Why are rich countries exporting high quality goods?*

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Abstract

This paper proposes a model of international trade and quality differentiation that explains why rich countries produce and export goods of higher quality than poor countries. The model is based on ideas from the business literature on quality management. One insight of this literature is that interaction with consumers is an important part of successful quality management as it allows firms to receive feedback on the quality, deficiencies and possible improvements of their products.
The model assumes that firms incur a cost for any feedback-generating interaction with consumers. The cost is proportional to the number of consumers, whereas feedback is proportional to the quantity consumed. Because consumers in rich countries consume more per capita, interaction with consumers is less costly for firms located in rich countries. As this gives rich countries a comparative advantage in the production of high quality goods, rich countries will produce and export goods of higher quality than poorer countries.

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1 Introduction

Recent empirical research has found ample evidence that richer countries export goods of a higher quality than poorer countries. Schott (2004), Hummels and Klenow (2005), Hallak (2006a) and Fieler (2007) find that export unit values within product categories increase with exporter per capita income and argue that quality differences are necessary to explain the observed variations in unit values. A different approach that avoids using prices as perfect proxies for quality is taken by Hallak and Schott (2008) and Khandelwal (2008). They use variations in the trade balance respectively market shares to identify a country's export quality and find that richer countries export goods of a higher quality. A different strand exists in the marketing literature. The country-of-origin effects literature takes a psychological approach and evaluates the perception of consumers of goods produced in different countries. One finding of this literature is that the development level of the country of origin has a strong impact on perceived quality.[1]

Traditional workhorse models of international trade largely ignore the existence of quality differences across countries and are thus unable to explain why rich countries export goods of a higher quality. Those models that feature quality differentiation can be roughly divided into models based on productivity differences across countries; models based on endowment differences across countries; and the Linder hypothesis. Models based on productive differences typically assume that rich countries have a comparative advantage in high quality goods.[2] Models based on endowment difference assume that skilled labor is more productive in the production of high quality goods. Then, assuming that rich countries are abundant in skilled labor, rich countries have a comparative advantage in the production of high quality goods.[3] Both types of models do not explain why rich countries have a comparative advantage in the production of high quality goods, that is, why productivity differences exists respectively why skilled labor is more productive in the production of high quality goods. One aspect of the Linder hypothesis is that close proximity to a market for a good gives firms a comparative advantage in the development and production of this good. With rich countries having a larger market for high quality goods, firms in rich countries will have a comparative advantage in the production of high quality goods.[4]

This paper proposes an alternative explanation for why rich countries export high quality goods. The model is in the spirit of the Linder hypothesis and is based on the idea that firms interact with consumers, and that this interaction

[1]See Verlegh and Steenkamp (1999) for a survey of the country-of-origin literature.

[2]Examples for this strand of literature are Flam and Helpman (1987) and Antras (2005).

[3]Examples for this strand of literature are Stokey (1991) and Murphy and Shleifer (1997)

[4]See Linder (1961) for the original formulation of the Linder hypothesis and Hallak (2006b) for a recent empirical test of the hypothesis.

generates information that allows firms to improve the quality of their products. Examples for this interaction include consumer surveys, repair and return data, consumer complaints, product reviews in the press or informal interaction between consumers and managers or engineers. The framework does not only apply to interactions between firms and consumers, but also between firms and their intermediate suppliers or internal customers within a firm.

The idea that consumer feedback allows firms to improve the quality of their products is not new. The business literature on quality management has long recognized the importance of consumer feedback for product quality. For example, the House of Quality also known as Quality Function Deployment (QFD) approach developed by Yoji Akao emphasizes the importance of conveying the "voice of the customer" throughout the design and manufacturing process.[5] Customer feedback, either from consumers, distributors or downstream customers is also an essential feature of the Total Quality Management (TQM) approach developed by Feigenbaum.[6] That product quality is positively influenced by customer feedback is supported by several studies. For example, Miller (1992) described the successful strategy of the Norand corporation to obtain continuous feedback from customers and to use the feedback to improve product quality. Reichheld and Sasser (1990) describes how companies use the feedback of defecting customers to improve the quality of their products or their services. Sethi (2000) finds that quality is positively related to the influence of customers on the product development process, whereas Garvin (1984) finds that consumer feedback plays an important role in the quality improvement process of the most successful Japanese room air conditioner manufacturers. Forza and Filippini (1998) test several TQM practices with data from 43 Italian manufacturing plants. They find that two measures of quality performance, customer satisfaction and quality conformity are strongly linked with the TQM practices involvement with customers regarding quality, information exchange with customers about quality, and attention to and contact with customers for product design.

The model presented in this paper assumes that firms interact with consumers to learn about product deficiencies and possible improvements to the quality of their products. Interaction with consumers is associated with a cost that is proportional to the number of consumer. The cost of interacting with domestic consumers is lower than the cost of interacting with foreign consumers. The feedback received is proportional to total consumption of all varieties produced within an industry, not only the firm's own variety. This simplifying assumption allows to avoid the issue of firms producing more of their own variety in order to receive more feedback. The amount of feedback is thus external to the firm, making the model similar to models with national external economies to scale at the indus-

[5]For the original work see Akao (1994). For a recent overview of the House of Quality framework see Hauser and Clausing (1988)

[6]For the original work see Feigenbaum (1951). For recent overviews of the TQM framework see Flynn, Schroeder and Sakakibara (1994) and Ahire, Landeros and Golhar (1995).

try level.[7] As in richer countries consumers consume more per capita, interaction with consumers is less costly for firms located in rich countries. Consequently rich countries have a comparative advantage in the production of high quality goods and richer countries will export goods of a higher quality than poorer countries. The model thus introduces a new source of comparative advantage, which is not related to technology or factor endowments, but the demand side of the economy. The rest of this paper is organized as follows. The following section introduces the setup of the model for the case of a closed economy. The model is solved for a closed economy and then extended for the case of an open economy with two countries. The predictions of the open economy case are then discussed and compared to the empirical facts established in the existing literature on the quality of a country's exports.

2 The setup of the model

There are two goods, y_t and x_t, available in period $t = 1$ and $t = 2$. Good y_t is a homogenous numéraire good with price equal to one. Good $x_t(v)$ comes in a continuum of varieties indexed by v. Varieties of the differentiated good can have different quality levels. In the first period quality is fixed and equal to one. In the second period firms choose a profit-maximizing quality level $Q_t(v)$.

2.1 Preferences

There are L identical households with preferences given by the following intertemporal Dixit-Stiglitz utility function:

$$\begin{aligned} U &= (1-\beta)\left[\log y_1 + \frac{1}{1+\gamma}\log y_2\right] \\ &+ \beta\left[\frac{1}{1-1/\epsilon}\log\int x_1(v)^{1-1/\epsilon}dv + \frac{1}{1+\gamma}\frac{1}{1-1/\epsilon}\log\int \left(Q_2(v)x_2(v)\right)^{1-1/\epsilon}dv\right], \end{aligned}$$

where $0 < \beta < 1$ is the share of income spend on the differentiated good, $\gamma > 0$ is the discount factor and $\epsilon > 1$ is the elasticity of substitution across varieties. The budget constraint is

$$I_1 + \frac{1}{1+r}I_2 = y_1 + \frac{1}{1+r}y_2 + \int p_1(v)x_1(v)dv + \frac{1}{1+r}\int p_2(v)x_2(v)dv,$$

[7] See Kemp (1955) and Chipman (1970) for the pioneering contributions and Krugman (1995) and Choi and Yu (2002) for recent literature survey.

where I_t is income in period t, r the interest rate and $p_t(v)$ is the price of a variety in period t. Each household is endowed with z efficient units of labor. Labor is supplied inelastically in a perfectly competitive labor market. The wage is determined in the homogenous good sector. Assuming that the homogenous good is produced with constant returns to scale and a labor requirement of one the wage is equal to one. Hence household income is given by $I_t = z$.
As is shown in the appendix, these preferences imply the following demand function for the differentiated good:

$$x_t(v) = A_t p_t(v)^{-\epsilon} Q_t(v)^{\epsilon-1}.$$

The demand level A_t is independent of v and is given by

$$A_t = \left[\int p_t(v)x_t(v)dv\right] / \left[\int \left(p_t(v)/Q_t(v)\right)^{1-\epsilon} dv\right]$$

This implies that the price respectively quality elasticity of demand is given by

$$-\frac{dx_t}{dp_t}\frac{p_t}{x_t} = \epsilon \qquad \text{and} \qquad \frac{dx_t}{dQ_t}\frac{Q_t}{x_t} = \epsilon - 1$$

2.2 Technology

There is a continuum of firms, with each firm being the sole producer of variety v. In every period a firm may decide to enter and to produce or to exit and not to produce. If the firm chooses to produce, it bears a fixed overhead cost f_x and constant marginal costs a_x. These costs are identical across firms. Quality is fixed at $Q_1(v) = 1$ in the first period. In the second period firms can improve the quality of their variety.
Improving quality is a two-stage process. First, the firm receives feedback from consumers by paying a cost of $a_{Q(v)}L$, where $a_{Q(v)}$ can be interpreted as the amount paid by the firm to interact with one consumer. Feedback received from one consumer is assumed to be proportional to $a_{Q(v)}$ and the consumer's total consumption of all varieties in the first period, that is $\int x_1(v)/L dv$. This implies that for a firm producing a specific variety feedback received from consumers of a different variety is equivalent to feedback received from consumers of the own variety. Furthermore, assuming that the number of varieties is sufficiently large, feedback received by one firm is independent of the firm's own production in the first period.
These simplifying assumptions allow to avoid the issue of firms producing more of their own variety in order to receive more feedback. The amount of feedback is thus external to the firm, making the model similar to models with external economies to scale at the industry level. Despite some similarities the model

differs from the existing literature on external economies of scale in several respects. Firstly, the external economies of scale apply to the quality improvement technology, not the production technology. Secondly, the external economies of scale depend on consumption and not output. Lastly, external economies of scale depend on per capita consumption, and not total consumption. Apart from that these two assumptions allow to solve the model, they can be justified by that firms are not only learning from the feedback received from consumers of their own variety, but also from feedback on the quality of their competitors varieties. With L consumers the feedback received by a firm paying a cost of $a_{Q(v)}L$ is given by $a_{Q(v)} \int x_1(v)dv$. The feedback is used by the firm to improve quality according to the following technology:

$$Q_2(v) = \left(1 + a_{Q(v)} \int x_1(v)dv\right)^{\eta},$$

where $0 < \eta < 1$. To ensure a unique solution, η is furthermore assumed to be restricted by $\eta < 1/(\epsilon - 1)$.

3 The closed economy version

3.1 Profit Maximization

The firm's profit maximization problem can be solved by backward induction. Firm profits in the second period are given by

$$\pi_2(v) = (p_2(v) - a_x)\, x_2(v) - a_Q L - f_x,$$

where $Q_2(v) = \left(1 + a_{Q(v)} \int x_1(v)dv\right)^{\eta}$. Maximizing profits yields that firms set prices according to

$$p_2(v) = \frac{\epsilon \cdot a_x}{\epsilon - 1}.$$

Furthermore, firms determine the profit maximizing amount of consumer feedback, and thus the optimal quality level, by by setting the cost of interacting with consumers according to

$$a_{Q(v)}L = \begin{cases} \eta a_x \cdot x_2(v) - \frac{L}{\int x_1(v)dv} & \text{if } \eta a_x \cdot x_2(v) - \frac{L}{\int x_1(v)dv} \geq 0 \\ 0 & \text{if } \eta a_x \cdot x_2(v) - \frac{L}{\int x_1(v)dv} < 0 \end{cases}.$$

Firm profit in the first period are given by

$$\pi_1(v) = (p_1(v) - a_x)\, x_1(v) - f_x.$$

Maximizing profits yields that firms set prices according to the pricing rule as in the second period, that is

$$p_1(v) = \frac{\epsilon \cdot a_x}{\epsilon - 1}.$$

3.2 Market Equilibrium

As entry and exit is free, and as the firm's profits in the second period are independent of whether the firm was producing in the first period or not, firms are making zero profits in every period. Thus the following zero-profit conditions must hold in the first period:

$$\frac{a_x}{\epsilon - 1} x_1(v) - f_x = 0.$$

The number of firms in the first period is denoted by n_1 and is determined according to

$$\int p_1(v)x_1(v) = n_1 p_1(v) x_1(v),$$

what gives that the number of firms is given by

$$n_1 = \frac{\int p_1(v)x_1(v)}{p_1(v)x_1(v)} = \frac{\int p_1(v)x_1(v)}{\epsilon f_x}.$$

As firm profits are zero in both periods and as all households are identical, no intertemporal borrowing and lending takes place. Hence the interest rate is equal to the discount factor, that is $1 + r = 1 + \gamma$, and in both periods total consumption of the differentiated good is equal to the share of income spend on the differentiated good, that is $\int p_t(v)x_t(v) = \beta z L$. The firm size and the number of firms in the first period are thus given by

$$x_1 = \frac{f_x}{a_x}(\epsilon - 1) \qquad \text{and} \qquad n_1 = \frac{\beta z L}{\epsilon f_x}.$$

The zero-profit condition for the second period is given by

$$\frac{a_x}{\epsilon - 1} x_2 - \left(\eta a_x \cdot x_2 - \frac{L}{\int x_1(v)dv} \right) \cdot \mathbb{I}_{\{\eta a_x \cdot x_2 - \frac{L}{\int x_1(v)dv} \geq 0\}} - f_x = 0,$$

where $\mathbb{I}$ is an indicator variable that takes a value of one if

$$\eta a_x \cdot x_2(v) - \frac{L}{\int x_1(v)dv} \geq 0$$

and a value of zero otherwise. After substituting and rearranging one gets

$$\frac{a_x}{\epsilon-1}x_2 = f_x + \left(\eta a_x \cdot x_2 - \frac{a_x}{\beta z}\frac{\epsilon}{\epsilon-1}\right) \cdot \mathbb{I}_{\{\eta a_x \cdot x_2(v) - \frac{a_x}{\beta z}\frac{\epsilon}{\epsilon-1} \geq 0\}}.$$

As is shown in the appendix there exists a unique x_2 such that this zero-profit condition holds. Firms will invest in quality improvement, that is will choose a strictly positive a_Q, if and only if

$$\frac{1}{\eta}\frac{1}{\beta z}\frac{\epsilon}{\epsilon-1} \leq \frac{f_x}{a_x}(\epsilon-1).$$

All else equal, firms in richer countries, indicated by a higher z, will be more likely to invest into improvement of the quality of their variety. The firm size and the number of firms in the second period are given by

$$\begin{aligned} x_2 &= \frac{1}{1-\eta(\epsilon-1)}\left(\frac{f_x}{a_x}(\epsilon-1) - \frac{\epsilon}{\beta z}\right), \\ n_2 &= (1-\eta(\epsilon-1))\frac{\beta^2 z^2 L(\epsilon-1)}{\beta z f_x \epsilon(\epsilon-1) - a_x \epsilon^2}, \end{aligned}$$

and quality by

$$Q_2 = (1 + a_Q n_1 x_1)^\eta = \left(\frac{\eta(\epsilon-1)}{1-\eta(\epsilon-1)}\left(\beta z \cdot \frac{f_x}{a_x}\frac{\epsilon-1}{\epsilon} - 1\right)\right)^\eta.$$

If firms decide not to invest into quality improvements, that is if they choose $a_Q = 0$, the firm size, the number of firms and quality are the same as in the first period, that is

$$x_2 = \frac{f_x}{a_x}(\epsilon-1), \qquad n_2 = \frac{\beta z L}{\epsilon f_x} \qquad \text{and} \qquad Q_2 = 1.$$

Henceforth it will be assumed that the parameters are such that firms will always want to invest into quality improvements, that is

$$\frac{1}{\eta}\frac{1}{\beta z}\frac{\epsilon}{\epsilon-1} \leq \frac{f_x}{a_x}(\epsilon-1).$$

Concentrating on country characteristics this means that it will be assumed that per capita income z is sufficiently large to rule out the case of firms not choosing to improve the quality of their products, that is firms choosing $a_Q = 0$.

3.3 Comparative Statics

In the first period the firm size is independent and the number of firms is increasing in per capita income and population size as

$$\frac{dx_1}{dz}\frac{z}{x_1} = 0, \; \frac{dx_1}{dL}\frac{L}{x_1} = 0 \quad \text{and} \quad \frac{dn_1}{dz}\frac{z}{n_1} = 1, \; \frac{dn_1}{dL}\frac{L}{n_1} = 1.$$

From the first to the second period the firm size will increase and the number of firms will decrease as

$$\begin{aligned} x_2 &= \frac{1}{1-\eta(\epsilon-1)}\left(\frac{f_x}{a_x}(\epsilon-1) - \frac{\epsilon}{\beta z}\right) \geq \frac{f_x}{a_x}(\epsilon-1) = x_1, \\ n_2 &= (1-\eta(\epsilon-1))\frac{\beta^2 z^2 L(\epsilon-1)}{\beta z f_x \epsilon(\epsilon-1) - a_x \epsilon^2} \leq \frac{\beta z L}{\epsilon f_x} = n_1. \end{aligned}$$

The average firm size is larger in the second period as firms incur an additional fixed cost of $a_Q L$, that is the cost of interacting with consumers. To recoup the fixed cost firms have to sell larger quantities in the second period, what implies that the firm size increases and the number of firms decreases.
In the second period firms can increase quality by investing $a_Q(v)L$. As interaction costs are incurred per household, total interaction costs are proportional to population size. Feedback received by firms depends on total consumption and hence feedback is increasing in market size. If the larger market size is due to a larger population L the the increase in feedback is offset by larger interaction costs. If the larger market size is due to higher per capita income z feedback will be higher, but interaction costs will stay constant. Thus firms in rich countries where per capita consumption is higher will either receive more feedback for the same cost or will receive the same feedback for a lower cost than firms in poor countries.
Quality will be increasing in per capita income and will be independent of population size as

$$\begin{aligned} \frac{dQ_2}{dz}\frac{z}{Q_2} &= \eta\frac{\beta z \frac{f_x}{a_x}\frac{\epsilon-1}{\epsilon}}{\beta z \frac{f_x}{a_x}\frac{\epsilon-1}{\epsilon} - 1}, \quad \text{with} \quad 0 < \frac{dQ_2}{dz}\frac{z}{Q_2} < 1, \\ \frac{dQ_2}{dL}\frac{L}{Q_2} &= 0. \end{aligned}$$

If per capita income increases, market size and hence total consumption of the differentiated good in the first period increases. Firms will be willing to pay more for interacting with consumers in order to receive more feedback and improve quality. If the population size increases, total interaction costs increase, without an accompanying increase in feedback. To offset this increase in interaction costs, firms will choose a lower a_Q. Consequently one has

$$\begin{aligned} \frac{da_Q L}{dz}\frac{z}{a_Q} &= \frac{a_x \epsilon}{\eta\beta z f_x(\epsilon-1)^2 - a_x\epsilon} > 0, \\ \frac{da_Q L}{dL}\frac{L}{a_Q} &= 0. \end{aligned}$$

The firm size is increasing in per capita income and independent of the population size as

$$\begin{aligned} \frac{dx_2}{dz}\frac{z}{x_2} &= \frac{a_x \epsilon}{\beta z f_x(\epsilon-1) - a_x\epsilon} > 0, \\ \frac{dx_2}{dL}\frac{L}{x_2} &= 0. \end{aligned}$$

The number of firms increases less than proportional to per capita income and proportional to the population size as

$$\begin{aligned} \frac{dn_2}{dz}\frac{z}{n_2} &= \frac{\beta z f_x \epsilon(\epsilon-1) - 2a_x\epsilon^2}{\beta z f_x \epsilon(\epsilon-1) - a_x\epsilon^2} < 1 \\ \frac{dn_2}{dL}\frac{L}{n_2} &= 1. \end{aligned}$$

This implies that, controlling for the size of the economy, the number of firms is smaller in rich countries.

4 The open economy

There are two countries, Home and Foreign, that are indexed by the superscripts $i = H, F$. Per capita income and population size are given by z^i respectively L^i. In contrast to the preceding section, there are M differentiated goods, denoted by $x_{mt}(v)$. Income shares are given by β_m for the differentiated good and by $1 - \sum_m \beta_m$ for the homogenous good. Transportation costs for the differentiated goods and the homogenous good are assumed to be zero. The inclusion of the homogenous good guarantees balanced trade and factor price equalization as long as the homogenous good sector is sufficiently large.
In what follows the Dornbusch-Fischer-Samuelson framework is used.[8] Marginal costs a_x^m are assumed to differ across industries and countries, and to be the same for all varieties within an industry. For Home marginal costs are increasing in

[8] See Dornbusch, Fischer and Samuelson (1977)

the index of the industry, and for Foreign marginal costs are decreasing in the index of the industry, that is

$$a^H_{x_m} < a^H_{x_{m+1}} \quad \text{and} \quad a^F_{x_m} > a^F_{x_{m+1}}, \quad m = 1, 2, \dots M-1.$$

Interaction costs with foreign consumers are assumed to be prohibitively high, implying that firms can receive feedback only from domestic consumers. This simplifying assumption can be justified by that interaction with foreign consumers involves additional costs due to a different language or culture, legal restrictions or a larger distance and higher cross-border communication costs. Under this assumption the amount of consumer feedback depends only on domestic consumption of all varieties within an industry, that is for a firm in industry m in country i the cost of interacting with domestic consumers is $a_Q L^i$ and the feedback received is $a_Q \int x^i_{m1}(v)dv$.
External economies of scale are thus national and at the industry level. An important policy implication is that protective trade barriers are harmful as they lower domestic consumption. This stands in contrast to the policy implications of national external economies of scale where economies scale depends on output and not on consumption. This policy implication holds as long as the assumption is maintained that feedback received by a firm from consumers of the firm's own variety is equivalent to the feedback received from consumers of other varieties. Relaxing this assumption would potentially result in different policy implications.

4.1 Trade Pattern in the First Period

Firms are free to produce any variety $x_{mt}(v)$. There is no Armington-assumption and hence variety x_{mt} produced in country i is identical to the same variety produced in country j. Consumers will buy the variety that is cheaper in quality-adjusted terms. That is, a consumer in country i will buy variety $x_{mt}(v)$ domestically if

$$\frac{p^i_{x_{mt}(v)}}{Q^i_{x_{mt}(v)}} \leq \frac{p^j_{x_{mt}(v)}}{Q^j_{x_{mt}(v)}}.$$

In the first period quality is fixed and equal to one. Thus in the first period a consumer in country H will buy domestically if $p^H_{x_{m1}(v)} \leq p^F_{x_{m1}(v)}$ and will buy from the foreign country otherwise.
For all industries m in which $a^H_{x_m} < a^F_{x_m}$, domestic consumption in Home will consist only of domestic varieties.[9] For all other industries m domestic consumption will consists only of varieties imported from Foreign. Furthermore, Home

[9]Suppose that there exists a foreign firm making a positive profit by exporting variety x_m to Home. Then, with $a^H_{x_m} < a^F_{x_m}$, a firm in Home could sell the same variety for a lower price and still make a positive profit.

will export to Foreign in all industries for which $a_{x_m}^H < a_{x_m}^F$.
As marginal costs are increasing in the index of the industry for Home and decreasing in the index of the industry for Foreign, it follows that there is an m_1 such that for all industries $m \leq m_1$ Home will be the only producer and that for all industries $m > m_1$ Foreign will be the only producer in the world market.

4.2 Trade Pattern in the Second Period

In the remainder of this paper it will be assumed, without loss of generality, that Home is richer than Foreign, that is $z^H > z^F$. Trade pattern are more difficult to determine for the second period as firms are competing in two dimensions, quality and price. Suppose for example that in an industry $a_{x_m}^F < a_{x_m}^H$. Although disadvantaged by higher marginal costs of production, firms in Home might have a cost advantage in improving quality if $z^H > z^F$. In this case it might be possible that the trade pattern changes and Home will be the only producer and exporter in the industry.
Home will be the only producer of a variety $x(v)$ whenever firms in Home have lower average costs of producing Qx effective units of the variety than firms in Foreign. Suppose that a firm in Foreign makes nonnegative profits by selling the amount x at price p^F and quality level Q^F. A firm in Home could sell the same amount at the price p^H and the quality Q^H if $p^H/Q^H < p^F/Q^F$. As firms in Home have lower average costs of producing Qx effect units this firm would make a strictly positive profit as

$$\frac{p^H}{Q^H} \geq \frac{a_x^H x + a_Q L^H + f_x}{Q^H x^H}. \tag{1}$$

Thus Home will be the only producer of the variety. Conversely, Foreign will be the only producer whenever firms in Foreign have lower average costs of producing Qx effect units than firms in Home.
As is shown in the appendix, there is an $\widehat{m}$, such that for all industries $m \leq \widehat{m}$ Home is the only producer, and for all industries $m \geq \widehat{m}$ Foreign is the only producer. Furthermore, $\widehat{m}$ has a lower bound m_1 that implies $a_x^H < a_x^F$ and an upper bound m_2 that implies $a_x^H > a_x^F (z^H)^{\frac{\eta}{1-\eta}}/(z^F)^{\frac{\eta}{1-\eta}}$. Furthermore, $\widehat{m}$ is increasing in Home's per capita income z^H and decreasing in Foreign's per capita income z^F.

4.3 Market equilibrium

The market equilibrium for the open economy can be summarized as follows. In the first period Home is the only producer and the exporter in all industries

$m \leq m_1$. The firm size and the number of firms in Home are given by

$$x^H_{m1} = \frac{f_x}{a^H_x}(\epsilon - 1) \qquad \text{and} \qquad n^H_{m1} = \frac{\beta_m z^H L^H + \beta_m z^F L^F}{\epsilon f_x}.$$

Foreign is the only producer and the exporter in all industries $m > m_1$. The firm size and the number of firms in Foreign are given by

$$x^F_{m1} = \frac{f_x}{a^F_x}(\epsilon - 1) \qquad \text{and} \qquad n^F_{m1} = \frac{\beta_m z^H L^H + \beta_m z^F L^F}{\epsilon f_x}.$$

In the first period total consumption of all varieties in an industry in Home and Foreign is given by

$$\int x^i_{m1}(v)dv = \frac{\beta_m z^i L^i}{a^k_x}\frac{\epsilon - 1}{\epsilon}, \quad i = \{H, F\}, \quad a^k_x = \min\{a^H_x, a^F_x\}.$$

In the second period Home is the only producer and the exporter for all industries $m \leq m_1$, that is industries for which $a^H < a^F$. The firm size, the number of Home firms and the quality level are given by

$$\begin{aligned}
x^H_{m2} &= \frac{1}{1 - \eta(\epsilon - 1)}\left(\frac{f_x}{a^H_x}(\epsilon - 1) - \frac{\epsilon}{\beta_m z^H}\right) \\
n^H_{m2} &= (1 - \eta(\epsilon - 1))\frac{\beta^2_m z^H (z^H L^H + z^F L^F)(\epsilon - 1)}{\beta_m z^H f_x \epsilon(\epsilon - 1) - a^H_x \epsilon^2} \\
Q^H_m &= \left(\frac{\eta(\epsilon - 1)}{1 - \eta(\epsilon - 1)}\left(\beta_m z^H \frac{f_x}{a^H_x}\frac{\epsilon - 1}{\epsilon} - 1\right)\right)^\eta.
\end{aligned}$$

Home is also the only producer and the exporter for all $m_1 < m \leq \widehat{m}$. The firm size, the number of Home firms and the quality level are given by

$$\begin{aligned}
x^H_{m2} &= \frac{1}{1 - \eta(\epsilon - 1)}\left(\frac{f_x}{a^H_x}(\epsilon - 1) - \frac{a^F_x}{a^H_x}\frac{\epsilon}{\beta_m z^H}\right) \\
n^H_{m2} &= (1 - \eta(\epsilon - 1))\frac{\beta^2_m z^H (z^H L^H + z^F L^F)(\epsilon - 1)}{\beta_m z^H f_x \epsilon(\epsilon - 1) - a^F_x \epsilon^2} \\
Q^H_m &= \left(\frac{\eta(\epsilon - 1)}{1 - \eta(\epsilon - 1)}\left(\beta_m z^H \frac{f_x}{a^F_x}\frac{\epsilon - 1}{\epsilon} - 1\right)\right)^\eta.
\end{aligned}$$

For all $m > \widehat{m}$ Foreign is the only producer and exporter. The firm size, the number of Foreign firms and the quality level are given by

$$
\begin{aligned}
x_{m2}^F &= \frac{1}{1-\eta(\epsilon-1)}\left(\frac{f_x}{a_x^F}(\epsilon-1)-\frac{\epsilon}{\beta_m z^F}\right) \\
n_{m2}^F &= (1-\eta(\epsilon-1))\frac{\beta_m^2 z^F(z^H L^H+z^F L^F)(\epsilon-1)}{\beta_m z^F f_x \epsilon(\epsilon-1)-a_x^F\epsilon^2} \\
Q_m^F &= \left(\frac{\eta(\epsilon-1)}{1-\eta(\epsilon-1)}\left(\beta_m z^F\frac{f_x}{a_x^F}\frac{\epsilon-1}{\epsilon}-1\right)\right)^{\eta}.
\end{aligned}
$$

4.4 Comparative Statics

In what follows changes in per capita income and population size and their effect on the firm size, the number of firms, and the quality level are examined.
As is shown in the appendix, $\widehat{m}$ is increasing in Home's per capita income z^H and decreasing in Foreign's per capita income z^F. This implies that conditional on the initial distribution of productivities, richer countries produce and export in more industries than poorer countries. Within industries, the firm size and the quality level are increasing in a country's per capita income and independent of a country's population size.
Computing the first derivative of the number of firms with respect to per capita income and population size gives

$$
\begin{aligned}
\frac{dn^i}{dz^i}\frac{z^i}{n^i} &= \frac{z^i L^i}{z^i L^i+z^j L^j}-\frac{a_x^k\epsilon}{\beta_m z^i f_x(\epsilon-1)-a_x^k\epsilon}<1 \\
\frac{dn^i}{dL^i}\frac{L^i}{n^i} &= \frac{z^i L^i}{z^i L^i+z^j L^j}<1,
\end{aligned}
$$

where $a_x^k=\min\{a_x^H,a_x^F\}$. Whereas the number of firms is increasing in population size, the effect of a rising income on the number of firms is ambiguous. Firstly, with rising income the market size and thus the number of firms increases. The magnitude of this effect depends on the relative size of the economy compared to the rest of the world. Secondly, with rising income firms will choose a higher quality level. In order to recoup the higher fixed costs associated with a higher quality level firm size has to increase, thus reducing the number of firms. The magnitude of this effect depends on the per capita income level and the size of the market for the industry, indicated by the expenditure share parameter β_m. Which of these two effects dominates is ambiguous and depends on the values of the parameters.

5 Model predictions

5.1 Exports

The total value of exports of the differentiated goods from Home to Foreign in the second period is given by

$$\sum_{m=1}^{\widehat{m}} p_{m2} \cdot n_{m2}^{H} \cdot \left(x_{m2}^{H} - c_{m2}^{H}\right) = \sum_{m=1}^{\widehat{m}} \beta_m z^F L^F,$$

where c_{m2}^{H} denotes domestic consumption of a variety. Balanced trade requires that the value of Home's exports equals the value of Foreign's imports. With the homogenous good y ensuring that trade is balanced this implies

$$y^H + \sum_{m=1}^{\widehat{m}} \beta_m z^F L^F = \sum_{m=\widehat{m}+1}^{M} \beta_m z^H L^H,$$

where y^H denotes Home's net exports of the homogenous good.
Total real exports of the differentiated goods from Home to Foreign in the second period are given by

$$\sum_{m=1}^{\widehat{m}} n_{m2}^{H} \cdot \left(x_{m2}^{H} - c_{m2}^{H}\right) = \sum_{m=1}^{\widehat{m}} \beta_m \frac{z^F L^F}{a_x^H(m)} \frac{\epsilon - 1}{\epsilon}.$$

Conversely, Home's real imports from Foreign are given by

$$\sum_{m=\widehat{m}+1}^{M} n_{m2}^{F} \cdot \left(x_{m2}^{F} - c_{m2}^{F}\right) = \sum_{m=\widehat{m}+1}^{M} \beta_m \frac{z^H L^H}{a_x^F(m)} \frac{\epsilon - 1}{\epsilon}.$$

Instead of a formal test of the model the following sections will compare the model predictions with results established in the existing literature.

5.2 Extensive Margin

Hummels and Klenow (2005) use cross-country data to examine how export margins vary with country characteristics. They decompose the total value of a country's exports into the intensive margin, which accounts for volumes exported; into the extensive margin, which accounts for the number of categories in which a country is exporting; and the quality margin, which accounts for higher export

prices due to higher quality. They find that larger economies export more, and that the extensive margin accounts for more than sixty percent of the greater exports of larger economies. Furthermore, the extensive margin is more important for richer countries than for countries with a large number of workers.
This result does partly conform to the model predictions. The model correctly predicts that richer countries will export in more categories, although this prediction is conditional on the initial distribution of productivities. The model fails to predict that countries with a large number of workers also export in more categories. Here the model predicts that the number of categories in which a country is exporting is independent of the population size, and, assuming that population size and the number of workers are highly correlated, independent of the number of workers.[10]

5.3 Intensive Margin

Although the model predicts that larger economies export more within industry categories, this relationship is only due to the assumption of balanced trade. As larger economies import more, balanced trade requires that they export more. Holding the number of categories constant, larger economies export more by increasing net exports of the homogenous good. Looking only at differentiated goods, the intensive margin vanishes as export quantities of the differentiated good depend only on foreign demand given by $\beta_m z^F L^F$. This prediction is at odds with the finding of Hummels and Klenow (2005) that within categories larger economies export higher quantities, and that this effect is more pronounced for country with a larger number of workers.

5.4 Quality Margin

The model predicts that quality is increasing in income per capita, but does not vary with population size. Conventional data on international trade and exports does not allow to observe quality directly. Hummels and Klenow (2005) use data on export prices and export quantities at a highly disaggregated six-digit level to measure quality indirectly. Potentially within-category variety can explain that a country exports higher quantities at higher prices. Hummels and Klenow show that under plausible assumptions the fact that richer countries export at higher prices can be only explained by that richer countries export goods of a higher

[10]Note that this is not exactly true if one includes the homogenous good in the category count. Suppose that initially a country is a net importer of the homogenous good. As the size of the population and thus of the economy increases, imports, and to ensure balanced trade, exports rise. At some point this causes the country to begin exporting the homogenous good, thus raising the extensive margin by one additional industry.

quality.
Although the model predicts that richer countries export goods of a higher quality, the theoretical model predictions cannot be directly compared to the findings of Hummels and Klenow. In order to solve the model it was assumed that marginal costs are such that a country either produces and exports or does not produce and only imports in any given industry.[11] This makes it impossible to compare prices and export quantities directly. Theoretically one could compare prices implied by the markup on marginal costs. For all industries $m < \widehat{m}$ the richer country Home will export at price $p^H = \epsilon \cdot a_x^H/(\epsilon - 1)$. If Foreign would produce in these industries it would be at the price $p^F = \epsilon \cdot a_x^F/(\epsilon - 1)$. Then for all industries $m \leq m_1$, Home exports at lower prices $p^H < p^F$ and for all $m_1 < m \leq \widehat{m}$, Home exports at higher prices $p^H > p^F$. Depending on the distribution of marginal costs a_x^i and the income share parameter β_m and the construction of the price index the model will conform to the finding of Hummels and Klenow that richer countries export goods of a higher quality as indicated by higher export prices.
The predictions of the model are easier to reconcile with the findings of Hallak and Schott (2008) and Khandelwal (2008). These two papers follow an alternative approach that avoids using prices as a perfect proxy for quality. They exploit the fact that conditional on the price consumers will prefer the higher quality good, leading to a larger market share for high quality goods. Hallak and Schott and Khandelwal show that richer countries export higher quality goods, and that higher quality is only partly reflected in higher export prices, consistent with the predictions of the model presented in this paper.

5.5 Conclusions

This paper developed a model in which country differences in per capita income give rich countries a comparative advantage in the production of high quality goods. In line with the Linder hypothesis and in contrast to the workhorse models of international trade comparative advantage and the trade pattern is not driven by differences in technology or factor endowments, but by differences in per capita income and per capita demand across countries. High per capita demand reduces the cost for firms of interacting with consumers, thus making it less costly for them to improve the quality of their products. Comparing the model predictions to empirical results in the existing literature shows that the model is partially successful in explaining real world trade patterns.
A key assumption of the model is that the cost of interacting with consumers is proportional to the number of consumers, and that consumer feedback is proportional to the number of units consumed. Furthermore, it is assumed that the cost

[11]This assumption rules out the case where quality-adjusted prices are equal for Home and Foreign, that is, $p^H/Q^H = p^F/Q^F$. Then households are indifferent between cheap low-quality goods and expensive high-quality goods.

of interacting with consumers in foreign countries is prohibitively high. Relaxing these two assumptions is a potentially fruitful area of further investigation. These include, but are not limited to, the following. Allowing the cost of interacting with consumers to vary with distance instead of national borders could give rise to economies of agglomeration such that firms in rich and densely populated areas have a comparative advantage in the production of high quality goods. Another possible extension is to allow the cost of interacting with consumers to vary with the characteristics of the industry such as for example average quantity purchased by consumers or how intensely a good is used. For example, some industries such as the aircraft industry are characterized by that relatively few customers purchase relatively large quantities. Other industries, such as for example the mobile phone industry sell relatively small quantities to a large number of customers, but at the same time profit from that the good is heavily used by consumers on a daily basis. Lastly, allowing firms to reduce the cost of interacting with foreign consumers by setting up branches or subsidiaries in foreign countries could yield interesting predictions and would add a new motivation for foreign direct investment to the literature.

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A Derivation of the aggregate demand function

Solving the utility maximization problem gives

$$\begin{aligned}(1-\beta)\frac{1}{y_1} &= \lambda \\ (1-\beta)\frac{1}{1+\gamma}\frac{1}{y_2} &= \lambda\frac{1}{1+r}\end{aligned}$$

as the first-order conditions for the homogenous good y_t, and

$$\begin{aligned}\beta\frac{1}{\int x_1(v)^{1-1/\epsilon}dv}x_1(v)^{-1/\epsilon} &= \lambda p_1(v) \\ \beta\frac{1}{\int \left(Q_2(v)x_2(v)\right)^{1-1/\epsilon}dv}\frac{1}{1+\gamma}Q_2(v)^{1-1/\epsilon}x_2(v)^{-1/\epsilon} &= \lambda\frac{1}{1+r}p_2(v)\end{aligned}$$

as the first-order conditions for the differentiated good $x_t(v)$. Solving the first-order condition for $x_1(v)$ and multiplying with $p_1(v)$ results in

$$p_1(v)x_1(v) = \left(\frac{1}{\lambda}\frac{\beta}{\int x_1(v)^{1-1/\epsilon}dv}\right)^{\epsilon}p_1(v)^{1-\epsilon}.$$

Integrating over all varieties $x_1(v)$ gives

$$\int p_1(v)x_1(v)dv = \left(\frac{1}{\lambda}\frac{\beta}{\int x_1(v)^{1-1/\epsilon}dv}\right)^{\epsilon}\int p_1(v)^{1-\epsilon}dv.$$

Rearranging gives that λ is

$$\lambda = \left(\frac{\beta}{\int x_1(v)^{1-1/\epsilon}dv}\right)\left(\frac{\int p_1(v)^{1-\epsilon}dv}{\int p_1(v)x_1(v)dv}\right)^{1/\epsilon}.$$

Substituting λ back into the first-order condition for $x_1(v)$ gives that the demand function for $x_1(v)$ is

$$x_1(v) = \frac{\int p_1(v)x_1(v)dv}{\int p_1(v)^{1-\epsilon}dv}p_1^{-\epsilon} = A_1 p_1^{-\epsilon}.$$

Analogously, solving the first-order condition for $x_2(v)$ and multiplying with $p_2(v)$ results in

$$p_2(v)x_2(v) = \left(\frac{1+r}{1+\gamma}\right)^{\epsilon}\left(\frac{1}{\lambda}\frac{\beta}{\int \left(Q_2(v)x_2(v)\right)^{1-1/\epsilon}dv}\right)^{\epsilon}Q_2^{\epsilon-1}p_2(v)^{1-\epsilon}.$$

Integrating over all varieties $x_2(v)$ gives

$$\int p_2(v)x_2(v)dv = \left(\frac{1+r}{1+\gamma}\right)^{\epsilon}\left(\frac{1}{\lambda}\frac{\beta}{\int (Q_2(v)x_2(v))^{1-1/\epsilon}\, dv}\right)^{\epsilon}\int Q_2^{\epsilon-1}p_2(v)^{1-\epsilon}dv.$$

Rearranging gives that λ is

$$\lambda = \left(\frac{1+r}{1+\gamma}\right)\left(\frac{\beta}{\int (Q_2(v)x_2(v))^{1-1/\epsilon}\, dv}\right)\left(\frac{\int Q_2^{\epsilon-1}p_2(v)^{1-\epsilon}dv}{\int p_2(v)x_2(v)dv}\right)^{1/\epsilon}.$$

Substituting λ back into the first-order condition for $x_2(v)$ gives the demand function for $x_2(v)$:

$$x_2(v) = \left(\frac{\int p_2(v)x_2(v)dv}{\int (p_2(v)/Q_2(v))^{1-\epsilon}\, dv}\right)^{1/\epsilon} p_2^{-\epsilon}Q_2(v)^{\epsilon-1} = A_2 p_2^{-\epsilon}Q_2(v)^{\epsilon-1}.$$

B Uniqueness of x_2

There exists only one x_2 such that the zero-profit condition

$$\frac{a_x}{\epsilon-1}x_2 = f_x + \left(\eta a_x\cdot x_2 - \frac{a_x}{\beta z}\frac{\epsilon}{\epsilon-1}\right)\cdot \mathbb{I}_{\{\eta a_x\cdot x_2(v)-\frac{a_x}{\beta z}\frac{\epsilon}{\epsilon-1}\geq 0\}}$$

holds. The left-hand side is a strictly increasing linear function in x_2, with slope $\frac{a_x}{\epsilon-1}$ and intercept 0. For all $x_2 < \frac{1}{\eta}\frac{1}{\beta z}\frac{\epsilon}{\epsilon-1}$, the right-hand side is a constant taking the value f_x. For all $x_2 \geq \frac{1}{\eta}\frac{1}{\beta z}\frac{\epsilon}{\epsilon-1}$ the right-hand side is a linear function with slope ηa_x. Note that $\eta a_x \leq \frac{a_x}{\epsilon-1}$ by assumption. Hence, if $\frac{1}{\eta}\frac{1}{\beta z}\frac{\epsilon}{\epsilon-1} > \frac{f_x}{a_x}(\epsilon-1)$, $a_Q L = 0$ and $x_2 = \frac{f_x}{a_x}(\epsilon-1)$. If $\frac{1}{\eta}\frac{1}{\beta z}\frac{\epsilon}{\epsilon-1} \leq \frac{f_x}{a_x}(\epsilon-1)$, $a_Q L \geq 0$ and $x_2 = \frac{1}{1-\eta(\epsilon-1)}(\frac{f_x}{a_x}(\epsilon-1)-\frac{\epsilon}{\beta z})$.

C Trade pattern in the second period

Note that throughout this appendix $z^H > z^F$. Furthermore, the assumptions $\frac{f_x}{a_x^i}(\epsilon-1) \geq \frac{1}{\eta}\frac{1}{\beta_m z^i}\frac{\epsilon}{\epsilon-1}$ and $\eta < \frac{1}{\epsilon-1}$ will be frequently used.
First consider the case $a_x^H < a_x^F$. Suppose that there is a firm in Foreign earning nonnegative profits by selling the amount x at price p^F and quality Q^F. This firm would incur a cost of $a_Q^F L^F$ for improving quality. For the same quality level a firm in Home would have to incur a cost of only $a_Q^H L^H = \frac{z^F}{z^H}a_Q^F L^F < a_Q^F L^F$.

Hence firms in Home could sell the same amount with the same quality at a lower price and make a strictly positive profit. It follows that for all $a_x^H < a_x^F$ Home will be the only producer.
For the case $a_x^F < a_x^H$, Home will be the only producer of variety $x(v)$ whenever firms in Home have lower average costs of producing Qx effective units of variety $x(v)$ than firms in Foreign. Average costs for firms in country i are given by

$$AC^i = \frac{a_x^i x + a_Q L^i + f_x}{Q^i(x)x}.$$

Minimizing average costs with respect to Q for a given output level x implies that

$$Q^i(x) = \begin{cases} \left(\frac{a_x^i x + f_x - \frac{a_x^F}{\beta_m z^i}\frac{\epsilon}{\epsilon-1}}{\frac{a_x^F}{\beta_m z^i}\frac{\epsilon}{\epsilon-1}} \frac{\eta}{1-\eta} \right)^\eta & \text{if } \left(\frac{a_x^i x + f_x - \frac{a_x^F}{\beta_m z^i}\frac{\epsilon}{\epsilon-1}}{\frac{a_x^F}{\beta_m z^i}\frac{\epsilon}{\epsilon-1}} \frac{\eta}{1-\eta} \right)^\eta > 1 \\ 1 & \text{otherwise,} \end{cases}$$

or alternatively

$$Q^i(x) = \begin{cases} \left(\frac{a_x^i x + f_x - \frac{a_x^F}{\beta_m z^i}\frac{\epsilon}{\epsilon-1}}{\frac{a_x^F}{\beta_m z^i}\frac{\epsilon}{\epsilon-1}} \frac{\eta}{1-\eta} \right)^\eta & \text{if } x > \frac{1}{a_x^i}\left(\frac{1}{\eta}\frac{a_x^F}{\beta_m z^i}\frac{\epsilon}{\epsilon-1} - f_x \right) = \widetilde{x}^i \\ 1 & \text{otherwise.} \end{cases}$$

Hence for all $x > \widetilde{x}^i$ one has $Q^i(x) > 1$. Furthermore $\widetilde{x}^F > \widetilde{x}^H$ as

$$\begin{aligned} \frac{1}{a_x^F}\left(\frac{1}{\eta}\frac{a_x^F}{\beta_m z^F}\frac{\epsilon}{\epsilon-1} - f_x\right) &> \frac{1}{a_x^H}\left(\frac{1}{\eta}\frac{a_x^F}{\beta_m z^H}\frac{\epsilon}{\epsilon-1} - f_x\right) \\ \Leftrightarrow \frac{f_x}{a_x^F}(\epsilon-1)\left(\eta a_x^F z^H - \eta a_x^H z^H\right) &> \frac{\epsilon}{\beta_m z^F}\left(a_x^F z^F - a_x^H z^H\right) \\ \Leftrightarrow \eta a_x^F z^H - \eta a_x^H z^H &> a_x^F z^F - a_x^H z^H \\ \Leftrightarrow a_x^H z^H - \eta a_x^H z^H &> a_x^F z^F - \eta a_x^F z^F > a_x^F z^F - \eta a_x^F z^H \\ \Leftrightarrow a_x^H z^H &> a_x^F z^F. \end{aligned}$$

If Home is the only producer of variety $x(v)$ the equilibrium firm size in Home is $x^H = \frac{1}{1-\eta(\epsilon-1)}(\frac{f_x}{a_x^H}(\epsilon-1) - \frac{a_x^F}{a_x^H}\frac{\epsilon}{\beta_m z^H})$. Conversely, if Foreign is the only producer of variety $x(v)$ the equilibrium firm size in Foreign is $x^F = \frac{1}{1-\eta(\epsilon-1)}(\frac{f_x}{a_x^F}(\epsilon-1) - \frac{\epsilon}{\beta_m z^F})$. Then $x^H > x^F$ as

$$\begin{aligned} \frac{1}{1-\eta(\epsilon-1)}\left(\frac{f_x}{a_x^H}(\epsilon-1) - \frac{a_x^j}{a_x^H}\frac{\epsilon}{\beta_m z^H}\right) &> \frac{1}{1-\eta(\epsilon-1)}\left(\frac{f_x}{a_x^F}(\epsilon-1) - \frac{a_x^j}{a_x^F}\frac{\epsilon}{\beta_m z^F}\right) \\ \Leftrightarrow \frac{f_x}{a_x^F}(\epsilon-1)\left(a_x^F z^H - a_x^H z^H\right) &> \frac{\epsilon}{\beta_m z^F}\left(a_x^F z^F - a_x^H z^H\right), \end{aligned}$$

where the latter inequality follows from that $\frac{f_x}{a_x^F}(\epsilon-1) > \frac{\epsilon}{\beta_m z^F}$.
Combining this with the fact that $\widetilde{x}^F > \widetilde{x}^H$ gives $x^H > x^F > \widetilde{x}^F > \widetilde{x}^H$ as

$$\begin{aligned} x^F = \frac{1}{1-\eta(\epsilon-1)}\left(\frac{f_x}{a_x^F}(\epsilon-1) - \frac{\epsilon}{\beta_m z^F}\right) &> \frac{1}{a_x^F}\left(\frac{1}{\eta}\frac{a_x^F}{\beta_m z^F}\frac{\epsilon}{\epsilon-1} - f_x\right) = \widetilde{x}^F \\ \Leftrightarrow \frac{f_x}{a_x^F}(\epsilon-1)\left(1-\eta+\frac{1}{\epsilon-1}\right) > \frac{f_x}{a_x^F}(\epsilon-1) &> \frac{1}{\eta(\epsilon-1)}\frac{\epsilon}{\beta_m z^F}. \end{aligned}$$

Hence for both x^H and x^F one has $Q > 1$. Average costs for country i at output level $x > \widetilde{x}^F$ are then given by

$$AC^i(x) = \left(\frac{a_x^i x + f_x - \frac{a_x^F}{\beta_m z^i}\frac{\epsilon}{\epsilon-1}}{1-\eta}\right)^{1-\eta}\left(\frac{1}{\eta}\frac{a_x^F}{\beta_m z^i}\frac{\epsilon}{\epsilon-1}\right)^{\eta}\frac{1}{x},$$

where $AC^i(x)$ denotes country i's average cost of producing Qx effective units at output level x.
Home will be the only producer of variety $x(v)$ whenever $AC^H(x^H) < AC^F(x^H)$ and $AC^H(x^F) < AC^F(x^F)$. As Home is always free to produce and sell the larger amount $x^H > x^F$, Home is also the only producer if $AC^H(x^H) < AC^F(x^H)$ and $AC^H(x^F) > AC^F(x^F)$. Foreign will be the only producer of variety $x(v)$ whenever $AC^F(x^F) < AC^H(x^F)$ and $AC^F(x^H) < AC^H(x^H)$. If $AC^F(x^F) < AC^H(x^F)$ and $AC^F(x^H) > AC^H(x^H)$, Home will be the only producer, as was shown above.
Hence Home will be the only producer if $AC^H(x^H) < AC^F(x^H)$ and Foreign will be the only producer otherwise.[12] Substituting x^H and x^F into the average cost function and simplifying gives that Home will be the only producer if

$$a_x^H < \underbrace{a_x^F\frac{\widehat{z}^H}{\widehat{z}^F}\left[1+\frac{1-\eta(\epsilon-1)}{\epsilon-1}\left(1-\frac{\widehat{z}^H}{\widehat{z}^F}\frac{\frac{f_x}{a_x^F}(\epsilon-1)-\frac{\epsilon}{\beta_m z^F}}{\frac{f_x}{a_x^F}(\epsilon-1)-\frac{\epsilon}{\beta_m z^H}}\right)\right]^{-1}}_{:=A(a_x^F;z^H,z^F)},$$

where $\widehat{z}^i$ denotes $(z^i)^{\frac{\eta}{1-\eta}}$. The term inside square brackets is strictly larger than one as

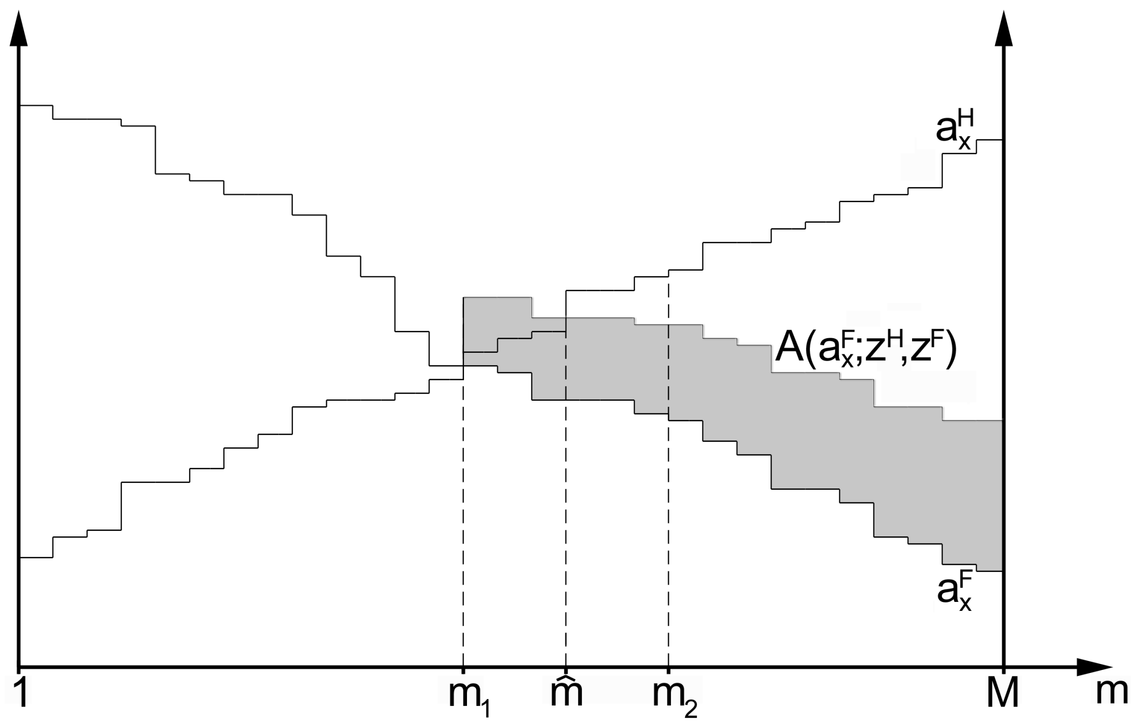
$$\frac{\widehat{z}^H}{\widehat{z}^F}\left(\frac{f_x}{a_x^F}(\epsilon-1)-\frac{\epsilon}{\beta_m z^F}\right) < \left(\frac{f_x}{a_x^F}(\epsilon-1)-\frac{\epsilon}{\beta_m z^H}\right). \tag{2}$$

It follows that for all a_x^H, a_x^F such that $a_x^H \geq a_x^F\frac{\widehat{z}^H}{\widehat{z}^F}$ this inequality does not hold and that hence Foreign is the only producer. Furthermore, it can be shown that for $a_x^H = a_x^F$ this inequality holds and Home is the only producer.

[12]It is assumed that marginal costs a_x^i are such that the cases $AC^H(x^H) = AC^F(x^H)$ $AC^H(x^F) = AC^F(x^F)$ can be ruled out.

Hence for all a_x^H, a_x^F such that $a_x^H \leq a_x^F$ Home is the only producer and for all a_x^H, a_x^F such that $a_x^H \geq a_x^F \frac{\widehat{z}^H}{\widehat{z}^F}$ Foreign is the only producer.
It remains to derive the trade pattern for all a_x^H, a_x^F such that $a_x^F < a_x^H < a_x^F \frac{\widehat{z}^H}{\widehat{z}^F}$. Marginal costs a_x^H and a_x^F are increasing in the index of the industry, m, for Home, and decreasing for Foreign. Then for all $m \leq m_1$, Home is the only producer as $a_x^H \leq a_x^F$. For all $m \geq m_2 > m_1$, Foreign is the only producer as $a_x^H \geq a_x^F \frac{\widehat{z}^H}{\widehat{z}^F}$.
Computing the first derivative of the right-hand side of the inequality above shows that $A(a_x^F; z^H, z^F)$ is increasing in a_x^F, and hence decreasing in m. This implies that there is an $m_1 < \widehat{m} < m_2$ such that for all $m \leq \widehat{m}$, the inequality holds and Home is the only producer, and for all $m > \widehat{m}$, the inequality does not hold and Foreign is the only producer.
Furthermore, treating, without loss of generality, a_x^H and a_x^F as differentiable functions of m and computing the implicit derivative of $\widehat{m}$ with respect to z^H and z^F shows that $\widehat{m}$ is increasing in z^H and decreasing in z^F.

Figure 1: Trade pattern in the second period



Notes: This graph depicts the bilateral trade pattern between Home and Foreign in the first and the second period. The total number of differentiated good industries is denoted by M. Individual industries and their productivity in Home and Foreign are denoted by m respectively a_x^H and a_x^F. As defined in the appendix, $A(a_x^F; z^H, z^F)$ denotes an "adjusted" industry productivity for the second period that accounts for the higher quality produced at lower costs in rich countries. A country will export in a industry if the productivity respectively for the second period their "adjusted" productivity in this industry is lower than for the other country. In period 1 Home will export in all industries $m \leq m_1$ and in period 2 in all industries $m \leq \widehat{m}$, with $\widehat{m}$ restricted by m_1 respectively m_2.

Table I: Model predictions

Export margins →	Quality	Extensive	Intensive	Price	Quantity
	Q	$\widehat{m}$	$np(x-c)$	p	$n(x-c)$
Per capita income z	+	+	0	0/+	0
Population size L	0	0	0	0	0
Firm characteristics →	Firm Size	Number of firms			
	x	n			
Per capita income z	+	−/+			
Population size L	0	+			

Note: Entries are model predictions for how export margins and firm characteristics change with respect to per capita income and population size of the exporting country in the second period. Quality is denoted by Q, the number of industries in which a country is exporting by $\widehat{m}$ and the good price by p. The number of firms within an industry and thus the number of varieties is denoted by n, firm size and thus output of a variety by x and domestic consumption of a variety by c. How the price and number of firms change with per capita income is ambiguous and depends on the parameter values.