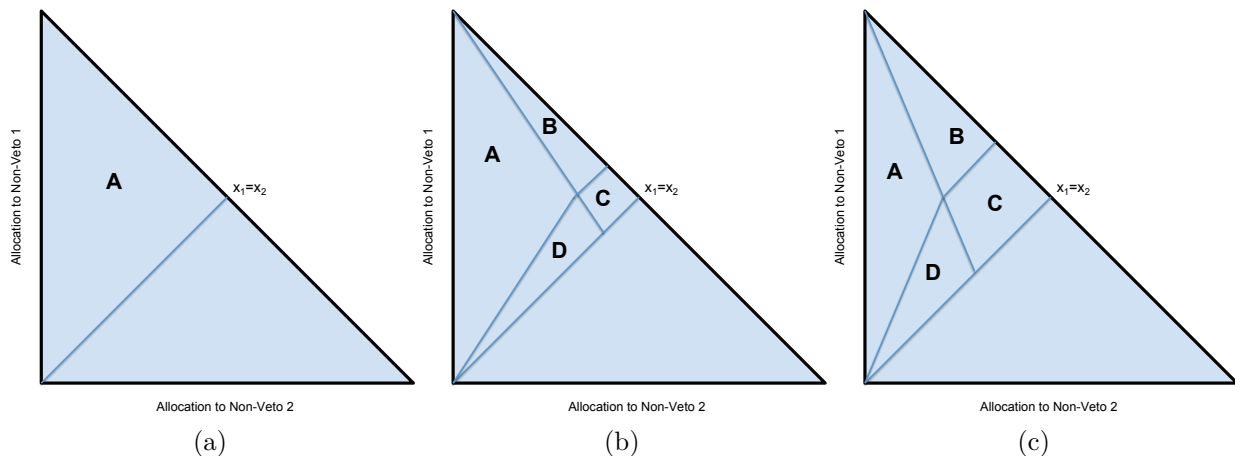


Figure 7: Partition of Δ into regions with different equilibrium strategies for allocations where $s_1 \geq s_2$: (a) $\delta=0$; (b) $\delta = \delta_1 > 0$; (c) $\delta = \delta_2 > \delta_1$. In A and B veto proposer builds a mwc with non-veto 2; in C and D veto proposer mixes between coalition partners; in B, and C veto gets 0 when he is not proposing.

in region A of Figure 7, the veto player always includes the poorer non-veto player in his coalition and he always receives a positive allocation when he is not proposing.⁹

The crucial step in the proof of Proposition 1 is verifying the optimality of proposal strategies. While the Appendix contains the details, here I sketch the key passages of the proof. Define the demand of agent i as the amount that makes i indifferent between the status quo \mathbf{s} and a new division $\mathbf{z} \in \bar{\Delta}$, as before. The proof then proceeds in three steps. First, I prove that, for each agent i , $i = v, 1, 2$, $U_i(\mathbf{x})$ is continuous and increasing in x_i for all $\mathbf{x} \in \bar{\Delta}$. This proves that—among acceptable allocations in $\bar{\Delta}$ —the proposer prefers the one that gives him the highest share of the dollar. Second, I show that the demand of the poorer non-veto player is (weakly) smaller than the demand of the richer non-veto player for any $\mathbf{s} \in \Delta$. This shows that the veto proposer never has an incentive to propose only to the richer non-veto player. Third, I show that the sum of the demands of the veto player and any non-veto player is less than or equal to one for any status quo allocation in Δ . This means that there always exists an acceptable allocation in $\bar{\Delta}$ that guarantees the proposer

⁹In the Appendix, I give the exact statement of the equilibrium proposal and voting strategies for each region of the simplex, and show that these strategies and the associated value functions constitute part of a symmetric MPE.

at least his demand or more. This, together with the monotonicity in the first step, proves that no feasible allocation $\mathbf{x} \in \Delta \setminus \bar{\Delta}$ gives the proposer a higher $U_i(\mathbf{x})$ than his preferred allocation in $\bar{\Delta}$.

I can now state the main result of the paper:

Proposition 2. *There exists a symmetric MPE in which, irrespective of the discount factor and the initial division of the dollar, the status quo policy eventually gets arbitrarily close to the veto player’s ideal point, that is $\forall \varepsilon > 0$ there exists T such that $\forall t \geq T$ the veto player’s allocation in the status quo is greater than or equal to $1 - \varepsilon$.*

Proof. The result derives from the features of the MPE characterized in the proof of Proposition 1. In this MPE, once we reach allocations in the absorbing set $\bar{\Delta}$, which happens after at most one period, the veto player is able to increase his share whenever he has the power to propose, and keeps a constant share when not proposing. For any ε and any starting allocation \mathbf{s}^0 , there exists a number of proposals by the veto player—which depends on δ —such that the veto player’s allocation in the status quo will be at least $1 - \varepsilon$ for all subsequent periods. Let this number of proposals be $n^*(\varepsilon, \delta, \mathbf{s}^0)$. Since each player has a positive probability of proposing in each period, the probability that in infinitely many periods the veto player proposes less than $n^*(\varepsilon, \delta, \mathbf{s}^0)$ is zero. ■

Proposition 3 addresses the speed of convergence to complete appropriation of the dollar by the veto player.

Proposition 3. *In the symmetric MPE characterized in the proof of Proposition 1, if legislators are impatient, $\delta = 0$, it takes at most two rounds of proposals by the veto player to converge to the irreducible absorbing state where $s_v = 1$. If legislators are patient, $\delta \in (0, 1)$, convergence to this absorbing state does not happen in a finite number of bargaining periods, and the higher the discount factor the slower the convergence.*

Proof. This result follows directly from the equilibrium demand of the poorer non-veto player in the absorbing set $\bar{\Delta}$, $d_{nv}(\mathbf{s}, \delta) = \frac{\delta}{3-2\delta} \bar{s}_{nv}$. When $\delta = 0$, this demand is zero. This means that, when the status quo is in $\bar{\Delta}$ —a set that is reached in at most one period—the

poorer non-veto supports any proposal by the veto player. The veto player can thus pass his ideal outcome as soon $\mathbf{s} \in \overline{\Delta}$ and he proposes. On the other hand, when $\delta \in (0, 1)$, this is not possible, and the poorer non-veto player always demands a positive share of the dollar to support any allocation that makes the veto player richer. The convergence in this case is only asymptotic as the non-veto player's demand is always positive as long as the allocation to the richer non-veto is positive, that is as long as the poorer veto player does not have the whole dollar in the status quo.¹⁰ ■

Finally, I prove that the equilibrium in Proposition 1 is well-behaved, in the sense that proposal strategies are weakly continuous in the status quo, \mathbf{s} .

Proposition 4. *The continuation value functions, V_i , and the expected utility functions, U_i , induced by the equilibrium in Proposition 1 are continuous.*

In the Appendix, I show that in equilibrium a small change in the status quo implies a small change in proposal strategies and, by extension, to the equilibrium transition probabilities. An immediate implication of the continuity of transition probabilities is the fact that continuation functions and expected utility are continuous.

4 Justifying Equilibrium Selection

The legislative game studied in this paper is an infinite horizon dynamic game with a plethora of subgame perfect equilibria and, thus, an equilibrium-selection issue. As standard in the literature on dynamic legislative bargaining, I have refined away equilibria involving stage-dominated or non-stationary strategies.¹¹ Even so, it is still possible that other Markovian equilibria of this game exist. In this Section, I characterize sufficient conditions for full appropriation by the veto player to be the unique equilibrium outcome in any MPE, and

¹⁰Notice that when the initial division of the dollar—which is assumed to be exogenous—assigns the whole dollar to the veto player, then the status quo will never be changed and the veto player gets the whole dollar in every period.

¹¹In large legislatures, non-stationary strategies that depend on the history are implausible because of legislators turnout, extraordinary commitment, coordination, and/or communications requirements. Nevertheless, see the discussion in Section 6 for an example of non-stationary strategies that can support more equitable long-run outcomes.

I offer some compelling reasons to select the particular equilibrium identified in Section 3. Before we state the sufficient conditions for uniqueness of the equilibrium outcome, we need two additional definitions:

Definition 3. (Continuity) *An MPE is continuous if the continuation value functions, V_i , induced by equilibrium strategies are continuous.*

Definition 4. (Weak Monotonicity) *An MPE of this game is weakly monotonic if the value functions associated to this equilibrium for each non-veto player are strictly increasing in the share assigned to that player, keeping the share assigned to the veto player constant.*

Definition 3 is a standard continuity assumption. While continuity of equilibrium strategies and value functions is generally not assured in dynamic legislative bargaining games this is an appealing characteristic of the MPE characterized in Section 3, as proved in Proposition 4. Regarding the *weak monotonicity* property in Definition 4, notice that it is much less stringent than a regular monotonicity condition on equilibrium value functions, satisfied when the value functions of the dynamic game are strictly increasing in the share to oneself (similarly to the utilities of the stage game). Weak monotonicity is implied by regular monotonicity, but the reverse is not true: weak monotonicity requires V_i to be strictly increasing in x_i only along the ray where the resources allocated to the veto player do not change, but does not restrict the change in the value function when moving in other directions on the simplex. In other words, an MPE satisfies the weak monotonicity condition if a forward-looking non-veto player is always worse off when transferring some of his share to the other non-veto player.

For example, the MPE characterized in Section 3 satisfies regular monotonicity only for δ low enough.¹² For δ high enough, however, the value functions associated with this MPE are not necessarily increasing in the share to oneself and they can be decreasing when both

¹²For $\delta = 0$, the value functions of the infinite horizon game retain the same properties of the stage game utilities, and are therefore strictly increasing in the share to oneself. Since the value functions of the MPE characterized in Section 3 are continuous in δ , regular monotonicity holds for δ low enough.

the share to oneself and the share to the veto player increase at the same time. The MPE in Section 3, however, satisfies weak monotonicity for any δ .

We are now ready to state Proposition 5:

Proposition 5. *Full appropriation of the dollar by the veto player is the unique absorbing state of the dynamic legislative bargaining game with veto power in any continuous and weakly monotonic Markov perfect Nash equilibrium.*

Proposition 5 states that full appropriation of the endowment by the veto player is the unique long run equilibrium outcome of this dynamic game in a class of Markovian equilibria. The intuition behind Proposition 5 is that, given continuity and weak monotonicity of the value functions, a proposing veto player will always be willing and able to pass a policy that gives him a higher share of the endowment, unless he already holds the whole dollar.

In addition to generating the unique equilibrium outcome from a reasonable class of equilibria, the MPE characterized in Section 3 is robust to changes in the legislators' time horizon and degree of patience. First, regarding the time horizon, it can be shown that any finite horizon version of this dynamic legislative bargaining game has a unique subgame perfect equilibrium in stage undominated voting strategies. While the characterization of this equilibrium for the T-period problem is cumbersome, it can be proven that, in this equilibrium, only a minimal winning coalition of legislators receives a positive share of the pie, the veto player never accepts a reduction to his allocation (with the possible exception of the first period, if the initial status quo is in the interior of the simplex), and the veto player can always increase his allocation when proposing, with his coalition partner demanding a premium to go along with this proposal.¹³ In this sense, the MPE characterized in Section 3 is the limit of the unique subgame perfect equilibrium of the finite game as the number of periods played goes to infinity.

Second, regarding the robustness to legislators' discount factors, this MPE is *well behaved* as shown by Propositions 1 and 4: it exists for any $\delta \in [0, 1)$ and for any $\mathbf{s} \in \Delta$, and it is

¹³The equilibrium analysis for T=2 is available from the author.

continuous in δ and \mathbf{s} . This MPE, moreover, converges smoothly to the unique SPE of the stage game as $\delta \rightarrow 0$.

Finally, while the model I study is the simplest one to account for veto power in committees and it abstracts from the specific institutional details of real world legislative forums featuring a veto player, the incentives the model lays bare are representative of tensions we observe in real world institutions. Consider, for example, the struggle between the U.S. Congress and the U.S. President in allocating federal resources through the yearly budget process, and how the balance of power between the two has evolved over time. A key insight of the model presented in this paper is that the power to veto, when coupled with the power to set the agenda, offers a strong leverage to his holder, who can slowly reduce the resources available to other agents. The U.S. Constitution establishes Congress as the body with controlling power over the federal budget and, while it gives the President the power to veto legislative decisions, it does not assign any budgetary authority to the executive branch. This changed with the Budget and Accounting Act of 1921 (BAA), which required the President to submit a budget to Congress on an annual basis. Although Congress technically retained nearly complete control over the budget process, the BAA also introduced the Office of Management and Budget, which granted the executive branch a monopoly on budgetary information and a strong advantage in setting the agenda (Gailmard and Patty 2012).

Between the 1920s and the 1970s, the scale and scope of the federal budget expanded but—especially between the late 1950s and the early 1970s—the share of the pie absorbed by entitlement spending grew disproportionately, with discretionary spending being progressively scaled down. The result was a general lack of flexibility in fiscal policy to undertake new policy objectives, particularly targeted transfers to local constituencies (Wildavsky 1986, Wildavsky and Caiden 1988). Eventually, the increased policymaking power of the President and the reduced resources available for discretionary spending culminated in a fiscal conflict with Congress. The result of this conflict was the 1974 Congressional Budget and Impound-

ment Act (CBA), which created a set of institutional changes designed to help Congress regain power over the budget process (Pfiffner 1979). The CBA not only limited the ability of the President to impound funds already appropriated by Congress (that is, it reduced the power to veto of the President) but, more importantly, it created the Congressional Budget Office to give Congress independent economic analysis and reduce the dependence of the budget process on the proposals of the executive branch (that is, it decreased the proposal power of the President).

5 Robustness and Extensions

According to the main result, presented in the previous section, the veto player is eventually able to steer the status quo policy arbitrarily close to his ideal point, and fully appropriate all the resources. In this section, I explore three institutional measures that could, in principle, reduce the leverage of the veto player and promote more equitable outcomes: reducing the recognition probability of the veto player, expanding the committee by increasing the number of veto players, and randomly re-assigning veto power in each period. While the first institutional arrangement decreases the agenda setting power of the veto player, the other two introduce competition in the use of veto power.

5.1 Heterogeneous Recognition Probabilities

The previous section assumed that the probabilities of being recognized as proposer are symmetric and history invariant. However, veto players may be outsiders who have lesser ability to set the agenda. For example, the U.S. President has no formal power to propose new legislation and, even if he is able to influence the agenda through like-minded representatives in Congress, his proposal power is lower than any individual member of Congress. In other settings, the veto player has a privileged position to set the agenda, for example committee chairs in the U.S. Congress. In this section, I relax the assumption of symmetric recognition

probabilities, and find that the veto player is still able to eventually appropriate all resources, as long as his recognition probability is positive, and that convergence to this outcome is slower the lower is this probability.

In particular, denote by p_v the probability the veto player is recognized as the proposer in each period, with $p_{nv} = \frac{1-p_v}{2}$ being the probability a non-veto player is recognized. Proposition 5 shows that there exists a MPE equilibrium of this dynamic game that has the same features as the one characterized in the previous section: all proposals entail positive distribution to only a minimal winning coalition and the status quo allocation converges to the ideal point of the veto player as long as $p_v > 0$.

Proposition 6. *With different recognition probabilities of veto and non-veto players, there exists a symmetric MPE in which, irrespective of the initial division of the dollar and the discount factor, the status quo policy eventually gets arbitrarily close to the veto player's ideal point, as long as $p_v > 0$. With the exception of at most the first period, the convergence to the absorbing state is faster the higher is the proposal power of the veto player.*

As in the case with even recognition probabilities, this result hinges on the fact that, once an allocation is in the absorbing set $\bar{\Delta}$ —the set of allocations where at least one non-veto gets zero—the veto player is able to increase his share whenever he proposes.

The proposal power of the veto player influences the speed of convergence to his ideal outcome both directly and indirectly. The direct effect is given by the change in the frequency at which the veto player can increase his allocation—which happens only when he proposes.

The indirect effect is given by the change in the amount the veto player can extract from the non-veto players when he proposes. The probability of recognition of the veto player affects the continuation value of the status quo policy for all legislators, and thus it affects how much they demand to support a policy change. In particular, as p_v increases, non-veto players are less likely to be recognized at time $t + 1$ and, thus, they are less concerned about their future coalition building costs. This reduces the premium the poorer non-veto player demands from the veto player to support an allocation that increases his share.

The proof of Proposition 6 shows that, when $\bar{\Delta}$ is reached, the demand of the poorer non-veto player is

$$d_{nv} = \frac{\delta(1 - p_v)}{2 - \delta(1 + p_v)} \bar{s}_{nv}$$

where \bar{s}_{nv} is the allocation to the richer non-veto player in the status quo. This demand is strictly greater than $s_{nv} = 0$ as long as $\bar{s}_{nv} > 0$, $\delta > 0$, and $p_v < 1$. Under these conditions, the poorer non-veto player demands a premium, $d_{nv} \geq s_{nv} = 0$, from the veto player. This premium is monotonically decreasing in p_v . Thus, with a higher p_v , the veto player is more likely to increase his share in each period, and he can also extract more from the non-veto players when he is the proposer.

When the initial allocation is in the interior of the simplex, and the proposer in the first period is a non-veto player, p_v has a second, indirect, effect on the demand of the veto player. The continuation value of moving to an allocation in $\bar{\Delta}$ for the veto player increases with p_v as he is more likely to be the proposer in $t + 1$ and enjoy the reduction in coalition building costs. This increases his willingness to give up a fraction of his share in order to move into the absorbing set, when a non-veto player is proposing in the initial period.

The proof of Proposition 6 shows that the demand of the veto player is:

$$d_v = \max\left\{s_v - \frac{2p_v\delta}{2 - \delta(1 + p_v)} s_{nv}, 0\right\}$$

where s_v is the allocation of the veto player in the status quo, and s_{nv} is the allocation of the poorer non-veto player in the status quo. This demand is monotonically decreasing in p_v . That is, the reduction the veto player is willing to accept to move the status quo from $s \in \Delta \setminus \bar{\Delta}$ into $\bar{\Delta}$ is increasing in his proposal power.

5.2 Multiple Veto Players

Next, I consider a committee with more than one veto player and show that the presence of multiple veto players with opposing preferences does not prevent the complete expropriation

of the resources initially allocated to non-veto players. However, the presence of other legislators with veto power reduces the amount a veto proposer can extract.

In particular, I study a setting with two veto players and two non-veto players where recognition probabilities are identical and a proposal passes only if it is approved by the two veto players and at least one non-veto player.¹⁴ Proposition 7 shows that this dynamic game has a symmetric MPE—where symmetry applies to legislators of the same type, veto or non-veto—in which the two veto players eventually extract all the surplus regardless of the initial allocation and the discount factor.

Proposition 7. *In the game with two veto players, there exists a symmetric MPE in which, irrespective of the discount factor and the initial division of the dollar, the sum of the allocations to the two veto players eventually gets arbitrarily close to one. In the absorbing state, the share to each veto player is strictly larger than his starting share, unless \mathbf{s}^0 is an absorbing state.*

As with only one veto player, the result hinges on the fact that a veto proposer can pass an allocation that increases his allocation at the expenses of the richer non-veto player. However, there is an important difference: the veto proposer now has to allocate to both the other veto player and the poorer non-veto player more than what they receive in the status quo, that is, he has to pay both of them a premium in order to increase his allocation.

The other legislators demand this premium because a higher current allocation to one veto player increases his future demand and, thus, decreases the extent to which other legislators can exploit the power to propose in $t + 1$, as with only one veto player. The proposing veto player, who builds a minimal winning coalition with the other veto player and the poorer non-veto player, has to share part of the amount he expropriates from the richer non-veto player with his coalition partners, in order to offset this loss and gain their

¹⁴Ideally, I would answer the question above studying a game with an arbitrary number of legislators n and veto players $k \leq n$. However, as the dimensionality of the state space increases analytical tractability is quickly lost. Adding one veto player allows me to gain a valuable insight on the issue of multiple veto players but preserves the analytical tractability of the model, even if the set of possible legislative outcomes passes from \mathbb{R}^2 to \mathbb{R}^3 .

vote.¹⁵ Since each veto player will always be part of a minimal winning coalition, both veto players enjoy a ratchet effect in their allocations, regardless of the identity of the proposer along the equilibrium path.

In the proof of Proposition 7, I show that, once allocations are in $\bar{\Delta}$, the demand of the veto player who is not proposing, denoted by d_v , and the demand of the poorer non-veto player, denoted by d_{nv} , are as follows:

$$\begin{aligned} d_v &= s_v + \frac{4\delta(1-\delta)}{16-16\delta+3\delta^2} \overline{s_{nv}} \\ d_{nv} &= \frac{\delta}{4-3\delta} \overline{s_{nv}} \end{aligned}$$

where $\overline{s_{nv}}$ is the allocation to the richer non-veto in the status quo. Some properties of these two demands are worth noting. First, both the non-proposing veto player and the poorer non-veto player asks for a premium, that is, $d_v > s_v$ and $d_{nv} > \underline{s_{nv}} = 0$, as long as $\delta > 0$ and $\overline{s_{nv}} > 0$. Second, out of the two veto players, the one that is proposing will get a greater share of the resources expropriated from the richer non-veto player. Finally, as for the case with one veto player, the premium demanded by coalition partners is increasing in legislators' patience and in the fraction of the dollar in the hands of non-veto players.

5.3 Rotating Veto Power

In the basic setting, as well as in the extensions already discussed, veto power is permanently assigned to one or more legislators. This section considers an alternative institutional arrangement where veto power is randomly assigned to a legislator in each period, in a similar—but independent—way as proposal power. In this case, the policy converges in finite time to an absorbing set where, in each period, either the proposer or the veto player get the entire dollar.

¹⁵Even I do not formally address the framework with heterogenous probabilities of recognition, note that, by the same logic explored in Section 4.1, this result would still hold if the two veto players had different probabilities of recognition, as long as both probabilities were strictly positive.

This setting is a significant departure from the basic setup, and the existence proof from Proposition 1 does not hold. In this section, I establish a Markov equilibrium of the dynamic game with random veto power, when the space of possible agreements is restricted to minimal-winning coalitions, $\mathbf{x} \in \Delta_2$, that is, the edges of the simplex, where at most two legislators have a positive share.¹⁶ The restriction to minimal winning coalitions simplifies considerably the analysis and it is a sensible conjecture on the properties of equilibria of the unrestricted game, given existing results for similar dynamic bargaining games.¹⁷

In the Appendix, I prove that a MPE of the restricted game with rotating veto power exists.¹⁸ This equilibrium is summarized by two properties. First, for every status quo, optimal proposals coincide with the feasible allocations that maximize the proposer's share of the surplus. Second, players with zero in the status quo allocation are willing to accept proposals that also allocate them zero, regardless of the identity of the proposer. This second feature is in line with the voting strategies of impatient agents and, contrary to what happens in the setting with permanent veto power, it is preserved when agents are patient. In an equilibrium with these features, the status quo policy converges to an absorbing set where the proposer can allocate the whole dollar to himself, unless he is the only legislator who gets nothing in the status quo, or unless the veto player is another legislator with a positive share in the status quo.

Proposition 8. *In the game with rotating veto power and feasible allocations $\mathbf{s} \in \Delta_2$, there exists a symmetric MPE in which, irrespective of the discount factor and the initial division of the dollar, eventually either the proposer or the veto player extract the whole dollar in all periods.*

This result is very intuitive, in light of the features discussed above. With the restriction

¹⁶Note that Δ_2 does not coincide with the partition $\bar{\Delta}$ defined in Section 3, as $\bar{\Delta}$ does not include those allocations where the two non-veto players have a positive allocation and the veto player has zero.

¹⁷Note that this is a restriction on the game, and not simply an equilibrium refinement. For dynamic games where minimal winning coalitions arise in equilibrium, see, among others, Kalandrakis (2004, 2009), and Battaglini and Coate (2007, 2008), apart from the results discussed in this paper.

¹⁸Note that, if minimal winning coalition proposals are optimal also in the unrestricted game where all $\mathbf{s} \in \Delta$ are feasible, this MPE coincides with a MPE of the unrestricted game, at least for all periods $t > 1$.

to minimal-winning coalitions, we have $s_i = 0$ for some i , possibly different across periods, in all periods. Let Δ_1 be the set of allocations where one legislator gets everything, that is, the vertices of the simplex. Now, first consider allocations in which exactly two legislators have a positive share of the dollar, i.e. $\mathbf{s} \in \Delta_2 \setminus \Delta_1$. If, in equilibrium legislator i with $s_i = 0$ does not object to new divisions of the dollar \mathbf{z} with $z_i = 0$, we have three possibilities:

1. if $j \neq i$ is recognized in period $t + 1$ and the veto power is in the hands of either i or j , a coalition of i and j vote for a proposal that allocates the whole dollar to j ;
2. if $j \neq i$ is recognized in period $t + 1$ and the veto power is in the hands of the third legislator $l \neq j \neq i$, the proposer cannot extract the whole dollar because l will object to it;
3. if i is the proposer, regardless of the identity of the veto player, he will propose the most favorable allocation in Δ_2 , as he is not able to allocate the whole dollar to himself.¹⁹

Once we transition to an allocation in Δ_1 , where one legislator gets everything, the implemented policy will always be in Δ_1 , and it will either be unchanged or move to another vertex of the simplex. Call i the legislator with the whole dollar in $\mathbf{s} \in \Delta_1$. If i has the proposal or veto power, which happens with probability $5/9$, the policy does not change. If i has neither power, then the proposer can extract the whole dollar, and will do so. Convergence to the equilibrium absorbing set of policy outcomes is fast, with a maximum expected time before absorption equal to one and a half periods.²⁰ This implies that—when legislators are patient—permanent veto power promotes less extreme outcomes than rotating veto power. To see why remember that, with permanent veto power, the convergence to the veto player’s ideal outcome happens in infinitely many periods, and—along the equilibrium path—the veto has to share the resources with one non-veto player.²¹

¹⁹Note that, when the other two legislators have nearly equal allocations, legislator i with $s_i = 0$ mixes between coalition partners. See the proof in the Appendix for details.

²⁰Absorption is not deterministic as it depends on the identity of the proposer recognized in each period.

²¹Note that the equilibrium of the game with rotating veto power is similar to the equilibrium of the game without veto power studied by Kalandrakis (2004): it has the same absorbing set—the vertices of the

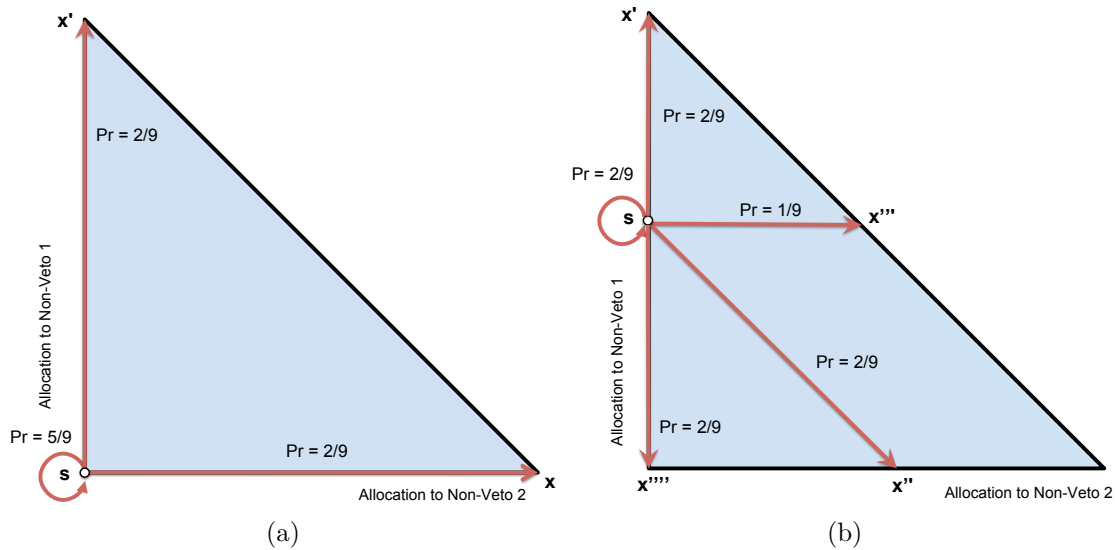


Figure 8: Transition probabilities with temporary veto power: (a) for $s \in \Delta_1$; (b) When $s \in \Delta_2 \setminus \Delta_1$

Figure 8 represents the transition probabilities for allocations s in the absorbing set Δ_1 , and in the complementary set of minimal winning coalition allocation. To understand the transition probabilities consider that, ex ante, each legislator has a $1/3$ chance of being selected as the proposer and independent $1/3$ chance of being assigned the power to veto. This means that, ex-ante, with probability $1/9$ an agent has both the power to veto and to propose, with probability $2/9$ he has the power to veto but not to propose, with probability $2/9$ he has the power to propose but not to veto, and, finally, with probability $4/9$ he has neither power.

6 Discussion

This paper studies the distributive consequences of veto power in a legislative bargaining game with an evolving status quo policy. As the importance of the right to block a decision crucially depends on the status quo, a static analysis cannot draw general conclusions about the effect of veto power on gridlock and policy capture by the veto player. Instead of making

simplex—and similar dynamics, the main difference being a greater status quo inertia with rotating veto power.

ad hoc assumptions on the status quo policy, I study veto power by exploring the inherently dynamic process via which the location of the current status quo is determined. I prove that there exists a Markov Perfect Equilibrium of this dynamic game such that the veto player is eventually able to extract all resources, irrespective of the discount factor, the probability of proposing, and the initial allocation of resources. This result shows that the right to veto is extremely powerful, especially if coupled with proposal power. This is true even when other legislators are patient, and take into account the loss in future bargaining power implied by making concessions to the veto player in the current period.

This paper is the first to derive theoretical predictions on the consequences of veto power in a dynamic setting. While the results certainly add to our understanding of the incentives present in real world legislatures, the setup is intentionally very simple and uses a number of specific assumptions. In the remainder of this section, I discuss some directions for further research.

Extension to General Number of Legislators. I have limited the analysis to legislatures with two non-veto players and, at most, two veto players. It would certainly be interesting to extend the asymptotic result of full appropriation by the veto player(s) to legislatures with an arbitrary number of veto and non-veto legislators. However, the existence proofs for the equilibria proposed in this paper rely on constructing the equilibrium strategies, and the associated continuation values, for any allocation of the dollar, $\mathbf{s} \in \Delta$. It is a very challenging task to extend this existence result and to characterize a Markov equilibrium with a higher number of legislators, as the dimensionality of the state space increases and tractability is quickly lost. Future research could explore the dynamics of a larger legislature using numerical methods, a solution often adopted in the literature on dynamic models with endogenous status quo (Baron and Herron 2003, Penn 2009, Battaglini and Palfrey 2011, Duggan, Kalandrakis and Manjunath 2008).

Extension to Concave Utilities. The equilibrium I characterize exhibits proposals where at most the members of a minimal winning coalition get a positive share of the dollar. An open question is whether there exist other Markovian equilibria of this game where universal coalitions prevail. One possible avenue for future research is to relax the assumption that legislators' utilities are linear in stage payoffs. It would be interesting to assess whether the equilibrium with minimal winning coalitions is robust to concavity in legislators' utilities, and whether equilibria without minimal winning coalitions may arise when stage payoffs are sufficiently concave. Indeed, Battaglini and Palfrey (2011) have recently explored such equilibria in the context of simple majority without a veto player. Using numerical methods, they find Markov equilibria in which players share the surplus in all periods when stage preferences exhibit sufficient concavity.

Non-Markovian Equilibria. I have focused on Markov perfect equilibria where agents' strategies depend only on the status quo policy. However, this legislative game is an infinite horizon dynamic game with many subgame perfect equilibria, and the Markovian assumption of stationary strategies is very restrictive. As noted in the seminal paper of Baron and Ferejohn (1989), these bargaining games usually have other subgame perfect equilibria that can sustain more equitable outcomes through the use of history-dependent strategies, that is, punishment and rewards for past actions. Bowen and Zahran (2009) explored this avenue without a veto player. They show the existence of non-Markovian equilibria in which players share the surplus as long as the legislators are neither too patient nor too impatient. Interestingly—and related to the previous point—this alternative equilibrium does not survive when players are risk neutral. In the Appendix, I propose strategy profiles for the dynamic game analyzed in this paper such that the initial allocation is an absorbing state and, thus, there is no convergence to full appropriation by the veto player, as long as the discount factor is high enough, and the two non-veto players receive enough at the beginning of the game.

Extension to Different Policy Domains. This study analyzes a divide-the-dollar game where legislators' preferences are purely conflicting. This is a natural starting point to analyze the consequences of veto power in a dynamic setting as it lays bare the incentives at work. However, there are two important reasons to extend the policy space beyond the pure distributive case. First, many applications, and policy domains, are better modeled with a spatial setting where legislators' preferences are partially aligned. Second, the pure distributive setting leaves little room to ask whether giving a legislator the power to veto is desirable from the societal point of view as, with linear utilities, all outcomes are Pareto-efficient. The welfare consequences, and the normative implications of introducing a veto player can be better analyzed in a setting with less conflicting preferences. One interesting possibility for future research is to analyze the consequences of veto power in a dynamic setting with a unidimensional policy space, and single peaked legislators' preferences over outcomes.²² An alternative way of exploring a setting with a lower degree of conflict could be to study a different divide-the-dollar game where the dollar can also be allocated to a public good.

Empirical Tests of Theoretical Predictions. The theory provides sharp empirical implications: the ratchet effect for the allocation of the veto player, the monotonic convergence to his ideal point, and the comparative statics on the discount factor, the recognition probabilities, the number of veto players, and the nature of the veto right (permanent vs rotating). One important goal of future research is to assess the empirical validity of these theoretical predictions, in particular with the use of laboratory experiments, which have some important advantages over field data when studying a highly structured dynamic environment such as the one in this paper (Battaglini and Palfrey 2011, Battaglini, Nunnari and Palfrey 2011).

²²A similar setting is studied by Baron (1996) in the context of simple majority without veto power.

Appendix

Proof of Proposition 1

The results of Proposition 1 follow from the existence of a symmetric MPE with the following minimal winning coalition proposal strategies for all $\mathbf{s} \in \Delta$, where $s_1 \geq s_2$:

- Case A $\left(s_1 \leq 1 - \frac{3-\delta}{3-2\delta}s_2, s_1 \geq \frac{3-\delta}{3-2\delta}s_2 \right)$:

$$\begin{aligned} \mathbf{x}^v &= [1 - d_2, 0, d_2], \mathbf{x}^1 = [d_v, 1 - d_v, 0], \mathbf{x}^2 = [d_v, 0, 1 - d_v] \\ d_v &= s_v - \frac{\delta s_2}{3 - 2\delta} \\ d_2 &= \frac{\delta}{3 - 2\delta}s_1 + \frac{(3 - \delta)}{(3 - 2\delta)}s_2 \end{aligned}$$

- Case B $\left(s_1 > 1 - \frac{3-\delta}{3-2\delta}s_2, s_1 \geq \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2}s_2 + \frac{\delta^2}{(3-\delta)^2} \right)$:

$$\begin{aligned} \mathbf{x}^v &= [1 - d_2, 0, d_2], \mathbf{x}^1 = [d_v, 1 - d_v, 0], \mathbf{x}^2 = [d_v, 0, 1 - d_v] \\ d_v &= 0 \\ d_2 &= \frac{9 - 12\delta + 3\delta^2}{(3 - 2\delta)^2}s_2 + \frac{\delta}{(3 - 2\delta)} \end{aligned}$$

- Case C $\left(s_1 > 1 - \frac{3-\delta}{3-2\delta}s_2, s_1 < \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2}s_2 + \frac{\delta^2}{(3-\delta)^2} \right)$:

$$\begin{aligned} \mathbf{x}^v &= \begin{cases} [1 - d_2, d_2, 0] & \text{w/ Pr} = \mu_v^C \\ [1 - d_2, 0, d_2] & \text{w/ Pr} = 1 - \mu_v^C \end{cases}, \mathbf{x}^1 = [d_v, 1 - d_v, 0], \mathbf{x}^2 = [d_v, 0, 1 - d_v] \\ d_v &= 0 \\ d_2 &= \frac{9 - 12\delta + 3\delta^2}{(3 - 2\delta)^2}s_2 + \frac{\delta}{(3 - 2\delta)} \\ \mu_v^C &= \frac{(-27 + 36\delta - 15\delta^2 + 2\delta^3)s_1}{2\delta((9 - 12\delta + 3\delta^2)s_2 - 2\delta^2 + 3\delta)} + \frac{(27 - 27\delta + 3\delta^2 + \delta^3)s_2 + 3\delta^2 - 2\delta^3}{2\delta((9 - 12\delta + 3\delta^2)s_2 - 2\delta^2 + 3\delta)} \end{aligned}$$

- Case D $s \left(1 \leq 1 - \frac{3-\delta}{3-2\delta}s_2, s_1 < \frac{3-\delta}{3-2\delta}s_2 \right) :$

$$\mathbf{x}^v = \begin{cases} [1 - d_2, d_2, 0] & \text{w/ Pr} = \mu_v^D \\ [1 - d_2, 0, d_2] & \text{w/ Pr} = 1 - \mu_v^D \end{cases}, \mathbf{x}^1 = [d_v, 1 - d_v, 0], \mathbf{x}^2 = [d_v, 0, 1 - d_v]$$

$$d_v = s_v - \frac{\delta s_2}{3 - 2\delta}$$

$$d_2 = \frac{\delta}{3 - 2\delta}s_1 + \frac{(3 - \delta)}{(3 - 2\delta)}s_2$$

$$\mu_v^D = \frac{3(-3 + 2\delta)s_1 + (3 - \delta)s_2}{2\delta(\delta s_1 + (3 - \delta)s_2)}$$

It is tedious but straightforward to check that, if players play the proposal strategies in cases A-D and these proposals pass, their continuation values are as follows:

- Case A

$$v_v(\mathbf{s}) = \frac{1}{1 - \delta} - \frac{2 - \delta}{(3 - \delta)(1 - \delta)}s_1 - \frac{1}{(1 - \delta)}s_2 \quad (8)$$

$$v_1(\mathbf{s}) = \frac{(3 - 3\delta + \delta^2)}{(3 - \delta)^2(1 - \delta)}s_1 + \frac{(3 - \delta)}{(3 - \delta)^2(1 - \delta)}s_2 \quad (9)$$

$$v_2(\mathbf{s}) = \frac{(3 - 2\delta)}{(3 - \delta)^2(1 - \delta)}s_1 + \frac{(6 - 5\delta + \delta^2)}{(3 - \delta)^2(1 - \delta)}s_2 \quad (10)$$

- Case B

$$v_v(\mathbf{s}) = \frac{1}{(1 - \delta)(3 - \delta)} - \frac{(3 - 4\delta + \delta^2)}{(3 - 2\delta)(1 - \delta)(3 - \delta)}s_2$$

$$v_1(\mathbf{s}) = \frac{(3\delta - 4\delta^2 + \delta^3)}{(3 - \delta)^2(1 - \delta)(3 - 2\delta)}s_2 + \frac{(9 - 15\delta + 9\delta^2 - 2\delta^3)}{(3 - \delta)^2(1 - \delta)(3 - 2\delta)}$$

$$v_2(\mathbf{s}) = \frac{(3 - 2\delta)}{(3 - \delta)^2(1 - \delta)} + \frac{(3 - 4\delta + \delta^2)}{(3 - \delta)^2(1 - \delta)}s_2$$

- Case C

$$\begin{aligned}
v_v(\mathbf{s}) &= \frac{1}{(1-\delta)(3-\delta)} - \frac{(3-4\delta+\delta^2)}{(3-2\delta)(1-\delta)(3-\delta)}s_2 \\
v_1(\mathbf{s}) &= \frac{(-9+18\delta-11\delta^2+2\delta^3)}{2\delta(3-\delta)(1-\delta)(3-2\delta)}s_1 + \frac{(9-15\delta+7\delta^2-\delta^3)}{2\delta(3-\delta)(1-\delta)(3-2\delta)}s_2 + \frac{6\delta-7\delta^2+2\delta^3}{2\delta(3-\delta)(1-\delta)(3-2\delta)} \\
v_2(\mathbf{s}) &= \frac{(9-18\delta+11\delta^2-2\delta^3)}{2\delta(3-\delta)(1-\delta)(3-2\delta)}s_1 + \frac{(-9+21\delta-15\delta^2+3\delta^3)}{2\delta(3-\delta)(1-\delta)(3-2\delta)}s_2 + \frac{6\delta-7\delta^2+2\delta^3}{2\delta(3-\delta)(1-\delta)(3-2\delta)}
\end{aligned}$$

- Case D

$$\begin{aligned}
v_v(\mathbf{s}) &= \frac{1}{1-\delta} - \frac{2-\delta}{(3-\delta)(1-\delta)}s_1 - \frac{1}{(1-\delta)}s_2 \\
v_1(\mathbf{s}) &= \frac{(-3+6\delta-2\delta^2)}{2\delta(3-\delta)(1-\delta)}s_1 + \frac{1}{2\delta(1-\delta)}s_2 \\
v_2(\mathbf{s}) &= \frac{(3-2\delta)}{2\delta(3-\delta)(1-\delta)}s_1 + \frac{(-3+7\delta-2\delta^2)}{2\delta(1-\delta)(3-\delta)}s_2
\end{aligned}$$

On the basis of these continuation values, we obtain players' expected utility functions, $U_i(\mathbf{x}) = x_i + \delta V_i(\mathbf{x})$. The reported demands are in accordance with Definition 2. In particular, d_i , $i = 1, 2$ and d_v can be easily derived from the following equations:

$$\begin{aligned}
s_i + \delta V_i(\mathbf{s}) &= d_i + \delta V_i([1-d_i, d_i, 0]) \\
s_v + \delta V_v(\mathbf{s}) &= d_v + \delta V_v([d_v, 1-d_v, 0])
\end{aligned}$$

The demands for non-veto player 1 are never part of a proposed allocation and have therefore been omitted in the statement of the equilibrium proposal strategies above but we will use them in the remainder of the proof. In cases C and D, the mixing of the veto player is such that $d_1 = d_2$. In the other two cases, d_1 is as follows:

- Case A $\left(s_1 \leq 1 - \frac{3-\delta}{3-2\delta}s_2, s_1 \geq \frac{3-\delta}{3-2\delta}s_2 \right)$:

$$d_1 = \frac{(4\delta^2 - 12\delta + 9)}{(3-2\delta)^2}s_1 + \frac{(3\delta - \delta^2)}{(3-2\delta)^2}s_2$$

- Case B $\left(s_1 > 1 - \frac{3-\delta}{3-2\delta}s_2, s_1 \geq \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2}s_2 + \frac{\delta^2}{(3-\delta)^2} \right)$:

$$d_1 = \frac{(27 - 63\delta + 51\delta^2 - 17\delta^3 + 2\delta^4)}{(3 - 2\delta)^3}s_1 + \frac{(3\delta^2 - 4\delta^3 + \delta^4)}{(3 - 2\delta)^3}s_2 + \frac{9\delta - 15\delta^2 + 9\delta^3 - 2\delta^4}{(3 - 2\delta)^3}$$

Furthermore, all reported non-degenerate mixing probabilities are well defined. On the basis of the expected utility functions, U_i , we can then construct equilibrium voting strategies, $A_i^*(\mathbf{s}) = \{\mathbf{x} | U_i(\mathbf{x}) \geq U_i(\mathbf{s})\}$, $i = v, 1, 2$, for all $\mathbf{s} \in \Delta$. These voting strategies obviously satisfy equilibrium condition (4). Then, to prove Proposition 1 it suffices to verify equilibrium condition (5). To do so, we make use of five lemmas. We seek to establish an equilibrium with proposals that allocate a positive amount to at most one non-veto player. Lemma 1 shows that the expected utility function for these proposals satisfies the following continuity and monotonicity properties. Lemma 2 proves that minimal winning coalition proposals are optimal among the set of feasible proposals in $\bar{\Delta}$. Lemma 3 establishes that the equilibrium demands of the veto player and one non-veto player sum to less than unity and that the demands of the two non-veto players are (weakly) ordered in accordance to the ordering of allocations under the state \mathbf{s} . Lemma 4 then establishes that the proposal strategies for legislators $i = v, 1, 2$ in Proposition 1 maximize $U_i(\mathbf{x})$ over all $\mathbf{x} \in W(\mathbf{s}) \cap \bar{\Delta}$; these proposals would then maximize $U_i(\mathbf{x})$ over all $\mathbf{x} \in W(\mathbf{s})$ if there is no $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\bar{\Delta}$ that accrues i higher utility. We establish that this is indeed the case in Lemma 5.

Lemma 1. *Consider a symmetric Markov Perfect strategy profile with expected utility $U_i(\mathbf{s})$, $\mathbf{s} \in \bar{\Delta}$, determined by the continuation values in equations (8)-(10). Then, for all $\mathbf{x} = (x, 1 - x, 0) \in \Delta$ (a) $U_i(\mathbf{x})$, $i = v, 1, 2$ is continuous and differentiable with respect to x , (b) $U_v(\mathbf{x})$ increases with x , while $U_1(\mathbf{s})$ and $U_2(\mathbf{s})$ does not increase with x .*

Proof. An allocation $\mathbf{x} = (x, 1 - x, 0) \in \Delta$ belongs to case A in Proposition 2. Therefore we can write $U_i(\mathbf{x}) = x_i + \delta V_i(\mathbf{x})$ as follows:

$$U_v(\mathbf{x}) = x + \frac{\delta}{1 - \delta} - \frac{\delta(2 - \delta)}{(3 - \delta)(1 - \delta)}(1 - x) \quad (11)$$

$$U_1(\mathbf{x}) = 1 - x + \delta \frac{(3 - 3d + \delta^2)}{(3 - \delta)^2(1 - \delta)}(1 - x) \quad (12)$$

$$U_2(\mathbf{x}) = \delta \frac{(3 - 2\delta)}{(3 - \delta)^2(1 - \delta)}(1 - x) \quad (13)$$

$U_i(\mathbf{x})$ is linear and continuous in x for $i = v, 1$, establishing part (a) of the Lemma. Regarding part (b):

$$\begin{aligned} \frac{\partial U_v(\mathbf{x})}{\partial x} &= 1 + \frac{\delta(2 - \delta)}{(3 - \delta)(1 - \delta)} > 0 \\ \frac{\partial U_1(\mathbf{x})}{\partial x} &= - \left(1 + \delta \frac{(3 - 3d + \delta^2)}{(3 - \delta)^2(1 - \delta)} \right) < 0 \\ \frac{\partial U_2(\mathbf{x})}{\partial x} &= -\delta \frac{(3 - 2\delta)}{(3 - \delta)^2(1 - \delta)} < 0 \end{aligned}$$

$\frac{\partial U_v(\mathbf{x})}{\partial x} > 0$ for any $\delta \in [0, 1)$, since both the numerator and the denominator of $\frac{\delta(2 - \delta)}{(3 - \delta)(1 - \delta)}$ are positive for any $\delta \in [0, 1)$; $\frac{\partial U_1(\mathbf{x})}{\partial x} < 0$ for any $\delta \in [0, 1)$, since both the numerator and the denominator of $\frac{(3 - 3d + \delta^2)}{(3 - \delta)^2(1 - \delta)}$ are positive for any $\delta \in [0, 1)$; and $\frac{\partial U_2(\mathbf{x})}{\partial x} < 0$ for any $\delta \in [0, 1)$, since both the numerator and the denominator of $\frac{(3 - 2\delta)}{(3 - \delta)^2(1 - \delta)}$ are positive for any $\delta \in [0, 1)$.

■

By the definition of demands and the monotonicity established in part (b) of Lemma 1 we immediately deduce:

Lemma 2. *Consider a symmetric Markov Perfect strategy profile with expected utility, $U_i(\mathbf{x})$, for $\mathbf{x} \in \bar{\Delta}$, $i = v, 1, 2$, given by (11)-(13). Every minimal winning coalition proposal of the veto player $x(v, i, d_i(\mathbf{s}))$, $i = \{1, 2\}$ is such that $x(v, i, d_i(\mathbf{s})) \in \arg \max\{U_v(\mathbf{x}) | \mathbf{x} \in \bar{\Delta}, U_i(\mathbf{x}) \geq U_i(\mathbf{s})\}$; similarly, every minimal winning coalition proposal of a non-veto player $x(i, v, d_v(\mathbf{s}))$, $i = \{1, 2\}$ is such that $x(i, v, d_v(\mathbf{s})) \in \arg \max\{U_i(\mathbf{x}) | \mathbf{x} \in \bar{\Delta}, U_v(\mathbf{x}) \geq U_v(\mathbf{s})\}$.*

Lemma 3. For all $\mathbf{s} \in \Delta$, the demands reported in Proposition 1 are such that (a) $s_i \geq s_j \Rightarrow d_i \geq d_j$, $i, j = 1, 2$, and (b) $d_i + d_v \leq 1$, $i = 1, 2$.

Proof. Part (a). Since we focus on the half of the simplex in which $s_1 \geq s_2$, we want to prove that $d_1 \geq d_2$. In cases C and D the mixed strategy of the veto player is such that $d_1 = d_2$, so we focus on cases A and B.

- Case A:

$$\begin{aligned} \frac{(4\delta^2 - 12\delta + 9)}{(3 - 2\delta)^2} s_1 + \frac{(3\delta - \delta^2)}{(3 - 2\delta)^2} s_2 &\geq \frac{\delta}{3 - 2\delta} s_1 + \frac{(3 - \delta)}{(3 - 2\delta)} s_2 \\ s_1 &\geq \frac{3 - \delta}{3 - 2\delta} s_2 \end{aligned}$$

- Case B:

$$\begin{aligned} &\frac{(27 - 63\delta + 51\delta^2 - 17\delta^3 + 2\delta^4)}{(3 - 2\delta)^3} s_1 + \frac{(3\delta^2 - 4\delta^3 + \delta^4)}{(3 - 2\delta)^3} s_2 + \frac{9\delta - 15\delta^2 + 9\delta^3 - 2\delta^4}{(3 - 2\delta)^3} \\ &\geq \frac{9 - 12\delta + 3\delta^2}{(3 - 2\delta)^2} s_2 + \frac{\delta}{(3 - 2\delta)} \\ s_1 &\geq \frac{27 - 27\delta + 3\delta^2 + \delta^3}{(3 - 2\delta)(3 - \delta)^2} s_2 + \frac{\delta^2}{(3 - \delta)^2} \end{aligned}$$

Part (b). Since we focus on the half of the simplex in which $s_1 \geq s_2$, by part (a) of the same Lemma, it is enough to prove that $d_1 + d_v \leq 1$.

- Case A:

$$\begin{aligned} s_v - \frac{\delta s_2}{(3 - 2\delta)} + \frac{(4\delta^2 - 12\delta + 9)}{(3 - 2\delta)^2} s_1 + \frac{(3\delta - \delta^2)}{(3 - 2\delta)^2} s_2 &\leq 1 \\ s_v + s_1 + \frac{\delta^2}{(3 - 2\delta)^2} s_2 &\leq 1 \end{aligned}$$

which holds for any $\delta \in [0, 1)$, because $s_v + s_1 + s_2 = 1$ and $\frac{\delta^2}{(3 - 2\delta)^2} \in [0, 1)$. To see this notice that $\frac{\delta^2}{(3 - 2\delta)^2}$ is monotonically increasing in δ and is equal to 1 when $\delta = 1$.

- Case B:

$$\frac{(27 - 63\delta + 51\delta^2 - 17\delta^3 + 2\delta^4)s_1}{(3 - 2\delta)^3} + \frac{(3\delta^2 - 4\delta^3 + \delta^4)s_2}{(3 - 2\delta)^3} + \frac{9\delta - 15\delta^2 + 9\delta^3 - 2\delta^4}{(3 - 2\delta)^3} \leq 1$$

Notice that $\frac{(27-63\delta+51\delta^2-17\delta^3+2\delta^4)}{(3-2\delta)^3} \geq \frac{(3\delta^2-4\delta^3+\delta^4)}{(3-2\delta)^3}$ for any $\delta \in [0, 1)$, so the LHS has an upper bound when $s_1 = 1$ and $s_2 = 0$. Therefore, we can prove the following inequality:

$$\begin{aligned} \frac{(27 - 63\delta + 51\delta^2 - 17\delta^3 + 2\delta^4)}{(3 - 2\delta)^3} + \frac{9\delta - 15\delta^2 + 9\delta^3 - 2\delta^4}{(3 - 2\delta)^3} &\leq 1 \\ \frac{(3 - 2\delta)^3}{(3 - 2\delta)^3} &\leq 1 \end{aligned}$$

- Case C:

$$\begin{aligned} \frac{9 - 12\delta + 3\delta^2}{(3 - 2\delta)^2} s_2 + \frac{\delta}{(3 - 2\delta)} &\leq 1 \\ s_2 &\leq \frac{3 - 2\delta}{3 - \delta} \end{aligned}$$

which holds for any $\delta \in [0, 1)$, since $s_v + s_1 + s_2 = 1$ and $\frac{3-2\delta}{3-\delta} \leq 1$ for any $\delta \in [0, 1)$. To see this notice that $\frac{3-2\delta}{3-\delta}$ is monotonically decreasing in δ and it is equal to 1 when $\delta = 0$.

- Case D:

$$\begin{aligned} s_v - \frac{\delta s_2}{3 - 2\delta} + \frac{\delta}{3 - 2\delta} s_1 + \frac{(3 - \delta)}{(3 - 2\delta)} s_2 &\leq 1 \\ s_v + s_2 + \frac{\delta}{3 - 2\delta} s_1 &\leq 1 \end{aligned}$$

which holds for any $\delta \in [0, 1)$ because $s_v + s_1 + s_2 = 1$ and $\frac{\delta}{3-2\delta} \in [0, 1)$. To see this notice that $\frac{\delta}{3-2\delta}$ is monotonically increasing in δ and is equal to 1 when $\delta = 1$. ■

We now show that equilibrium proposals are optimal over feasible alternatives in $\bar{\Delta}$.

Lemma 4. $\mu_i[\mathbf{z}|\mathbf{s}] > 0 \Rightarrow \mathbf{z} \in \arg \max\{U_i(\mathbf{x})|\mathbf{x} \in W(s) \cap \bar{\Delta}\}$, for all $\mathbf{z}, \mathbf{s} \in \Delta$.

Proof. All equilibrium proposals take the form of minimal winning coalition proposals: $\mathbf{x}(v, j, d_j(\mathbf{x}))$ when the veto player is proposing and $\mathbf{x}(j, v, d_v(\mathbf{x}))$ when a non-veto player is proposing. Also, whenever $\mu_v[\mathbf{x}(v, 1, d_1)|\mathbf{s}] > 0$ and $\mu_v[\mathbf{x}(v, 2, d_2)|\mathbf{s}] > 0$, we have $d_1 = d_2$ so that $U_v(\mathbf{x}(v, 1, d_1)) = U_v(\mathbf{x}(v, 2, d_2))$. Thus, in view of Lemma 2 it suffices to show that if $\mu_i[\mathbf{x}(i, j, d_j)|\mathbf{s}] = 1$, then $U_i(\mathbf{x}(i, j, d_j)) = U_i(\mathbf{x}(i, h, d_h))$, $h \neq i, j$, i.e. proposer i has no incentive to coalesce with player h instead of j . This is immediate for a non-veto player, since only coalescing with the veto player guarantees the possibility to change the state. To show that - for the veto player - if $\mu_v[\mathbf{x}(v, j, d_j)|\mathbf{s}] = 1$, then $U_v(\mathbf{x}(v, j, d_j)) = U_v(\mathbf{x}(v, h, d_h))$, $j \neq h$, it suffices to show $d_h \geq d_j$ by part (b) of Lemma 1. In Proposition 1 we have $s_1 \geq s_2$, (by part (a) of Lemma 3) $d_1 \geq d_2$, and when $d_1 \neq d_2$, we have $\mu_v[\mathbf{x}(v, 1, d_1)|\mathbf{s}] = 0$ which gives the desired result. ■

We conclude the proof by showing that optimum proposal strategies cannot belong in $\Delta/\bar{\Delta}$. In particular, we show that if an alternative in $\Delta/\bar{\Delta}$ beats the status quo by majority rule, then for any player i we can find another alternative in $\bar{\Delta}$ that is also majority preferred to the status quo and improves i 's utility.

Lemma 5. *Assume $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\bar{\Delta}$; then for any $i = v, 1, 2$ there exists $\mathbf{y} \in W(\mathbf{s}) \cap \bar{\Delta}$ such that $U_i(\mathbf{y}) \geq U_i(\mathbf{s})$.*

Proof. Consider first the veto player, $i = v$. Let $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\bar{\Delta}$. Consider first the case $\mathbf{x} \in A_v^*(\mathbf{s})$. Then, \mathbf{x} is weakly preferred to s by v and at least one i , $i = 1, 2$. Now set $\mathbf{y} = \mathbf{x}(v, j, d_j(\mathbf{x}))$, where $d_j(\mathbf{x})$ is the applicable demand from Proposition 1. We have $U_j(\mathbf{x}(v, j, d_j(\mathbf{x}))) \geq U_j(\mathbf{x})$, by the definition of demand. From part (b) of Lemma 3 have $d_v(\mathbf{x}) + d_j(\mathbf{x}) \leq 1$ and as a result $x_v(v, j, d_j(\mathbf{x})) = 1 - d_j(\mathbf{x}) \geq d_v(\mathbf{x})$; hence, $U_v(\mathbf{x}(v, j, d_j(\mathbf{x}))) \geq U_v(\mathbf{x})$, which follows from the weak monotonicity in part (b) of Lemma 1. Thus, $\mathbf{y} = \mathbf{x}(v, j, d_j(\mathbf{x})) \in W(\mathbf{s})$ (because is supported by v and j), and we have completed the proof for this case. Now consider the case $\mathbf{x} \notin A_v^*(\mathbf{s})$, i.e. $U_v(\mathbf{s}) > U_v(\mathbf{x})$. Part (a) of Lemma 3 ensures that $d_v(\mathbf{s}) + d_j(\mathbf{s}) \leq 1$, hence proposal $\mathbf{y} = \mathbf{x}(v, j, d_j(\mathbf{s}))$ has $x_v(v, j, d_j(\mathbf{s})) = 1 - d_j(\mathbf{s}) \geq d_v(\mathbf{s})$. Then $U_v(\mathbf{y}) \geq U_v(\mathbf{s}) > U_v(\mathbf{x})$, and $\mathbf{y} \in W(\mathbf{s}) \cap \bar{\Delta}$.

Now consider a non veto player, $i = 1, 2$. Let $\mathbf{x} \in W(\mathbf{s}) \cap \Delta / \bar{\Delta}$. Consider first the case $\mathbf{x} \in A_i^*(\mathbf{s})$. Then, \mathbf{x} is weakly preferred to s by v and (at least) i . Now set $\mathbf{y} = \mathbf{x}(i, v, d_v(\mathbf{x}))$, where $d_v(\mathbf{x})$ is the applicable demand from Proposition 1. We have $U_v(\mathbf{x}(i, v, d_v(\mathbf{x}))) \geq U_v(\mathbf{x})$, by the definition of demand. From part (b) of Lemma 3 have $d_v(\mathbf{x}) + d_i(\mathbf{x}) \leq 1$ and as a result $x_i(i, v, d_v(\mathbf{x})) = 1 - d_v(\mathbf{x}) \geq d_i(\mathbf{x})$; hence, $U_i(\mathbf{x}(i, v, d_v(\mathbf{x}))) \geq U_i(\mathbf{x})$, which follows from the weak monotonicity in part (b) of Lemma 1. Thus, $\mathbf{y} = \mathbf{x}(i, v, d_v(\mathbf{x})) \in W(\mathbf{s}) \cap \bar{\Delta}$ (because is supported by v and i), and we have completed the proof for this case. Finally, consider the case $\mathbf{x} \notin A_i^*(\mathbf{s})$, i.e. $U_i(\mathbf{s}) > U_i(\mathbf{x})$. Part (a) of Lemma 3 ensures that $d_v(\mathbf{s}) + d_i(\mathbf{s}) \leq 1$, hence proposal $\mathbf{y} = \mathbf{x}(i, v, d_v(\mathbf{s}))$ has $x_i(i, v, d_v(\mathbf{s})) = 1 - d_v(\mathbf{s}) \geq d_i(\mathbf{s})$. Then $U_i(\mathbf{y}) \geq U_i(\mathbf{s}) > U_i(\mathbf{x})$, and $\mathbf{y} \in W(\mathbf{s}) \cap \bar{\Delta}$, which completes the proof. ■

As a result of Lemmas 4 and 5, equilibrium proposals are optima over the entire range of feasible alternatives. It then follows that proposal strategies in Cases A-D of Proposition 2 satisfy the equilibrium condition (5) which completes the proof. ■

Proof of Proposition 4

The result of Proposition 4 follows once we establish that the proposal strategies in the equilibrium from Proposition 1 are weakly continuous in the status quo \mathbf{s} , i.e. that in equilibrium a small change in the status quo implies a small change in proposal strategies and, by extension, to the equilibrium transition probabilities.

Lemma 6. *The equilibrium proposal strategies μ_i^* in the proof of Proposition 1 are such that for every $\mathbf{s} \in \Delta$ and every sequence $\mathbf{s}_n \in \Delta$ with $\mathbf{s}_n \rightarrow \mathbf{s}$, $\mu_i^*[\cdot | \mathbf{s}_n]$ converges weakly to $\mu_i^*[\cdot | \mathbf{s}]$.*

Proof: The equilibrium is such that $\mu_i^*[\cdot | \mathbf{s}]$ $i = 1, 2$ has mass on only one point $\mathbf{x}(i, v, d_v(\mathbf{s}))$ and that $\mu_v^*[\cdot | \mathbf{s}]$ has mass on at most two points $\mathbf{x}(v, 1, d_1(\mathbf{s}))$, and $\mathbf{x}(v, 2, d_2(\mathbf{s}))$. It suffices to show that these proposals (when played with positive probability) and associated mixing probabilities are continuous in s (see Kalandrakis (2004) and Billingsley (1999)). Continuity holds in the interior of Cases A-D in Proposition 1, so it remains to check the boundaries

of these cases. In order to distinguish the various applicable functional forms we shall write d_i^w and $\mu_v^w[\cdot|\mathbf{s}]$ where $w = \{A, B, C, D\}$ identifies the case for which the respective functional form applies.

- Boundary of Cases A and B: at the boundary (as in the interior of the two cases) we have $\mu_v^A[\mathbf{x}(v, 1, d_2)|\mathbf{s}] = \mu_v^B[\mathbf{x}(v, 1, d_2)|\mathbf{s}] = 0$; at the boundary we have $s_1 = 1 - \frac{3-\delta}{3-2\delta}s_2$, then:

$$\begin{aligned} d_v^A &= d_v^B = 0 \\ d_1^A &= d_1^B = 1 - \frac{9 - 12\delta - 3\delta^2}{(3 - 2\delta)^2} s_2 \\ d_2^A &= d_2^B = \frac{9 - 12\delta - 3\delta^2}{(3 - 2\delta)^2} s_2 + \frac{\delta}{(3 - 2\delta)} \end{aligned}$$

- Boundary of Cases B and C: at the boundary we have $s_1 = \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2} s_2 + \frac{\delta^2}{(3-\delta)^2}$; then:

$$\begin{aligned} \mu_v^B[\mathbf{x}(v, 1, d_2)|\mathbf{s}] &= \mu_v^C[\mathbf{x}(v, 1, d_2)|\mathbf{s}] = 0 \\ d_v^B &= d_v^C = 0 \\ d_1^B &= d_1^C = \frac{9 - 12\delta + 3\delta^2}{(3 - 2\delta)^2} s_2 + \frac{\delta}{(3 - 2\delta)} \\ d_2^B &= d_2^C = \frac{9 - 12\delta - 3\delta^2}{(3 - 2\delta)^2} s_2 + \frac{\delta}{(3 - 2\delta)} \end{aligned}$$

- Boundary of Cases C and D: at the boundary we have $s_1 = 1 - \frac{3-\delta}{3-2\delta}s_2$; then:

$$\begin{aligned} \mu_v^C[\mathbf{x}(v, 1, d_2)|\mathbf{s}] &= \mu_v^D[\mathbf{x}(v, 1, d_2)|\mathbf{s}] = \frac{3(-3 + 2\delta)((-2 + 2s_2)\delta + 3 - 6s_2)}{2\delta((3s_2 - 2)\delta^2 + (-12s_2 + 3)\delta + 9s_2)} \\ d_v^C &= d_v^D = 0 \\ d_1^C &= d_1^D = d_2^C = d_2^D = \frac{9 - 12\delta - 3\delta^2}{(3 - 2\delta)^2} s_2 + \frac{\delta}{(3 - 2\delta)} \end{aligned}$$

- Boundary of Cases D and A: at the boundary we have $s_1 = \frac{3-\delta}{3-2\delta}s_2$; then:

$$\begin{aligned}\mu_v^D[\mathbf{x}(v, 1, d_2)|\mathbf{s}] &= \mu_v^A[\mathbf{x}(v, 1, d_2)|\mathbf{s}] = 0 \\ d_v^D &= d_v^A = s_v - \frac{\delta s_2}{3 - 2\delta} \\ d_1^D &= d_1^A = d_2^D = d_2^A = \frac{(3 - \delta)^2}{(3 - 2\delta)^2} s_2\end{aligned}$$

■

Proof of Proposition 5

We want to show that $\mathbf{x} = (x_v, x_1, x_2)$ is not an absorbing policy, unless $\mathbf{x} = (1, 0, 0)$. It is straightforward to prove that $(1, 0, 0)$ is absorbing. We can show that no other policy $\mathbf{x} \neq (1, 0, 0)$ is absorbing by contradiction. Assume $\mathbf{x} \neq (1, 0, 0)$ is absorbing. Then $EU_v(\mathbf{x})$, the expected utility of the veto player from \mathbf{x} , is $x_v/(1 - \delta)$. First, notice that the veto player would be better off with a different policy \mathbf{y} , such that $y_v > x_v$. This is because, moving to \mathbf{y} the veto player can guarantee to himself an expected utility of at least $y_v/(1 - \delta)$ (by vetoing any future proposed change). To run into a contradiction and show the desired result, we therefore need to show that the veto player, when proposing, can always find a non-veto player who is at least indifferent between the status quo policy \mathbf{x} and a new policy \mathbf{y} such that $y_v > x_v$. Assume that, in \mathbf{x} , $x_2 > x_1 \geq 0$ (this is without loss of generality, because we can switch labels for all other policies different than $(1, 0, 0)$). Consider the policy $\mathbf{x}' = (x_v, x_1 + x_2, 0)$. By the weak monotonicity condition, this policy is strictly preferred to \mathbf{x} by non-veto player 1 (the veto player gets the same amount and he gets more). By continuity of the value functions, there always exists $\epsilon > 0$ such that EU_1 of the policy $\mathbf{x}'' = (x_v + \epsilon, x_1 + x_2 - \epsilon, 0)$ is weakly greater than $EU_1(\mathbf{x})$. This means that the veto player is able to and willing to move to a different policy, when proposing (an event that happens with positive probability), and that, thus, $\neq(1, 0, 0)$ cannot be absorbing. ■

Proof of Proposition 6

As before, we focus on the allocations in which $s_1 \geq s_2$. The other cases are symmetric. Consider the following equilibrium proposal strategies (all supported by a minimal winning coalition) and demands (as defined in the proof of Proposition 1):

- CASE A: $s_1 \geq 1 - \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)}s_2$; $s_1 \geq \frac{2p_v\delta^2+10\delta-8-3\delta^2-2p_v\delta+p_v^2\delta^2}{(2-\delta(1+p_v))(1-p_v)\delta}s_2 + \frac{4-4p_v\delta-4\delta+\delta^2+p_v^2\delta^2+2p_v\delta^2}{(2-\delta(1+p_v))(1-p_v)\delta}$

$$\begin{aligned}\mathbf{x}^v &= [1 - d_2^A, 0, d_2^A], \mathbf{x}^1 = [d_v^A, 1 - d_v^A, 0], \mathbf{x}^2 = [d_v^A, 0, 1 - d_v^A] \\ d_v^A &= s_v - \frac{2p_v\delta}{2 - (1 + p_v)\delta} s_2 \\ d_2^A &= \frac{\delta(1 - p_v)}{2 - \delta(1 + p_v)} s_1 + \frac{2 - \delta(1 - p_v)}{2 - \delta(1 + p_v)} s_2 \\ d_1^A &= \frac{-4p_v\delta + 4 + 2p_v\delta^2 - 4\delta + p_v^2\delta^2 + \delta^2}{(2 - \delta(1 + p_v))^2} s_1 + \frac{-p_v^2\delta^2 - \delta^2 - 2p_v\delta + 2\delta + 2p_v\delta^2}{(2 - \delta(1 + p_v))^2} s_2\end{aligned}$$

- CASE B:

$$s_1 < 1 - \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)}s_2; s_1 \geq \frac{-2\delta^3 p_v^2 + p_v^3 \delta^3 + p_v \delta^3 + \delta^2 + p_v^2 \delta^2 - 2p_v \delta^2 - 4\delta + 4}{(p_v \delta - \delta + 2)(p_v \delta - 1)(-2 + p_v + p_v \delta)} s_2 + \frac{-p_v \delta^3 - 2p_v^2 \delta^2 + p_v^3 \delta^3 + 2p_v \delta^2}{(p_v \delta - \delta + 2)(p_v \delta - 1)(-2 + p_v + p_v \delta)}$$

$$\begin{aligned}\mathbf{x}^v &= [1 - d_2^B, 0, d_2^B], \mathbf{x}^1 = [d_v^B, 1 - d_v^B, 0], \mathbf{x}^2 = [d_v^B, 0, 1 - d_v^B] \\ d_v^B &= 0 \\ d_2^B &= \frac{-2p_v\delta^2 + 2\delta^2 + 2p_v\delta - 6\delta + 4}{(2 - \delta(1 + p_v))^2} s_2 + \frac{p_v^2\delta^2 - \delta^2 - 2p_v\delta + 2\delta}{(2 - \delta(1 + p_v))^2} \\ d_1^B &= \frac{-16\delta + 10\delta^2 + 2p_v\delta^4 + 2p_v^3\delta^3 - 10p_v\delta^3 - 2p_v^2\delta^2}{(2 - \delta(1 + p_v))^3} s_1 + \dots \\ &+ \frac{16p_v\delta^2 + 2\delta^3 p_v^2 + 8 - 2\delta^3 - 2p_v^3\delta^4 - 8p_v\delta}{(2 - \delta(1 + p_v))^3} s_1 + \dots \\ &+ \frac{2p_v\delta^2 - 4p_v^2\delta^2 - 6p_v\delta^3 - 4p_v^2\delta^4 + 4p_v\delta^2 + 8p_v^2\delta^3 - 2p_v^3\delta^3 - 2p_v^3\delta^4}{(2 - \delta(1 + p_v))^3} s_2 + \dots \\ &+ \frac{4\delta - 4p_v\delta - 2p_v\delta^4 - 4\delta^2 - 5\delta^3 p_v^2 + 7p_v\delta^3 - 3p_v^3\delta^3 + \delta^3 + 8p_v^2\delta^2 - 4p_v\delta^2 + 2p_v^3\delta^4}{(2 - \delta(1 + p_v))^3}\end{aligned}$$

- CASE C:

$$s_1 < 1 - \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)}s_2; s_1 < \frac{-2\delta^3 p_v^2 + p_v^3 \delta^3 + p_v \delta^3 + \delta^2 + p_v^2 \delta^2 - 2p_v \delta^2 - 4\delta + 4}{(p_v \delta - \delta + 2)(p_v \delta - 1)(-2 + p_v + p_v \delta)}s_2 + \frac{-p_v \delta^3 - 2p_v^2 \delta^2 + p_v^3 \delta^3 + 2p_v \delta^2}{(p_v \delta - \delta + 2)(p_v \delta - 1)(-2 + p_v + p_v \delta)}$$

$$\mathbf{x}^v = \begin{cases} [1 - d_2^C, d_2^C, 0] & \text{w/ Pr} = \mu_v^C \\ [1 - d_2^C, 0, d_2^C] & \text{w/ Pr} = 1 - \mu_v^C \end{cases}, \mathbf{x}^1 = [d_v^C, 1 - d_v^C, 0], \mathbf{x}^2 = [d_v^C, 0, 1 - d_v^C]$$

$$d_v^C = 0$$

$$d_1^C = d_2^C = \frac{-2p_v \delta^2 + 2\delta^2 + 2p_v \delta - 6\delta + 4}{(2 - \delta(1 + p_v))^2} s_2 + \frac{p_v^2 \delta^2 - \delta^2 - 2p_v \delta + 2\delta}{(2 - \delta(1 + p_v))^2}$$

$$\mu_v^C = \frac{(-3p_v^2 \delta^2 + 3p_v \delta^2 - 3\delta - 3p_v \delta + 6)s_1}{2(-2 + \delta + p_v \delta)\delta d_2^C} + \frac{(3p_v^2 \delta^2 - 3p_v \delta^2 + 3\delta + 3p_v \delta - 6)s_2}{2(-2 + \delta + p_v \delta)\delta d_2^C} + \dots$$

$$+ \frac{\delta^2 d_2^C p_v + \delta^2 d_2^C - 2\delta d_2^C}{2(-2 + \delta + p_v \delta)\delta d_2^C}$$

• CASE D: $s_1 \geq 1 - \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)}s_2; s_1 < \frac{2p_v \delta^2 + 10\delta - 8 - 3\delta^2 - 2p_v \delta + p_v^2 \delta^2}{(2-\delta(1+p_v))(1-p_v)\delta} s_2 + \frac{4-4p_v \delta - 4\delta + \delta^2 + p_v^2 \delta^2 + 2p_v \delta^2}{(2-\delta(1+p_v))(1-p_v)\delta}$

$$\mathbf{x}^v = \begin{cases} [1 - d_2^D, d_2^D, 0] & \text{w/ Pr} = \mu_v^D \\ [1 - d_2^D, 0, d_2^D] & \text{w/ Pr} = 1 - \mu_v^D \end{cases}, \mathbf{x}^1 = [d_v^D, 1 - d_v^D, 0], \mathbf{x}^2 = [d_v^D, 0, 1 - d_v^D]$$

$$d_v^D = s_v - \frac{2p_v \delta}{2 - (1 + p_v)\delta} s_2$$

$$d_1^D = d_2^D = \frac{\delta(1 - p_v)}{2 - \delta(1 + p_v)} s_1 + \frac{2 - \delta(1 - p_v)}{2 - \delta(1 + p_v)} s_2$$

$$\mu_v^D = \frac{(-3p_v^2 \delta^2 + 3p_v \delta^2 - 3\delta - 3p_v \delta + 6)s_1}{2(-2 + \delta + p_v \delta)\delta d_2^D} + \frac{(3p_v^2 \delta^2 - 3p_v \delta^2 + 3\delta + 3p_v \delta - 6)s_2}{2(-2 + \delta + p_v \delta)\delta d_2^D} + \dots$$

$$+ \frac{\delta^2 d_2^D p_v + \delta^2 d_2^D - 2\delta d_2^D}{2(-2 + \delta + p_v \delta)\delta d_2^D}$$

where μ_v^C and μ_v^D are the probabilities that the veto player coalesces with non-veto 1 in cases C, and D respectively. These are well defined probability in $[0,1]$ such that $d_1^C = d_2^C$ and $d_1^D = d_2^D$, or such that $s_1 + \delta v_1(\mathbf{s}, \mu_v, d_2) = s_2 + \delta v_2(\mathbf{s}, \mu_v, d_2)$.

It is tedious but straightforward to show that these equilibrium strategies and the associated value functions are part of a symmetric MPE, using the same strategy employed in the proof of Proposition 1. ■

Proof of Proposition 7

In this case an allocation is $\mathbf{s} = [s_{v1}, s_{v2}, s_1, s_2]$, where s_{vi} , $i = 1, 2$, denote the share to a veto player and s_j , $j = 1, 2$, denote the share to a non-veto player. In the remainder of the proof, we focus on the allocations in which $s_1 \geq s_2$ and $s_{v1} \geq s_{v2}$. The other cases are symmetric. The equilibrium I characterize is similar to the one from Proposition 1 and the steps behind the proof are the same. In particular, we partition the state space into regions where the veto proposer mixes or not between coalition partners and regions where the “demand” of a veto player to the proposal of a non-veto (as defined in the proof of Proposition 1) is bounded at zero. Since there are two veto players we have 6 regions, 3 where the veto proposers do not mix and three where they do (in order to keep the demand of the two non-veto players equal). The three regions with no mixing are characterized by A) $d_{v1} \geq d_{v2} > 0$; B) $d_{v1} > 0$ and $d_{v2} = 0$; and C) $d_{v1} = d_{v2} = 0$. In these regions, a veto proposer coalesces with non-veto player 2 with probability 1. The remaining three regions are analogous with the difference that the veto proposer coalesces with non-veto player 1 with probability $\mu \in [0, 1]$.

Consider the following equilibrium proposal strategies (all supported by a minimal winning coalition) and demands (as defined in the proof of Proposition 1):

- CASE A: $s_{v1} \geq s_{v2} \geq \frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2$; $s_1 \geq \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8}s_2$

$$\mathbf{x}^{v1} = [1 - d_{v2}^A - d_2^A, d_{v2}^A, 0, d_2^A], \mathbf{x}^{v2} = [d_{v1}^A, 1 - d_{v1}^A - d_2^A, 0, d_2^A]$$

$$\mathbf{x}^1 = [d_{v1}^A, d_{v2}^A, 1 - d_{v1}^A - d_{v2}^A, 0], \mathbf{x}^2 = [d_{v1}^A, d_{v2}^A, 0, 1 - d_{v1}^A - d_{v2}^A]$$

$$d_{v1}^A(v) = s_{v1} + \frac{4\delta(1-\delta)}{16-16\delta+3\delta^2}s_1 + \frac{(3\delta^5-9\delta^4+72\delta^3-248\delta^2+320\delta-128)\delta}{(-4+3\delta)(\delta-4)(\delta^2+6\delta-8)(\delta-2)}s_2$$

$$d_{v1}^A(nv) = s_{v1} - \frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2$$

$$d_{v2}^A(v) = s_{v2} + \frac{4\delta(1-\delta)}{16-16\delta+3\delta^2}s_1 + \frac{(3\delta^5-9\delta^4+72\delta^3-248\delta^2+320\delta-128)\delta}{(-4+3\delta)(\delta-4)(\delta^2+6\delta-8)(\delta-2)}s_2$$

$$d_{v2}^A(nv) = s_{v2} - \frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2$$

$$d_2^A = \frac{\delta}{4-3\delta}s_1 + \frac{4\delta^3+48\delta-18\delta^2-32}{3\delta^3+14\delta^2-48\delta+32}s_2$$

$$d_1^A = -\frac{\delta^7-62\delta^5+5\delta^6-72\delta^4+736\delta^3+3072\delta-3264\delta^2-1024}{(32-56\delta+32\delta^2-7\delta^3+\delta^4)(-4+3\delta)(\delta^2+6\delta-8)}s_1 + \dots$$

$$-\frac{4\delta^7-752\delta^4-58\delta^6+330\delta^5-256\delta^2+736\delta^3}{(32-56\delta+32\delta^2-7\delta^3+\delta^4)(-4+3\delta)(\delta^2+6\delta-8)}s_2$$

- CASE B: $s_{v1} \geq \frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)} s_2 \geq s_{v2}; s_1 \geq \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8} s_2$

$$\mathbf{x}^{v1} = [1 - d_{v2}^B - d_2^B, d_{v2}^B, 0, d_2^B], \mathbf{x}^{v2} = [d_{v1}^B, 1 - d_{v1}^B - d_2^B, 0, d_2^B]$$

$$\mathbf{x}^1 = [d_{v1}^B, d_{v2}^B, 1 - d_{v1}^B - d_{v2}^B, 0], \mathbf{x}^2 = [d_{v1}^B, d_{v2}^B, 0, 1 - d_{v1}^B - d_{v2}^B]$$

$$\begin{aligned} d_{v1}^B(v) &= s_{v1} + \frac{\delta(18\delta^6 - 428\delta^4 + 816\delta^3 + 384\delta^2 - 1792\delta + 1024)}{4(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} s_1 + \dots \\ &+ \frac{\delta(21\delta^6 - 80\delta^5 - 108\delta^4 + 1296\delta^3 - 3136\delta^2 + 3072\delta - 1024)}{4(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} s_2 + \dots \\ &+ \frac{\delta(18\delta^6 - 284\delta^4 + 864\delta^3 - 1088\delta^2 + 512\delta)}{4(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} s_{v1} + \dots \\ &+ \frac{\delta(284\delta^4 + 1088\delta^2 - 512\delta - 18\delta^6 - 864\delta^3)}{4(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} \end{aligned}$$

$$\begin{aligned} d_{v1}^B(nv) &= s_{v1} - \frac{\delta(-22\delta^2 - 32 + 48\delta + 3\delta^3)}{(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} s_1 - \frac{(-8\delta + 2\delta^2 + 2\delta^3)\delta}{(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} s_2 + \dots \\ &- \frac{(-22\delta^2 - 32 + 48\delta + 3\delta^3)\delta}{(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} s_{v1} - \frac{(32 - 48\delta - 3\delta^3 + 22\delta^2)\delta}{(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} \end{aligned}$$

$$\begin{aligned} d_{v2}^B(v) &= s_{v2} + \frac{\delta(18\delta^6 - 428\delta^4 + 816\delta^3 + 384\delta^2 - 1792\delta + 1024)}{4(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} s_1 + \dots \\ &+ \frac{\delta(21\delta^6 - 80\delta^5 - 108\delta^4 + 1296\delta^3 - 3136\delta^2 + 3072\delta - 1024)}{4(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} s_2 + \dots \\ &+ \frac{\delta(18\delta^6 - 284\delta^4 + 864\delta^3 - 1088\delta^2 + 512\delta)}{4(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} s_{v2} + \dots \\ &+ \frac{\delta(284\delta^4 + 1088\delta^2 - 512\delta - 18\delta^6 - 864\delta^3)}{4(-4 + 3\delta)^2(\delta + 4)(\delta - 2)} \end{aligned}$$

$$d_{v2}^B(nv) = 0$$

$$\begin{aligned} d_2^B &= -\frac{16\delta^2 - 5\delta^4 + 6\delta^5 - 16\delta^3}{(-4 + 3\delta)^2(\delta + 4)} s_1 - \frac{-576\delta + 400\delta^2 + 256 - 25\delta^4 + 7\delta^5 - 64\delta^3}{(-4 + 3\delta)^2(\delta + 4)} s_s + \dots \\ &- \frac{-14\delta^4 + 6\delta^5 - 28\delta^3 + 96\delta^2 - 64\delta}{(-4 + 3\delta)^2(\delta + 4)} s_{v1} - \frac{-6\delta^5 + 64\delta + 14\delta^4 - 96\delta^2 + 28\delta^3}{(-4 + 3\delta)^2(\delta + 4)} \end{aligned}$$

$$\begin{aligned} d_1^B &= -\frac{44544\delta^2 + 8320\delta^4 + 784\delta^5 + 8192 - 30720\delta}{(32 - 56\delta + 32\delta^2 - 7\delta^3 + \delta^4)(\delta + 4)(-4 + 3\delta)^3} s_1 + \dots \\ &- \frac{-30336\delta^3 - 47\delta^8 + 6\delta^9 - 1016\delta^6 + 265\delta^7}{(32 - 56\delta + 32\delta^2 - 7\delta^3 + \delta^4)(\delta + 4)(-4 + 3\delta)^3} s_1 + \dots \\ &- \frac{-2048\delta + 8192\delta^2 + 10624\delta^4 - 4160\delta^5 - 13184\delta^3 - 74\delta^8 + 7\delta^9 + 412\delta^6 + 227\delta^7}{(32 - 56\delta + 32\delta^2 - 7\delta^3 + \delta^4)(\delta + 4)(-4 + 3\delta)^3} s_2 + \dots \\ &- \frac{-492\delta^6 + 4640\delta^4 - 656\delta^5 - 8320\delta^3 + 6656\delta^2 - 2048\delta - 56\delta^8 + 6\delta^9 + 262\delta^7}{(32 - 56\delta + 32\delta^2 - 7\delta^3 + \delta^4)(\delta + 4)(-4 + 3\delta)^3} s_{v1} + \dots \\ &- \frac{-6\delta^9 + 2048\delta + 8320\delta^3 + 492\delta^6 - 6656\delta^2 - 4640\delta^4 - 262\delta^7 + 656\delta^5 + 56\delta^8}{(32 - 56\delta + 32\delta^2 - 7\delta^3 + \delta^4)(\delta + 4)(-4 + 3\delta)^3} \end{aligned}$$

- CASE C: $\frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2 \geq s_{v1} \geq s_{v2}; s_1 \geq \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8}s_2$

$$\mathbf{x}^{v1} = [1 - d_{v2}^C - d_2^C, d_{v2}^C, 0, d_2^C], \mathbf{x}^{v2} = [d_{v1}^C, 1 - d_{v1}^C - d_2^C, 0, d_2^C]$$

$$\mathbf{x}^1 = [d_{v1}^C, d_{v2}^C, 1 - d_{v1}^C - d_{v2}^C, 0], \mathbf{x}^2 = [d_{v1}^C, d_{v2}^C, 0, 1 - d_{v1}^C - d_{v2}^C]$$

$$d_{v1}^C(v) = s_{v1} \left(1 + \frac{\delta(-36\delta^3 + 240\delta^2 - 448\delta + 256)}{8(-4 + 3\delta)^2(\delta - 4)} \right) + \frac{\delta(9\delta^4 - 54\delta^3 + 8\delta^2 + 160\delta - 128)}{8(-4 + 3\delta)^2(\delta - 4)}s_1 + \dots$$

$$\frac{\delta(12\delta^4 - 134\delta^3 + 368\delta^2 - 384\delta + 128)}{8(-4 + 3\delta)^2(\delta - 4)}s_2 + \frac{\delta(-9\delta^4 - 104\delta^2 + 54\delta^3 + 64\delta)}{8(-4 + 3\delta)^2(\delta - 4)}$$

$$d_{v1}^C(nv) = 0$$

$$d_{v2}^C(v) = s_{v2} \left(1 + \frac{\delta(-36\delta^3 + 240\delta^2 - 448\delta + 256)}{8(-4 + 3\delta)^2(\delta - 4)} \right) + \frac{\delta(9\delta^4 - 54\delta^3 + 8\delta^2 + 160\delta - 128)}{8(-4 + 3\delta)^2(\delta - 4)}s_1 + \dots$$

$$\frac{\delta(12\delta^4 - 134\delta^3 + 368\delta^2 - 384\delta + 128)}{8(-4 + 3\delta)^2(\delta - 4)}s_2 + \frac{\delta(-9\delta^4 - 104\delta^2 + 54\delta^3 + 64\delta)}{8(-4 + 3\delta)^2(\delta - 4)}$$

$$d_{v2}^C(nv) = 0$$

$$d_2^C = -\frac{3\delta^3 - 4\delta^2}{2(-4 + 3\delta)^2}s_1 - \frac{4\delta^3 - 28\delta^2 + 56\delta - 32}{2(-4 + 3\delta)^2}s_2 - \frac{-3\delta^3 + 10\delta^2 - 8\delta}{2(-4 + 3\delta)^2}$$

$$d_1^C = -\frac{3\delta^7 - 1024 - 4160\delta^2 + 2544\delta^3 - 824\delta^4 + 3328\delta - 25\delta^6 + 160\delta^5}{2(-4 + 3\delta)^2(32 - 56\delta + 32\delta^2 - 7\delta^3 + \delta^4)}s_1 + \dots$$

$$-\frac{4\delta^7 - 56\delta^6 + 308\delta^5 + 256\delta - 896\delta^2 + 1232\delta^3 - 848\delta^4}{2(-4 + 3\delta)(32 - 56\delta + 32\delta^2 - 7\delta^3 + \delta^4)^2}s_2 + \dots$$

$$-\frac{-256\delta + 768\delta^2 + 544\delta^4 - 174\delta^5 + 31\delta^6 - 3\delta^7 - 912\delta^3}{2(-4 + 3\delta)^2(32 - 56\delta + 32\delta^2 - 7\delta^3 + \delta^4)}$$

- CASE D: $s_{v1} \geq s_{v2} \geq \frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2; s_1 < \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8}s_2$

$$\mathbf{x}^{v1} = \begin{cases} [1 - d_2^D - d_{v2}^D, d_{v2}^D, d_2^D, 0] & \text{w/ Pr} = \mu_v^D \\ [1 - d_2^D - d_{v2}^D, d_{v2}^D, 0, d_2^D] & \text{w/ Pr} = 1 - \mu_v^D \end{cases}$$

$$\mathbf{x}^{v2} = \begin{cases} [d_{v1}^D, 1 - d_2^D - d_{v1}^D, d_2^D, 0] & \text{w/ Pr} = \mu_v^D \\ [d_{v1}^D, 1 - d_2^D - d_{v1}^D, 0, d_2^D] & \text{w/ Pr} = 1 - \mu_v^D \end{cases}$$

$$\mathbf{x}^1 = [d_{v1}^D, d_{v2}^D, 1 - d_{v1}^D - d_{v2}^D, 0], \mathbf{x}^2 = [d_{v1}^D, d_{v2}^D, 0, 1 - d_{v1}^D - d_{v2}^D]$$

$$d_{v1}^D = d_{v1}^A$$

$$d_{v2}^D = d_{v2}^A$$

$$d_1^D = d_2^D = d_2^A$$

- CASE E: $s_{v1} \geq \frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2 \geq s_{v2}; s_1 < \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8}s_2$

$$\begin{aligned}
\mathbf{x}^{v1} &= \begin{cases} [1 - d_2^C - d_{v2}^C, d_{v2}^C, d_2^C, 0] & \text{w/ Pr} = \mu_v^E \\ [1 - d_2^C - d_{v2}^C, d_{v2}^C, 0, d_2^C] & \text{w/ Pr} = 1 - \mu_v^E \end{cases} \\
\mathbf{x}^{v2} &= \begin{cases} [d_{v1}^C, 1 - d_2^C - d_{v1}^C, d_2^C, 0] & \text{w/ Pr} = \mu_v^E \\ [d_{v1}^C, 1 - d_2^C - d_{v1}^C, 0, d_2^C] & \text{w/ Pr} = 1 - \mu_v^E \end{cases} \\
\mathbf{x}^1 &= [d_{v1}^A, d_{v2}^A, 1 - d_{v1}^A - d_{v2}^A, 0], \mathbf{x}^2 = [d_{v1}^A, d_{v2}^A, 0, 1 - d_{v1}^A - d_{v2}^A] \\
d_{v1}^E &= d_{v1}^B \\
d_{v2}^E &= d_{v2}^B \\
d_1^C &= d_2^C = d_2^B
\end{aligned}$$

- CASE F: $\frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2 \geq s_{v1} \geq s_{v2}; s_1 < \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8}s_2$

$$\begin{aligned}
\mathbf{x}^{v1} &= \begin{cases} [1 - d_2^F - d_{v2}^F, d_{v2}^F, d_2^F, 0] & \text{w/ Pr} = \mu_v^F \\ [1 - d_2^F - d_{v2}^F, d_{v2}^F, 0, d_2^F] & \text{w/ Pr} = 1 - \mu_v^F \end{cases} \\
\mathbf{x}^{v2} &= \begin{cases} [d_{v1}^F, 1 - d_2^F - d_{v1}^F, d_2^F, 0] & \text{w/ Pr} = \mu_v^F \\ [d_{v1}^F, 1 - d_2^F - d_{v1}^F, 0, d_2^F] & \text{w/ Pr} = 1 - \mu_v^F \end{cases} \\
\mathbf{x}^1 &= [d_{v1}^F, d_{v2}^F, 1 - d_{v1}^F - d_{v2}^F, 0] \\
\mathbf{x}^2 &= [d_{v1}^F, d_{v2}^F, 0, 1 - d_{v1}^F - d_{v2}^F] \\
d_{v1}^F &= d_{v1}^C \\
d_{v2}^F &= d_{v2}^C \\
d_1^F &= d_2^F = d_2^C
\end{aligned}$$

where μ_v^J is the probability that a veto proposer coalesces with non-veto 1 in case J , $d_{vi}^J(v)$ is the demand of veto player i when the proposer is the other veto in case J , and $d_{vi}^J(nv)$ is the demand of veto player i when the proposer is a non-veto in case J . Notice

that μ_v^J are well defined probability in $[0,1]$ such that $d_1^i = d_2^i$, $i = D, E, F$, or such that $s_1 + \delta v_1(\mathbf{s}, \mu_v, d_2) = s_2 + \delta v_2(\mathbf{s}, \mu_v, d_2)$. It is tedious but straightforward to show that these equilibrium strategies and the associated value functions are part of a symmetric MPE, using the same strategy employed in the proof of Proposition 1. In particular, the crucial steps will be 1) showing that in the absorbing set where one non-veto player receives zero, the expected utility of all agents are weakly increasing in their current allocation (the main passage in proving that the proposed proposals are optimal among all minimal winning coalition proposal), and 2) showing that the sum of the demands of a minimal winning coalition is always weakly smaller than 1 (meaning that there always exists a minimal winning coalition proposal that makes the proposer at least as well off as he is in a status quo where everyone has a positive share). ■

Proof of Proposition 8

By assumption, we are restricting the set of possible legislative outcomes to allocations on the edges of the simplex, i.e. to $\mathbf{s} \in \Delta_2$. I focus on allocations where $s_1 \geq s_2 \geq s_3 = 0$ (the other cases being symmetric). Since the endowment is 1 and $s_3 = 0$, we can reduce the problem to one dimension replacing $s_2 = 1 - s_1$ and focusing on allocations where $s_1 \geq 1/2$. Consider the proposal and voting strategies that would be part of an equilibrium with perfectly impatient agents: each agent, when proposing, tries to maximize his current allocation (i.e. he proposes the “acceptable” allocation, $\mathbf{x} \in W(\mathbf{s})$, that give him the greatest share) and each agent votes yes to any proposal that gives him as much as he gets in the status quo. I want to show that these strategies and the associated value functions are part of an equilibrium even when agents are patient. First of all, consider the allocations in the absorbing set $\mathbf{s} \in \Delta_1$ where one agent gets the whole dollar. Denote with \bar{V}_0 the continuation value from an allocation $\mathbf{s} \in \Delta_1$ where the agent gets nothing, and \bar{V}_1 the continuation value from an allocation $\mathbf{s} \in \Delta_1$ where the agent gets the whole dollar. We can derive \bar{V}_0 and \bar{V}_1 ,

using the transition probabilities discussed in Section 5.3:

$$\begin{aligned}
\bar{V}_1 &= \frac{5}{9}(1 + \delta\bar{V}_1) + \frac{4}{9}(0 + \delta\bar{V}_0) \\
\bar{V}_0 &= \frac{7}{9}(0 + \delta\bar{V}_0) + \frac{2}{9}(1 + \delta\bar{V}_1) \\
\implies \bar{V}_1 &= \frac{5 - 3\delta}{3(3 - 4\delta + \delta^2)} \\
\implies \bar{V}_0 &= \frac{2}{3(3 - 4\delta + \delta^2)}
\end{aligned}$$

Using these continuation values, the probability of being selected as veto and as proposer, and the conjectured strategies discussed in Section 5.3, we can derive the continuation values for any allocation $\mathbf{s} \in \Delta_2$. It is tedious but straightforward to verify that the value functions are as follows:

- CASE A: $s_1 \geq \frac{134\delta^4 + 7371\delta^2 - 1665\delta^3 + 6561 - 12393\delta}{27(\delta^2 + 3 - 4\delta)(8\delta^2 - 99\delta + 162)}$

$$v_1^A(\mathbf{s}) = -\frac{-4374\delta + 2790\delta^2 + 39\delta^4 + 2187 - 642\delta^3}{3(\delta - 1)(\delta - 3)(13\delta^3 - 207\delta^2 + 729\delta - 729)}s_1 + \dots$$

$$-\frac{1458 - 100\delta^3 - 1620\delta + 630\delta^2}{3(\delta - 1)(\delta - 3)(13\delta^3 - 207\delta^2 + 729\delta - 729)}$$

$$v_2^A(\mathbf{s}) = \frac{39\delta^4 - 696\delta^3 - 5508\delta + 2916 + 3249\delta^2}{3(\delta - 1)(\delta - 3)(13\delta^3 - 207\delta^2 + 729\delta - 729)}s_1 + \dots$$

$$-\frac{-39\delta^4 - 4374 + 7047\delta + 743\delta^3 - 3753\delta^2}{3(\delta - 1)(\delta - 3)(13\delta^3 - 207\delta^2 + 729\delta - 729)}$$

$$v_3^A(\mathbf{s}) = \frac{-108\delta^2 + 81\delta + 27\delta^3}{3(\delta - 1)(\delta - 3)(13\delta^3 - 207\delta^2 + 729\delta - 729)}s_1 + \dots$$

$$+\frac{-85\delta^3 + 243\delta + 243\delta^2 - 729}{3(\delta - 1)(\delta - 3)(13\delta^3 - 207\delta^2 + 729\delta - 729)}$$

$$d_1^A = \frac{35\delta^4 - 1998\delta^3 + 8424\delta^2 - 13122\delta + 6561}{81(\delta^2 + 3 - 4\delta)(\delta^2 - 15\delta + 27)}s_1 + \frac{-53\delta^4 + 126\delta^3 - 81\delta^2}{81(\delta^2 + 3 - 4\delta)(\delta^2 - 15\delta + 27)}$$

$$d_2^A = s_2$$

$$d_3^A = 0$$

- CASE B: $s_1 < \frac{134\delta^4 + 7371\delta^2 - 1665\delta^3 + 6561 - 12393\delta}{27(\delta^2 + 3 - 4\delta)(8\delta^2 - 99\delta + 162)}$

$$\begin{aligned}
v_1^B(\mathbf{s}) &= -\frac{24\delta^4 - 393\delta^3 + 1746\delta^2 - 2835\delta + 1458}{3(\delta - 1)(\delta - 3)\delta(8\delta^2 - 99\delta + 162)}s_1 - \frac{-10\delta^3 - 207\delta^2 - 729 + 810\delta}{3(\delta - 1)(\delta - 3)\delta(8\delta^2 - 99\delta + 162)} \\
v_2^B(\mathbf{s}) &= \frac{24\delta^4 - 393\delta^3 + 1746\delta^2 - 2835\delta + 1458}{3(\delta - 1)(\delta - 3)\delta(8\delta^2 - 99\delta + 162)}s_1 + \frac{-24\delta^4 + 403\delta^3 - 1539\delta^2 + 2025\delta - 729}{3(\delta - 1)(\delta - 3)\delta(8\delta^2 - 99\delta + 162)} \\
v_3^B(\mathbf{s}) &= -\frac{2(7\delta^2 - 27)}{(\delta - 1)(\delta - 3)\delta(8\delta^2 - 99\delta + 162)} \\
\mu_3^B &= -\frac{216\delta^4 - 3537\delta^3 + 15714\delta^2 - 25515\delta + 13122}{2(52\delta^3 + 207\delta^2 - 972\delta + 729)\delta}s_1 + \dots \\
&\quad - \frac{-134\delta^4 + 1665\delta^3 - 7371\delta^2 + 12393\delta - 6561}{2(52\delta^3 + 207\delta^2 - 972\delta + 729)\delta} \\
d_1^B &= d_2^B = s_2 \\
d_3^B &= 0
\end{aligned}$$

where μ_3^B is the probability that legislator 3 chooses legislator 1 as coalition partner when he is both the proposer and the veto player. The difference between Case A and Case B lies in whether the legislator who receives zero in the status quo mixes between coalition partners or not (when he is both the proposer and the veto player). As discussed in Section 5.3, when the other two legislators have similar allocations, coalescing always with the “poorer” one would not constitute an equilibrium because, for some states, the “richer” legislator would be “cheaper”. When legislator 3 uses pure strategies and always coalesces with legislator 2, legislator 1 demands more than legislator 2 as long as $s_2 \leq d_1 + \delta v_2(1 - d_1)$ (or $s_1 + \delta v_1 = s_2 + \delta v_2$). This gives us the boundary between the two cases, $s_1 \geq \frac{134\delta^4 + 7371\delta^2 - 1665\delta^3 + 6561 - 12393\delta}{27(\delta^2 + 3 - 4\delta)(8\delta^2 - 99\delta + 162)}$.

Lemma 7. Consider a symmetric Markov Perfect strategy profile with expected utility $U_i(\mathbf{s})$, $\mathbf{s} \in \Delta_2$, determined by the continuation values above. Then, for all $\mathbf{x} = (x, 1 - x, 0) \in \Delta_2$, $U_1(\mathbf{x})$ does not decrease with x , while $U_2(\mathbf{x})$ does not increase with x .

Proof. Denote $\hat{x} = \frac{134\delta^4 + 7371\delta^2 - 1665\delta^3 + 6561 - 12393\delta}{27(\delta^2 + 3 - 4\delta)(8\delta^2 - 99\delta + 162)}$. Then we have:

$$U_1(x, 1 - x, 0) = \begin{cases} 1 + \delta \frac{5-3\delta}{3(3-4\delta+\delta^2)} & \text{if } x = 1 \\ x + \delta v_1^A(s_1 = x) & \text{if } x \in (\hat{x}, 1) \\ x + \delta v_1^B(s_1 = x) & \text{if } x \in (1/2, \hat{x}) \\ x + \delta v_2^B(s_1 = 1 - x) & \text{if } x \in (1 - \hat{x}, 1/2) \\ x + \delta v_2^A(s_1 = 1 - x) & \text{if } x \in (0, 1 - \hat{x}) \\ \delta \frac{2}{3(3-4\delta+\delta^2)} & \text{if } x = 0 \end{cases}$$

Notice that we have $\frac{\partial U_1(\mathbf{x})}{x} > 0$ for all pieces of the function and for any $\delta \in [0, 1)$.

Symmetry completes the proof for $U_2(\mathbf{x})$. ■

The optimality of the conjectured proposal and voting strategies for states $\mathbf{s} \in \Delta_2$ follows from the monotonicity established in Lemma 6. ■

Non-Markov Equilibria

I propose strategy profiles such that the initial allocation can be supported as the outcome of a Subgame Perfect Nash Equilibrium (SPNE) and, thus, there is no convergence to full expropriation by the veto player. This SPNE exists as long as the discount factor is high enough and the two non-veto players receive enough. In particular, I want to prove that:

Proposition 9. *For any $\mathbf{s} \in \Delta$ such that $\min_{j=1,2} s_j \geq 1/4$, there is a $\bar{\delta}(\mathbf{s})$ such that for $\delta > \bar{\delta}(\mathbf{s})$ the initial division of the dollar can be supported as the outcome of a Subgame Perfect Nash Equilibrium of the game.*

The idea behind the proof is the following: if a non-veto player accepts a proposal that expropriates the other non-veto player, we switch to a punishment phase in which we reverse to the MPE characterized above. The discount factor needed to support this outcome depends on the share granted to the two non-veto legislators at the beginning of the game:

the lower the allocation an agent receives in the initial status quo, the more profitable a deviation.

Proof. To support the initial allocation \mathbf{s}^0 as the outcome of a Subgame Perfect Nash Equilibrium, employ the following strategy configuration:

1. whenever a member is recognized, he proposes the status quo allocation \mathbf{s}^0 and everyone supports it;
2. if a proposer deviates by proposing $\mathbf{z} \neq \mathbf{s}^0$, every non-veto player j votes against the proposal;
3. if a non-veto player j deviates by voting contrary to the strategies above, from the following period on we reverse to the MPE equilibrium proposal and voting strategies characterized in Section 3.

The strategies for the punishment phase are clearly a SPNE as shown in the proof of Proposition 1 (MPE being one of the many SPNEs of this game). We need to show that, under certain conditions on \mathbf{s}^0 and δ , the non-veto players have no profitable deviation from the equilibrium strategy on the equilibrium path. The payoff to a non-veto player if she follows the equilibrium strategy is:

$$V_{EQ}^j(\mathbf{s}) = \frac{s_j}{1 - \delta}$$

The payoff to deviating and proposing or voting in favor an allocation $\mathbf{z} \neq \mathbf{s}^0$ is given by:

$$V_{DEV}^j(\mathbf{x}) = x_j + \delta v_{MPE}^j(\mathbf{x})$$

where $v_{MPE}^j(\mathbf{x})$ is the value function from the MPE characterized in the proof of Proposition 1. The most profitable deviation when proposing is a proposal that assigns the whole dollar to oneself (if this is in the acceptance set of the veto player). Similarly, the most profitable

deviation when voting is to accept a veto player's proposal that assigns the whole dollar to oneself. In both cases the expected utility from the deviation is as follows (assuming the deviator is agent 2):

$$V_{DEV}^j(0, 1, 0) = 1 + \delta \frac{3 - 3\delta + \delta^2}{(3 - \delta)^2(1 - \delta)}$$

When is the payoff from the equilibrium strategies higher than the payoff from the most profitable deviation?

$$\begin{aligned} \frac{s_j}{1 - \delta} &\geq 1 + \delta \frac{3 - 3\delta + \delta^2}{(3 - \delta)^2(1 - \delta)} \\ s_j &\geq \frac{(3 - 2\delta)^2}{(3 - \delta)^2} \end{aligned}$$

Since this condition has to hold for both non-veto players, we conclude that an equilibrium where the initial status quo is never changed can be supported by a SPNE if the following condition holds:

$$\min_{i=1,2} s_i^0 \geq \frac{(3 - 2\delta)^2}{(3 - \delta)^2}$$

The right-hand side is a linear and decreasing function of δ , and it is equal to 1 when $\delta = 0$ and to 1/4 when $\delta = 1$. This means that there exists a discount factor for which the proposed strategies can support the initial status quo allocation forever, only as long both non-veto player have at least 1/4 of the dollar each at the beginning of the game. ■

References

- Bailey, S.D. 1969. *Voting in the Security Council*. Indiana University Press.
- Banks, J.S. and J. Duggan. 2000. “A Bargaining Model of Collective Choice.” *The American Political Science Review* 94(1):73–88.
- Banks, J.S. and J. Duggan. 2006. “A General Bargaining Model of Legislative Policy-Making.” *Quarterly Journal of Political Science* 1(1):49–85.
- Baron, D.P. 1996. “A Dynamic Theory of Collective Goods Programs.” *The American Political Science Review* 90(2):316–330.
- Baron, D.P., D. Diermeier and P. Fong. 2011. “A Dynamic Theory of Parliamentary Democracy.” *Economic Theory* In press.
- Baron, D.P. and J.A. Ferejohn. 1989. “Bargaining in Legislatures.” *The American Political Science Review* 83(4):1181–1206.
- Baron, D.P. and M.C. Herron. 2003. “A Dynamic Model of Multidimensional Collective Choice.” *Computational Models in Political Economy* pp. 13–47.
- Barseghyan, L., M. Battaglini and S. Coate. 2010. “Fiscal Policy Over the Real Business Cycle: A Positive Theory.” mimeo.
- Battaglini, M. and S. Coate. 2007. “Inefficiency in Legislative Policymaking: A Dynamic Analysis.” *The American Economic Review* 97(1):118–149.
- Battaglini, M. and S. Coate. 2008. “A Dynamic Theory of Public Spending, Taxation and Debt.” *American Economic Review* 98(1):201–36.
- Battaglini, M., S. Nunnari and T.R. Palfrey. 2011. “Legislative Bargaining and the Dynamics of Public Investment.” mimeo.

- Battaglini, M. and T.R. Palfrey. 2011. “The Dynamics of Distributive Politics.” *Economic Theory* In press.
- Bernheim, B.D., A. Rangel and L. Rayo. 2006. “The Power of the Last Word in Legislative Policy Making.” *Econometrica* 74(5):1161–1190.
- Billingsley, P. 1999. *Convergence of Probability Measures*. Wiley-Interscience.
- Blum, Y.Z. 2005. “Proposals for UN Security Council Reform.” *The American Journal of International Law* 99(3):632–649.
- Bourantonis, D. 2005. *The History and Politics of UN Security Council Reform*. Routledge.
- Bowen, T.R. and Z. Zahran. 2009. “On Dynamic Compromise.” mimeo.
- Cameron, C.M. 2000. *Veto Bargaining: Presidents and the Politics of Negative Power*. Cambridge University Press.
- Cho, S.J. 2005. “A Dynamic Model of Parliamentary Democracy.” mimeo.
- Diermeier, D. and R.B. Myerson. 1999. “Bicameralism and Its Consequences for the Internal Organization of Legislatures.” *The American Economic Review* 89(5):1182–1196.
- Diermeier, Daniel and Pohan Fong. 2011. “Legislative Bargaining with Reconsideration.” *The Quarterly Journal of Economics* 126(2):947–985.
- Duggan, J. 2011. “Coalitional Bargaining Equilibria.” Wallis Working Papers WP62, University of Rochester - Wallis Institute of Political Economy.
- Duggan, J. and T. Kalandrakis. 2010. “Dynamic Legislative Policy Making.” mimeo.
- Duggan, J., T. Kalandrakis and V. Manjunath. 2008. “Dynamics of the Presidential Veto: A Computational Analysis.” *Mathematical and Computer Modelling* 48(9-10):1570–1589.

- Dziuda, W. and A. Loeper. 2010. "Ongoing Negotiation with Endogenous Status Quo." mimeo.
- Epple, D. and M.H. Riordan. 1987. "Cooperation and Punishment Under Repeated Majority Voting." *Public Choice* 55(1):41–73.
- Fassbender, B. 1998. *UN Security Council Reform and the Right of Veto: a Constitutional Perspective*. Vol. 32 Martinus Nijhoff Publishers.
- Gailmard, S. and J.W. Patty. 2012. *Learning While Governing: Information, Accountability, and Executive Branch Institutions*. The University of Chicago Press.
- Gomes, A. and P. Jehiel. 2005. "Dynamic Processes of Social and Economic Interactions: On the Persistence of Inefficiencies." *Journal of Political Economy* 113(3):626–667.
- Groseclose, T. and N. McCarty. 2001. "The politics of blame: Bargaining before an audience." *American Journal of Political Science* pp. 100–119.
- Ingberman, D.E. 1985. "Running Against the Status Quo: Institutions for Direct Democracy Referenda and Allocations Over Time." *Public Choice* 46(1):19–43.
- Kalandrakis, A. 2004. "A Three-Player Dynamic Majoritarian Bargaining Game." *Journal of Economic Theory* 116(2):294–322.
- Kalandrakis, T. 2009. "Minimum Winning Coalitions and Endogenous Status Quo." *International Journal of Game Theory* pp. 1–27.
- Krehbiel, K. 1998. *Pivotal Politics: A Theory of US Lawmaking*. University of Chicago Press.
- Lagunoff, R. 2008. "Markov Equilibrium in Models of Dynamic Endogenous Political Institutions." mimeo.

- Lagunoff, R. 2009. "Dynamic Stability and Reform of Political Institutions." *Games and Economic Behavior* 67(2):569–583.
- Leech, D. and R. Leech. 2004. "Voting Power in the Bretton Woods Institutions." *Warwick Economic Working Paper No. 718* .
- Maskin, E. and J. Tirole. 2001. "Markov Perfect Equilibrium:: I. Observable Actions." *Journal of Economic Theory* 100(2):191–219.
- Matthews, S.A. 1989. "Veto Threats: Rhetoric in a Bargaining Game." *The Quarterly Journal of Economics* 104(2):347.
- McCarty, N. M. 2000a. "Proposal Rights, Veto Rights, and Political Bargaining." *American Journal of Political Science* 44(3):506–522.
- McCarty, N.M. 2000b. "Presidential Pork: Executive Veto Power and Distributive Politics." *The American Political Science Review* 94(1):117–129.
- Penn, E.M. 2009. "A Model of Farsighted Voting." *American Journal of Political Science* 53(1):36–54.
- Pfiffner, J.P. 1979. *The President, the Budget, and Congress: Impoundment and the 1974 Budget Act*. Westview Press.
- Primo, D.M. 2006. "Stop Us Before We Spend Again: Institutional Constraints on Government Spending." *Economics & Politics* 18(3):269–312.
- Rapkin, D.P. and J.R. Strand. 2006. "Reforming the IMF's Weighted Voting System." *The World Economy* 29(3):305–324.
- Riker, W.H. 1962. *The Theory of Political Coalitions*. Yale University Press.
- Russell, R.B. 1958. *A History of the United Nations Charter: the Role of the United States, 1940-1945*. Brookings Institution.

- Tsebelis, G. 2002. *Veto Players: How Political Institutions Work*. Princeton University Press.
- Weiss, T.G. 2003. "The illusion of UN Security Council reform." *Washington Quarterly* 26(4):147–161.
- Wildavsky, A.B. 1986. *Budgeting: A Comparative Theory of Budgetary Processes*. Transaction Publishers.
- Wildavsky, A.B. and N. Caiden. 1988. *The New Politics of the Budgetary Process*. Scott, Foresman Boston.
- Winter, E. 1996. "Voting and Vetoing." *The American Political Science Review* 90(4):813–823.
- Woods, N. 2000. "The Challenge of Good Governance for the IMF and the World Bank Themselves." *World Development* 28(5):823–841.