

# RIP to HIP: The Data Reject Heterogeneous Labor Income Profiles \*

Dmytro Hryshko<sup>†</sup>  
University of Alberta

## Abstract

Idiosyncratic labor incomes are typically modeled either by stochastic processes featuring heterogeneous income profiles (HIP) or restricted income profiles (RIP). The HIP assumes that individual labor income grows deterministically at an unobserved rate and contains a persistent but stationary component, while the RIP assumes that income contains a random walk, a stationary component, and no unobserved deterministic growth-rate component. I show that if idiosyncratic labor income contains a persistent component, a deterministic household-specific trend, and a random walk component, then all of the components can be identified. Using data on idiosyncratic labor income growth from the Panel Study of Income Dynamics, I find that the estimated variance of deterministic income growth is zero, i.e., the HIP model can be rejected. The RIP model with a permanent component cannot be rejected. This result is important for an appropriate choice of modeling the heterogeneity in individual incomes and calibrating/estimating macro models with incomplete insurance markets and heterogeneous agents.

KEYWORDS: Idiosyncratic income processes, heterogeneity, labor income risk.

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\*I benefited from conversations with and/or comments from Adolf Buse, Flavio Cunha, Fatih Guvenen, Georgii Kambourov, Greg Kaplan, Iouri Manovskii, René Morissette, Giuseppe Moscarini, Bent Sørensen, Henry van Egteren, and Gianluca Violante. For useful comments, I thank conference participants at the 2008 SED Meetings in Boston, and the 2008 CEA Meetings in Vancouver.

<sup>†</sup>University of Alberta, Department of Economics, 8-14 HM Tory Building. Edmonton, Alberta, Canada, T6G 2H4. E-mail: [dhryshko@ualberta.ca](mailto:dhryshko@ualberta.ca). Phone: 780-4922544. Fax: 780-4923300.

# 1 Introduction

Individuals and households face substantial amounts of idiosyncratic labor market risk. Layoffs, health shocks, bonuses, promotions, demotions, and time-varying returns to the individual skills valued by labor market contribute towards fluctuating individual labor incomes. Idiosyncratic labor income risk, absent perfectly functioning credit and insurance markets, affects individual and aggregate welfare. The importance of risk in real life is mirrored by its importance in modern macro models featuring agents with heterogeneous income fortunes.

Two different approaches to modeling individual and household labor income risks currently stand out.<sup>1</sup> The first approach, with a long-standing tradition, models each individual's income growing at the individual-specific, deterministic rate, with the level of income affected by a stochastic component with moderate persistence. Since each individual's labor income profile, even in the absence of shocks, is unique, I label this model, following Guvenen (2007a), the "Heterogeneous Income Profiles" (HIP) model. The second approach models idiosyncratic labor income as the sum of a permanent random walk component, the shocks to which persist for the entire working lifetime of an individual, and a mean-reverting stationary component, the shocks to which die out quickly. Since this model abstracts from the deterministic growth-rate heterogeneity, I label it the "Restricted Income Profiles" (RIP) model. Even though variants of the RIP are currently a preferred choice in macro models, there is no consensus in the labor income processes literature on which income model best fits the earnings data. As Guvenen (2007b) concludes: "... it is fair to say that this literature has not produced an unequivocal verdict." This paper is a step towards finding a verdict in favor of the RIP model.

I start with a general income model that encompasses the RIP and HIP models. I then conduct a Monte Carlo study to explore identification of different income processes found in the literature, obtained when certain restrictions on this general process are imposed. I find that if the true income process is the RIP with a permanent random walk component and an econometrician estimates the misspecified HIP model instead, he will typically find statistically significant amounts of the growth-rate heterogeneity, of magnitudes comparable with those in the literature. I show that the general income process composed of a deterministic growth rate, a permanent random walk, and transitory components can be identified when the earnings data used in estimation are in first differences. The results of a Monte Carlo study confirm that the

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<sup>1</sup>See Table 1 and its discussion in Section 2 for details.

parameters of this general process should be precisely recovered. I then proceed by estimating the model utilizing labor income data for male household heads from the Panel Study of Income Dynamics (PSID). I find that the estimate of the variance of the deterministic growth-rate component is zero, while the variance of the shock to the random walk component is significant and substantial. Hence, the data utilized in this paper favor the RIP model with a permanent random walk component and a mean-reverting persistent process.

The results of this paper are important as they contribute to understanding a number of issues. First, they speak to the economists' choices for modeling of household consumption, savings, and wealth. If the correct model for idiosyncratic labor income is the HIP, one needs to model individuals sequentially learning about their own labor income profiles to jointly fit the features of consumption and income data. Guvenen (2007b) is an example of such a model that successfully explains the profile of consumption inequality observed in U.S. micro data and the co-movement of the life-cycle profiles of earnings and consumption for households with different levels of schooling. If a substantial variation in incomes is due to permanent and persistent shocks, as is found in this paper, an appropriate model for household choices of consumption, savings, and wealth is an incomplete markets model with uninsurable persistent and/or permanent shocks. Castañeda, Díaz-Giménez, and Ríos-Rull (2003), utilizing such a model, successfully explain the U.S. wealth and earnings inequality; Scholz, Seshadri, and Khitatrakun (2006) explain more than 80% of the 1992 cross-sectional variation of household wealth observed in data from the Health and Retirement Study. Krebs (2003) is an example of a model where permanent idiosyncratic risk, absent in the estimations of the HIP processes but found to be substantial in this and some other papers,<sup>2</sup> reduces economic growth and individual welfare. De Santis (2007) develops a model where log-individual consumption is a random walk due to permanent uninsurable idiosyncratic income shocks and shows that such a model can potentially produce large welfare gains from eliminating business cycles.

Second, the results of this paper speak to the literature on the importance of initial conditions at the start of the individual's working career versus life-cycle shocks for the lifetime inequality in earnings and welfare (for recent contributions, see Storesletten, Telmer, and Yaron (2004a) and Huggett, Ventura, and Yaron (2007)). If household incomes contain a random walk and persistent components, the marginal propensity to consume from the permanent shock should be

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<sup>2</sup>The prominent papers that find substantial amounts of permanent idiosyncratic labor income risk are Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Moffitt and Gottschalk (1995).

close to one, and this reaction should translate one-for-one into consumption inequality among a cross-section of households with similar labor market experience. The contribution of the life-cycle labor income shocks will be understated if one models household incomes as the HIP, since, as shown in this paper, initial conditions at the start of the individual's working career—determined by the variance of initial incomes and the growth-rate heterogeneity—will capture the variation in incomes due to permanent shocks. Third, the contribution of the variance due to persistent components towards the rising earnings inequality observed in the U.S. will be underestimated if the random walk component is ignored.<sup>3</sup>

Lastly, the idiosyncratic labor income process, best fitting the data utilized in this paper, places restrictions on the models attempting to endogenize labor incomes. A fruitful starting point can be the model in Krebs (2003), where, in equilibrium, permanent shocks to individual human capital translate into permanent shocks to individual labor incomes. Perhaps it could be profitable to adopt distinct forms of human capital: human capital, shocks to which are permanent (e.g., disability shocks, or idiosyncratic returns to such general skills as computer skills), and human capital, shocks to which are dying out fast (e.g., temporary illness, or returns to skills that are non-transferable across occupations or even employers in the same occupation).

From a policy perspective, it also matters whether the true income process is the HIP or RIP. If an objective of the policymaker is to reduce consumption inequality and the true idiosyncratic income process is the HIP with a stochastic component of moderate persistence, the policymaker may want to implement policies that subsidize human capital investments by disadvantaged; self-insurance will be a sufficient shield against the shocks of moderate persistence. If, however, the true income process is the RIP with substantial permanent shocks, an appropriate policy, in addition to the above-mentioned, is to educate the public about risk-sharing instruments provided by credit institutions, stock, and insurance markets.

The rest of the paper is structured as follows. In Section 2, I present a Monte Carlo study of income processes found in the literature and introduce the HIP and RIP models. I estimate income processes on simulated data both in levels and first differences. I also discuss identification of the models containing a random walk and deterministic growth-rate components when data used for estimation are in first differences. In Section 3, I first describe the data I use and then present the empirical results. Section 4 concludes.

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<sup>3</sup>See Baker and Solon (2003), which elaborates on this issue using Canadian earnings data.

## 2 A Monte Carlo Study

In this section, I present the income processes estimated in the literature and perform a Monte Carlo study to explore identification of those income processes.

Let the true income process be:

$$y_{iht} = \alpha_i + \beta_i h + p_{iht} + \tau_{iht} + u_{iht,me} \quad (1)$$

$$p_{iht} = p_{ih-1t-1} + \xi_{iht} \quad (2)$$

$$\tau_{iht} = \theta(L)\epsilon_{iht}, \quad (3)$$

where  $y_{iht}$  is the idiosyncratic log-income of individual  $i$  with  $h$  years of labor market experience at time  $t$ ;  $\beta_i$  is individual  $i$ 's growth rate of income;  $\alpha_i$  is individual  $i$ 's initial level of income;  $p_{iht}$  is the permanent stochastic component of income;  $\xi_{iht}$  is a mean-zero shock to the permanent component;  $\tau_{iht}$  is the transitory stochastic component of income;  $\epsilon_{iht}$  is a mean-zero shock to the transitory component;  $u_{iht,me}$  is a mean-zero measurement error;  $L$  is the lag operator so that  $L^k x_t = x_{t-k}$ ,  $\forall k = 0, \pm 1, \pm 2, \dots$ ; and  $\theta(L)$  is a moving average polynomial in  $L$ .

The income process outlined in equations (1)–(3) encompasses most of the income processes estimated in the literature.<sup>4</sup> In Table 1, I list the most cited studies of idiosyncratic labor income processes for individuals or households, along with the specific restrictions on the process in (1)–(3) imposed in those studies.<sup>5</sup> Hause (1980), Lillard and Weiss (1979), and more recently Guvenen (2007a) estimate the income process that is driven by “deterministic effects,”  $\alpha_i$  and  $\beta_i$ ; an AR(1) transitory component affected each period by the transitory shock,  $\epsilon_{iht}$ ; and measurement error,  $u_{iht,me}$ . I label this process the HIP. Meghir and Pistaferri (2004) and

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<sup>4</sup>Baker and Solon (2003) estimate a similar process but allow the contribution of the deterministic component— $\alpha_i + \beta_i h$ —towards individual earnings to vary by calendar year, and the variance of the transitory shock by age. Haider (2001) assumes away the permanent component, models the transitory component as an ARMA(1,1) process, and allows the contribution of the deterministic component to vary by calendar year. These modifications of the model in (1)–(3) are largely done to fit the time-varying cross-sectional variances and covariances observed in earnings data. Most of the studies in the literature, to account for time-varying variances and covariances, allow instead for time-varying variances of stochastic (transitory and, if present, permanent) disturbances. This is the strategy I adopt in Section 3.2.

<sup>5</sup>Note that even though, say, Guvenen (2007a) does not model the permanent stochastic component of income explicitly, he allows a root of the autoregressive representation of  $\tau_{iht}$  to be one. The studies not modeling the permanent component explicitly find that the largest root of the stochastic component is below unity. They interpret this as the absence of the random walk component in idiosyncratic labor income, i.e., as if  $p_{iht} = 0$  for all  $t$ .

Carroll and Samwick (1997) are the prominent examples of the studies that assume the presence of a random walk and transitory components in idiosyncratic income but assume away (or present some evidence against) the deterministic idiosyncratic growth-rate component. I label this process the RIP. My ultimate goal is to determine whether the process containing random walk, transitory and deterministic components can be identified empirically.

## 2.1 Simulation Details

To see whether different processes are identified, I conduct a Monte Carlo study. I simulate data for 3,000 individuals “observed” for at most 30 periods using the data generating process of equations (1)–(3). I purposefully do not create a balanced panel data set—to mimic the patterns of the PSID data, which I will later use in empirical analysis. The PSID may contain at most 30 consecutive records on income for each head of household (from the 1968–1997 waves), but, since many heads of household first enter the labor market in different years and because of attrition and non-response, many heads contribute one or more observations on labor income.

The details of simulations are as follows. I assume that  $\alpha_i$  and  $\beta_i$  are mean-zero, possibly correlated normally distributed fixed effects, with which the head is endowed when he enters the labor market. I further assume that  $\xi_{iht}$  is an i.i.d. mean-zero shock to the permanent component of income normally distributed with the variance equal to  $\sigma_\xi^2$ ;  $\epsilon_{iht} \sim iidN(0, \sigma_\epsilon^2)$ ;  $u_{iht,me} \sim iidN(0, \sigma_{u,me}^2)$ ; and  $\tau_{iht}$  is either a moving average process of order 1 or an autoregressive process of order 1. I use these particular representations of the transitory component of earnings for the following reasons. First, RIP studies, such as Abowd and Card (1989) and Meghir and Pistaferri (2004), find that the growth rate in male earnings can be represented by a moving average process of order 2, suggesting that the transitory component is a moving average process of order 1. Second, HIP studies, such as Lillard and Weiss (1979) and Guvenen (2007a), model the transitory component as an autoregressive process of order 1. The estimated AR(1) process is easy to deal with in computational models featuring incomplete insurance markets and agents with idiosyncratic earnings histories, as argued in Guvenen (2007a). Third, a moving average process of order 1 with the moving average parameter of a small magnitude is hard to distinguish from an autoregressive process of order 1.<sup>6</sup>

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<sup>6</sup>If the true transitory process is  $\tau_{it} = (1 + \theta L)\epsilon_{it}$ , it can be represented by an infinite order autoregressive process,  $\tau_{it} = \theta\tau_{it-1} - \theta^2\tau_{it-2} + \theta^3\tau_{it-3} - \dots + \epsilon_{it}$ , and approximated by  $\tau_{it} = \theta\tau_{it-1} + v_{it}$ , where  $v_{it} = -\theta^2\tau_{it-2} + \theta^3\tau_{it-3} - \dots + \epsilon_{it}$ . Galbraith and Zinde-Walsh (1994) show that low order autoregressive approximations of an MA(1) process—of order 1 up to order 3—with a moving average parameter of 0.5 and less in absolute value

In the first year of a simulated data set, I observe a cross section of households whose heads' labor market experience ranges from one to 30 years, 100 of each type; heads with one year of experience in the first sample year contribute 30 observations towards the final sample, while heads with 30 years of experience are observed in the first year only. In the second year, since all of the heads have one more year of experience and those with 30 years of experience exit the sample, I add 100 households whose heads just enter the labor market and have only one year of experience. I repeat these steps until I simulate a data set with the time dimension of 30 years. This procedure ensures that there are 100 households whose heads' experience levels range from one to 30 observed in each year, and the cross-sectional mean of experience is constant across the years of each simulated data set. For each estimated income model, I report the results based on 100 simulated samples. The models are identified by fitting the theoretical autocovariances to the autocovariances in the simulated data. Estimation is performed using the minimum distance method, with the identity weighting matrix.<sup>7</sup> I now turn to estimation results for different simulated income processes.

## 2.2 Models Estimated on Data in Levels

Most of the HIP models are estimated on income data in levels while most of the RIP models are estimated on income data in first differences. The income process (1)–(3) provides restrictions on the variances and autocovariances for the data both in levels and first differences. In this section, I present estimation results on simulated data in levels. In the next section, I first simulate data in levels, then transform them to first differences, estimate the models, and present the results of those estimations.

First, I simulate the income process that consists of the deterministic effects and measurement error; i.e., I set  $p_{iht}$  and  $\tau_{iht}$  to zero for all individuals and years. This exercise helps determine how well the distribution of initial conditions is identified under the best circumstances, when the cross-sectional variances and covariances are not affected by stochastic permanent and transitory components. I set the true variance of the individual-specific growth rate,  $\sigma_\beta^2$ , to 0.0004, the true variance of the individual-specific intercept,  $\sigma_\alpha^2$ , to 0.03, the correlation between them to  $-0.3$ , and the variance of measurement error,  $\sigma_{u,me}^2$ , to 0.04. The results of estimation are

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perform the best in terms of minimizing the mean squared error.

<sup>7</sup>Altonji and Segal (1996) showed that an identity weighting matrix is the best choice for weighting the moments while estimating models of autocovariance structures on micro data with small samples. Most of the papers listed in Table 1, guided by this result, utilize this weighting matrix.

presented in column (1) of Table 2. The variance of the individual-specific growth rate and the variance of measurement error are well identified and tightly estimated. The variance of the intercept and the correlation between the intercept and individual-specific growth effects are biased downward. Indeed, the estimated standard deviation of the intercept is 25% below its true value. Poor identification of the correlation and the variance of the intercept is due to the fact that many combinations of these two parameters lead to a similar fit of the auto-covariance function in simulated data. When taken to the data, this poor identification may create numerical problems while calculating the standard errors of the parameters.

In column (2), the true income process is the same while I set the covariance between  $\alpha_i$  and  $\beta_i$  to zero when estimating the model. The purpose of this exercise is to see whether the variance of the individual-specific growth rate,  $\beta_i$ , the main disagreement of the HIP and RIP models, is well identified (if present) when one assumes away the correlation between individual-specific fixed effects. Again, the variance of the individual-specific growth rate and the variance of measurement error are well identified even though the estimated model is misspecified. The variance of the intercept is substantially biased downward.

In column (3), I set the true correlation between the individual-specific growth rate and intercept to zero. Estimation recovers the true variances of the growth rate, intercept, and measurement error very well. In the following, I will estimate the variances of the individual-specific fixed effects setting the true correlation between them to zero.<sup>8</sup> One should keep in mind that the estimated variance of the individual-specific intercept will be biased downward if the true correlation between the fixed effects is non-zero in empirical data.

Next, I estimate income processes that contain a random walk component, deterministic individual-specific fixed effects, and measurement error. As before, I assume that  $\sigma_\beta^2 = 0.0004$ ,  $\sigma_\alpha^2 = 0.03$ , and  $\sigma_{u,me}^2 = 0.04$ ; I also set  $\tau_{iht} = 0$  for all individuals and years. The results are presented in Table 3. In columns (1) and (2), the true variance of the shock to the random walk component,  $\sigma_\xi^2$ , is equal to 0.02 and 0.01, respectively. I label these processes the HIP with random walk components with high- and low-variance permanent shocks, respectively. The purpose of these estimations is to see how well the model is identified if it contains a random walk component, a special case of the models estimated by Baker (1997), Guvenen (2007a),

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<sup>8</sup>In Section 3, I find, for PSID data, that the variance of individual-specific growth rates is equal to zero; the correlation between individual-specific intercepts and growth rates is not identified in this case and can, therefore, be set to zero.



Hause (1980), and Lillard and Weiss (1979). Estimation recovers well the variances of the random walk shock and measurement error; the estimated variance of the individual-specific intercept is slightly biased downward. The persistence of the stochastic permanent component, measured by an autoregressive coefficient, is underestimated. Since the contribution of the random walk component towards the empirical autocovariance function is “downward-biased,” estimation places a larger weight on the deterministic growth-rate component—this results in an upward biased estimate of the variance of  $\beta_i$ . The null hypothesis of no permanent component, however, will be rejected, on average, if the true process contains the individual-specific growth rate and permanent components and no stochastic transitory components.

The literature estimating RIP, say, Meghir and Pistaferri (2004), presents some evidence that labor income contains a permanent random walk component and a mean-reverting stationary component but no individual-specific growth-rate component.

In Table 4, I show the results of estimations on simulated data that are generated in accordance with the RIP but are estimated as the HIP processes. Specifically, I assume that the true income process contains a random walk component, an AR(1) component, measurement error, and individual-specific intercepts. In column (1), I assume that an autoregressive coefficient of the transitory component is equal to 0.25. In column (3), I set an autoregressive coefficient to 0.50—a more persistent transitory component of earnings. In both columns, I set the variance of the shock to the transitory component to 0.04, and the variance of the permanent shock to 0.02. I estimate this process by fitting the autocovariance function from simulated data to the autocovariance function of the HIP process that contains individual-specific intercepts, the growth-rate component, and a potentially non-stationary autoregressive component. Estimation fails to recover all of the coefficients. The variance of the individual-specific intercepts is downward biased, while the variance of measurement error is substantially biased upward. Importantly, the estimated persistence of an autoregressive component can be as low as 0.84 and the variance of the deterministic growth-rate component can be as high as 0.0008, while the true value of the latter is equal to zero. These estimates are consistent with the estimates of the HIP process in the literature: significant estimates of the growth-rate heterogeneity, and estimates of the persistence of an autoregressive component below unity. In columns (2) and (4), I present the results of the same estimations assuming that the variance of measurement error is known and is equal to 0.04. The estimated persistence of an autoregressive component is even lower,

at about 0.74 for the process with a permanent random walk component and an autoregressive component with lower persistence, and at about 0.76 for the process with a permanent random walk component and an autoregressive process with higher persistence. The estimated variance of idiosyncratic deterministic growth rates is even higher, while their true value is equal to zero.

In Figure 1, I plot the theoretical autocovariance functions for the RIP process with  $\sigma_\beta^2 = 0.00$ ,  $\sigma_\xi^2 = 0.02$ ,  $\phi = 0.25$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_{u,me}^2 = 0.04$ ,  $T = H = 30$  and the HIP process, whose parameters are estimated on the data simulated by the RIP with the parameters just specified ( $\hat{\sigma}_\beta^2=0.0009$ ,  $\hat{\phi}=0.742$ ,  $\hat{\sigma}_\epsilon^2=0.038$ ,  $\hat{\sigma}_\xi^2=0.0$ ,  $\hat{\sigma}_\alpha^2=0.01$ , and  $\sigma_{u,me}^2=0.04$ )—see Table 4, column (2) for details. As can be seen from the graph, the autocovariance functions are hardly distinguishable. Thus, it is challenging, if possible at all, to distinguish between the HIP and RIP processes using income data in levels.

Abowd and Card (1989), MaCurdy (1982), and Meghir and Pistaferri (2004) present some evidence that the transitory component is a moving average process of at most order 1. In Table 5, therefore, I show the estimates of the misspecified HIP when the true transitory component is a moving average process of order 1, while the estimated stochastic component is an autoregressive process of order 1. In column (1), the true moving average coefficient is 0.25, while in column (3) it is equal to 0.50. The results in Table 5 are qualitatively similar to those reported in Table 4.

Summing up, the HIP model can be reasonably identified using data in levels if the true income process contains a permanent random walk component, the individual-specific intercept and growth rate, and no transitory component. If, however, the true income process contains a persistent transitory component along with a random walk component and no deterministic growth rate, an econometrician estimating the misspecified HIP model will find significant and substantial growth-rate heterogeneity, and the persistence of an autoregressive component below unity.

### 2.3 Models Estimated on Data in First Differences

In the previous section, I presented some Monte Carlo evidence on identification of the HIP and RIP income processes when data used for estimation are in levels. In this section, I present estimation results on simulated data transformed into first differences. I first discuss identification of the processes containing a random walk component, a transitory component, a deterministic

growth-rate component, and measurement error.

Note that both the variances and autocovariances estimated from data in levels contain contributions from the growth-rate heterogeneity, the permanent component, and the mean-reverting persistent component (see Appendix A). It is therefore challenging to identify all the components utilizing data in levels. The autocovariance function for the data in first differences, however, can be used to identify the growth-rate heterogeneity and random walk components, if both are present in the data. Permanent shocks will contribute only to the diagonal elements of the autocovariance function, i.e., the variances, while the growth-rate heterogeneity will contribute, in addition, towards all the off-diagonal elements of the autocovariance function. This information can be used to identify all the components as is shown in detail below.

### 2.3.1 Identification

In this section, I provide the intuition behind identification of income processes that contain individual-specific growth rates, a permanent random walk and mean-reverting transitory components when the data used for estimation are in first differences. In the next section, I confirm identification using the minimum distance method, which utilizes all the available information in the autocovariance structure of the data.

#### Income Processes with Deterministic Growth-Rate Heterogeneity and a Random Walk Component

In first differences, the process (1)–(3) is:

$$\Delta y_{it} = \beta_i + \xi_{it} + \theta(L)\Delta\epsilon_{it} + \Delta u_{it,me}, \quad (4)$$

where  $\Delta \equiv 1 - L$ .

For simplicity, assume that the transitory component is a moving average process of order 1, i.e.,  $\tau_{iht} = (1 + \theta L)\epsilon_{iht}$ .<sup>9</sup> The theoretical autocovariance moments,  $\gamma_k = E[\Delta y_{it}\Delta y_{it-k}]$ , of this process are:

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<sup>9</sup>Absent the growth-rate heterogeneity, the income process in first differences is a moving average process of order 2. This is consistent with the results in Abowd and Card (1989) and Meghir and Pistaferri (2004).

$$\gamma_0 = \sigma_\xi^2 + \sigma_\beta^2 + (1 + (1 - \theta)^2 + \theta^2)\sigma_\epsilon^2 + 2\sigma_{u,me}^2 \quad (5)$$

$$\gamma_1 = \sigma_\beta^2 - (\theta - 1)^2\sigma_\epsilon^2 - \sigma_{u,me}^2 \quad (6)$$

$$\gamma_2 = \sigma_\beta^2 - \theta\sigma_\epsilon^2 \quad (7)$$

$$\gamma_k = \sigma_\beta^2, \quad k \geq 3. \quad (8)$$

The empirical variance-covariance matrix contains  $T(T+1)/2$  unique moments. The variance of deterministic growth,  $\sigma_\beta^2$ , can be identified from the following vector of moments:

$$E[\Delta y_{it}\Delta y_{it+k}] = \sigma_\beta^2 \mathbf{1}, \quad k = 3, \dots, T-t, \quad t = 1, \dots, T-k, \quad (9)$$

where  $\mathbf{1}$  is a vector of ones of the row dimension  $(T-3)(T-2)/2$ . Empirical analogs of the moments  $\gamma_0$ ,  $\gamma_1$ , and  $\gamma_2$  can be further used to identify three out of the other four parameters:  $\sigma_\epsilon^2$ ,  $\sigma_\xi^2$ ,  $\sigma_{u,me}^2$ , and  $\theta$ . To identify the variances of transitory and permanent shocks and the moving average coefficient, one needs to restrict the variance of measurement error.

The asymptotic variance of the scaled mean of a mean-zero stationary process,  $\lim_{T \rightarrow \infty} E \left[ \frac{1}{\sqrt{T}} \sum_{t=1}^T \Delta y_{it} \right]^2$ , will be equal to  $\sum_{k=-\infty}^{\infty} \gamma(k)$ , or the sum of the variance and twice the sum of the non-zero autocovariances, if the autocovariances are absolutely summable.<sup>10</sup> This requirement is violated for the HIP process since the higher-order autocovariances are all equal to  $\sigma_\beta^2$ . One may, however, calculate the sample variance of the (scaled) sample mean. For a sample with a finite time dimension equal to  $T$  and a covariance-stationary process, this variance will be equal to  $\frac{1}{T} [T\gamma_0 + 2(T-1)\gamma_1 + 2(T-2)\gamma_2 + 2(T-3)\gamma_3 + \dots + 2\gamma_{T-1}]$ .

For the process in equation (5)–(8), the variance is equal to:

$$E \left[ \frac{1}{\sqrt{T}} \sum_{t=1}^T \Delta y_{it} \right]^2 = \sigma_\xi^2 + T\sigma_\beta^2 + \frac{2}{T} [\sigma_\epsilon^2(1 + \theta^2) + \sigma_{u,me}^2] \approx \sigma_\xi^2 + T\sigma_\beta^2, \quad (10)$$

where the quality of approximation is better in samples with larger time dimension.

<sup>10</sup>See, e.g., Hamilton (1994) Chapter 7 for a proof. This moment identifies the long-run variance of  $\{\Delta y_{it}\}$ .

This moment, together with the moment in equation (9), can be used to estimate the variance of permanent shocks,  $\sigma_\xi^2$ .

If  $\tau_{iht} = (1 - \phi L)^{-1} \epsilon_{iht}$ , i.e., the transitory component is an AR(1) process, the theoretical autocovariance moments of the income process in first differences are:<sup>11</sup>

$$\gamma_0 = \sigma_\xi^2 + \sigma_\beta^2 + \frac{2}{1 + \phi} \sigma_\epsilon^2 + 2\sigma_{u,me}^2 \quad (11)$$

$$\gamma_1 = \sigma_\beta^2 - \frac{1 - \phi}{1 + \phi} \sigma_\epsilon^2 - \sigma_{u,me}^2 \quad (12)$$

$$\gamma_k = \sigma_\beta^2 - \phi^{k-1} \frac{1 - \phi}{1 + \phi} \sigma_\epsilon^2, \quad k \geq 2. \quad (13)$$

Note that  $\sigma_\beta^2$  should be identified from higher-order autocovariances—when the contribution of the transitory component towards the autocovariances approaches zero. One may expect better identification of the growth-rate heterogeneity for the income processes with transitory components of lower persistence. The moment condition in equation (10), for a sufficiently large time dimension of the sample, can be used to identify the sum of the variance of permanent shocks and the variance of the growth-rate heterogeneity, scaled by the time dimension of the sample, for any mean-reverting transitory income process, inclusive of an AR(1) process.

Thus, if the income process contains individual-specific growth rates and intercepts, a permanent random walk component, a mean-reverting transitory component, and measurement error, it is possible to identify the variance of permanent shocks and the variance of the deterministic growth-rate heterogeneity. I will confirm this intuition in estimations using simulated data in Section 2.3.2.

### **Income Processes with a Random Walk Component but No Deterministic Growth-Rate Heterogeneity—the RIP Processes**

What if the true variance of the growth-rate heterogeneity is zero and the income process contains a random walk component but an econometrician estimates the HIP model instead?

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<sup>11</sup>Guvenen (2007a) estimates the parameters of this transitory income process since an AR(1) process is easy to deal with in computational macro models and parsimonious enough to fit the autocovariance structure of earnings dynamics in micro data. The population autocovariance moments in equations (11)–(13) are for the data with the time dimension approaching infinity. As shown below, models with the transitory component modeled as an AR(1) and simulated data with the time dimension of 30 periods for individuals with finite labor market experience are well identified by matching the sample autocovariance moments to the population moments in equations (11)–(13).

For the income model in equation (4) with  $\sigma_\beta^2 = 0$  and the transitory component modeled as an MA(1), the asymptotic variance of the (scaled) sample mean is:<sup>12</sup>

$$\lim_{T \rightarrow \infty} E \left[ \frac{1}{\sqrt{T}} \sum_{t=1}^T \Delta y_{it} \right]^2 = \sigma_\xi^2. \quad (14)$$

The empirical analog of equation (14), for a covariance-stationary process, can be estimated from  $\frac{1}{T} [T\gamma_0 + 2(T-1)\gamma_1 + 2(T-2)\gamma_2] = \sigma_\xi^2 + \frac{2}{T} [\sigma_\epsilon^2(1 + \theta^2) + \sigma_{u,me}^2]$ .<sup>13</sup> The estimated moment will be closer to  $\sigma_\xi^2$  for a larger time dimension of the data,  $T$ . If, however, the random walk is ignored in estimation, the theoretical autocovariance function is non-zero beyond order 2 and is equal to  $\sigma_\beta^2$ . The moment in equation (14) will be estimated as  $\frac{1}{T} [T\gamma_0 + 2(T-1)\gamma_1 + 2(T-2)\gamma_2 + \dots + 2\gamma_{T-1}] = \frac{1}{T} \hat{\sigma}_\beta^2 [T + 2(T-1) + 2(T-2) + \dots + 4 + 2] + \frac{2}{T} [\hat{\sigma}_\epsilon^2(1 + \hat{\theta}^2) + \hat{\sigma}_{u,me}^2] = T\hat{\sigma}_\beta^2 + \frac{2}{T} [\hat{\sigma}_\epsilon^2(1 + \hat{\theta}^2) + \hat{\sigma}_{u,me}^2]$ , where  $\hat{\sigma}_\epsilon^2$ ,  $\hat{\sigma}_{u,me}^2$ , and  $\hat{\theta}$  will differ from their true values.

Equating the two moments, one can show that if the true data generating process consists of a random walk, a persistent moving average component, and measurement error, and an econometrician estimates the (misspecified) HIP instead, the variance of the deterministic growth component will be approximately equal to:

$$\hat{\sigma}_\beta^2 \approx \frac{1}{T} \sigma_\xi^2. \quad (15)$$

Major micro data sets in the U.S. have no more than 30 years of consecutive observations on individual labor income. Thus, if the true variance of permanent shocks is equal to 0.02 and  $T = 30$ , then the variance of the deterministic growth will be estimated at about 0.0007—within the bounds of the typical estimates of the HIP in the literature.

The same logic holds if the transitory stochastic component of income is an AR(1) process. If the true income process is the RIP with a permanent random walk component, the empirical analog of the moment in equation (14) is equal to  $\frac{1}{T} [T\gamma_0 + 2(T-1)\gamma_1 + 2(T-2)\gamma_2 + \dots +$

<sup>12</sup>If the transitory income component is an AR(1) process, the moment condition (14) will be the same; it will also identify  $\sigma_\xi^2$  only.

<sup>13</sup>The population moment condition, for the transitory process modeled as an MA(1), will be equal to  $\gamma_0 + 2\gamma_1 + 2\gamma_2$ , and can be also estimated from  $E \left[ \Delta y_{it} \sum_{k=-2}^{k=2} \Delta y_{it+k} \right]$ , the moment used by Meghir and Pistaferri (2004) to uncover the variance of the permanent shock.

$2\gamma_{T-1}] = \sigma_\xi^2 + \frac{2\sigma_\epsilon^2}{1+\phi} - \frac{2}{T} \frac{1-\phi}{1+\phi} \sum_{j=1}^{T-1} \phi^{j-1} + \frac{2}{T} \sigma_{u,me}^2$ . If the random walk is ignored and the HIP is

estimated instead, the moment will be estimated as  $T\hat{\sigma}_\beta^2 + \frac{2\hat{\sigma}_\epsilon^2}{1+\hat{\phi}} - \frac{2}{T} \frac{1-\hat{\phi}}{1+\hat{\phi}} \sum_{j=1}^{T-1} \hat{\phi}^{j-1} + \frac{2}{T} \hat{\sigma}_{u,me}^2$ . Thus, equation (15) should approximately hold for any mean-reverting transitory process, provided the true income process contains a random walk component and the estimated process is the HIP.

### **Income Processes with Deterministic Growth-Rate Heterogeneity but No Random Walk Components—the HIP Processes**

What if the true income process is the HIP but an econometrician estimates the RIP model instead? If the transitory component is a moving average process of order 1, the econometrician will match the sample autocovariance moments to the misspecified population autocovariance moments, estimating  $\sigma_\xi^2$ ,  $\theta$ , and  $\sigma_\epsilon^2$ . The model restricts the population autocovariance moments of order 3 and higher to zero, while the true population moments will be equal to  $\sigma_\beta^2$ . Heuristically, the matching procedure will look for  $\sigma_\xi^2$ ,  $\theta$ ,  $\sigma_\epsilon^2$  that minimize the squared distance between  $T$  sample and theoretical zero-order autocovariances,  $2(T-1)$  first-order autocovariances, and  $2(T-2)$  second-order autocovariances. The estimate of the moment  $E \left[ \frac{1}{\sqrt{T}} \sum_{t=1}^T \Delta y_{it} \right]^2$  for the misspecified model will be equal to  $\hat{\sigma}_\xi^2 + \frac{2}{T} \left[ \hat{\sigma}_\epsilon^2 (1 + \hat{\theta}^2) + \hat{\sigma}_{u,me}^2 \right]$ , while the estimate of the same moment for the true model, with zero restrictions placed on the autocovariances above order 2, will be equal to  $\frac{1}{T} \sigma_\beta^2 (5T-6) + \frac{2}{T} \left[ \sigma_\epsilon^2 (1 + \theta^2) + \sigma_{u,me}^2 \right]$ . Thus, the estimated variance of permanent shocks will be approximately equal to  $\frac{1}{T} \sigma_\beta^2 (5T-6)$ . If, e.g., the true variance of deterministic growth-rate heterogeneity is equal to 0.0004, and the time dimension of the sample is 30 periods, the variance of permanent shocks will be approximately estimated at 0.00192, even though the true variance of these shocks is equal to zero. Similar arguments apply to the case when the transitory component is an AR(1) process—the estimated variance of permanent shocks will be non-zero when the true income process contains the deterministic idiosyncratic trends and an autoregressive transitory component, but an econometrician estimates the RIP process.

### **2.3.2 Simulation Results**

In this section, I present the results of estimations of income processes using simulated data in first differences.

In Table 6 column (1), the true income process contains the individual-specific intercept, a random walk component, a transitory moving average component of order 1, and measurement error. The true variance of the random walk shock is equal to 0.02, and the moving average parameter is equal to 0.25—the process with a stationary stochastic component of low persistence. The process is estimated as the HIP containing a deterministic growth-rate component and an unrestricted moving average component of order 1. Since the variance of measurement error is not identified in this case, I set it to the true variance while estimating the model. The variance of the shock to the transitory component is estimated at about 0.05, while the moving average parameter is estimated at about 0.28, both significant at the 1% level. Importantly, when the random walk component is ignored in estimation, the long-run persistence of the process is captured instead by the variance of the deterministic growth, with the estimated value substantially and significantly away from its true value of zero. This is the estimate one can expect given the time series dimension of 29 periods (see Section 2.3.1).

In column (2), I estimate the process that contains the individual-specific intercepts and growth rates, a random walk component, a stationary moving average component of order 1, and measurement error. The parameters and their true values that can be estimated using data in first differences are:  $\sigma_\beta^2 = 0.0004$ ,  $\sigma_\xi^2 = 0.02$ ,  $\theta = 0.25$ , and  $\sigma_\epsilon^2 = 0.04$ . All of these parameters are recovered without any biases and precisely by the equally weighted minimum distance method.

In columns (3) and (4), I present similar results for the income processes with  $\sigma_\xi^2 = 0.02$  and  $\theta = 0.50$ . Again, if the random walk component is ignored in estimation, the growth-rate heterogeneity captures the long-run variance of income growth—column (3). The estimated variance of the growth-rate component, in accordance with equation (15), should be approximately equal to  $0.02/29 \approx 0.0007$ , the exact match of the estimate in column (3). If both the growth-rate and random walk components are present and accounted for in estimation, all of the structural parameters are recovered extremely well—column (4).

Next, I assume that the true income process contains the individual-specific intercept, measurement error, a permanent random walk component with the variance equal to 0.02, and a transitory component modeled as an autoregressive process of order 1 with the autoregressive coefficient equal to 0.25 (0.50) and the variance of the transitory shock equal to 0.04 (0.04)—column (1) (column (3)) of Table 7. In columns (2) and (4), the income processes, in addition,



contain the deterministic growth-rate component with the variance equal to 0.0004. The results are qualitatively similar to those in Table 6. If the transitory component is modeled as an autoregressive process, it is possible, in addition, to identify the variance of measurement error.<sup>14</sup>

Guvenen (2007a), in a simulation exercise, shows that the tests of higher-order autocovariances equal to zero will falsely reject the growth-rate heterogeneity even when the true income process contains idiosyncratic growth rates. This test was previously used by MaCurdy (1982). Table 8 confirms this result. I first create 1,000 samples generated in accordance with the model in Table 6 column (4), which contains deterministic idiosyncratic growth rates, a permanent random walk component, and a transitory moving average process with the moving average parameter equal to 0.50. For each simulated sample, I calculate the empirical autocovariance function. The results in the first column are the averages of the autocovariances of a given order across 1,000 simulated samples; standard errors, in parentheses, are calculated as the standard deviations of these estimates across 1,000 simulated samples. As can be seen from the column, only autocovariances of orders 0, 1, and 2 are significant. The rest are insignificant, even though the magnitude of the autocovariances of orders 3 and higher will correctly identify the magnitude of the variance of the deterministic growth-rate heterogeneity. In the second column of Table 8, I perform the same exercise for the income process that contains the transitory component modeled as an AR(1) process with persistence equal to 0.50. The autocovariance function is significant only from order 0 to order 4, inclusive; the contribution of the transitory component towards the autocovariance function dissipates quickly and higher-order autocovariances will, on average, correctly identify the size of the growth-rate heterogeneity. Intuitively, what matters for identification of the growth-rate heterogeneity is the average magnitude of higher-order autocovariances. The minimum distance procedure uses the entire autocovariance function and its sample variability to uncover correctly and precisely the variance of the deterministic growth-rate heterogeneity—Table 6, column (4) and Table 7, column (4).

Summing up, when using data in first differences it is feasible to identify the variance of the

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<sup>14</sup>I also performed estimations of misspecified RIP income processes, when the true income processes contain deterministic growth-rate heterogeneity while the estimated models contain a random walk component and no idiosyncratic deterministic growth rates. The results, not reported here, largely confirm identification arguments outlined above. E.g., when the true income processes are such that the variance of deterministic growth-rate heterogeneity,  $\sigma_{\beta}^2$ , is equal to 0.0004, the transitory process is modeled as an MA(1) process with the moving average parameter equal to 0.25, and the estimated models ignore the growth-rate heterogeneity, the estimated variance of random walk shocks is equal to 0.0091. This is in line with the theoretical prediction calculated from the formula  $\frac{1}{T}\sigma_{\beta}^2(5T - 6)$ , with  $T=29$ .

shock to the random walk component, the variance of the shock to the transitory component, and the variance of the growth-rate heterogeneity. If, however, the process contains a random walk permanent component and no deterministic growth-rate component and the model is estimated as the HIP, estimation will capture the long-run variance of income growth due to permanent shocks with a significant estimate of the growth-rate heterogeneity. The magnitude of the estimate will depend on the true variance of the permanent shock and the time dimension of a data set.

### 3 Empirical Results

In this section, I estimate time series processes for idiosyncratic labor incomes of male household heads from the PSID. I first describe the data I utilize.

#### 3.1 Data

I use income and demographic data from the 1968–1997 waves of the PSID. I select male household heads of ages 20–64, with labor market experience between one and 40 years.<sup>15</sup> I exclude data for households from the Survey of Economic Opportunity (SEO) sub-sample, which oversamples the poor. The measure of income utilized is the head’s labor income from all sources, inclusive of the labor part of farm and business income. Income data in the PSID refer to the previous calendar year; I adjust them appropriately by the consumer price index for all items normalized to 100 in 1982–1984. I set income observations to missing if the head’s labor income in any year is below 1,000 dollars in 1982–1984 prices. I further drop observations for the years when the change of log-labor income in adjacent years is above two in absolute value. The measure of the idiosyncratic head’s labor income in each year is the head’s residual from a cross-sectional regression of log-labor income on the full set of experience and education dummies.<sup>16</sup> This regression specifying the deterministic component common to all heads is general enough and assumes that returns to the head’s experience and education are affected by the aggregate

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<sup>15</sup>The head  $i$ ’s potential experience in year  $t$  is calculated as  $\max(0, age_{it} - \max(education_{it}, 12) - 6)$ .

<sup>16</sup>In the 1969–1974 family files of the PSID a continuous measure of education is either absent or the files contain just a few observations. For these waves, I impute years of schooling to the heads using a categorical measure of education that is widely available. Specifically, I assign two years of schooling to those heads who indicate they finished zero to five grades of schooling; seven years—if they finished six to eight grades; 10 years—for nine to eleven grades; 12 years—for 12 grades, or 12 grades plus some non-academic training; 14 years—for college with no degree; 16 years—for college with a bachelor’s degree; 17 years—if they finished college, advanced or professional degree.

state of the economy, i.e., differ by year. My main sample contains data for 4,551 heads with at least three, not necessarily consecutive, observations on idiosyncratic labor income and at least two observations on idiosyncratic labor income growth. Some basic summary statistics for the main sample are presented in Table 15.

### 3.2 Results

The models are estimated by fitting the empirical autocovariance function to the theoretical autocovariance function, utilizing the identity weighting matrix, i.e., by the equally weighted minimum distance method.

In Table 9, I present the results of estimations of income processes utilizing the data on idiosyncratic log-labor income in levels. Column (1) contains the results of estimation of the HIP process, the income process that contains the individual-specific intercepts and growth rates, an autoregressive process of order 1, and measurement error. The point estimates of the parameters are very similar to the ones in Guvenen (2007a), Table 1 row (4). The variance of the deterministic growth-rate heterogeneity is estimated at 0.0003 and is significant. One standard deviation in idiosyncratic growth rate translates into almost two percentage points difference in the earnings growth rate over the life-cycle—a very large effect. The variance of initial log-income levels (intercepts) is estimated at about 0.03 but is insignificant; the initial income levels and growth rates are negatively correlated with the estimated correlation coefficient of about  $-0.37$ ; an autoregressive component is found to be persistent yet the persistence is substantially below unity; the variance of the shock to the persistent component is estimated at 0.05, and the variance of measurement error at about 0.07. Note that both the variance of the individual-specific initial incomes and the covariance between the deterministic components are insignificant. This may indicate that they are either poorly identified (see discussion in Section 2.2), or that these effects are not present in the sample I utilize.<sup>17</sup> In column (2), I restrict the correlation to zero and re-estimate the model. The estimate of an autoregressive coefficient becomes slightly larger and the variance of the initial individual-specific incomes is estimated at zero. The latter result is consistent with the simulation results in Table 2 column (2). In Table 9 column (3), I restrict the variance of the individual-specific growth rates to zero. This model corresponds to the model for household idiosyncratic incomes estimated in Storesletten, Telmer, and Yaron (2004b) and

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<sup>17</sup>See also Guvenen (2007a), Table 1, rows (4)–(8). He was not able to find significant  $\hat{\sigma}_\alpha^2$  and  $\hat{\sigma}_{\alpha\beta}$  in all of his estimations as well.

Hubbard, Skinner, and Zeldes (1994).<sup>18</sup> The estimate of an autoregressive parameter increases to 0.91, and the variance of measurement error increases to 0.09. These results are expected for a model of income processes that contain both a random walk component and a stationary mean-reverting component.

Next, I present the results for estimations on the same sample and labor income data in first differences—Table 10.<sup>19</sup>

In column (1), I estimate the HIP process, which ignores the potentially important random walk component in idiosyncratic labor incomes. The variance of individual-specific growth rates is estimated at 0.0007, significant at the 1% level. The estimate of the variance of measurement error is substantially below the estimates from the specifications in levels and is equal to about 0.02.

In column (2), I allow for both a random walk and a deterministic growth-rate component in earnings. Monte Carlo results and theoretical arguments spelled out in Section 2.3.1 indicated that, if both these components are present, the process should be empirically identified. In column (2), the estimate for the variance of the individual-specific growth rates binds at zero while the estimate of the variance of the shock to the random walk component is equal to 0.036 and is significant at the 1% level. An autoregressive parameter of the transitory process is estimated at 0.40, capturing the fast decline of the empirical autocovariance function of labor income growth rates beyond the first order. The estimate of the variance of measurement error is, perhaps, too low.

In column (3), I set the variance of measurement error to the value estimated in column (1) and re-estimate the process of column (2).<sup>20</sup> The main results hold: the estimated variance of the growth-rate heterogeneity is zero, while the estimated variance of the random walk shock is substantial and significant.

In column (4), I estimate the model of column (1), assuming that the transitory component is a moving average process of order 1. In this case, the variance of measurement error is not

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<sup>18</sup>Hubbard, Skinner, and Zeldes (1994), in addition, restrict  $\sigma_\alpha^2$  to zero.

<sup>19</sup>Most of the studies listed in Table 1 allow for time-dependent variances of permanent and/or persistent shocks. My estimates of these parameters in Tables 9–11 should be interpreted as the unconditional variances of transitory and permanent shocks.

<sup>20</sup>The variance of idiosyncratic income growth in the sample is equal to 0.16. If measurement error is i.i.d., it contributes twice its variance towards the variance of idiosyncratic income growth. For this choice of the variance of measurement error, the share of total variance of idiosyncratic income growth “explained” by measurement error is  $(2 * 0.023)/0.16 \approx 0.29$ , or 29%. This share is higher than the 27% found in Bound and Krueger (1991) using matched earnings data from the Current Population Survey and Social Security payroll tax records, and the 22% found in Bound, Brown, Duncan, and Rodgers (1994) using the PSID Validation Study.

identified; I therefore fix it at 0.023—the estimate of column (1). The results do not change qualitatively: ignoring the random walk component, the estimated variance of the growth-rate heterogeneity is large and highly significant. In column (5), the results, again, indicate that including the random walk component into the model leads to a substantial estimate of the variance of the random walk shock and an estimate of zero for the growth-rate heterogeneity. In accordance with the discussion in Section 2.3.1, the variance of the growth-rate heterogeneity should be estimated at  $\sigma_{\xi}^2/T$  if the random walk component is ignored. Since the estimate of  $\sigma_{\xi}^2$  in column (5) is equal to 0.05 and the time dimension of the sample is 29 periods,  $\hat{\sigma}_{\beta}^2$  should be equal to 0.0017, a slight overestimate of the value in column (4).

Previous results are based on a sample of heads with at least two observations on idiosyncratic labor income growth. For robustness, I re-estimate the models in Table 10 on a sample of heads who contribute at least 20, not necessarily consecutive, records on idiosyncratic labor income growth. This sample consists of 1,034 heads. The results are presented in Table 11. The estimated variances of different components are somewhat smaller; otherwise, the results are qualitatively similar.

The variance of the growth-rate heterogeneity is largely identified from the off-diagonal elements of the empirical autocovariance matrix. The number of heads contributing towards the empirical autocovariance  $\hat{\gamma}_k$  is, in general, smaller the larger the lag length  $k$  is, which separates the head’s income observation at time  $t$  from the income observation at time  $t + k$ . Placing an equal weight on all the variances and autocovariances in estimation may bias an estimate of the growth-rate heterogeneity towards zero if higher-order empirical autocovariances are very close to zero as, indeed, is found in empirical data.<sup>21</sup> To take care of this concern, following Guvenen (2007a), I re-estimate the models in Table 11 utilizing only the first 10 empirical autocovariances and all the variances in estimation—Table 12. The main results remain unaltered.

Most of the studies reviewed in Table 1 allow for time-varying variances of disturbances to better fit the autocovariance function in data. In Table 13, I re-estimate the models of Table 10, columns (1)–(3) on my main sample of 4,551 male heads, allowing for time-varying variances. I report the time averages of the estimated variances and their standard errors. In column (1), I estimate the HIP. Compared with the results in Table 10 column (1), I find

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<sup>21</sup>Note that the estimations performed on simulated data are based on unbalanced panels, with only 100 simulated heads out of 3,000 contributing towards estimation of the autocovariance of order 29, 200 towards estimation of the autocovariance of order 28, etc. Even in that case the models are well identified; see, e.g., Table 7, columns (2) and (4).

somewhat higher estimates for the variance of the deterministic growth-rate heterogeneity and the (average) variance of shocks to the autoregressive component, and lower estimates of the persistence and the variance of measurement error.

In column (2), I restrict the variance of measurement error to its estimate from Table 10, column (1). This results in a higher estimate of the persistence parameter and a lower estimate of the variance of shocks to the autoregressive component; the estimate of the variance of the growth-rate heterogeneity is left unchanged.

In column (3), I repeat the analysis, introducing the permanent variance into the models. Since the variances of time-varying persistent shocks are identified from a larger set of moments, I first restrict the variance of the permanent shock to be the same across the sample periods. The results are very similar to those in Table 10, column (2), the estimation that uncovers the unconditional means of the variances of stochastic disturbances. Importantly, the variance of the growth-rate heterogeneity is estimated at zero and the variance of the permanent shock is estimated at about 0.04.

In column (4), I perform the same estimation, setting the variance of measurement error to 0.023. As expected from previous results, the persistence of an autoregressive component increases while the (average) variance of the shocks to the autoregressive component falls.

Finally, in column (5), I allow for time-varying variances of the shocks to the permanent and transitory components. The (average) variances are similar to those in column (4). The full sets of the transitory and permanent variances, along with their standard errors, are presented in Table 14.

In Figure 2 and Figure 3, I plot the resulting time series of the estimated variances of permanent and transitory shocks, respectively. The permanent variation in heads' incomes was increasing in the late 1970s and early 1980s, leveled off throughout most of the 1980s, and started falling at the end of the 1980s. The pattern of the variances of permanent shocks mirrors that in Meghir and Pistaferri (2004), for their pooled sample and their measure of household idiosyncratic income constructed from the combined head's and wife's labor incomes. It is also qualitatively similar to the hump-shaped pattern of the permanent volatility of household incomes in the 1980s reported in Blundell, Pistaferri, and Preston (2008). The variance of transitory shocks to the heads' incomes was hump-shaped in the 1970s, increased in the early 1980s, flattened out in the mid-1980s and increased again in the early 1990s. For the 1980s, a

similar pattern of transitory variances can be found in Blundell, Pistaferri, and Preston (2008).

For robustness, I estimate the models of Table 10 using separate samples for heads who did not finish high school (“high school dropouts”), finished high school but did not finish college (“high school graduates”), and those who have a bachelor’s degree or more (“college graduates”). In some estimations, when I fix the variance of measurement error, I assume that it is equal to 0.023 in all sub-samples. The results are in Tables B-1–B-3. When the random walk component is ignored in estimation, the variance of the deterministic growth-rate heterogeneity is always substantial and statistically significant (columns (1) and (4) of those tables); the estimated AR(1) persistence of the stochastic component is moderate, ranging from about 0.82 for the college sample to 0.64 for high school graduates. Regardless of the choice of a model for the transitory component, the variance of transitory shocks to the persistent (but mean-reverting) component is estimated to be the highest for high school dropouts, yet these shocks exhibit the lowest persistence for the dropouts; college graduates are hit by transitory shocks with the smallest variance. Interestingly, the estimated variance of permanent shocks is the highest for the sample of college graduates. Qualitatively similar results can be found in Meghir and Pistaferri (2004), Table III.

Summarizing, for the samples utilized in this study, it appears that I can reject the HIP model. The RIP model with a permanent random walk component and a transitory mean-reverting component cannot be rejected.<sup>22</sup>

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<sup>22</sup>There is some evidence, not relying on estimation of income processes, interpreted by some researchers as favoring income models with heterogeneous income profiles. Haider and Solon (2006) and Böhlmark and Lindquist (2006) study the association between current and lifetime income over the life cycle for U.S. and Swedish samples, respectively. Specifically, they focus on the life-cycle variation in the slope coefficient from the following regression:  $y_{ia} = \beta_a V_i + \epsilon_{ia}$ , where  $y_{ia}$  is individual  $i$ ’s log-income at age  $a$ ,  $V_i$  is individual  $i$ ’s log-lifetime income, calculated as (the log of) the annuity value of the discounted sum of annual real incomes observed for individual  $i$ , and  $\epsilon_{ia}$  is individual  $i$ ’s regression error at age  $a$ . Haider and Solon (2006) find that  $\beta_a$  is estimated at about 0.20 at age 19, steadily increases afterwards, equals one at age 34 and levels off for the rest of the life cycle. Böhlmark and Lindquist (2006), for a much larger Swedish sample, find that  $\hat{\beta}_a$  starts at about 0.20 at age 19, crosses one at age 34 and peaks at 1.45 at age 48. The latter authors interpreted this result as evidence favoring the presence of heterogeneous income profiles—income is low in the beginning of the life cycle and is well below the lifetime income (which is estimated to be time-invariant by the authors); income then steadily grows until it exceeds the lifetime income in the later part of the life cycle. This result is, however, also true for income processes that contain random walks and do not have deterministic idiosyncratic trends. Using the estimates of the RIP process in this paper, I was able to replicate, in simulations not reported here, the pattern of  $\hat{\beta}_a$ ’s found in Böhlmark and Lindquist (2006). The intuition behind this result is the following. Note that  $\hat{\beta}_a = \frac{\text{cov}(y_{ia}, V_i)}{\text{var}(V_i)}$ . While the denominator is constant over the life cycle, the cross-sectional covariance between current incomes and lifetime incomes will be growing over the life cycle since current incomes will accumulate random walk shocks over the life cycle and will, therefore, co-vary more strongly with lifetime incomes, which aggregate all the permanent shocks to individual incomes over the entire life cycle.

## 4 Conclusion

I estimate idiosyncratic labor income processes on simulated and empirical data. The main results of a Monte Carlo study are the following. If idiosyncratic labor income contains a random walk and a deterministic growth-rate component, the estimated persistence of the stochastic component modeled as an AR(1) is close to one. If, however, the stochastic component of idiosyncratic earnings consists of a random walk and a mean-reverting component, and there is no growth-rate heterogeneity and an econometrician estimates the HIP, the estimated persistence can be modest and the variance of the deterministic growth-rate heterogeneity can be substantial and significant—as is found in the HIP studies. When data are in first differences, it is possible to identify a general process containing all the elements of the HIP and RIP models. The most important elements are the growth-rate heterogeneity and the variance of a random walk component. For simulated data in first differences I show that both these elements, if present, should be recovered precisely in empirical estimations. The results on simulated data in first differences confirm another important finding of this paper: if the true income process is the sum of a random walk and persistent components, i.e., the RIP, and the random walk is ignored in estimation, the misspecified HIP model recovers significant and substantial growth-rate heterogeneity and modest persistence.

I use data for male household heads from the 1968–1997 waves of the PSID to estimate idiosyncratic labor income processes. I find that the estimated variance of the deterministic growth-rate heterogeneity is zero; i.e., the HIP model can be rejected. The RIP model, with permanent random walk and mean-reverting components, cannot be rejected. I find that the estimated variance of the permanent component is significant and substantial. Thus, the results of the paper favor the view that the observed variation in idiosyncratic income growth rates over the life cycle is entirely due to the shocks of different “durability.”

The results of this paper are important for understanding a number of issues. Among them are the choice of an appropriate model of the heterogeneity in individual and household idiosyncratic incomes used in macro models; the importance of shocks versus initial conditions for the life-cycle profiles of earnings and welfare inequality; and the importance of shocks versus initial conditions for the time series of earnings and consumption inequality. The process, best fitting the data utilized in this paper, places restrictions on the models that make earnings an endogenous variable.



In this paper, I utilize only income data to identify the variances of idiosyncratic permanent and transitory shocks. Perhaps, more accurate estimates of the variances could be obtained by jointly studying consumption and income data. For recent attempts at this approach see Hryshko (2007) and Blundell, Pistaferri, and Preston (2008) (in the context of RIP), and Guvenen and Smith (2008) (in the context of HIP).

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FIGURE 1: THE AUTOCOVARANCE FUNCTION FOR RIP AND HIP PROCESSES

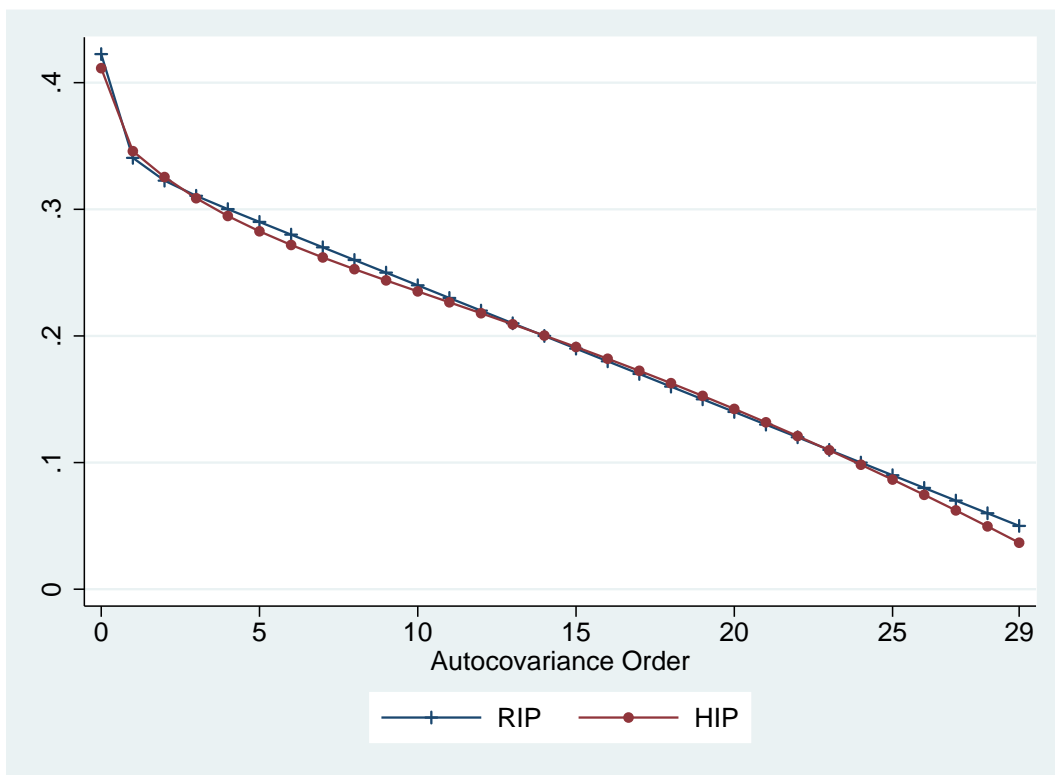


FIGURE 2: THE VARIANCE OF PERMANENT SHOCKS BY YEAR

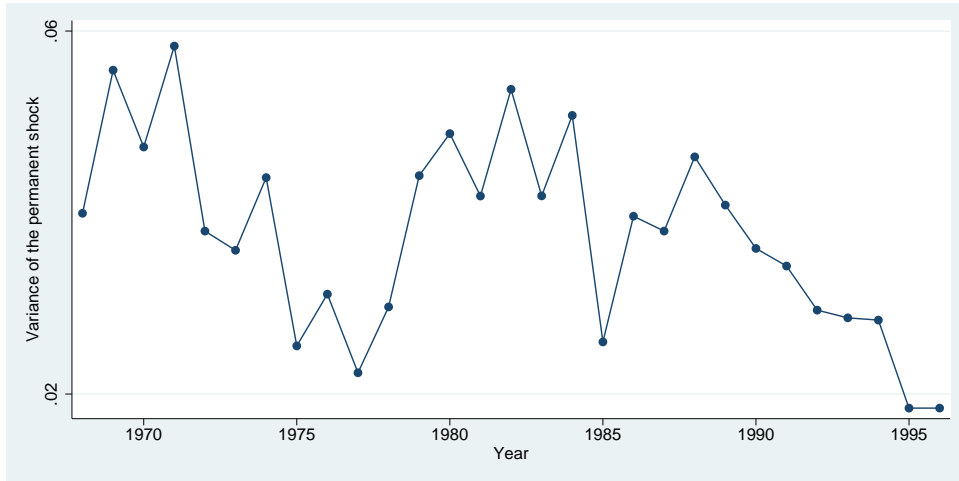


FIGURE 3: THE VARIANCE OF TRANSITORY SHOCKS BY YEAR

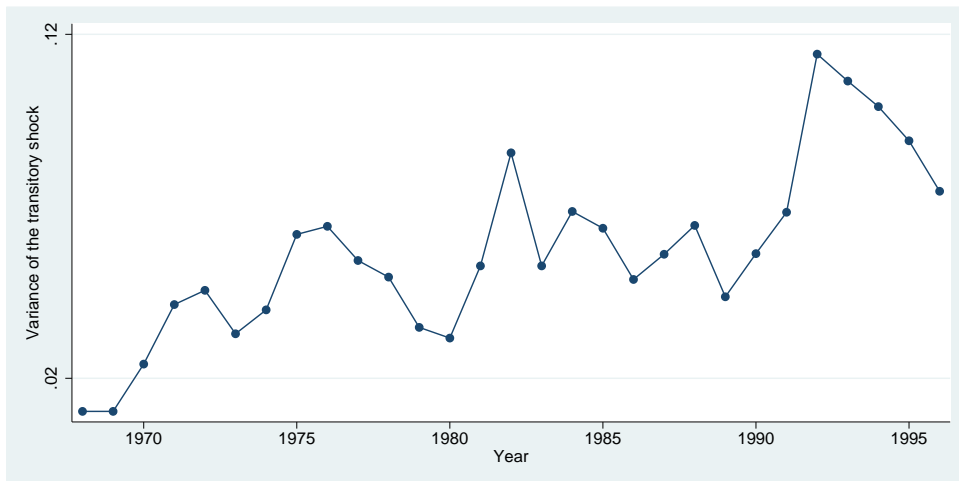


TABLE 1: INCOME PROCESSES ESTIMATED IN THE LITERATURE.

Study	Restrictions <sup>a</sup>	Data Transformation	Data/Data Set
Abowd and Card (1989)	$\beta_i = 0$ ; $\tau_{iht}$ is MA(1)	First differences	head's labor income/ PSID: 1969–1979 waves; other data sets
Baker (1997)	$p_{iht} = 0, \forall t$ ; $\tau_{iht}$ is ARMA(1,1)/ ARMA(1,2)/AR(1)	Levels/first differences	head's labor income/ PSID: 1968–1987 waves
Baker and Solon (2003)	none; $\tau_{iht}$ is AR(1)	Levels	earnings of Canadian men, 1976–1992
Carroll and Samwick (1997)	$\beta_i = 0$ ; $\tau_{iht}$ is W.N./MA(1)	First differences	household non-capital income/ PSID: 1987–1987 waves
Guvenen (2007a)	$p_{iht} = 0, \forall t$ ; $\tau_{iht}$ is AR(1)	Levels	head's labor income/ PSID: 1968–1993 waves
Haider (2001)	$p_{iht} = 0, \forall t$ ; $\tau_{iht}$ is ARMA(1,1)	Levels	head's labor income/ PSID: 1968–1992 waves
Hause (1980)	$p_{iht} = 0, \forall t$ ; $\tau_{iht}$ is AR(1)	Levels	labor income/ Swedish white collar workers, 1964–1969
Lillard and Weiss (1979)	$p_{iht} = 0, \forall t$ ; $\tau_{iht}$ is AR(1)	Levels	labor income/ NSF Register (scientists): 1960–1970
Moffitt and Gottschalk (1995) <sup>b</sup>	$\beta_i = 0$ ; $\tau_{iht}$ is ARMA(1,1)	Levels	head's wage/salary income/ PSID: 1970–1988 waves
MaCurdy (1982)	$\beta_i = 0$ ; $\tau_{iht}$ is MA(1)/MA(2), W.N., ARMA(1,1)	Levels/first differences	head's labor income/ PSID: 1968–1977 waves
Meghir and Pistaferri (2004)	$\beta_i = 0$ ; $\tau_{iht}$ is MA(1)	First differences	head's labor income/ PSID: 1968–1993 waves
Storesletten, Telmer, and Yaron (2004b)	$\beta_i = 0$ ; $p_{iht} = 0, \forall t$ ; $\tau_{iht}$ is AR(1)	Levels	head's and wife's labor and transfer income/ PSID: 1968–1993 waves

Notes: <sup>a</sup>Restrictions imposed on the income process:  $y_{iht} = \alpha_i + \beta_i h + p_{iht} + \tau_{iht} + u_{iht,me}$ , where  $p_{ih+1t+1} = p_{iht} + \xi_{ih+1t+1}$ ,  $\tau_{iht}$  is a mean-reverting stochastic process. <sup>b</sup>The authors' preferred specification.

TABLE 2: ESTIMATES OF THE HIP WITHOUT A PERSISTENT COMPONENT. SIMULATED DATA IN LEVELS.

	(1)	(2)	(3)
$\hat{\sigma}_\beta^2$	0.0004 (0.00002)	0.0004 (0.00001)	0.0004 (0.00002)
$\hat{\sigma}_\alpha^2$	0.017 (0.009)	0.0008 (0.002)	0.03 (0.006)
$\hat{\sigma}_{\alpha\beta}$	-0.0006 (0.0003)	0.00 —	0.00 —
$\widehat{corr}_{\alpha\beta}$ (implied)	-0.246 (0.117)	0.00 —	0.00 —
$\hat{\sigma}_{u,me}^2$	0.04 (0.0008)	0.04 (0.0009)	0.04 (0.0009)

*Notes:* In column (1) the true income process is:  $y_{iht} = \alpha_i + \beta_i h + u_{iht,me}$ , where  $(\alpha_i, \beta_i) \sim iidN(0, \Omega)$ ,  $u_{iht,me} \sim iidN(0, \sigma_{u,me}^2)$ , with  $\sigma_\alpha^2 = \Omega_{11} = 0.03$ ,  $\sigma_\beta^2 = \Omega_{22} = 0.0004$ ,  $corr_{\alpha\beta} = -0.30$ ,  $\sigma_{u,me}^2 = 0.04$ . In column (2), the true income process is the same, while I set the covariance between  $\alpha_i$  and  $\beta_i$  to zero when estimating the model. In column (3), the true income process is as in columns (1) and (2), with the true  $corr_{\alpha\beta} = 0$ . Models are estimated by the equally weighted minimum distance method. Standard errors in parentheses calculated as the standard deviations of the estimates across 100 model simulations.

TABLE 3: ESTIMATES OF THE HIP WITH A RANDOM WALK COMPONENT. SIMULATED DATA IN LEVELS.

	$\sigma_\beta^2=0.0004, \sigma_\xi^2=0.02$	$\sigma_\beta^2=0.0004, \sigma_\xi^2=0.01$
	(1)	(2)
$\hat{\sigma}_\beta^2$	0.00058 (0.0003)	0.00047 (0.0002)
$\hat{\sigma}_\alpha^2$	0.028 (0.016)	0.028 (0.013)
$\hat{\rho}$	0.986 (0.025)	0.985 (0.039)
$\hat{\sigma}_\xi^2$	0.019 (0.004)	0.0098 (0.003)
$\hat{\sigma}_{u,me}^2$	0.04 (0.002)	0.04 (0.001)

*Notes:* The true income process is:  $y_{iht} = \alpha_i + \beta_i h + p_{iht} + u_{iht,me}$ , where  $(1 - \rho L)p_{ih+1t+1} = \xi_{ih+1t+1}$ ,  $\beta_i \sim iidN(0, \sigma_\beta^2)$ ,  $\alpha_i \sim iidN(0, \sigma_\alpha^2)$ ,  $u_{iht,me} \sim iidN(0, \sigma_{u,me}^2)$ ,  $\xi_{iht} \sim iidN(0, \sigma_\xi^2)$ , with  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\beta^2 = 0.0004$ ,  $corr_{\alpha\beta} = 0.00$ ,  $\sigma_{u,me}^2 = 0.04$ ,  $\rho = 1$ . Models are estimated by the equally weighted minimum distance method. Standard errors in parentheses calculated as the standard deviations of the estimates across 100 model simulations.



TABLE 4: ESTIMATES OF THE MISSPECIFIED HIP: SIMULATED DATA IN LEVELS, AR(1) STATIONARY COMPONENT.

	Low Pers., $\sigma_\beta^2=0.00$		High Pers., $\sigma_\beta^2=0.00$	
	(1)	(2)	(3)	(4)
$\hat{\sigma}_\beta^2$	0.0006 (0.0003)	0.0009 (0.00008)	0.0008 (0.0002)	0.0009 (0.00008)
$\hat{\sigma}_\alpha^2$	0.021 (0.016)	0.01 (0.012)	0.017 (0.017)	0.012 (0.013)
$\hat{\sigma}_\xi^2$	0.00 —	0.00 —	0.00 —	0.00 —
$\hat{\phi}$	0.915 (0.048)	0.742 (0.062)	0.840 (0.056)	0.764 (0.042)
$\hat{\sigma}_\epsilon^2$	0.02 (0.003)	0.038 (0.003)	0.032 (0.004)	0.043 (0.003)
$\hat{\sigma}_{u,me}^2$	0.072 (0.002)	0.04 —	0.059 (0.005)	0.04 —

*Notes:* The true income process is:  $y_{iht} = \alpha_i + p_{iht} + (1 - \phi L)^{-1} \epsilon_{iht} + u_{iht,me}$ , where  $p_{ih+1t+1} = p_{iht} + \xi_{ih+1t+1}$ ,  $\alpha_i \sim iidN(0, \sigma_\alpha^2)$ ,  $u_{iht,me} \sim iidN(0, \sigma_{u,me}^2)$ ,  $\xi_{iht} \sim iidN(0, \sigma_\xi^2)$ ,  $\epsilon_{iht} \sim iidN(0, \sigma_\epsilon^2)$ , with  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_{u,me}^2 = 0.04$ . In the first and second columns,  $\phi = 0.25$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_\xi^2 = 0.02$ ; in the third and fourth columns,  $\phi = 0.50$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_\xi^2 = 0.02$ . The estimated income process is:  $y_{iht} = \alpha_i + \beta_i h + (1 - \phi L)^{-1} \tilde{\epsilon}_{iht} + u_{iht,me}$ . Models are estimated by the equally weighted minimum distance method. Standard errors in parentheses calculated as the standard deviations of the estimates across 100 model simulations.

TABLE 5: ESTIMATES OF THE MISSPECIFIED HIP: SIMULATED DATA IN LEVELS, MA(1) STATIONARY COMPONENT.

	Low Pers., $\sigma_\beta^2=0.00$		High Pers., $\sigma_\beta^2=0.00$	
	(1)	(2)	(3)	(4)
$\hat{\sigma}_\beta^2$	0.0005 (0.00035)	0.0009 (0.00007)	0.0007 (0.00027)	0.0009 (0.00005)
$\hat{\sigma}_\alpha^2$	0.026 (0.016)	0.0081 (0.01)	0.019 (0.017)	0.0068 (0.009)
$\hat{\sigma}_\xi^2$	0.00 —	0.00 —	0.00 —	0.00 —
$\hat{\phi}$	0.929 (0.06)	0.715 (0.069)	0.862 (0.074)	0.689 (0.052)
$\hat{\sigma}_\epsilon^2$	0.021 (0.003)	0.039 (0.004)	0.026 (0.004)	0.047 (0.004)
$\hat{\sigma}_{u,me}^2$	0.074 (0.003)	0.04 —	0.071 (0.005)	0.04 —

*Notes:* The true income process is:  $y_{iht} = \alpha_i + p_{iht} + (1 + \varphi L)\epsilon_{iht} + u_{iht,me}$ , where  $p_{ih+1t+1} = p_{iht} + \xi_{ih+1t+1}$ ,  $\alpha_i \sim iidN(0, \sigma_\alpha^2)$ ,  $u_{iht,me} \sim iidN(0, \sigma_{u,me}^2)$ ,  $\xi_{iht} \sim iidN(0, \sigma_\xi^2)$ ,  $\epsilon_{iht} \sim iidN(0, \sigma_\epsilon^2)$ , with  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_{u,me}^2 = 0.04$ . In the first and second columns,  $\varphi = 0.25$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_\xi^2 = 0.02$ ; in the third and fourth columns,  $\varphi = 0.50$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_\xi^2 = 0.02$ . The estimated income process is:  $y_{iht} = \alpha_i + \beta_i h + (1 - \phi L)^{-1}\tilde{\epsilon}_{iht} + u_{iht,me}$ . Models are estimated by the equally weighted minimum distance method. Standard errors in parentheses.

TABLE 6: ESTIMATES OF THE HIP: SIMULATED DATA IN FIRST DIFFERENCES, MA(1) STATIONARY COMPONENT.

	Low Pers. Trans. Comp.		High Pers. Trans. Comp.	
	$\sigma_\beta^2=0.00$ (1)	$\sigma_\beta^2=0.0004$ (2)	$\sigma_\beta^2=0.00$ (3)	$\sigma_\beta^2=0.0004$ (4)
$\hat{\sigma}_\beta^2$	0.0007 (0.00004)	0.0004 (0.00008)	0.0007 (0.00005)	0.0004 (0.00008)
$\hat{\sigma}_\xi^2$	0.00 —	0.02 (0.001)	0.00 —	0.02 (0.001)
$\hat{\theta}$	0.28 (0.01)	0.25 (0.01)	0.475 (0.01)	0.501 (0.013)
$\hat{\sigma}_\epsilon^2$	0.052 (0.0006)	0.04 (0.001)	0.052 (0.0005)	0.04 (0.001)
$\sigma_{u,me}^2$	0.04 —	0.04 —	0.04 —	0.04 —

*Notes:* In the first and third columns, the true income process is:  $y_{iht} = \alpha_i + p_{iht} + (1 + \theta L)\epsilon_{iht} + u_{iht,me}$ , with  $(1 - L)p_{ih+1t+1} = \xi_{ih+1t+1}$ , and  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\xi^2 = 0.02$ ,  $\theta = 0.25$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_{u,me}^2 = 0.04$  in the first column;  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\xi^2 = 0.01$ ,  $\theta = 0.50$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_{u,me}^2 = 0.04$  in the third column. In the first and third columns, the estimated income process is:  $y_{iht} = \alpha_i + \beta_i h + (1 + \tilde{\theta}L)\tilde{\epsilon}_{iht} + u_{iht,me}$ . In the second and fourth columns, the true income process is:  $y_{iht} = \alpha_i + \beta_i h + p_{iht} + (1 + \theta L)\epsilon_{iht} + u_{iht,me}$ , with  $(1 - L)p_{ih+1t+1} = \xi_{ih+1t+1}$ , and  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\beta^2 = 0.0004$ ,  $\sigma_\xi^2 = 0.02$ ,  $\theta = 0.25$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_{u,me}^2 = 0.04$  in the second column;  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\beta^2 = 0.0004$ ,  $\sigma_\xi^2 = 0.02$ ,  $\theta = 0.50$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_{u,me}^2 = 0.04$  in the fourth column. Prior to estimation, simulated data are transformed to first differences; models are estimated by the equally weighted minimum distance method. Standard errors in parentheses calculated as the standard deviations of the estimates across 100 model simulations.

TABLE 7: ESTIMATES OF THE HIP: SIMULATED DATA IN FIRST DIFFERENCES, AR(1) STATIONARY COMPONENT.

	Low Pers. Trans. Comp.		High Pers. Trans. Comp.	
	$\sigma_\beta^2=0.00$ (1)	$\sigma_\beta^2=0.0004$ (2)	$\sigma_\beta^2=0.00$ (3)	$\sigma_\beta^2=0.0004$ (4)
$\hat{\sigma}_\beta^2$	0.0006 (0.00005)	0.0004 (0.0001)	0.0006 (0.00005)	0.0004 (0.0001)
$\hat{\sigma}_\xi^2$	0.00 —	0.02 (0.002)	0.00 —	0.02 (0.003)
$\hat{\varphi}$	0.60 (0.05)	0.243 (0.095)	0.682 (0.022)	0.488 (0.063)
$\hat{\sigma}_\epsilon^2$	0.044 (0.003)	0.046 (0.016)	0.054 (0.002)	0.04 (0.003)
$\hat{\sigma}_{u,me}^2$	0.054 (0.003)	0.034 (0.016)	0.044 (0.001)	0.039 (0.003)

*Notes:* In the first and third columns, the true income process is:  $y_{iht} = \alpha_i + p_{iht} + (1 - \varphi L)^{-1} \epsilon_{iht} + u_{iht,me}$ , with  $(1 - L)p_{ih+1t+1} = \xi_{ih+1t+1}$ , and  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\xi^2 = 0.02$ ,  $\varphi = 0.25$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_{u,me}^2 = 0.04$  in the first column;  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\xi^2 = 0.02$ ,  $\varphi = 0.50$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_{u,me}^2 = 0.04$  in the third column. In the first and third columns, the estimated income process is:  $y_{iht} = \alpha_i + \beta_i h + (1 - \tilde{\varphi} L)^{-1} \tilde{\epsilon}_{iht} + u_{iht,me}$ . In the second and fourth columns, the true income process is:  $y_{iht} = \alpha_i + \beta_i h + p_{iht} + (1 - \varphi L)^{-1} \epsilon_{iht} + u_{iht,me}$ , with  $(1 - L)p_{ih+1t+1} = \xi_{ih+1t+1}$ , and  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\beta^2 = 0.0004$ ,  $\sigma_\xi^2 = 0.02$ ,  $\varphi = 0.25$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_{u,me}^2 = 0.04$  in the second column;  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\beta^2 = 0.0004$ ,  $\sigma_\xi^2 = 0.02$ ,  $\varphi = 0.50$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_{u,me}^2 = 0.04$  in the fourth column. Prior to estimation, simulated data are transformed to first differences; models are estimated by the equally weighted minimum distance method. Standard errors in parentheses calculated as the standard deviations of the estimates across 100 model simulations.

TABLE 8: AUTOCOVARIANCES FOR INCOME PROCESSES WITH GROWTH-RATE HETEROGENEITY AND A RANDOM WALK COMPONENT.

Order	$\sigma_\beta^2=0.0004, \sigma_\xi^2=0.02$ $\tau_{iht} \sim \text{MA}(1), \theta = 0.50$	$\sigma_\beta^2=0.0004, \sigma_\xi^2=0.02$ $\tau_{iht} \sim \text{AR}(1), \phi = 0.50$
0	0.16007 (0.00085)	0.15356 (0.00083)
1	-0.04962 (0.00061)	-0.05299 (0.00061)
2	-0.01959 (0.00067)	-0.00633 (0.00065)
3	0.00039 (0.00068)	-0.00296 (0.00066)
4	0.00045 (0.00072)	-0.00129 (0.00069)
5	0.00037 (0.00073)	-0.00043 (0.00072)
6	0.00039 (0.00073)	-0.00002 (0.00075)
7	0.00040 (0.00083)	0.00016 (0.00074)
8	0.00039 (0.00085)	0.00032 (0.00081)
9	0.00044 (0.00088)	0.00038 (0.00087)
10	0.00041 (0.00093)	0.00037 (0.00088)
11	0.00039 (0.00094)	0.00036 (0.00093)
12	0.00039 (0.00101)	0.00038 (0.00103)
13	0.00040 (0.00110)	0.00045 (0.00109)
14	0.00045 (0.00119)	0.00035 (0.00115)
15	0.00031 (0.00121)	0.00039 (0.00119)
16	0.00047 (0.00136)	0.00044 (0.00124)
17	0.00032 (0.00141)	0.00037 (0.00136)
18	0.00040 (0.00154)	0.00040 (0.00154)
19	0.00038 (0.00176)	0.00035 (0.00172)
20	0.00038 (0.00194)	0.00040 (0.00193)

*Notes:* In the first column, the true income process is:  $y_{iht} = \alpha_i + p_{iht} + (1 + \theta L)\epsilon_{iht} + u_{iht,me}$ , with  $(1 - L)p_{ih+1t+1} = \xi_{ih+1t+1}$ , and  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\xi^2 = 0.02$ ,  $\theta = 0.50$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_{u,me}^2 = 0.04$ . In the second column, the true income process is:  $y_{iht} = \alpha_i + \beta_i h + p_{iht} + (1 - \phi L)^{-1}\epsilon_{iht} + u_{iht,me}$ , with  $(1 - L)p_{ih+1t+1} = \xi_{ih+1t+1}$ , and  $\sigma_\alpha^2 = 0.03$ ,  $\sigma_\beta^2 = 0.0004$ ,  $\sigma_\xi^2 = 0.02$ ,  $\phi = 0.50$ ,  $\sigma_\epsilon^2 = 0.04$ ,  $\sigma_{u,me}^2 = 0.04$ . Simulated data are transformed to first differences. Autocovariances of a given order are the averages of the autocovariances in simulated data across 1,000 simulations. Standard errors in parentheses calculated as the standard deviations of the estimated autocovariances of a given order across 1,000 model simulations.

TABLE 9: ESTIMATES OF INCOME PROCESSES. PSID DATA IN LEVELS.

	(1)	(2)	(3)
$\hat{\sigma}_\beta^2$	0.0003 (0.0001)	0.0003 (0.00008)	0.00 —
$\hat{\sigma}_\alpha^2$	0.025 (0.40)	0.00 (0.03)	0.06 (0.02)
$\hat{\sigma}_{\alpha\beta}$	-0.001 (0.01)	0.00 —	0.00 —
$\hat{\phi}$	0.769 (0.031)	0.80 (0.03)	0.91 (0.02)
$\hat{\sigma}_\epsilon^2$	0.05 (0.004)	0.047 (0.003)	0.036 (0.003)
$\hat{\sigma}_{u,me}^2$	0.071 (0.004)	0.075 (0.003)	0.091 (0.004)

*Notes:* The estimated income process is:  $y_{iht} = \alpha_i + \beta_i h + (1 - \phi L)^{-1} \epsilon_{iht} + u_{iht,me}$ . Models are estimated by the equally weighted minimum distance method. Sample consists of 4,551 male household heads. Standard errors in parentheses.

TABLE 10: ESTIMATES OF INCOME PROCESSES. PSID DATA IN FIRST DIFFERENCES.

	R.W. & AR(1)			R.W. & MA(1)	
	(1)	(2)	(3)	(4)	(5)
$\hat{\sigma}_\beta^2$	0.0007 (0.00005)	0.00 (0.0001)	0.00 (0.0001)	0.001 (0.00005)	0.00 (0.0001)
$\hat{\sigma}_\xi^2$	0.00 —	0.036 (0.003)	0.03 (0.004)	0.00 —	0.052 (0.002)
$\hat{\phi}$	0.659 (0.02)	0.392 (0.059)	0.56 (0.03)	0.355 (0.012)	0.33 (0.02)
$\hat{\sigma}_\epsilon^2$	0.092 (0.004)	0.072 (0.007)	0.063 (0.004)	0.068 (0.002)	0.038 (0.002)
$\hat{\sigma}_{u,me}^2$	0.023 (0.002)	0.009 (0.007)	0.023 —	0.023 —	0.023 —

*Notes:* In columns (1)–(3), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 - \phi L)^{-1} \epsilon_{iht} + u_{iht,me}$ , where  $p_{ih+1t+1} = p_{iht} + \xi_{ih+1t+1}$ . In columns (4)–(5), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 + \phi L) \epsilon_{iht} + u_{iht,me}$ . Models are estimated by the equally weighted minimum distance method. Sample consists of 4,551 male household heads. Standard errors in parentheses.

TABLE 11: ESTIMATES OF INCOME PROCESSES. PSID DATA IN FIRST DIFFERENCES; 20 OR MORE OBSERVATIONS PER HEAD.

	R.W. & AR(1)			R.W. & MA(1)	
	(1)	(2)	(3)	(4)	(5)
$\hat{\sigma}_\beta^2$	0.00055 (0.00004)	0.00 (0.0001)	0.00 (0.0002)	0.00083 (0.00005)	0.00 (0.0001)
$\hat{\sigma}_\xi^2$	0.00 —	0.029 (0.004)	0.023 (0.005)	0.00 —	0.04 (0.003)
$\hat{\phi}$	0.657 (0.033)	0.379 (0.097)	0.56 (0.05)	0.355 (0.021)	0.33 (0.04)
$\hat{\sigma}_\epsilon^2$	0.073 (0.005)	0.059 (0.01)	0.052 (0.005)	0.055 (0.003)	0.031 (0.003)
$\hat{\sigma}_{u,me}^2$	0.023 (0.003)	0.01 (0.01)	0.023 —	0.023 —	0.023 —

*Notes:* In columns (1)–(3), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 - \phi L)^{-1} \epsilon_{iht} + u_{iht,me}$ , where  $p_{ih+1t+1} = p_{iht} + \xi_{ih+1t+1}$ . In columns (4)–(5), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 + \phi L) \epsilon_{iht} + u_{iht,me}$ . Models are estimated by the equally weighted minimum distance method. Sample consists of 1,034 male household heads. Standard errors in parentheses.



TABLE 12: ESTIMATES OF INCOME PROCESSES. PSID DATA IN FIRST DIFFERENCES; 20 OR MORE OBSERVATIONS PER HEAD; VARIANCES AND FIRST 10 AUTOCOVARIANCES USED.

	R.W. & AR(1)			R.W. & MA(1)	
	(1)	(2)	(3)	(4)	(5)
$\hat{\sigma}_\beta^2$	0.001 (0.0002)	0.00 (0.0004)	0.00 (0.0005)	0.0015 (0.0002)	0.00 (0.0002)
$\hat{\sigma}_\xi^2$	0.00 —	0.029 (0.006)	0.023 (0.008)	0.00 —	0.04 (0.003)
$\hat{\phi}$	0.640 (0.035)	0.379 (0.115)	0.561 (0.07)	0.355 (0.021)	0.33 (0.04)
$\hat{\sigma}_\epsilon^2$	0.073 (0.005)	0.059 (0.01)	0.052 (0.008)	0.054 (0.003)	0.031 (0.003)
$\hat{\sigma}_{u,me}^2$	0.022 (0.004)	0.01 (0.01)	0.023 —	0.023 —	0.023 —

*Notes:* In columns (1)–(3), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 - \phi L)^{-1} \epsilon_{iht} + u_{iht,me}$ , where  $p_{ih+1t+1} = p_{iht} + \xi_{ih+1t+1}$ . In columns (4)–(5), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 + \phi L) \epsilon_{iht} + u_{iht,me}$ . Models are estimated by the equally weighted minimum distance method. Sample consists of 1,034 male household heads. Standard errors in parentheses.

TABLE 13: ESTIMATES OF INCOME PROCESSES. PSID DATA IN FIRST DIFFERENCES.  
TIME-VARYING VARIANCES.

	(1)	(2)	(3)	(4)	(5)
$\hat{\sigma}_\beta^2$	0.0008 (0.00006)	0.0008 (0.00006)	0.00 (0.0001)	0.00 (0.0001)	0.00 —
$\hat{\sigma}_\xi^2$	0.00 —	0.00 —	0.037 (0.003)	0.037 (0.004)	0.038 <sup>a</sup> (0.01)
$\hat{\phi}$	0.605 (0.02)	0.650 (0.013)	0.410 (0.041)	0.525 (0.03)	0.516 (0.029)
$\hat{\sigma}_\epsilon^2$	0.096 <sup>a</sup> (0.009)	0.091 <sup>a</sup> (0.009)	0.071 <sup>a</sup> (0.01)	0.057 <sup>a</sup> (0.009)	0.057 <sup>a</sup> (0.01)
$\hat{\sigma}_{u,me}^2$	0.018 (0.003)	0.023 —	0.01 (0.005)	0.023 —	0.023 —

*Notes:* <sup>a</sup>Variances differ by calendar year; average variances and average standard errors reported. The estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 - \phi L)^{-1} \epsilon_{iht} + u_{iht,me}$ , where  $p_{ih+1t+1} = p_{iht} + \xi_{ih+1t+1}$ . Models are estimated by the equally weighted minimum distance method. Sample consists of 4,551 male household heads. Standard errors in parentheses.

TABLE 14: THE VARIANCES OF PERMANENT AND TRANSITORY SHOCKS BY YEAR.

Year	Trans. shock	St. err.	Perm. shock	St. err.
1968	0.01038 <sup>a</sup>	0.01124	0.03992	0.01319
1969	0.01038 <sup>a</sup>	0.00807	0.05569	0.01151
1970	0.02413	0.00842	0.04722	0.00841
1971	0.04144	0.00906	0.05835	0.01138
1972	0.04558	0.00896	0.03796	0.00838
1973	0.03294	0.00769	0.03584	0.00833
1974	0.03989	0.00824	0.04386	0.00821
1975	0.06184	0.00916	0.02531	0.00673
1976	0.06418	0.00986	0.03101	0.00765
1977	0.05425	0.00920	0.02235	0.00724
1978	0.04942	0.00826	0.02960	0.00723
1979	0.03480	0.00744	0.04407	0.00830
1980	0.03170	0.00768	0.04870	0.00899
1981	0.05268	0.00941	0.04182	0.00883
1982	0.08555	0.01151	0.05358	0.01083
1983	0.05267	0.00947	0.04184	0.00829
1984	0.06850	0.00990	0.05071	0.00909
1985	0.06362	0.00903	0.02575	0.00801
1986	0.04873	0.00813	0.03960	0.00775
1987	0.05606	0.00915	0.03796	0.00873
1988	0.06444	0.00971	0.04614	0.00982
1989	0.04370	0.00745	0.04083	0.00774
1990	0.05625	0.00851	0.03605	0.00768
1991	0.06827	0.01032	0.03411	0.00977
1992	0.11424	0.01240	0.02925	0.01054
1993	0.10641	0.01120	0.02840	0.01004
1994	0.09898	0.01283	0.02815	0.00976
1995	0.08906	0.01507	0.01845 <sup>a</sup>	0.01349
1996	0.07437	0.01563	0.01845 <sup>a</sup>	0.02210

*Notes:* Estimates from the model in Table 13, column (5). <sup>a</sup>Variances are restricted in estimation to be equal in those years.

TABLE 15: SUMMARY STATISTICS.

Survey year	Age		Yrs. of schooling Mean	Labor income		Number of heads N
	Mean	Std.		Mean	Std. of log-income	
1968	38.55	9.89	11.85	25959	0.56	1285
1969	38.39	10.44	11.96	26476	0.60	1420
1970	38.19	10.84	12.02	26626	0.63	1536
1971	37.69	11.02	12.10	26072	0.65	1596
1972	37.38	11.21	12.16	26026	0.64	1673
1973	36.84	11.12	12.24	27118	0.60	1737
1974	36.52	11.03	12.32	27753	0.59	1788
1975	36.01	10.92	12.63	26704	0.62	1890
1976	35.89	10.89	12.70	25077	0.61	1918
1977	36.02	10.93	12.71	26079	0.62	1980
1978	36.07	10.86	12.76	26714	0.61	2020
1979	36.11	10.75	12.82	27479	0.62	2057
1980	36.19	10.67	12.87	27132	0.62	2101
1981	36.20	10.52	12.94	25552	0.65	2106
1982	36.31	10.39	13.01	24744	0.66	2122
1983	36.42	10.35	13.22	24984	0.73	2153
1984	36.56	10.19	13.22	25290	0.72	2152
1985	36.78	10.07	13.52	26533	0.70	2202
1986	36.99	9.96	13.53	26652	0.72	2221
1987	37.18	9.79	13.56	26992	0.73	2251
1988	37.38	9.66	13.56	27421	0.72	2271
1989	37.58	9.56	13.62	27975	0.73	2255
1990	37.76	9.45	13.63	27969	0.70	2272
1991	37.89	9.28	13.64	27381	0.72	2263
1992	38.15	9.16	13.66	27008	0.74	2288
1993	38.40	9.21	13.69	27973	0.75	2279
1994	38.60	9.22	13.70	28294	0.74	2469
1995	38.82	9.30	13.70	28350	0.74	2462
1996	39.54	9.19	13.71	29227	0.73	2382
1997	40.38	8.99	13.76	29659	0.71	2269

*Notes:* Mean head's labor income in 1982–1984 dollars.

## Appendix A: The Autocovariance Function and Estimation Details.

In this appendix, I present theoretical autocovariances and variances for the model (1)–(3) used to identify the model. For convenience, I reproduce the model equations here, assuming that the transitory component is an autoregressive process of order 1.

$$\begin{aligned} y_{iht} &= \alpha_i + \beta_i h + p_{iht} + \tau_{iht} + u_{iht,me} \\ p_{iht} &= p_{ih-1t-1} + \xi_{iht} \\ \tau_{iht} &= (1 - \phi L)^{-1} \epsilon_{iht}, \end{aligned}$$

where  $(\alpha_i, \beta_i) \sim iid(0, \Omega)$ , with  $\Omega_{11} = \sigma_\alpha^2$ ,  $\Omega_{12} = \Omega_{21} = \sigma_{\alpha\beta}$ ,  $\Omega_{22} = \sigma_\beta^2$ ;  $u_{iht,me} \sim iid(0, \sigma_{u,me}^2)$ ;  $\xi_{iht} \sim iid(0, \sigma_\xi^2)$ ;  $\epsilon_{iht} \sim iid(0, \sigma_\epsilon^2)$ . The moments used in matching estimations are:

$$\begin{aligned} var(y_{iht}) &= \sigma_\alpha^2 + \sigma_\beta^2 h^2 + 2\sigma_{\alpha\beta} h + \sigma_{u,me}^2 + var(\tau_{iht}) + var(p_{iht}), \quad t = 1, \dots, T, \quad h = 1, \dots, H \\ var(\tau_{i1t}) &= \sigma_\epsilon^2 \quad var(p_{i1t}) = \sigma_\xi^2, \quad t = 1, \dots, T \\ var(\tau_{ih1}) &= \sigma_\epsilon^2 \sum_{j=0}^{h-1} \phi^{2j} \quad var(p_{ih1}) = \sum_{j=0}^{h-1} \sigma_\xi^2, \quad t = 1, \quad h = 2, \dots, H \\ var(\tau_{iht}) &= \phi^2 var(\tau_{ih-1t-1}) + \sigma_\epsilon^2 \quad var(p_{iht}) = var(p_{ih-1t-1}) + \sigma_\xi^2, \quad t = 2, \dots, T, \quad h = 2, \dots, H \\ cov(y_{iht}, y_{ih+kt+k}) &= \phi^k var(\tau_{iht}) + var(p_{iht}) + \\ &+ \sigma_\alpha^2 + \sigma_{\alpha\beta}(2h+k) + \sigma_\beta^2 h(h+k), \quad k = 1, \dots, \min(H-h, T-t), \quad h = 1, \dots, H, \quad t = 1, \dots, T, \end{aligned}$$

where  $H$  is the maximum labor market experience in the sample, and  $T$  is the time dimension of the sample.

I am assuming that  $\tau_{i0t} = 0$  and  $p_{i0t} = 0$ , i.e., a head with no labor market experience entering the labor market at time  $t+1$  is “endowed” with zero permanent and transitory components of earnings.

For idiosyncratic labor income growth, the above model is:

$$\Delta y_{it} = \beta_i + \xi_{it} + (1 - \phi L)^{-1} \Delta \epsilon_{it} + \Delta u_{it,me}.$$

The autocovariance moments are shown in equations (11)–(13). If the transitory component is a moving average process of order 1, see the autocovariance function in the text in equations (5)–(8).

The empirical moments, taking into account that the data used in estimations are unbalanced, are calculated as:

$$\left( \sum_{i=1}^N \tilde{y}_i \tilde{y}_i' \right) / N_{tt'},$$

where  $\tilde{y}_i = (y_{i(h)1}, y_{i(h+1)2}, \dots, y_{i(h+T-1)T})$  if data are in levels; and  $\tilde{y}_i = (\Delta y_{i2}, \Delta y_{i3}, \dots, \Delta y_{iT})$  if data are in first differences;  $N$  is the total number of heads in the sample;  $N_{tt'}$  is a matrix with the row and column dimensions  $\frac{T(T+1)}{2}$ ;  $N_{11}$  is the number of heads contributing towards estimation of the variance in period 1 ( $t = 1, t' = 1$ );  $N_{12}$ —the number of heads contributing towards estimation of the first-order autocovariance between periods 1 and 2 ( $t = 1, t' = 2$ ), etc.<sup>23</sup> The vector of data moments used in estimation is  $m_{\frac{T(T+1)}{2}}^d = vech \left( \left( \sum_{i=1}^N \tilde{y}_i \tilde{y}_i' \right) / N_{tt'} \right)$ , where  $\frac{T(T+1)}{2}$  is the row dimension of the empirical moments' vector. The model parameters,  $\Theta$ , are recovered by minimizing a squared distance function  $[m(\Theta) - m^d]' I_{\frac{T(T+1)}{2}} [m(\Theta) - m^d]$ , where  $I_{\frac{T(T+1)}{2}}$  is an identity matrix with the row dimension  $\frac{T(T+1)}{2}$ .

Standard errors of the parameters are calculated as the square roots of the diagonal of  $(G'_\Theta G_\Theta)^{-1} G'_\Theta V G_\Theta (G'_\Theta G_\Theta)^{-1}$ , where  $G_\Theta = \frac{\partial}{\partial \Theta} [m(\hat{\Theta}) - m^d]$ , a vector with the row dimension  $\frac{T(T+1)}{2}$ , and the column dimension equal to the row

<sup>23</sup>Note that if the head's income is missing, say, in period 1, this head's contributions towards the variance at time 1 and all the sample autocovariances involving this period are zero.

dimension of the vector of estimated parameters;  $V$  is equal to  $[diag(vch(\sqrt{N_{tt'}}))]^{-1} \Omega [diag(vch(\sqrt{N_{tt'}}))]^{-1'}$ , where  $\Omega$  is a matrix of the fourth moments and  $diag(\cdot)$  is a diagonal matrix with the row and column dimension  $\frac{T(T+1)}{2}$ , with the diagonal elements equal to  $vch(\sqrt{N_{tt'}})$ .<sup>24</sup>

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<sup>24</sup>See MaCurdy (2007), Section 6.6.2 for more details on estimating time series models utilizing unbalanced panel data.

## Appendix B: Additional Results.

TABLE B-1: ESTIMATES OF INCOME PROCESSES FOR HIGH SCHOOL DROPOUTS. PSID DATA IN FIRST DIFFERENCES.

	R.W. & AR(1)			R.W. & MA(1)	
	(1)	(2)	(3)	(4)	(5)
$\hat{\sigma}_\beta^2$	0.0007 (0.00005)	0.00 (0.0004)	0.00 (0.0004)	0.001 (0.0003)	0.00 (0.0003)
$\hat{\sigma}_\xi^2$	0.00 —	0.042 (0.009)	0.039 (0.01)	0.00 —	0.049 (0.007)
$\hat{\phi}$	0.647 (0.089)	0.209 (0.253)	0.291 (0.089)	0.251 (0.034)	0.196 (0.056)
$\hat{\sigma}_\epsilon^2$	0.089 (0.014)	0.094 (0.09)	0.078 (0.013)	0.095 (0.006)	0.066 (0.008)
$\hat{\sigma}_{u,me}^2$	0.048 (0.01)	0.004 (0.09)	0.023 —	0.023 —	0.023 —

*Notes:* In columns (1)–(3), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 - \phi L)^{-1} \epsilon_{iht} + u_{iht,me}$ , where  $p_{ih+1t+1} = p_{iht} + \xi_{ih+1t+1}$ . In columns (4)–(5), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 + \phi L) \epsilon_{iht} + u_{iht,me}$ . Models are estimated by the equally weighted minimum distance method. Sample consists of 778 male household heads. Standard errors in parentheses.

TABLE B-2: ESTIMATES OF INCOME PROCESSES FOR HIGH SCHOOL GRADUATES. PSID DATA IN FIRST DIFFERENCES.

	R.W. & AR(1)			R.W. & MA(1)	
	(1)	(2)	(3)	(4)	(5)
$\hat{\sigma}_\beta^2$	0.0007 (0.00007)	0.00 (0.0001)	0.00 (0.0001)	0.001 (0.00008)	0.00 (0.0001)
$\hat{\sigma}_\xi^2$	0.00 —	0.032 (0.004)	0.027 (0.004)	0.00 —	0.048 (0.003)
$\hat{\phi}$	0.641 (0.028)	0.402 (0.074)	0.535 (0.038)	0.343 (0.016)	0.312 (0.028)
$\hat{\sigma}_\epsilon^2$	0.086 (0.005)	0.069 (0.009)	0.062 (0.005)	0.066 (0.002)	0.039 (0.003)
$\hat{\sigma}_{u,me}^2$	0.024 (0.003)	0.012 (0.009)	0.023 —	0.023 —	0.023 —

*Notes:* In columns (1)–(3), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 - \phi L)^{-1} \epsilon_{iht} + u_{iht,me}$ , where  $p_{ih+1t+1} = p_{iht} + \xi_{ih+1t+1}$ . In columns (4)–(5), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 + \phi L) \epsilon_{iht} + u_{iht,me}$ . Models are estimated by the equally weighted minimum distance method. Sample consists of 2,631 male household heads. Standard errors in parentheses.



TABLE B-3: ESTIMATES OF INCOME PROCESSES FOR COLLEGE GRADUATES. PSID DATA IN FIRST DIFFERENCES.

	R.W. & AR(1)			R.W. & MA(1)	
	(1)	(2)	(3)	(4)	(5)
$\hat{\sigma}_\beta^2$	0.0005 (0.0002)	0.00 (0.0005)	0.00 (0.0003)	0.001 (0.0002)	0.00 (0.0002)
$\hat{\sigma}_\xi^2$	0.00 —	0.037 (0.017)	0.04 (0.01)	0.00 —	0.068 (0.006)
$\hat{\phi}$	0.822 (0.03)	0.662 (0.15)	0.584 (0.089)	0.321 (0.03)	0.217 (0.084)
$\hat{\sigma}_\epsilon^2$	0.084 (0.007)	0.052 (0.011)	0.053 (0.01)	0.064 (0.004)	0.024 (0.005)
$\hat{\sigma}_{u,me}^2$	0.03 (0.004)	0.027 (0.006)	0.023 —	0.023 —	0.023 —

*Notes:* In columns (1)–(3), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 - \phi L)^{-1} \epsilon_{iht} + u_{iht,me}$ , where  $p_{ih+1t+1} = p_{iht} + \xi_{ih+1t+1}$ . In columns (4)–(5), the estimated income process is:  $y_{iht} = \alpha_i + p_{iht} + \beta_i h + (1 + \phi L) \epsilon_{iht} + u_{iht,me}$ . Models are estimated by the equally weighted minimum distance method. Sample consists of 450 male household heads. Standard errors in parentheses.