Business Cycles in the Equilibrium Model of Labor Market Search and Self-Insurance

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Abstract

The standard Mortensen-Pissarides model of search and matching is extended by introducing capital, risk-averse workers, labor-leisure choice and lack of complete markets to insure away unemployment shocks. The business cycle properties of the model with aggregate productivity shocks are explored, with an emphasis on labor market dynamics. In particular, I ask whether the model can replicate the large volatility of unemployment and vacancies observed in the U.S. economy. Shimer (2005) finds that the standard Mortensen-Pissarides model with productivity shocks of a plausible magnitude does not have a strong amplification mechanism and thus cannot generate the observed cyclical properties of unemployment and vacancies. I find that the answer is yes; the model can generate the observed large volatility of unemployment and vacancies, with a standard calibration. Two channels make the cyclical properties of the model different from Shimer's (2005) model, namely, the role of savings in mitigating the negative effect of the unemployment shock on current consumption and the additional utility for the unemployed from leisure. I show that both are crucial in the strong amplification mechanism of the baseline model. I also compare the business cycle properties of the model with those of the standard real business cycle model. The model endogenously produces both extensive and intensive margins of labor supply adjustments and thus is able to replicate some business cycle properties of the U.S. economy which the standard model cannot.

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1 Introduction

This paper has two main purposes. The first and the main purpose is to extend the standard labor search and matching model by incorporating capital and labor-leisure choice and to re-examine the puzzle presented by Shimer (2005) in the extended model.

The Mortensen-Pissarides search and matching model has become the standard theory of equilibrium unemployment (Mortensen and Pissarides (1994), Pissarides (2001)). However, Shimer (2005) pointed out that the Mortensen-Pissarides model with labor productivity shocks cannot replicate the volatility of the unemployment and vacancies observed in the U.S. data. The volatilities of the unemployment and vacancies in Shimer's (2005) model are about one-tenth of their volatilities in the U.S. data.

In the baseline model constructed below, workers can accumulate capital but the market is incomplete; workers cannot write a contract to completely insure away the unemployment risk, but they can self-insure by accumulating capital. In addition, workers in the model choose how many hours to work and how many hours to enjoy leisure. Comparing the properties of the baseline model and the model by Shimer (2005) is virtually the same as exploring the role played by the two features, namely, precautionary saving and labor-leisure choice, in the baseline model economy.

I find that if the model is calibrated in the standard way that a real business cycle model is calibrated, the model can generate a large volatility of unemployment and vacancies. In other words, the model has a strong amplifying mechanism. I further find that both the ability of workers to save and that labor-leisure choice are crucial in generating high volatilities of unemployment and vacancies. I show that models that lack one of the two features cannot generate the same strong amplification as in the baseline model economy. The reason why both can play a crucial role is similar to the finding in Hagedorn and Manovskii (2005). Both the option to save and prepare for future unemployment spells and the additional utility for the unemployed from longer leisure time help to increase the relative value of being unemployed. As Hagedorn and Manovskii (2005) show, if the value of being unemployed is close to the value of being employed, labor search and matching models can generate a large volatility in unemployment and vacancies.

The second purpose of the paper is to compare the business cycle properties of the baseline model with those of the standard real business cycle model. The current model can also be considered the standard real business cycle model extended by incorporating search and matching in the labor market. Since the current model generates fluctuations in both employment (extensive margin) and average hours worked (intensive margin), the model has the capacity to match a variety of cyclical properties of U.S. business cycles, especially those associated with the labor market. With the capacity of the baseline model in mind, the cyclical properties of the model are explored and compared with those of the standard real business cycle model. I show that the model does a good job of replicating many cyclical properties of the U.S. labor market that the standard real business cycle model cannot produce.

The rest of the paper is organized as follows. In Section 2, related literature is reviewed. Section 3 summarizes the cyclical properties of the U.S. economy. The model's performance is measured by the extent to which the model can replicate the properties. In Section 4, the baseline model is presented. Section 5 discusses the calibration of the model, and Section 6 offers a brief discussion of the computational methods. Appendix A gives details of the computation. Section 7 presents the main results of the paper and discusses them. Section 8 uses the baseline model to evaluate the effect of the declining volatility of the productivity shock on the cyclical properties of macroeconomic aggregates. Section 9 concludes.

2 Related Literature

Two lines of the literature are related to the current paper. One line is trying to solve the puzzle presented by Shimer (2005) by extending the basic Mortensen-Pissarides model. The other line extends the macroeconomic models by incorporating labor market search and matching. To the best of my knowledge, there is no other model that combines the incomplete market model with labor market search and matching in the general equilibrium framework.

The first line of research starts with Shimer (2005), who finds that the standard Mortensen-Pissarides model of labor market search and matching with aggregate shocks to labor productivity cannot generate a large volatility of unemployment and vacancies.

Hall (2005) claims that the problem lies in the Nash bargaining assumption in wage setting. He points out that if there is stickiness in real wage setting, instead of real wage elastically responding to changes in productivity, the labor search and matching model can produce a large volatility of unemployment and vacancies. If real wage is sticky, firms' profit responds more to the changes in productivity, which leads to the larger volatility of vacancy postings and, eventually, unemployment.

Hagedorn and Manovskii (2005) claim that the problem does not lie in the model itself but in the way the model is calibrated. They show that the model can be calibrated in such a way that the volatility of unemployment and vacancies matches the volatility in the U.S. data. Crucial to their calibration is how the period utility of being unemployed is compared with that of being employed. Shimer (2005) assumes, based on the average replacement ratio, a substantially lower period utility of being unemployed, compared with the calibration of Hagedorn and Manovskii (2005).

Costain and Reiter (2005a) propose that cohort-specific technology shocks can improve the model's performance. Their model easily achieves a higher volatility of unemployment and vacancies because the number of vacancies is sensitive to the value of *new* matches, not to the value of *existing* matches. The cohort-specific technology shocks can increase the volatility of the value of new matches, without affecting the volatility of the value of existing matches. In turn, the volatility of vacancies increases without increasing the volatility of output, which is much less sensitive to the volatility of the value of new matches.

Pries (2007) introduces a heterogeneity across workers' productivity and shows that the composition effect associated with the productivity of workers help to generate a high volatility of unemployment and vacancies.

In the other line of research related to the current paper, Andolfatto (1996) and Merz (1995) are the seminal papers. Both point out that the performance of real business cycle models can be improved by incorporating labor search and matching. However, both fail to generate the observed high volatility of unemployment and vacancies.

A recent paper by Jung (2006) constructs a version of the real business cycle model with labor market search and marching and a representative family and shows that the model can replicate the volatility of unemployment and vacancies with reasonable parameter values. The main difference in this paper is that the existence of a representative family is assumed to simplify the model, while in the current paper, markets are incomplete, and agents can only self-insure. In the current paper, the importance of allowing workers to self-insure by saving is emphasized, whereas there is no endogenous self-insurance in Jung (2006).

Regarding the solution method, the one developed by Krusell and Smith (1998) is used. The model developed in this paper can be considered an extension of their model in the sense that the formation and resolution of matches is endogenous in the current model, while it is exogenous in Krusell and Smith's (1998) model.

3 Data

Table 1 summarizes the cyclical properties of the U.S. economy between 1951 and 2005. All data are quarterly. The cyclical properties are computed using the log of the data after filtering with the Hodrick-Prescott (H-P) filter. Following the standard practice in the business cycle literature for quarterly data, a smoothing parameter of 1600 is used.[1]

The first block of the table (the first three rows) contains the cyclical properties of output and its major components, namely, consumption and investment. The second block of the table (the fourth to the seventh rows) contains the cyclical properties of unemployment, vacancies, their ratio and the probability that a worker in the unemployment pool finds a job. The data on the number of vacancies are constructed in the same way as in Shimer (2005); the help-wanted advertising index compiled by the Conference Board is used as a proxy for the number of vacancies. Monthly job finding probability is constructed in the way suggested by Shimer (2005). For quarterly data, the average of the monthly data in each quarter is used. The properties of these data are important because these are the ones Shimer (2005) claims the standard Mortensen-Pissarides model fails to replicate. The third and last block of the table (the last seven rows) summarizes cyclical properties of other major variables related to the labor

[1] Shimer (2005) uses a smoothing parameter of 10^5. According to Hornstein et al. (2005) (Table 1), there is no substantial difference in terms of the volatility of the important labor market variables relative to the volatility of labor productivity.

Table 1: Cyclical properties of U.S. economy: 1951:1-2005:4[1,2]

Variable	SD%	Relative SD%[3]	Auto-corr	Cross-Correlation of Output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Output	1.58	1.00	0.84	0.60	0.84	1.00	0.84	0.60
Consumption	1.25	0.79	0.83	0.68	0.82	0.85	0.70	0.46
Investment	7.19	4.57	0.80	0.57	0.76	0.88	0.72	0.48
Unemployment rate (u)	12.48	7.92	0.87	-0.35	-0.63	-0.84	-0.86	-0.73
Vacancies (v)	13.95	8.86	0.91	0.55	0.78	0.90	0.85	0.68
v/u ratio	25.91	16.45	0.90	0.46	0.72	0.89	0.87	0.72
Job finding probability	7.74	4.92	0.80	0.43	0.68	0.83	0.84	0.69
Compensation to employees	1.75	1.11	0.89	0.37	0.64	0.85	0.87	0.76
Labor share	1.07	0.68	0.72	-0.40	-0.34	-0.26	0.07	0.34
Total hours	1.74	1.10	0.89	0.44	0.69	0.87	0.87	0.75
Employment	1.00	0.63	0.89	0.33	0.59	0.80	0.86	0.78
Average weekly hours	0.51	0.32	0.77	0.59	0.70	0.71	0.52	0.28
Output per person	1.31	0.83	0.76	0.68	0.72	0.69	0.34	-0.01
Output per hour	1.04	0.66	0.69	0.54	0.55	0.52	0.17	-0.14
Compensation per hour	0.88	0.56	0.78	0.22	0.29	0.31	0.25	0.17

[1] Source: BEA (output and its components), BLS (labor market-related data), Conference Board (help-wanted advertising index used as the proxy for the number of vacancies).

[2] All data are quarterly between 1951:1 and 2005:4. Logs of the data are filtered using the H-P filter with a smoothing parameter of 1600.

[3] Relative to the standard deviation of output.

market. As I will show, the model constructed in the current paper can generate endogenously the fluctuations of the variables in the table.

The key features of the cyclical properties of the U.S. economy presented in Table 1 are summarized as follows:

1. Consumption (including durable and nondurable goods and services) is less volatile than output and strongly procyclical.

2. Investment is about five times as volatile as output and strongly procyclical.

3. The unemployment rate is about eight times as volatile as output and strongly countercyclical.

4. The number of vacancies posted is about nine times as volatile as output and strongly procyclical.

5. As a result of the above two properties, the ratio of vacancies over the unemployment rate is about 16 times as volatile as output and strongly procyclical.
6. Job finding probability is about five times as volatile as output and strongly procyclical.
7. Compensation to employees is as volatile as output and strongly procyclical.
8. Labor share (share of total income earned by employees) is as volatile as output and mildly countercyclical.
9. Total hours worked is as volatile as output and strongly procyclical. If the volatility of the total hours is disaggregated into the volatility of employment (extensive margin) and the volatility of hours per worker (intensive margin), the former accounts for two-thirds of the volatility of the total hours. The latter accounts for one-third.
10. Employment lags the cycle by about one quarter. This translates into a strong correlation between output and lagged total hours.
11. There is no lead or lag with respect to the average hours worked.
12. Since employment is procyclical but less volatile than output, output per person is moderately procyclical and less volatile than output.
13. Since total hours is more volatile than employment and both are procyclical, compensation per hour is less volatile than output per person and mildly procyclical.

The success of the model presented below is measured by how well the cyclical properties of the model replicate those in Table 1, in particular the cyclical properties of unemployment and vacancies.

4 Model

4.1 Preference

Time is discrete. The economy is populated by a mass of infinitely lived workers and firms. The total measure of the workers is normalized to unity. A worker maximizes his expected lifetime utility. The expected lifetime utility of a worker takes the following time-separable form:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right\} \tag{1}$$

where β is the time discount factor, and $\mathbb{E}_0$ is an expectation operator with information available at period 0. c_t is the consumption of the worker in period t. h_t is the leisure time enjoyed in period t. The period utility function $u(c, h)$ is assumed to be strictly increasing and strictly concave, and it satisfies Inada conditions with respect to each of c and h.

There are a large number of firms. Each firm is risk-neutral and maximizes the present value of its profit stream. The firms discount their future profits using the real interest rate.

4.2 Endowment

A worker is endowed with capital a_0 in the initial period and one unit of time each period. Workers can use their time for either work or leisure. Denote hours worked in period t as ℓ_t. Hours for leisure in period t is denoted as h_t. The following time constraint must be satisfied in any period:

$$h_t + \ell_t = 1 \tag{2}$$

4.3 Production Technology

In order to produce, a worker and a firm have to be matched. We call a matched worker employed ($e = 1$) and an unmatched worker unemployed ($e = 2$). A firm can also be either matched ($e = 1$) or unmatched ($e = 2$). Matched pairs have access to the following production technology:

$$Y = e^z F(K, L) \tag{3}$$

where K is capital input, L is labor input, and e^z is total factor productivity (TFP). z is stochastic and follows a first order autoregressive process. We will give a parameterized form for the process later. It is assumed that the function $F(K, L)$ satisfies constant returns to scale and is strictly increasing and strictly concave with respect to each input. Capital stock depreciates at a constant rate δ.

4.4 Job Turnover Technology

Workers without a job search for a job. I assume that there is no cost for job search. According to Reichling (2007), the average unemployed worker in 2005 spent as little as 3 minutes per day searching for a job. Therefore, no cost for job search is a reasonable approximation. There is no search intensity decision. The probability of finding a job is the same across all workers who are searching for a job. Unmatched firms search for a worker by posting a vacancy. The cost of posting a vacancy is represented by a parameter κ. The probability of finding a worker is the same across all firms that are searching for a worker.

Let the total number of workers that are employed and unemployed be denoted as N and U, respectively. Since the total number of workers is normalized to one, the number of employed workers N can be expressed as $N = 1 - U$. Let V be the total number of vacancies posted by unmatched firms.

For a matched pair, there is a constant probability λ that the match is dissolved. Because of the law of large numbers, when the measure of the employed workers (which is equal to the measure of matched pairs) is N, the measure of matches that dissolve can be expressed as λN. There is no on-the-job search. Once matched, the worker or the firm is not allowed to search for a new match until the current match dissolves.

It is assumed that a worker whose match is just dissolved can immediately join the unemployed pool and thus start searching for a new job. It implies that a worker whose match is dissolved but who immediately finds the next job does not experience an unemployed spell in the model. This timing assumption is needed to achieve a proper calibration. A period will be set as a quarter. If we force a worker in a dissolved match to stay unemployed for at least one period, the average duration of unemployment spells will be longer than one model period (one quarter). However, it is hard to reconcile the implied average duration of unemployment spells with the monthly average job finding probability of 45% (Shimer (2005)) and the average unemployment rate of 5.67%. To distinguish between the number of unemployed workers and the number of workers that are looking for a job, I use S for the measure of workers who are looking for a job. By definition:

$$S = U + \lambda N \tag{4}$$

It is assumed that the number of new matches M is a function of the total number of workers looking for a job (S) and the total number of vacancies posted (V). The function is called the aggregate matching function and is expressed as follows:

$$M = min(f_m(S, V), S, V) \tag{5}$$

4.5 Market Structure

Capital is exchanged in the competitive market. It implies that the rental price of capital is equal to the marginal product of capital net of depreciation in equilibrium. Labor can be supplied only in a matched pair. All the firms are assumed to be owned by all the workers. As a result, the total profit of the firms in each period, net of total costs for posting vacancies, is shared by the owners (workers) equally as a lump-sum transfer d_t.

The workers are not allowed to trade securities contingent on the state of the world and thus to insure against unemployment shocks. Partial insurance is provided by the public sector in the form of unemployment insurance, which is discussed in the next subsection. Apart from the partial public insurance, all that the workers can do is to save in the form of capital and self-insure. It is assumed that workers cannot take a short position with respect to capital. It is justified by the assumption that a worker can default on his loan at no cost or punishment.

4.6 Unemployment Insurance

The government runs an unemployment insurance program. Each period, the government taxes the labor income of all employed workers at a constant tax rate t_t and distributes the proceeds equally among the unemployed. Denote the amount of unemployment insurance benefit in period t as b_t. b_t and t_t are uniquely determined each period such that the following two conditions are simultaneously satisfied:

1. b_t is a constant fraction ξ of the average after-tax labor income of a worker in any given period. ξ is called the replacement ratio.

2. The government budget balances each period.

4.7 Recursive Formulation

Equilibrium is defined recursively. From now on, time subscripts are dropped and variables in the next period are denoted by prime. A worker can be characterized by employment status $e \in E = \{1, 2\}$ and the capital stock holding $a \in A = [0, \infty)$. For ease of notation, I define x as a probability measure over $\mathcal{X}$, which is a $\sigma - algebra$ generated by the set $X \equiv E \times A$. x is used to represent a type distribution of workers.

In any period, the aggregate state of the economy can be characterized by the currently realized shock to total factor productivity z, and the type distribution of workers, which is represented by the probability measure x. Notice that, since z follows a first order autoregressive process, the current z is sufficient to predict the future realizations of z.

Using the aggregate states, the total measure of employed and unemployed workers, N, U, the total measure of workers searching for a job S, the aggregate (or average, because the size of the total population is normalized to unity) stock holding A, and the number of matches M can be characterized by the following functions:

$$N(x) = \int_X \mathcal{I}_{e=1} dx \tag{6}$$

$$U(x) = \int_X \mathcal{I}_{e=2} dx \tag{7}$$

$$S(x) = U(x) + \lambda N(x) \tag{8}$$

$$A(x) = \int_X a \ dx \tag{9}$$

$$M(z, x) = min(f_m(S(x), V(z, x)), S(x), V(z, x)) \tag{10}$$

where $\mathcal{I}_{condition}$ is an indicator function that takes the value 1 if the *condition* is true, and 0 otherwise. $V(z, x)$ is the total number of vacancies posted, which is determined by the optimal entry decision of unmatched firms, and thus is a function of aggregate states. I will discuss more on $V(z, x)$ later.

Furthermore, these functions can be used to construct functions of matching probability for a worker searching for a job (f_w) and for an unmatched firm (f_j) as follows:

$$f_w(z,x) = \frac{M(z,x)}{S(x)} \tag{11}$$

$$f_j(z,x) = \frac{M(z,x)}{V(z,x)} \tag{12}$$

The transition function for the measure x is defined as follows:

$$x' = f_x(z,x) \tag{13}$$

The law of motion for the number of employed and unemployed workers can be defined using the functions defined above, as follows:

$$U' = f_u(z,x) = U(x) - M(z,x) + \lambda N(x) \tag{14}$$

$$N' = f_e(z,x) = N(x) + M(z,x) - \lambda N(x) \tag{15}$$

Finally, I will denote the real interest rate, labor productivity, worker's share of total surplus, dividend income from firms, unemployment insurance tax rate, and unemployment insurance benefit as functions $r(z,x)$, $p(z,x)$, $w(z,x)$, $d(z,x)$, $t(z,x)$, and $b(z,x)$, respectively.

4.8 Worker's Problem

Now we are ready to define the problem of a worker recursively. Given the law of motion of aggregate states, $f_x(z,x)$, and functions for interest rate, $r(z,x)$, dividends, $d(z,x)$, marginal product of labor, $p(z,x)$, unemployment insurance tax rate, $t(z,x)$, unemployment insurance benefit, $b(z,x)$, worker's share of surplus, $w(z,x)$, and the job finding probability, $f_w(z,x)$, the worker's problem can be recursively formulated as follows:

$$W(z,x,e,a) = \max_{c\geq 0, a'\geq 0, \ell\in[0,1]} \left\{ u(c, 1-\ell) + \beta \mathbb{E}_{z'|z} \sum_{e'} P^W_{ee'} W(z', x', e', a') \right\} \tag{16}$$

subject to

$$c + a' = y + (1 + r(z,x))a + d(z,x) \tag{17}$$

$$y = \begin{cases} p(z,x)\ell w(z,x)(1-t(z,x)) & \text{if } e = 1 \text{ (employed)} \\ b(z,x) & \text{if } e = 2 \text{ (unemployed)} \end{cases} \tag{18}$$

$$x' = f_x(z,x) \tag{19}$$

where

$$
\begin{aligned}
P_{11}^{W} &= 1 - \lambda(1 - f_w(z,x)) \\
P_{12}^{W} &= \lambda(1 - f_w(z,x)) \\
P_{21}^{W} &= f_w(z,x) \\
P_{22}^{W} &= 1 - f_w(z,x)
\end{aligned}
$$

One important assumption that should be mentioned here is that the worker's share out of total surplus $w(z,x)$ is taken as given in the problem above and does not depend on the individual state. A worker's share out of total surplus is assumed to be determined collectively by all workers and is shared by all workers regardless of the individual type. I will discuss this in more detail below.

Let $g_a(z,x,e,a)$, $g_c(z,x,e,a)$, and $g_\ell(z,x,e,a)$ be the optimal decision rules associated with the optimal value function that solves the problem above. Note that $g_\ell(z,x,e,a) = 0$ for $e = 2$, because the income of the unemployed does not depend on ℓ and marginal utility from leisure is strictly positive. Using the optimal decision rule with respect to hours worked, I can define the aggregate hours worked, $L(z,x)$, as follows:

$$
L(z,x) = \int_X g_\ell(z,x,e,a)dx \tag{20}
$$

4.9 Wage Determination

Wage, or a worker's share out of total surplus, is determined by the generalized Nash bargaining. It is the standard assumption in the Mortensen-Pissarides model. Moreover, bargaining is centralized or done collectively. Matched workers and matched firms collectively negotiate their shares out of total surplus every period. For the purpose of defining the centralized bargaining problem, I first construct *the representative worker* and *the representative firm* in the subsequent subsections.

I want to make three remarks here. First, the existence of the representative worker in the bargaining is used by Costain and Reiter (2005b). They call it a *union.* They assume that workers with different productivity form separate unions. The share of the workers differs across different groups of workers, but it is the same among the workers with the same individual productivity.

Second, in the standard Mortensen-Pissarides model, the assumption of the representative worker does not make any difference. This is because there's no heterogeneity across the employed workers. The only heterogeneity across workers is the current employment status (employed or unemployed).

Third, using the representative worker has two important benefits. First, since the surplus sharing rule depends on aggregate states, but not on individual states, the surplus sharing function is simpler. This helps to simplify the computation. Second, since individual workers are atomless, a worker cannot affect the bargaining outcome when bargaining is done collectively. When individual bargaining is assumed instead of the centralized bargaining, workers take into account the effect of their choices on the outcome of future bargaining. As a result, the value function of the worker might not be strictly concave, and the optimal decision for some individual state might not be unique. Reichling (2007) uses individual bargaining but has to assume that the workers are not allowed to take into account the effect of their action on the bargaining outcome to keep useful properties of the value function.

4.9.1 The Representative Worker

The representative worker weights the welfare of all the workers equally, negotiates with the representative firm and determines how to share total surplus. The representative worker's share out of total surplus is denoted as w. Once the shares out of total surplus are determined in the collective bargaining process, each worker takes the bargaining outcome as given when making his own decisions regarding consumption, savings, and hours for work and leisure.

The utility of the representative worker, given a bargaining outcome w, is defined as follows:

$$\widetilde{W}(z, x, 1, w) = \frac{1}{N(x)} \int_X \mathcal{I}_{e=1} \overline{W}(z, x, 1, a, w) \ dx \tag{21}$$

$$\widetilde{W}(z, x, 2) = \frac{1}{N(x)} \int_X \mathcal{I}_{e=1} W(z, x, 2, a) \ dx \tag{22}$$

where

$$\overline{W}(z, x, 1, a, w) = \max_{c \geq 0, a' \geq 0, \ell \in [0,1]} \left\{ u(c, 1-\ell) + \beta \mathbb{E}_{z'|z} \sum_{e'} P^W_{1e'} W(z', x', e', a') \right\} \tag{23}$$

subject to

$$c + a' = y + (1 + r(z, x))a + d(z, x) \tag{24}$$

$$y = p(z, x)\ell w(1 - t(z, x)) \tag{25}$$

$$x' = f_x(z, x) \tag{26}$$

Equations (21) and (22) integrate individual values over the type distribution of employed workers and normalize by the total measure of employed workers to construct the value of the representative worker. Equation (23) defines the value of an employed worker conditional on the current w. Notice that it is not necessary to define the value of an unemployed worker conditional on w because the value of an unemployed worker does not depend on the current w.

4.9.2 The Representative Firm

I assume that there is a representative firm in the economy. All the matched firms produce jointly and negotiate jointly with the representative worker over w. Since w represents the representative worker's share out of total surplus, the share of the representative firm is denoted as $(1 - w)$.

Given the interest rate function $r(z,x)$, law of motion for x, $f_x(z,x)$, aggregate labor supply function $L(z,x)$, and surplus sharing rule $w(z,x)$, the value of the matched ($e=1$) representative firm, $J(z,x,1)$, can be defined by the following Bellman equation:

$$J(z,x,1) = \max_{K\geq 0}\left\{\frac{j(z,x,K)}{N(x)} + \frac{1-\lambda}{1+r(z,x)}\mathbb{E}_{z'|z}J(z',x',1)\right\} \tag{27}$$

subject to

$$j(z,x,K) = (e^z F(K,L(z,x)) - (r(z,x)+\delta)K)(1-w(z,x)) \tag{28}$$

$$x' = f_x(z,x) \tag{29}$$

Notice that the formulation above already takes into account that the value of being unmatched, $J(z,x,2)$, is zero in equilibrium. $j(z,x,K)$ is the function for the aggregate profit of firms, conditional on K units of capital being rented. The current profit of the representative firm is $j(z,x,K)$ divided by the total measure of matched firms $N(x)$. The aggregate profit of firms is the current output minus the rental cost of capital, multiplied by the representative firm's share out of total surplus $(1-w(z,x))$. The optimal decision rule with respect to the rented capital is denoted as $K(z,x)$.

Notice that the problem faced by the representative firm is virtually static; the current choice of K does not affect future value of the representative firm. Therefore it is easy to see that the optimal choice with respect to K satisfies the following marginal condition:

$$r(z,x) = e^z F_K(K,L(z,x)) - \delta \tag{30}$$

Moreover, applying the Euler's theorem to the production function and the marginal condition (30), we can obtain a simple formula for the current profit, conditional on the optimal choice of K, as follows:

$$j(z,x,K(z,x)) = e^z F_L(K(z,x),L(z,x))L(z,x)(1-w(z,x)) = p(z,x)L(z,x)(1-w(z,x)) \tag{31}$$

This implies that the before-tax labor income of a worker who works for ℓ units of time is $p(z,x)\ell w(z,x)$.

The value of the representative firm conditional on the current w can be defined as follows:

$$\widetilde{J}(z,x,1,w) = \frac{p(z,x)L(z,x)(1-w)}{N(x)} + \frac{1-\lambda}{1+r(z,x)}\mathbb{E}_{z'|z}J(z',x',1) \tag{32}$$

subject to (29). Notice that the formula for the current profit is simplified using (31).

4.9.3 Surplus Sharing Rule

As argued in Hall (2005), the set of efficient sharing rules is large. To solve the model, it is necessary to set a sharing rule between the two agents even if we restrict our attention to the set of efficient sharing rules. Following the standard Mortensen-Pissarides model, the generalized Nash bargaining solution is used as the surplus sharing rule. The representative worker's share, w, is determined such that w solves the Nash bargaining problem with a given bargaining parameter for workers μ. Formally, the generalized Nash bargaining problem is defined as follows:

$$w(z,x) = \underset{w\in[0,1]}{\operatorname{argmax}} \left(\widetilde{W}(z,x,1,w) - \widetilde{W}(z,x,2)\right)^{\mu} \left(\widetilde{J}(z,x,1,w) - J(z,x,2)\right)^{1-\mu} \tag{33}$$

The term inside the first parenthesis is the surplus of the representative worker given a worker's share w. The term inside the second parenthesis is the surplus of the representative firm given w. Notice that $J(z,x,2) = 0$ in equilibrium. Therefore, the term inside the second parenthesis can be simplified to $\widetilde{J}(z,x,1,w)$. We cannot obtain a simpler characterization of the bargaining solution like in the standard Mortensen-Pissarides model because of the curvature of the workers' utility function.

4.10 Vacancy Posting

Denote the value of the representative firm when unmatched ($e = 2$) as $J(z,x,2)$. The value can be defined by the following Bellman equation:

$$J(z,x,2) = \max\left\{0, -\kappa + \frac{f_j(z,x)}{1+r(z,x)}\mathbb{E}_{z'|z}J(z',x',1)\right\} \tag{34}$$

subject to (29). The cost of posting a vacancy is κ. We assume free entry. It implies that unmatched firms keep entering the market by posting a vacancy until the value of posting a vacancy and searching for a worker is driven down to the value of the alternative for unmatched firms, which is zero. Using $J(z,x,2) = 0$, equation (34) can be simplified to the following:

$$\kappa = \frac{M(z,x)\ \mathbb{E}_{z'|z}J(z', f_x(z,x), 1)}{(1+r(z,x))\ V(z,x)} \tag{35}$$

Equation (35) implicitly characterizes the number of vacancies posted $V(z,x)$, which makes the value of an unmatched firm zero.

4.11 Equilibrium

Definition 1 (Recursive equilibrium)
A recursive equilibrium is a list of functions $W(z,x,e,a)$, $J(z,x,e)$, $g_c(z,x,e,a)$, $g_a(z,x,e,a)$, $g_\ell(z,x,e,a)$, $K(z,x)$, $d(z,x)$, $w(z,x)$, $V(z,x)$, $r(z,x)$, $p(z,x)$, $t(z,x)$, $b(z,x)$, $L(z,x)$, $A(x)$ *and* $f_x(z,x)$ *such that:*

1. *Given the pricing functions and the law of motion of aggregate states,* $W(z,x,e,a)$ *is a solution to the worker's problem, and* $g_c(z,x,e,a)$, $g_a(z,x,e,a)$ *and* $g_\ell(z,x,e,a)$ *are associated optimal decision rules for consumption, capital for the next period, and hours worked, respectively.*

2. *Given the pricing functions and the law of motion of aggregate states, $J(z,x,e)$ is a solution to the representative firm's problem and $K(z,x)$ is an associated optimal decision rule for the amount of capital rented.*

3. *The law of motion associated with x, $f_x(z,x)$, is consistent with the optimal decision rule for capital holding $g_a(z,x,e,a)$ and the law of motion associated with the employment status e.*

4. *$w(z,x)$ is a solution to the Nash bargaining problem between the representative worker and the representative firm.*

5. *Capital market clears. Formally:*

$$K(z,x) = A(x)$$

6. *Labor market clears, i.e., $L(z,x)$ is equal to the aggregate labor supply implied by $g_\ell(z,x,e,a)$.*

7. *The rental price of capital is equalized to the marginal product of capital minus depreciation.*

$$r(z,x) = e^z F_K(K(z,x), L(z,x)) - \delta$$

8. *Labor productivity is the marginal product of labor.*

$$p(z,x) = e^z F_L(K(z,x), L(z,x))$$

9. *The free entry of the firms yields zero profit in terms of the value of an unmatched firm, i.e., $J(z,x,2) = 0$. The number of vacancies posted $V(z,x)$ is determined such that the free entry condition is satisfied.*

10. *$t(z,x)$ and $b(z,x)$ are consistent with a balanced government budget and a constant replacement ratio ξ.*

11. *The dividend is distributed among workers as a lump-sum transfer. The dividend $d(z,x)$ is determined as follows:*

$$d(z,x) = -\kappa V(z,x) + j(z,x,K(z,x))$$

5 Calibration

5.1 Model Period

A period is a quarter, because this is the highest frequency at which the aggregate data of interest are available.

5.2 Preference

For the period utility function, the following functional form is used. This is one of the functional forms consistent with the existence of a balanced growth path, and it allows a non-unity relative risk aversion. The functional form exhibits constant relative risk aversion and non-separability between consumption and leisure.

$$u(c,h) = \frac{(c^{\psi}h^{1-\psi})^{1-\sigma}}{1-\sigma} \tag{36}$$

The risk aversion parameter σ is calibrated such that the coefficient of relative risk aversion is 1.5, which is a standard value in the literature. Notice that because of the aggregation between consumption and leisure, the parameter σ is not equal to the coefficient of relative risk aversion. Once ψ is chosen, σ is set such that $\psi(1-\sigma) = 1 - 1.5$.

The aggregation parameter ψ is calibrated endogenously, jointly with other parameters. The calibration procedure is further discussed later. Mainly, ψ is used to guarantee that the average hours worked in the steady state of the model is 0.33.

The time discount factor β is also calibrated endogenously. Mainly, β takes care of the aggregate capital stock.

5.3 Production Technology

The production function takes the Cobb-Douglas form as follows:

$$Y = e^{z}F(K,L) = e^{z}K^{\theta}L^{1-\theta} \tag{37}$$

θ is pinned down to match the average capital share of income in the U.S. economy (computed using data from the National Income and Product Accounts), which is 0.29. This is slightly lower than the commonly used value (around one-third), because the firms' profit is not included in the capital share. Instead, the firms' profit is part of labor's contribution to output.

It is assumed that the shock to TFP follows the following AR(1) process:

$$z' = \rho z + \epsilon \tag{38}$$

where $\epsilon \sim N(0, \sigma_{\epsilon}^2)$. ρ is set at 0.95 and σ_{ϵ} is set at 0.007. These are the values Cooley and Prescott (1995) estimate using the Solow residuals. The depreciation rate δ is calculated using the average ratio of total capital consumption over total capital stock (computed using NIPA data). δ is set at 0.014 (5.6% annual depreciation).

5.4 Job Turnover Technology

Following Shimer (2005), the following Cobb-Douglas function is used for the matching function:

$$f_m(S,V) = \gamma S^{\alpha}V^{1-\alpha} \tag{39}$$

Notice that, assuming $f_m(S,V) < \min(S,V)$, matching probabilities for a worker (f_w) and for a firm (f_j) take the following simple forms:

$$f_w(z,x) = \frac{f_m(S(x),V(z,x))}{S(x)} = \frac{\gamma S^{\alpha}(x)V^{1-\alpha}(z,x)}{S(x)} = \gamma\left(\frac{V(z,x)}{S(x)}\right)^{1-\alpha} \quad (40)$$

$$f_j(z,x) = \frac{f_m(S(x),V(z,x))}{V(z,x)} = \frac{\gamma S^{\alpha}(x)V^{1-\alpha}(z,x)}{V(z,x)} = \gamma\left(\frac{V(z,x)}{S(x)}\right)^{-\alpha} \quad (41)$$

There are variety of estimates for α. Shimer (2005) uses the unemployment data of the Bureau of Labor Statistics (BLS) and the help-wanted advertising index constructed by the Conference Board and obtains $\alpha = 0.72$. Hall (2005) uses Job Openings and Labor Turnover Survey (JOLTS) data on vacancies, unemployment, and job-finding probability and obtains $\alpha = 0.235$. Blanchard and Diamond (1989) use Current Population Survey (CPS) data to construct the data on unemployment and new matches and the help-wanted advertising index of the Conference Board and obtain $\alpha = 0.4$. As the baseline calibration, I use $\alpha = 0.5$.

The separation probability λ is set at 0.10, since it is consistent with the average tenure of a job in the data (2.5 years, or 10 quarters).

In order to achieve a quarterly job destruction probability of 0.10 and a steady state unemployment rate of 0.0567, the steady state quarterly matching probability for a worker has to be 0.625.

The average value of V is normalized to be equal to the average value of the unemployment rate, which is 0.0567. This normalization plus the average job finding probability of 0.625 yields $\gamma = 0.625$. In addition, the choice of the average V guarantees $M = f_m(S,V) < \min(S,V)$.

The parameter for the cost of posting a vacancy κ is pinned down such that in the steady state of the model, the representative firm's share of surplus is 3% of the total surplus. The firm's surplus being close to zero is supported by various empirical work. The target is the same as the one used in both Shimer (2005) and Hagedorn and Manovskii (2005). Using this procedure, $\kappa = 0.114$ is obtained.

5.5 Unemployment Insurance

There's no agreement on the target value of the replacement ratio for the following two reasons. First, a variety of numbers could be justified as the monetary value of being unemployed, mainly because the wage (before unemployment) distribution of unemployed workers is typically very different from the wage distribution of the entire labor force. Second, even if the monetary value of being unemployed is pinned down, if the model does not explicitly include leisure, the value of additional leisure when a worker is unemployed needs to be imputed and the replacement ratio needs to be adjusted to reflect the value of additional leisure when a worker is unemployed.

As for the first reason, the unemployment insurance benefit replaces around 60% of past earnings in the U.S. (Hornstein et al. (2005)). However, since those who are unemployed tend to earn less than the average worker, the ratio of the average unemployment insurance benefit and average earnings is substantially below 60%. According to the BLS, the ratio of the average weekly benefit and the average weekly wage is 35% between 1951 and 2004. Hornstein et al. (2005) claim that, according to the Organization for Economic Co-operation and Development (OECD), 20% is the upper bound of the replacement ratio of the U.S. if we take into account the lower average wage of unemployed workers.

As for the second reason, Shimer (2005) uses 40% as the value of being unemployed relative to being employed, including the imputed value of leisure. Alternatively, Hagedorn and Manovskii (2005) do not set a prior for the imputed value of leisure but instead calibrate the replacement ratio to match the fact that in response to a 1% increase in labor productivity, the wage increases by 0.5%. As a result, they come up with a replacement ratio, including the value of leisure, of 95%.

For the baseline calibration, the second issue is not a question because the value of leisure is explicitly modeled. The value of leisure is indirectly pinned down when the parameter ψ is pinned down such that average hours worked in the model are close to the average hours worked in the U.S. economy. Notice that there is no freedom in choosing the value of leisure. As for the first issue, we use the average of 0.2 (suggested by Hornstein et al. (2005)) and 0.35 (BLS data), which is 0.275.

Once the replacement ratio ξ is pinned down, the unemployment insurance tax rate $t(z,x)$ and benefit $b(z,x)$ are jointly determined such that *(i)* the level of benefit is consistent with the preset replacement ratio ξ, and *(ii)* the government budget balances in each period. Specifically, the following conditions must be simultaneously satisfied:

$$b(z,x) = \xi p(z,x)\ell(z,x)w(z,x)(1-t(z,x)) \tag{42}$$

$$U(x)b(z,x) = N(x)p(z,x)\ell(z,x)w(z,x)t(z,x) \tag{43}$$

5.6 Endogenously Calibrated Parameters

Three parameters are yet to be pinned down endogenously: *(i)* the time discount factor β, *(ii)* the aggregation parameter between consumption and leisure ψ, and *(iii)* the Nash bargaining parameter μ. In calibrating these parameters, a steady state economy is solved, by replacing the process for the technology shock by its unconditional mean. The parameters are calibrated so that the following three targets, all of which are averages of the U.S. data, are satisfied in the steady state. In other words, the three parameters are estimated using the simulated method of moments with a unit weighting matrix.

1. The capital output ratio is 12.70 (3.175 with annual output).

2. Hours worked are 0.33 of the total available time.

3. Workers' share out of the total profit is 0.97.

Table 2: Summary of the baseline calibration

Parameter	Description	Value
β	Time discount factor	0.9912
σ	Curvature parameter for the utility function	2.3972
ψ	Aggregation parameter between c and ℓ	0.3579
θ	Capital share of income	0.2900
δ	Quarterly depreciation rate of capital	0.0140
ρ	Persistence of TFP shock	0.9500
σ_ϵ	Standard deviation of shocks to TFP	0.0070
γ	Level parameter for matching function	0.6246
α	Curvature parameter of the matching function	0.5000
λ	Match separation probability	0.1000
ξ	Replacement ratio for UI benefit	0.2750
μ	Nash bargaining share parameter for worker	0.0386

The resulting values of the parameters, together with all the other parameters, are summarized in Table 2.

6 Computation

6.1 Approximate Equilibrium

To compute an equilibrium of the model, I use the solution method developed by Krusell and Smith (1998); I focus on the stationary stochastic recursive equilibrium and compute the approximation of the original equilibrium.

The key component of Krusell and Smith's (1998) approximation method is to use a finite set of statistics $\widetilde{x}$ of the type distribution x to represent x. As a result, instead of dealing with an infinitely dimensional object x, we only need to deal with a finite set of statistics of x. An interpretation of the approximation method is that agents in the model are allowed to use partial information $\widetilde{x}$ of the entire type distribution x when they make decisions. In this sense, the approximate equilibrium can be called the *equilibrium with partial information*.

The important finding of Krusell and Smith (1998) is that, in their model with both aggregate and uninsured idiosyncratic shocks, using a very small set of statistics that represent x is sufficient to achieve a good approximation. This insight helps greatly reduce the computational cost in the current model as it does in theirs.

The smallest set of statistics that represent x and are sufficient to compute all the prices needed to solve the problems of workers and firms contains A, aggregate capital stock held by workers, and N, total measure of employed workers. I use this smallest set of statistics to represent x. Formally:

$$\widetilde{x} = (A, N) \tag{44}$$

The model here is more challenging than the model used by Krusell and Smith (1998), for two reasons. First, since both the aggregate capital stock and the total measure of employed workers (or the total measure of unemployed workers) must be a part of the aggregate state variables, the number of aggregate state variables must be at least two, instead of one. Second, it is necessary to simultaneously find four, instead of one, forecasting functions that are consistent with the optimal decisions of workers and firms.

6.2 Computation of the Approximate Equilibrium

Computing the approximate equilibrium involves finding four functions, which map the aggregate state variables $(z, \widetilde{x}) = (z, A, N)$ into capital stock in the next period, A', number of vacancies posted, V, bargaining outcome, w, and aggregate labor supply, L, respectively. The four functions are taken as given in the optimization problems of workers and firms and must be consistent with the resulting optimal decisions.

Following Krusell and Smith (1998), the four forecasting functions are parameterized and thus characterized by a set of coefficients. Finding an equilibrium is basically finding a set of coefficients with which the forecasting functions are consistent with the decisions of workers and firms.

The value function is approximated using the shape-preserving spline developed by Schumaker (1983) with respect to individual capital holding a. An important benefit of using the shape-preserving spline is that it is costless to evaluate the derivative of the approximated value function. The optimal decision rules for future capital holding, consumption, and hours worked are approximated using piecewise-linear functions with respect to current capital holding. The problem of the workers is solved using the value function iteration.

Details of the computation, including a detailed discussion of how to construct the approximate equilibrium, are found in Appendix A.

7 Results

7.1 Aggregate Statistics of the Baseline Model

Table 3 compares the aggregate statistics of the U.S. economy and the calibrated baseline model economy. The basic message is that the calibration is successful. The baseline model achieves all the targets simultaneously.

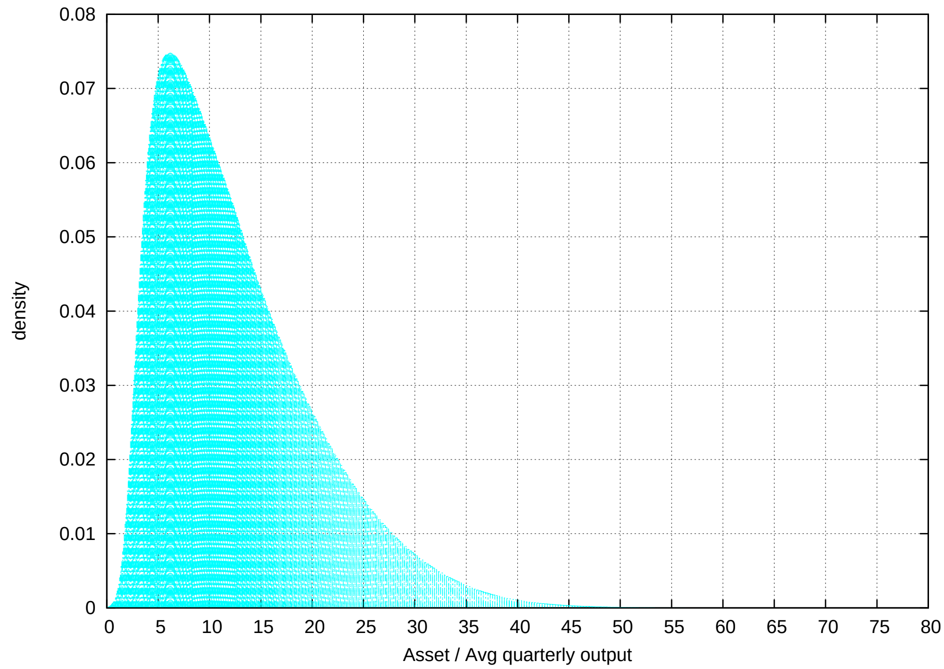

Figure 1: Density function of asset holding

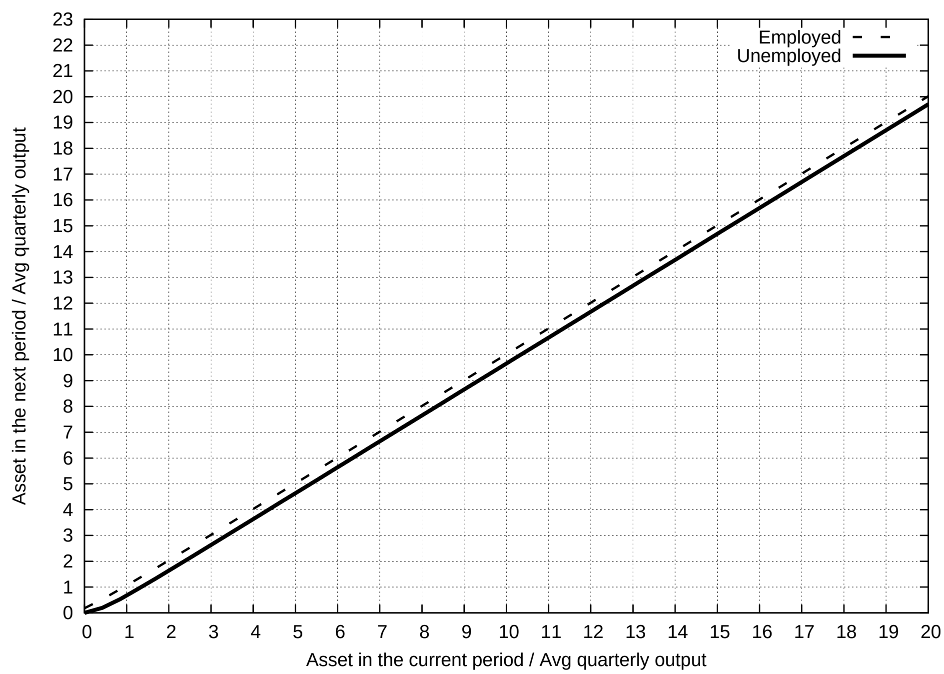

Figure 2: Optimal decision rule with respect to asset holding

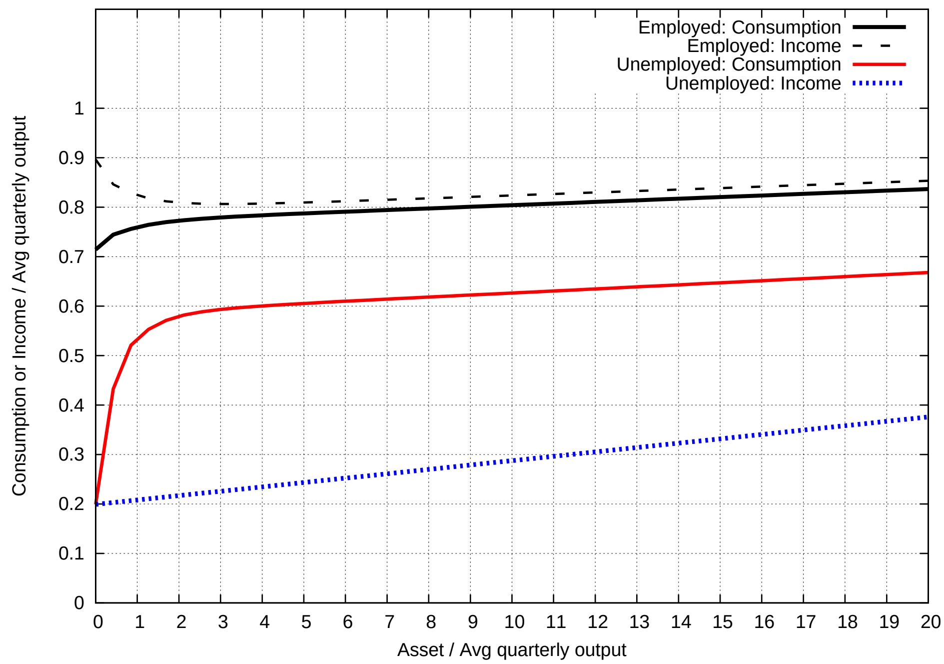

Figure 3: Optimal decision rule with respect to consumption

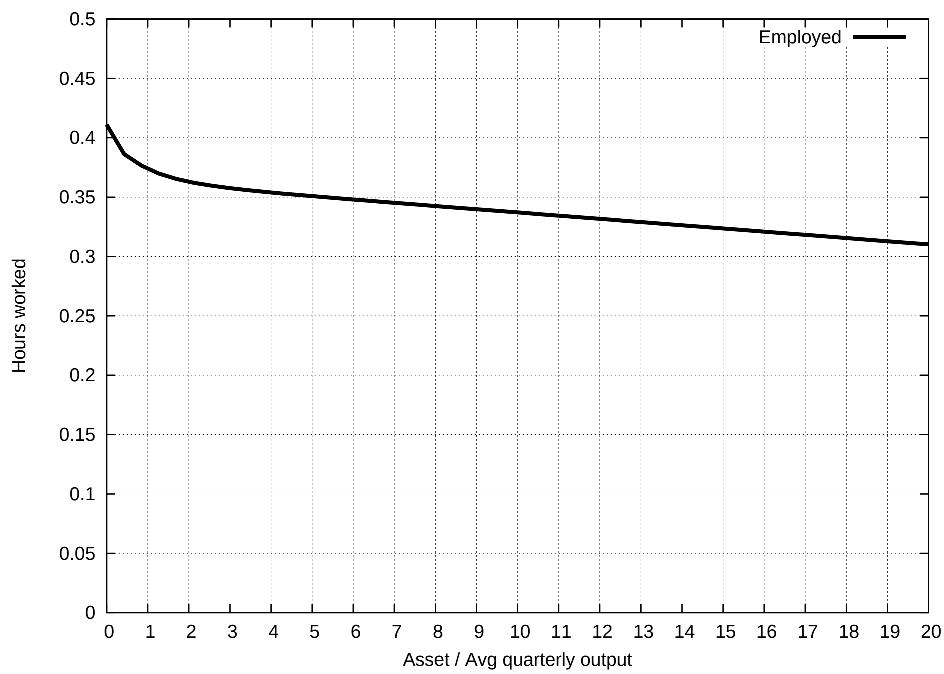

Figure 4: Optimal decision rule with respect to hours worked

Table 3: Aggregate statistics of the U.S. and baseline model economy

Statistics	U.S. economy	Model economy
Capital-output ratio[1]	3.175	3.175
Labor share of output	0.689	0.689
Capital share of output	0.290	0.290
Profit-surplus ratio	0.030	0.030
Average hours worked	0.330	0.330
Unemployment rate	0.0567	0.0567

[1] Annualized.

Figure 1 exhibits the density function with respect to the asset holding of workers. The model is populated by workers with different asset holdings, depending on employment history. Since the only idiosyncratic shock to an individual worker is the shock to employment status, the inequality with respect to wealth holding is mild compared with that of the U.S. economy. The Gini index for the U.S. economy is 0.80 according to Budría et al. (2002), whereas the Gini index for the baseline model economy is 0.33.

Figure 2 shows the optimal decision rule for asset holding. Employed workers increase asset holdings, while unemployed workers reduce asset holdings to sustain consumption even though current income from the unemployment insurance benefit is lower than income from working.

Figure 3 shows the optimal decision rule for consumption. Employed workers consume less than their income, which is consistent with the optimal savings decision shown in the previous figure. Among unemployed workers, those with very low asset holdings suffer a low level of consumption. Other unemployed workers can easily sustain a consumption level that would be optimal even without the borrowing constraint. There is a gap between the consumption of the employed and that of the unemployed for any given asset holding level, because unemployed workers enjoy additional utility from more leisure time and thus do not want to consume as much as employed workers.

Finally, figure 4 shows the optimal decision rule for hours worked by employed workers. Optimal working hours vary between 40% and 30% of the total available time. The optimal decision is downward sloping with respect to asset holding, because there is no difference in productivity level among employed workers. If there is a persistent difference in individual productivity, workers with higher productivity would tend to hold more assets. If the substitution effect due to high productivity and thus a high wage dominates, workers with a high asset holding level might be working more than workers with lower individual productivity.

Table 4: Cyclical properties of the baseline model economy[1]

Variable	SD%	Relative SD%[2]	Auto-corr	Cross-correlation of output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Output	1.50	1.00	0.83	0.58	0.83	1.00	0.83	0.58
Consumption	0.41	0.27	0.86	0.43	0.72	0.95	0.89	0.71
Investment	6.11	4.07	0.85	0.59	0.83	1.00	0.84	0.58
Unemployment rate (u)	11.25	7.48	0.83	-0.39	-0.62	-0.85	-0.99	-0.79
Vacancies (v)	8.31	5.53	0.49	0.64	0.78	0.78	0.30	0.05
v/u ratio	16.16	10.75	0.86	0.60	0.83	0.99	0.84	0.57
Job finding probability	5.16	3.44	0.71	0.66	0.86	0.94	0.61	0.33
Compensation to employees	1.14	0.76	0.86	0.54	0.80	0.99	0.88	0.64
Labor share	0.39	0.26	0.69	-0.64	-0.85	-0.95	-0.62	-0.35
Total hours	0.94	0.62	0.88	0.51	0.75	0.95	0.95	0.71
Employment	0.70	0.47	0.83	0.39	0.62	0.85	0.99	0.79
Average weekly hours	0.31	0.21	0.69	0.65	0.86	0.94	0.61	0.33
Output per person	0.98	0.65	0.60	0.60	0.82	0.93	0.56	0.32
Output per hour	0.67	0.45	0.56	0.57	0.80	0.91	0.53	0.31
Compensation per hour	0.31	0.21	0.41	0.45	0.68	0.78	0.38	0.22

[1] All data are in logs and filtered using the H-P filter with the smoothing parameter of 1600.
[2] Relative to the standard deviation of output.

7.2 Business Cycle Properties of the Baseline Model

Table 4 summarizes the business cycle properties of the baseline model economy. The table is the exact counterpart of Table 1, which exhibits the business cycle properties of the U.S. economy. For the sake of comparison, Table 5 reproduces Table 1.2 of Cooley and Prescott (1995), which shows the cyclical properties of the standard real business cycle model.

First, the standard deviation of output of the model (1.50%) is very close to that of the U.S. economy (1.58%). Since the shock to TFP that we feed into the model is estimated from the data, the closeness implies that the model has an amplification mechanism that is as strong as that in the U.S. economy. The similarity of the cyclical properties of the two economies is not only in the size of the volatility but also in the autocorrelation structure. Autocorrelation of output is 0.83 in the baseline model and 0.84 in the U.S. economy. Compared with the baseline model, the standard real business cycle model generates a lower volatility of output and a lower autocorrelation. The amplification mechanism of the standard real business cycle model is not as strong as in the baseline model.

Consumption in the model economy replicates the key properties of the U.S. business cycle

Table 5: Cyclical properties of the standard real business cycle model[1,2]

Variable	SD%	Relative SD%[3]	Cross-correlation of output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Output	1.35	1.00	0.44	0.70	1.00	0.70	0.44
Consumption	0.33	0.24	0.59	0.73	0.84	0.50	0.23
Investment	5.95	4.41	0.39	0.66	0.99	0.71	0.47
Total hours	0.77	0.60	0.37	0.65	0.99	0.72	0.48
Compensation per hour	0.61	0.45	0.51	0.73	0.98	0.65	0.38

[1] Replicated from Cooley and Prescott (1995), Table 1.2.

[2] All data are in logs and filtered using the H-P filter with the smoothing parameter of 1600.

[3] Relative to the standard deviation of output.

data; consumption is less volatile than output and strongly positively correlated with output. Quantitatively, though, the volatility of consumption relative to the volatility of output in the model (0.27) is smaller than the same statistics for the U.S. economy (0.79). In other words, the model exhibits an excess smoothness of consumption. This is a common property of the standard business cycle model. As shown in Table 5, the consumption volatility relative to the output volatility of the standard real business cycle model is 0.24, which is very close to the number obtained in the current model economy.

Investment in the model economy successfully replicates the key cyclical properties of the U.S. data both qualitatively and quantitatively. The volatility of investment is more than four times that of output (4.07% in the model, and 4.57% in the U.S. economy), and investment is extremely procyclical. As can be seen in Table 5, the cyclical properties of investment for the baseline model is similar to those of the standard real business cycle model.

The similarity of the cyclical properties of consumption and investment between the baseline model economy and the standard real business cycle model is consistent with what the optimal decision rules suggest. The workers self-insure very well so that, at the aggregate level, the behavior of consumption and investment in the baseline model with incomplete markets is similar to the standard real business cycle model with complete markets.

The volatility of total hours in the model relative to output volatility is 0.62. This is lower than in the U.S. data (1.10). However, the model correctly replicates the property of the data that total hours are strongly procyclical and somewhat lag the cycle by a quarter. Both the contemporaneous correlation between output and total hours and the correlation between output and total hours in the next quarter are the same – 0.95 – in the model. The numbers are somewhat lower for the U.S. economy (0.87) but again they are the same. As in Table 5, there is no lead or lag between output and hours worked in the standard real business cycle model.

While there is only the intensive margin of labor supply adjustment in the standard real business cycle model, the fluctuations of total hours can be broken down into the extensive and the intensive margins in the current model economy. The standard deviations of employment (extensive margin) and average hours worked (intensive margin) are 0.70 and 0.31, respectively, in the baseline model. In the U.S. economy the volatilities are 1.00 and 0.51. They are higher in the U.S. economy, but the relative size is about the same in both economies; in both economies, the volatility of employment is about twice as large as the volatility of average hours.

An important property of employment in the data is that employment lags the cycle by a quarter. In the U.S. economy, the contemporaneous correlation between output and employment is 0.80, whereas the correlation between output and employment in the next quarter is higher at 0.86. The model replicates the feature. The contemporaneous correlation in the model is 0.85, while the correlation between output and employment in the next quarter is 0.99.

Average hours do not lead or lag the cycle in both the model and U.S. economies. Contemporaneous correlation between output and average hours is 0.71 in the U.S. economy and 0.94 in the model economy.

Because of the procyclicality of average hours, productivity measured by output per person is more volatile than productivity measured by output per hour in the data. In particular, standard deviations of output per person and output per hour are 1.31 and 1.04, respectively, in the U.S. economy. The same relationship holds in the model economy, too. Standard deviations of output per person and output per hour are 0.98 and 0.67, respectively, in the model.

Labor share of income is constant over the business cycle in the standard real business cycle model, while labor share is volatile (standard deviation is 1.07%) and somewhat countercyclical (contemporaneous correlation is −0.26). Another advantage of the current model economy over the standard real business cycle model is that labor share fluctuates over the business cycle in the current model economy. The model economy replicates the features that labor share is less volatile than output and that labor share is countercyclical. However, quantitatively, the properties of the model economy do not match well their counterparts in the data. In the model economy, the standard deviation of labor share is 0.39%, which is lower than in the data. The contemporaneous correlation between output and labor share is −0.95, which is substantially higher than in the U.S. economy.

7.3 Cyclical Properties of Unemployment and Vacancies

The most important success of the baseline model is that the model economy replicates very high volatility of unemployment and vacancies. The standard deviation of the unemployment rate in the model economy is 11.25%, which is substantially higher than that of output (1.50%) and close to the U.S. economy counterpart (12.48%). For a comparison, Shimer (2005) finds that in the standard Mortensen-Pissarides model with productivity shocks, the standard deviation of

Table 6: Comparison of cyclical properties of unemployment and vacancies

Economy	σ_U	σ_V	$\sigma_{V/U}$	$\rho(U_{t-1}, V_t)$	$\rho(U_t, V_t)$
U.S.	12.48	13.95	25.91	-0.92	-0.92
Baseline model	11.25	8.31	16.16	-0.81	-0.35
Andolfatto (1996)	1.05	4.58	4.86	-0.65	-0.19
Merz (1995)	4.95	6.83	n.a.	-0.82	-0.15

unemployment is 0.9%, whereas the standard deviation is 19.0% in the U.S. data.[2] It is worth noting that the current model economy generates volatility of unemployment as high as in the data, while the level of unemployment insurance benefit is calibrated at the plausible level in the U.S. economy.

The model also captures other cyclical properties of unemployment. Autocorrelation in the model is 0.83, which is close to its U.S. economy counterpart (0.87). Unemployment is strongly countercyclical in both economies. The contemporaneous correlation between unemployment and output is -0.84 in the U.S. and -0.85 in the model. Finally, unemployment lags the cycle in both the model and U.S. economies. The correlation between output and unemployment in the next quarter is -0.86 in the U.S. and -0.99 in the model.

The standard deviation of the number of vacancies in the model is 8.31. It is significantly higher than the standard deviation of output (1.50), as in the U.S. data, but the absolute level is lower than the level in the U.S. economy (13.95). The number of vacancies in the data is much more strongly autocorrelated. Autocorrelation is 0.91 in the data, while it is 0.49 in the model economy. The number of vacancies is strongly procyclical in both economies. The contemporaneous correlation with output is 0.90 in the U.S. economy, while it is 0.78 in the model economy.

Job finding probability in the model is much more volatile than output and strongly procyclical, as in the data. The standard deviation is 5.16 in the model and 7.74 in the U.S. economy. The contemporaneous correlation between output and the job finding probability is extremely high at 0.94 in the model and 0.83 in the U.S. economy.

Although real business cycle models with labor market search and matching (Andolfatto (1996) and Merz (1995)) have similar success in terms of replicating various dimensions of the business cycle properties of the U.S. economy, the current model is better in achieving these while generating very volatile unemployment and vacancies. Table 6 summarizes the comparison. In the models by Andolfatto (1996) and Merz (1995), volatilities of unemployment and vacancies are substantially lower, and the contemporaneous correlation between the two (Beveridge curve) is low, compared with those of the U.S. economy. The current model economy exhibits a substan-

[2] The difference of the volatility of unemployment in the U.S. economy stems from the difference in the way the original data are detrended. Both use the H-P filter, but Shimer (2005) sets the smoothness parameter at 10^5, while I set the parameter at 1600, which is the standard choice in the real business cycle literature.

tially higher variation of unemployment and vacancies. Regarding the strong negative correlation between the two, the model does better than the previous models but is still lower than in the data.

How can the previous two real business cycle models with labor market search and matching successfully replicate many properties of U.S. business cycles? Costain and Reiter (2005b) analyze crucial assumptions in the models. Andolfatto (1996) relies on the assumption that the mean unemployment rate is 43%. The assumption is justified by assuming that all of the people out of the labor force are actually searching for a job as well. Since the proportion of the unemployed is as large as 43%, a small volatility of unemployment is enough to generate a strong amplification. Conversely, the strong amplification is not accompanied by a large volatility of unemployment. The success of the model of Merz (1995) relies on the extremely small size of the total surplus. A very small surplus helps generating a large volatility of vacancies. I will come back to the mechanism in the next subsection.

7.4 Discussion: Why High σ_U and σ_V?

The mechanism behind the high volatility of unemployment and vacancies in the current model is closely related to the mechanism pushed by Hagedorn and Manovskii (2005). I will go over the intuition of the results by Hagedorn and Manovskii (2005) first and discuss how the properties of the current model are related to their argument.

For the family of Mortensen-Pissarides models with the standard aggregate matching function, large volatilities of unemployment and vacancies are generated by a large volatility of firms' expected profit. When the expected profit of firms increases substantially, unmatched firms have a strong incentive to increase the number of vacancy postings, which yields a large increase in the number of matches and a corresponding substantial decline in unemployment.

When the volatility of total surplus of a match is calibrated using the data, and thus prevented from becoming very high, the following two conditions generate large volatility of firms' expected profit:

1. Firms' profit is a small proportion of the total surplus of a match.
2. A large part of the changes in the total surplus is distributed to firms.

The first condition would make firms' profit relatively (relative to the level of firms' profit) volatile, for a given volatility of firms' profit. In the limit, if the size of firms' profit is approaching zero, the relative volatility of firms' profit can be infinitely increased. However, an extremely small size of firms' profit is not consistent with the U.S. data. According to the empirical evidence referred to by Hagedorn and Manovskii (2005), the average ratio of firms' profit over the total surplus is about 3% in the U.S., the same number used for the calibration of the current model.

As the level of firms' profit is restricted by the U.S. data, a large fluctuation of firms' profit needs to be generated by volatile procyclical fluctuations of firms' share of total surplus. The answer by

Hall (2005) to the puzzle of Shimer (2005), which is the sticky wage, can be easily understood in this context. Suppose the wage is fixed, and the productivity of a match increases. Then workers do not benefit from the positive shock to productivity, because their wage is fixed. Instead, the additional surplus out of a match is distributed solely to the firms. Thus firms' share of total surplus becomes strongly procyclical, and the volatility of firms' profit becomes larger than the volatility of total surplus. In fact, other kinds of friction that prevent the wage (workers' share of total surplus) from flexibly responding to changes in the size of total surplus would work in the same way.

If the distribution of total surplus is determined according to Nash bargaining, firms' proportion of total surplus is determined by $1-\mu$. Since μ represents workers' share of total surplus, firms' profit becomes more volatile as μ becomes smaller. However, μ affects workers' average share of total surplus, which is about 0.97. How can μ be small while, on average, distributing 97% of total surplus to workers? The key is what μ affects. μ does not directly affect workers' share of total surplus but determines the relative size of the utility gain for workers. If workers, on average, receive 0.97 of total surplus but the utility gain from the share is small, μ takes a small value and a large proportion of total surplus is still distributed to workers.

What Hagedorn and Manovskii (2005) find is that, intuitively, if workers' utility from being unemployed is close to workers' utility from working, it is possible that workers' utility gain by working and receiving a majority of the total surplus is small. In particular, they find that if the instantaneous utility of being unemployed is 95% of the instantaneous utility of being employed and working, the model can satisfy low μ and a high workers' share of total surplus simultaneously, and thus the model exhibits a strong amplification.

Even though Shimer (2005) and Hagedorn and Manovskii (2005) use the same Mortensen-Pissarides model with aggregate shocks to match productivity, their models generate very different amplification properties. The reason is that while Shimer (2005) assumes that the instantaneous utility of being unemployed (including the value from leisure) is about 40% of the value of working, Hagedorn and Manovskii (2005) calibrate the parameter as 95%.

How are the results related to the current model? First, the parameter μ in the baseline model economy is calibrated at 0.0386. Since Nash bargaining is used as the surplus sharing rule, it has to be the case that, as in Hagedorn and Manovskii (2005), the value of the representative worker for being unemployed is close to the value of being employed. However, in the current model, the replacement ratio of the unemployment insurance benefit is set at 0.275, meaning that the monetary value of being unemployed is even lower than the number used by Shimer (2005) (0.40).

The next question is, how can $\mu = 0.0386$ be possible when the monetary value of unemployment is only 27.5% of the average labor income? There are two channels. First, contrary to the standard labor search and matching models, in the current model, workers save to prepare for future periods of unemployment. If workers save sufficiently, consumption, and thus period utility, does not drop even for the unemployed. Notice that in the standard labor search and matching model, workers are assumed to be risk neutral, and therefore, there is a one-to-one relationship between the change in income and the change in utility. This is not the case for the current

Table 7: Cyclical properties of the model economy without savings[1]

Variable	SD%	Relative SD%[2]	Auto-corr	Cross-correlation of output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Output	0.82	1.00	0.69	0.44	0.69	1.00	0.69	0.44
Consumption	1.01	1.24	0.69	0.44	0.69	1.00	0.69	0.44
Investment	0.02	0.03	0.43	0.46	0.67	0.92	0.38	0.13
Unemployment rate (u)	0.31	0.38	0.81	-0.18	-0.40	-0.66	-0.97	-0.83
Vacancies (v)	0.23	0.28	0.43	0.46	0.67	0.92	0.38	0.13
v/u ratio	0.44	0.54	0.85	0.37	0.63	0.94	0.88	0.66
Job finding probability	0.14	0.17	0.67	0.45	0.70	1.00	0.67	0.42
Compensation to employees	0.82	1.00	0.69	0.44	0.69	1.00	0.69	0.44
Labor share	0.01	0.01	0.68	0.36	0.57	0.82	0.71	0.51
Total hours	0.11	0.14	0.62	-0.45	-0.70	-0.99	-0.61	-0.35
Employment	0.02	0.02	0.81	0.18	0.40	0.66	0.97	0.83
Average weekly hours	0.12	0.15	0.70	-0.44	-0.69	-1.00	-0.70	-0.45
Output per person	0.80	0.99	0.68	0.45	0.69	1.00	0.68	0.43
Output per hour	0.93	1.14	0.68	0.44	0.69	1.00	0.68	0.43
Compensation per hour	0.93	1.14	0.69	0.44	0.69	1.00	0.68	0.43

[1] All data are in logs and filtered using the H-P filter with the smoothing parameter of 1600.
[2] Relative to the standard deviation of output.

model, since workers are assumed to be risk-averse and are allowed to save for precautionary purposes.

Second, the additional utility of leisure if unemployed is not included in the monetary value of being unemployed. Besides, the value is implicitly determined when the parameter ψ, which dictates the relative value of leisure, is pinned down such that workers spend, on average, 33% of their disposable time working. If the implicitly determined value of additional leisure for the unemployed is large, the amplification mechanism of the current model is expected to be similar to that of Hagedorn and Manovskii (2005).

7.5 Measuring the Contribution of Savings and Leisure Separately

To measure the contribution from each of the two channels, the role of savings and the role of extra utility from leisure when unemployed, separately, I implement two experiments. First, the baseline model is modified such that workers are not allowed to save. The model is re-calibrated such that the same targets as those used for the calibration of the baseline model economy are

Table 8: Cyclical properties of the model economy without leisure[1]

Variable	SD%	Relative SD%[2]	Auto-corr	Cross-correlation of output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Output	0.93	1.00	0.73	0.48	0.73	1.00	0.73	0.48
Consumption	0.19	0.20	0.80	0.24	0.53	0.88	0.77	0.64
Investment	4.47	4.82	0.73	0.51	0.74	1.00	0.70	0.43
Unemployment rate (u)	0.94	1.01	0.83	-0.21	-0.43	-0.69	-0.97	-0.85
Vacancies (v)	0.67	0.72	0.48	0.50	0.71	0.92	0.42	0.16
v/u ratio	1.33	1.44	0.86	0.40	0.66	0.95	0.90	0.68
Job finding probability	0.42	0.45	0.71	0.48	0.73	1.00	0.71	0.45
Compensation to employees	0.92	0.99	0.73	0.47	0.73	1.00	0.73	0.48
Labor share	0.01	0.01	0.42	-0.38	-0.52	-0.69	-0.32	-0.11
Total hours	0.06	0.06	0.83	0.21	0.43	0.69	0.97	0.85
Employment	0.06	0.06	0.83	0.21	0.43	0.69	0.97	0.85
Average weekly hours	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Output per person	0.89	0.96	0.70	0.48	0.73	1.00	0.70	0.44
Output per hour	0.89	0.96	0.70	0.48	0.73	1.00	0.70	0.44
Compensation per hour	0.88	0.95	0.70	0.48	0.73	1.00	0.70	0.44

[1] All data are in logs and filtered using the H-P filter with the smoothing parameter of 1600.
[2] Relative to the standard deviation of output.

matched in the modified model economy. To prevent workers from saving and thus preparing for future unemployment spells, but to preserve capital in the model, all workers are endowed with the average capital stock in the steady state of the baseline model, and they are forced to save the same amount. These assumptions help to keep steady state capital stock level in the modified economy, thus making the comparison with the baseline model economy reasonable, and still prevent workers from accumulating savings for precautionary purposes. Second, the baseline model is modified so that workers do not value leisure, but if they are employed, they are assumed to spend 33% of their disposable time working. Again, the model is re-calibrated.

Table 7 summarizes the cyclical properties of the model without saving. Most important, the model loses the strong amplification of the baseline model economy. The volatility of the unemployment rate is 0.31%, which is even lower than in Shimer's (2005) model (0.9%). The volatility of the number of vacancies is also quite low at 0.23%. Accordingly, the volatility of output (0.82) is almost the same as the volatility of the total factor productivity shock, which means that the model without saving has almost no amplification mechanism. In other words, the option of saving for workers is crucial for the results in the baseline model economy. Only the additional utility from leisure for the unemployed is not enough to keep the gap in value between the employed and the unemployed workers small so that the model exhibits a strong amplification. How about the bargaining power of the representative worker and the representative firm in the economy without the option of saving? μ in the modified economy is calibrated at 0.64. This is lower than the level in the calibration of Shimer (2005), but it is far from the value in the baseline model economy.

Table 8 summarizes the cyclical properties of the model without leisure. Most important, the model without the labor-leisure choice is subject to the puzzle of Shimer (2005) again. The standard deviation of unemployment is now 0.93%, which is very close to the number that Shimer (2005) obtains (0.9%). In other words, the strong amplification mechanism in the baseline model economy vanished again. Labor share of output in the modified model is now almost constant over the business cycle, implying that only a small share of changes in total surplus is distributed to firms over the business cycle. The basic message is that the strong amplification obtained from the baseline model crucially depends on the existence of the additional value of leisure for the unemployed. If the unemployed do not enjoy the additional time for leisure, the value for being unemployed falls so much that the low value of μ can no longer be sustained. The model without leisure requires $\mu = 0.794$ to achieve the workers' share of total surplus of 0.97. The value of μ in the modified economy is actually close to the value used in Shimer (2005), which is 0.72. In other words, allowing agents to save for precautionary purposes is not enough to close the gap of value between being employed and being unemployed and thus generate a strong amplification mechanism. It is necessary to let unemployed workers enjoy extra leisure time to achieve a very small difference of the value between the two employment statuses. More important, the value of leisure that is consistent with the standard calibration of the real business cycle model can generate the strong amplification.

In sum, the results from the two experiments confirm that both the additional utility for the unemployed from leisure and the option of saving are crucial in generating the strong amplification in the baseline model economy.

8 Application: Implications of Lower Volatility of TFP

It is well known that the business cycle volatility of output in the U.S. economy declined substantially around the early 1980s. The primary reason for this *Great Moderation* is still an open question. Arias et al. (2006) find that the volatility of shocks to TFP measured by the Solow residual declined as well and that the business cycle properties of the standard real business cycle model replicate well the changes that occurred in the U.S. economy, by feeding the estimated changes in shocks to TFP.

I implement the same experiment as in Arias et al. (2006) using the baseline model economy constructed in this paper. Because of the richness of the model, the experiment enables more detailed investigation of the changes in the business cycle properties, in response to the decline in the volatility of TFP.

Since the standard deviation of shocks to TFP is estimated to have declined by about 50% before and after the end of the 4th quarter of 1983, according to Arias et al. (2006), the baseline model is run again where the volatility of the shocks to TFP is reduced by half.[3]

Table 9 compares the changes in the cyclical properties of the U.S. economy and the two model economies. First, the changes in the standard deviation are very similar between the two model economies. In both economies, the changes are almost perfectly proportional to the changes in TFP. For the standard real business cycle model (Arias et al. (2006)), it is guaranteed, since the model is solved by linear-quadratic approximation. More surprising is that the pattern of changes is shared by the baseline model economy.

If the changes in the cyclical properties are compared between the U.S. economy and the baseline model economy, the changes in output, consumption, and investment in the U.S. economy are well captured in the model economy. On the other hand, while the volatility of total hours, employment, and average hours in the U.S. economy declined by $20 - 30\%$, the declines are about 50% in the model economy.

9 Conclusion

I extend the standard real business cycle model by incorporating labor market search and matching and incomplete markets. I find that the model, under the standard calibration, can replicate a high volatility of unemployment and vacancies, together with business cycle properties of macroeconomic aggregates and various labor market variables. I show that both of the channels – the option of saving and preparing for future unemployment spells and additional utility for the unemployed from leisure – are crucial in generating the strong amplification mechanism.

The model developed in the current paper can be extended in a variety of ways. Variants of the baseline model can be used to analyze welfare and macroeconomic consequences of changes in policy related to the labor market, since the current model is richer in capturing labor market dynamics. For example, the baseline model can be used to evaluate the effects of different tax policies on the labor market. Another interesting extension is to introduce richer heterogeneity, for example, differences in educational attainment, across workers. The extended model can be used to measure the heterogeneous effect of the welfare cost of business cycles for agents with different education levels and different employment histories.

[3] The autocorrelation parameter for TFP shocks is fixed between the two sub-periods, since constructed data on TFP imply that there is no significant change in the autocorrelation of TFP.

Table 9: Cyclical properties of the economy before and after 1983:4[1]

Variable	Percent standard deviation 1955:3-1983:4 (Early)	1983:4-2003:2 (Late)	Late/Early[2]
U.S. economy[3]			
Output	1.78	0.93	0.53
Consumption	1.38	0.80	0.57
Investment	7.66	4.41	0.58
Total hours[4]	1.58	1.12	0.71
Employment[4]	1.08	0.73	0.68
Hours per worker[4]	0.74	0.58	0.79
Total factor productivity	1.21	0.62	0.51
Arias et al. (2006)			
Output	1.80	0.87	0.49
Consumption	0.46	0.22	0.49
Investment	6.45	3.07	0.49
Total hours	1.43	0.69	0.49
Total factor productivity (input)	0.95	0.46	0.49
Baseline model economy			
Output	1.50	0.75	0.50
Consumption	0.41	0.20	0.50
Investment	6.11	3.00	0.49
Total hours	0.94	0.46	0.49
Employment	0.70	0.34	0.49
Hours per worker	0.31	0.16	0.50
Total factor productivity (input)	0.90	0.45	0.50

[1] All data are in logs and filtered using the H-P filter with the smoothing parameter of 1600.
[2] SD% for 1955:3-1983:4 divided by SD% for 1984:1-2003:2.
[3] Replicated from Arias et al. (2006).
[4] Household survey data.

References

Andolfatto, David, "Business Cycles and Labor-Market Search," *American Economic Review*, 1996, *86* (1), 112–132.

Arias, Andres, Gary D. Hansen, and Lee E. Ohanian, "Why Have Business Cycle Fluctuations Become Less Volatile?," *NBER Working Paper*, 2006.

Blanchard, Olivier J. and Peter Diamond, "The Beveridge Curve," *Brookings Papers on Economic Activity*, 1989, *1989* (1), 1–76.

Budría, S., Javier Díaz-Gimenez, Vincenzo Quadrini, and J.-V. Ríos-Rull, "Updated Facts on the U.S. Distributions of Earnings, Income and Wealth," *Federal Reserve Bank of Minneapolis Quarterly Review*, 2002, *26* (3), 2–35.

Cooley, Thomas F. and Edward C. Prescott, "Economic Growth and Business Cycles," in Thomas F. Cooley, ed., *Frontiers of Business Cycle Research*, Princeton, NJ: Princeton University Press, 1995, chapter 1.

Costain, James S. and Michael Reiter, "Business Cycles, Unemployment Insurance, and the Calibration of Matching Models," 2005a. Unpublished Manuscript, Universitat Pompeu Fabra.

_ **and** _ , "Stabilization Versus Insurance: Welfare Effects of Procyclical Taxation under Incomplete Markets," 2005b. Unpublished Manuscript, Universitat Pompeu Fabra.

Hagedorn, Marcus and Iourii Manovskii, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," 2005. Unpublished Manuscript, University of Pennsylvania.

Hall, Robert E., "Unemployment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 2005, *95* (1), 53–69.

Hornstein, Andreas, Per Krusell, and Giovanni L. Violante, "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism," *Federal Reserve Bank of Richmond Economic Quarterly*, 2005, *91* (3), 19–50.

Howard, Ronald A., *Dynamic Programming and Markov Process*, Cambridge, MA: MIT Press, 1960.

Jung, Philip, "Unemployment, Capital and Hours: On the Quantitative Performance of a DSGE Labor Market Model," 2006. Unpublished Manuscript.

Krusell, Per and Anthony Smith, "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 1998, *106*, 867–896.

Merz, Monika, "Search in the Labor Market and the Real Business Cycle," *Journal of Monetary Economics*, 1995, *36*, 269–300.

Mortensen, Dale T. and Christopher A. Pissarides, "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 1994, *61* (3), 397–415.

Pissarides, Christopher A., *Equilibrium Unemployment Theory*, 2nd ed., Cambridge, MA: MIT Press, 2001.

Pries, Michael J., "Heterogeneity and Labor Market Volatility in Matching Models," *Review of Economic Dynamics*, 2007.

Reichling, Felix, "Optimal Unemployment Insurance in Labor Market Equilibrium When Workers Can Self-Insure," 2007. Unpublished Manuscript.

Ríos-Rull, José-Víctor, "Computation of Equilibria in Heterogeneous-Agent Models," in Ramon Marimon and Andrew Scott, eds., *Computational Methods for the Study of Economic Dynamics*, Oxford: Oxford University Press, 1999, chapter 11.

Schumaker, Larry L., "On Shape Preserving Quadratic Spline Interpolation," *SIAM Journal on Numerical Analysis*, 1983, *20* (4), 854–864.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 2005, *95* (1), 25–49.

Tauchen, George, "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 1986, *20*, 177–181.

Appendix A Details of Computation

A.1 Approximate Equilibrium

Following the insight by Krusell and Smith (1998), the type distribution of workers, x, is represented by a set of its statistics $\widetilde{x}$. The smallest set of statistics that represent x and are sufficient to compute all the prices necessary to solve the problems of workers and firms consists of two statistics: *(i)* aggregate capital stock held by workers A, and *(ii)* total measure of employed workers N. I use this smallest set of statistics to represent x. Formally:

$$\widetilde{x} = (A, N) \tag{45}$$

This approximation makes it possible to store the functions of x in a computer, by replacing an infinitely dimensional object x with a finite set of statistics $\widetilde{x}$. As will be shown below, using (A, N) is sufficient to obtain a good approximation of the true equilibrium we are interested in.

Notice that, as usual, there is a trade-off between precision and computational time. Naturally, it is better to use more statistics associated with x to improve the precision of the approximation. Intuitively, if agents are allowed to use more information about x, the decisions of agents would be closer to the decisions in the true equilibrium where agents can use x in making decisions. On the other hand, it takes more time to solve the model when more statistics are used to represent x. My choice of $\widetilde{x} = (A, N)$ is based on the fact that it takes a long time to solve the model even if a smallest set of statistics is used.

The AR(1) process for the technology shock z is approximated with a first order Markov process using the approximation method developed by Tauchen (1986). Five abscissas are used for the baseline model economy. Increasing the number of abscissas from five does not change the cyclical properties of the model economy significantly. And the computational time is already very long even with five abscissas.

Using the aggregate state variables $(z, \widetilde{x}) = (z, A, N)$, the workers' value function, and the optimal decision rules are $W(z, \widetilde{x}, e, a)$, $g_a(z, \widetilde{x}, e, a)$, $g_c(z, \widetilde{x}, e, a)$, and $g_\ell(z, \widetilde{x}, e, a)$, respectively. The value function for the representative firm is $J(z, \widetilde{x}, e)$.

To solve the workers' problem, we need to have forecasting functions for aggregate capital in the next period, $A'(z, \widetilde{x})$, number of vacancies, $V(z, \widetilde{x})$, bargaining outcome, $w(z, \widetilde{x})$, and aggregate labor supply $L(z, \widetilde{x})$. With these forecasting functions, and the aggregate state variables, we can compute all the prices needed to solve the workers' problem. In addition, the forecasting functions give the law of motion of $\widetilde{x}$ as follows:

$$(A', N') = f_x(z, \widetilde{x}) = (A'(z, \widetilde{x}), N + f_m(1 - N + \lambda N, V(z, \widetilde{x})) - \lambda N) \tag{46}$$

Note that I assume that $f_m(S, V) = \min(f_m(S, V), S, V)$. This is satisfied with the way the model is calibrated.

The definition of the recursive equilibrium with partial information can be constructed by replacing x with $\widetilde{x}$ in the definition of the original equilibrium.

A.2 Data Structure

The value function of workers $W(z, \widetilde{x}, e, a)$ is interpolated in the dimension of continuous states $(\widetilde{x}, a)$. For $\widetilde{x} = (A, N)$, polynomial interpolation is used for each of A and N. Regarding a, the shape-preserving spline interpolation developed by Schumaker (1983) is used. The optimal decision rules of workers $g_c(z, \widetilde{x}, e, a)$, $g_a(z, \widetilde{x}, e, a)$, and $g_\ell(z, \widetilde{x}, e, a)$ are interpolated as well. Polynomial interpolation and piecewise-linear interpolation are used in the dimensions of $\widetilde{x}$ and a, respectively. The value of the representative firm $J(z, \widetilde{x}, e)$ is interpolated with polynomial interpolation with respect to $\widetilde{x}$.

The forecasting functions for aggregate capital stock in the next period, $A'(z, \widetilde{x})$, number of vacancy postings, $V(z, \widetilde{x})$, bargaining outcome, $w(z, \widetilde{x})$, and aggregate labor supply, $L(z, \widetilde{x})$ are parameterized using the following log-linear forms:

$$A'(z, \widetilde{x}) = \phi_z^{A0} + \phi_z^{A1} \log(A) + \phi_z^{A2} \log(N) \tag{47}$$
$$V(z, \widetilde{x}) = \phi_z^{V0} + \phi_z^{V1} \log(A) + \phi_z^{V2} \log(N) \tag{48}$$
$$w(z, \widetilde{x}) = \phi_z^{w0} + \phi_z^{w1} \log(A) + \phi_z^{w2} \log(N) \tag{49}$$
$$L(z, \widetilde{x}) = \phi_z^{L0} + \phi_z^{L1} \log(A) + \phi_z^{L2} \log(N) \tag{50}$$

For ease of notation, the set of all the coefficients associated with the four forecasting functions defined above is denoted as Φ.

Finally, the type distribution of workers is expressed by a density function $x(e, a)$. Piecewise-linear approximation is used with respect to the dimension of a. For more details, see Ríos-Rull (1999).

A.3 Algorithm

The following algorithm, which is based on Krusell and Smith (1998), is used to solve the model. For the initial condition for the value function of workers and firms, those obtained from the steady state economy are used.

1. Set a guess for the coefficients of forecasting function Φ_0.
2. Given Φ_0, solve the workers' optimization problem. This step includes the following sub-steps:
   (a) Set a guess for workers' value function $W_0(z, \widetilde{x}, e, a)$.
   (b) Given the future value $W_0(z, \widetilde{x}, e, a)$ and forecasting functions with coefficients Φ_0, update the value function using workers' Bellman equation. Denote the updated value function $W_1(z, \widetilde{x}, e, a)$. In finding the optimal decisions, first order conditions are used. A great benefit of using a finite element method with polynomials is that it is costless to compute the derivative of the approximated function. In the current case, since the

value function is approximated using Schumaker's (1983) shape-preserving spline, the derivative of the value function is obtained with almost zero cost in computation time. In addition, the shape-preserving spline preserves the monotonicity of the derivative of the value function, which makes the algorithm to find the optimal decision very stable.

(c) Compute a distance between $W_0(z, \widetilde{x}, e, a)$ and $W_1(z, \widetilde{x}, e, a)$ using some norm (sup-norm is used).

(d) If the distance is smaller than the predetermined tolerance level, this stage is done. Otherwise, update the value function using $W_0(z, \widetilde{x}, e, a) = W_1(z, \widetilde{x}, e, a)$ and go back to step (b).

(e) To speed up the iteration procedure, I use the policy iteration algorithm of Howard (1960). Howard's algorithm is easily used as we implement the value function iteration algorithm.

3. Given Φ_0, solve the representative firm's optimization problem. This step includes the following sub-steps.

   (a) Set a guess for the representative firm's value function $J_0(z, \widetilde{x}, e)$.

   (b) Given the future value $J_0(z, \widetilde{x}, e)$ and forecasting functions with coefficients Φ_0, update the value function using the representative firm's Bellman equation. Denote the updated value function $J_1(z, \widetilde{x}, e)$.

   (c) Compute a distance between $J_0(z, \widetilde{x}, e)$ and $J_1(z, \widetilde{x}, e)$ using some norm (sup-norm is used).

   (d) If the distance is smaller than the predetermined tolerance level, this stage is done. Otherwise, update the value function using $J_0(z, \widetilde{x}, e) = J_1(z, \widetilde{x}, e)$ and go back to step (b).

4. Using the optimal decision rules and the value functions obtained from the previous steps, run a simulation of 1100 periods. This step involves the following sub-steps:

   (a) Draw a sequence $\{z_t\}_{t=0}^{1100}$.

   (b) Set $t = 0$. Set an initial distribution of workers $x_t(e, a)$. The stationary distribution of the steady state version of the model is used as the initial $x_t(e, a)$.

   (c) Compute aggregate statistics (A_t, N_t) associated with $x_t(e, a)$.

   (d) Using the optimal decision rules already obtained in step 2 and the forecasting functions with coefficients Φ_0, compute the aggregate labor supply L_t and the aggregate capital stock in the next period A_{t+1}.

   (e) Compute the number of vacancies V_t using the representative firm's value function obtained in step 3 and the zero profit condition.

   (f) Compute the profit sharing rule w_t using the value of the representative worker and the representative firm.

(g) Update the type distribution to obtain $x_{t+1}(e,a)$.

(h) If $t = 1100$, stop. Otherwise, set $t = t+1$ and go back to step (c).

5. The simulation generates an artificial time series of $\{A_t, N_t, V_t, w_t, L_t\}_{t=1}^{1100}$. Drop the first 100 periods. Use an OLS regression to obtain the set of coefficients Φ_1 that best fit the simulated time series.

6. Compute a distance between Φ_0 and Φ_1 using some norm (sup-norm is used).

7. If the distance is smaller than the predetermined tolerance level, the iteration is done. Otherwise, update Φ and go back to step 2. In updating Φ, a conservative updating turns out to be beneficial for the stable implementation of the algorithm. For example, the coefficients for the forecasting function associated with A are updated as follows, with an updating parameter $\nu \in (0,1]$. The smaller ν is, the more conservative and thus more stable it is in updating, with more computational time as a cost.

$$\phi^{A0}_{z,\ new} = \nu\phi^{A0}_{z,0} + (1-\nu)\phi^{A0}_{z,1} \tag{51}$$
$$\phi^{A1}_{z,\ new} = \nu\phi^{A1}_{z,0} + (1-\nu)\phi^{A1}_{z,1} \tag{52}$$
$$\phi^{A2}_{z,\ new} = \nu\phi^{A2}_{z,0} + (1-\nu)\phi^{A2}_{z,1} \tag{53}$$

8. Once the equilibrium Φ is obtained, business cycle statistics of interest can be computed. This step involves the following sub-steps:

   (a) Draw a sequence of $\{z_t\}_{t=1}^{1100}$.

   (b) Set $x_0(e,a)$.

   (c) Using $x_0(e,a)$, $\{z_t\}_{t=0}^{1100}$, and the equilibrium Φ, run a simulation of 1100 periods.

   (d) Drop the first 100 periods. Using the artificial time series, compute business cycle statistics of interest.

   (e) Repeat the steps (a)-(d) for many times, with different realizations of $\{z_t\}_{t=0}^{1100}$. Compute the mean of the statistics of interest to eliminate sampling error.

A.4 Accuracy

One way to measure the accuracy of the approximated equilibrium is to measure the size of the regression error. If the regression error is large, the approximated forecasting function should be improved by either using more flexible functional form or expanding the set of statistics that represent the type distribution. Table 10 summarizes two measures of fit (adjusted R^2 and standard errors) from the baseline model economy.

Table 10: Measures of fit: baseline model economy

Regressand	z abscissa	Adjusted R^2	Standard error (%)
A'	1	0.9999994	0.00174
A'	2	0.9999995	0.00158
A'	3	0.9999995	0.00130
A'	4	0.9999996	0.00109
A'	5	0.9999996	0.00119
V	1	0.9990196	0.26451
V	2	0.9989218	0.25220
V	3	0.9988816	0.20370
V	4	0.9992322	0.14034
V	5	0.9993512	0.12705
w	1	0.9810171	0.02783
w	2	0.9763834	0.02708
w	3	0.9747930	0.02513
w	4	0.9718815	0.02286
w	5	0.9767984	0.02054
L	1	0.9978578	0.00822
L	2	0.9982236	0.00821
L	3	0.9979106	0.00724
L	4	0.9982326	0.00596
L	5	0.9984483	0.00625