VENTURE CAPITAL AND UNDERPRICING: CAPACITY CONSTRAINTS AND EARLY SALES*

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September 2008

Abstract

I present a new theory about underpricing and Venture Capital-backed (VC-backed) companies where the key feature is capacity constraints - Venture Capital firms can take only a limited number of new projects. This new theory predicts that younger Venture Capitalists rush to Initial Public Offerings (IPOs) and sell below the market price. Moreover, the model predicts the positive impact of hot issue markets and technological cost saving shocks on underpricing. The latter features are absent in existing models, and our findings are consistent with the data. Finally, the model presents a microfounded auction model to underpricing in IPOs with random arrival of potential buyers. It generates testable implications on the impact of time between registration to an IPO and firm age on underpricing. I present preliminary empirical results that support the theoretical predictions.

Keywords: IPO,Venture Capital, Underpricing, Capacity Constraints.

JEL Codes: G24, G11.

*I'd like to thank Jan Eeckhout Ken Burdett and Guido Menzio for several discussions and encouragement during this project. I'm also indebted with Andrew Postlewaite, Boyan Jovanovic, Randall Wright, Philipp Kircher, Ludo Visschers, Chao Fu, Teddy Kim and the participants of Penn Search and Matching workshop and Micro theory club for helpful comments and discussions. Finally, I'd like to thank Rafael Silveira for lengthy discussions at the beginning of this project. All remaining errors are mine.

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1 Introduction

There is an apparent puzzle in the recent empirical literature on venture capital: Even though IPOs of young companies are deeply underpriced, venture capital (VC) firms insist on taking these infant companies - which have lower revenues than non-VC backed firms and are less likely to be profitable - public. This pattern is still more evident if we look at young venture capital firms: these intermediaries take public companies that are on average two years younger and usually raise much less money on the IPO than their more mature counterparts.

In this paper, I present a model that addresses a new explanation for this puzzle. The main trade-off behind the IPO timing decision by a Venture Capital firm is the opportunity cost of turning down new projects against the benefit of holding on to existing projects and let them come to maturity yielding higher profits. The key to this trade off is that VC firms are capacity constrained, that is, they are able to handle only a limited number of firms. The benefits of waiting to go public come from the increase in the expected return on a given project, obtained by improving the project's quality and by increasing the number of potential buyers; therefore increasing the competition for shares offered at the IPO. In this framework, we show that VCs have a higher incentive to go public than entrepreneurs, going public earlier than non-VC backed firms in equilibrium.

The distinction between young and mature VC firms comes from how binding the Capacity Constraints are; young firms usually have fewer and less experienced manager and are able to handle less projects at the same time. As a result, they need to go public with younger projects on average than mature VC firms.

This model makes possible an analysis of different features of the market that could not be done by previous explanations based on reputation/signaling models. Phenomena such as hot issues markets - in which many companies go public in a short period of time, suffering a large underprice - can be seen as the result of cost saving technological shocks in the economy. It also allows us to investigate the impact of technological waves that change the market tightness - i.e., the measure of projects in the market compared to the measure of venture capitalists, and we can show that this impact depends on the interaction between the tightness and the costs to start a new partnership.

This model offers an interpretation for the VC's ability to raise larger funds after IPOs and the positive relation between underprice and size of next raised fund which is not present in previous literatures: an IPO, especially an underpriced one, means that a new high quality project has been found and hence investors should expect high returns in the near future. Therefore, in my interpretation with forward-looking investors and underpricing, an IPO is a signal of good news. This interpretation is absent in the existing literature.

Finally, in order to endogeneize the expected return obtained through an initial public offer, I present an IPO as an auction in which potential buyers arrive through time and the time until an IPO is decided by the VC/underwriter. This structure gives me an S-shaped expected return function. With more and more bidders, additional bidders raise expected revenue from competition less and less, hence the concavity. The convexity derives from the fact that there is no increase in expected revenue from the arrival of the first bidder since he obtains the entire surplus. I also obtain testable implications on the impact of time between registration to an IPO and firm age on underpricing - being this important as a first attempt to evaluate the impact of the road shows on underpricing. Our preliminary empirical results seem to agree with the predictions of the model.

The next section will discuss the main features of the VC market. In the third section, I present the setup of the model. The fourth section presents the equilibrium and main results, while the fifth section discusses the possible ways to endogeneize the expected return function. The sixth section shows empirical evidence about the correlation between measures of time to IPO and underpricing to support my results from the fifth section. The seventh section concludes the paper. All proofs are presented in Appendix A.

2 The Venture Capital Market

A Venture capital firm is a financial intermediary that takes investors' capital and invests it directly in portfolio companies. Its primary goal is to maximize its financial return by exiting investments through a sale or an Initial Public Offering (IPO).

Its payment is based on a profit-sharing arrangement, the usual being an 80-20 split: after returning all of the original investment to the external investors (called limited partners), the general partner (VC) keeps 20% of everything else.

VC firms are usually small (on average they have 10 senior partners) and handle a restricted amount of resources and few portfolio companies. According to Metrick (2006):

> *"VCs recognize that most of what they do is not scalable and there are limits on the total number of investments that they can make (...) firms are reluctant to increase fund sizes by very much".*

The estimated total committed capital in the industry is US$ 261 billion, which is managed by an estimated 9,239 VC professionals, meaning that the industry is managing about US$ 28 million per investment professional. Even the most famous VC funds usually only manage about US$ 50 million to US$ 100 million per professional.

Finally, the typical VC fund will invest in portfolio companies and draw down capital over its first five years, which are known as the investment period or commitment period. After the investment period is over, the VC can only continue to invest in its current portfolio companies. However, VC firms usually raise a news fund every few years, so that there is always at least one fund in the investment period at all times.

Venture capitalists retain extensive control rights, in particular rights to claim control on a contingent basis and the right to fire the founding management team; they keep hard claims in the form of convertible debt or preferred stock, underpinning the right to claim/control and abandon the project; staged financing and the inclusion of explicit performance benchmarks make it possible to fine-tune the abandonment decision.

In summary, venture capital firms are financial intermediaries that suffer from capacity constraints (they have limited human capital to manage their portfolio companies), receive a fraction of the profit which is realized once they exit the investment through competitive sale or IPO, have strong control over exit decisions, and keep looking for new opportunities.

According to Lee and Wahal (2004), venture capitalists generally take smaller and younger firms public. These firms have lower revenues than non-VC backed firms and are less likely to be profitable. They also found that VC-backed IPOs raise less cash than non-VC backed ones. The table below summarizes their findings from studying a sample of over 6,413 IPOs between 1980 and 2000 of which over 37% (2,383) are VC backed.

SEE TABLE 1

They also found evidence in favor of the grandstanding hypothesis, first proposed by Gompers (1996), which posits that since VC firms must periodically raise funds, they need to establish a reputation of being capable of taking portfolio companies public in order to obtain future fund-raising. The grandstanding hypothesis predicts that the relation between bringing companies public and fund-raising ability should be stronger for young venture capital firms. Each additional IPO attracts relatively more capital from investors for a young venture capital firm than for an old venture capital firm, since it changes investors' estimate of a young venture capitalist's ability more than it does their estimate of an old venture capitalist's ability. Therefore, less-established VC firms need to signal quality by taking portfolio companies public. As a result, they are more willing to bear the cost of greater underpricing.

SEE TABLE 2

Table 2 above summarizes some of this empirical evidence: Analyzing a sample of 433 venture-backed initial public offerings (IPOs) from January 1, 1978, through December 31, 1987, Gompers (1996) found that IPO companies financed by young venture capitalists are nearly two years younger and more underpriced when they go public than companies backed by older venture capital firms. He also found that these young VCs spend on average 14 months less on the IPO company's board of directors and hold smaller percentage equity stakes at the time of IPO than the stakes held by established venture firms. The offerings also differ in magnitude. The equity stake retained by managers and employees after the offering is significantly larger for firms backed by the less experienced venture capitalists. In addition the dollars raised in the IPOs by firms with seasoned venture investors is larger. Both results are consistent with Leland and Pyle (1977), who argue that lower quality managers must retain larger equity stakes and raise less money to obtain any external financing.

Testing the grandstanding hypothesis, Lee and Wahal (2004) found that the flow of capital into the lead VC firm is positively related to VC age, the number of previous IPOs done by the VC firm, and underpricing, implying that there is a benefit to bearing the cost of underpricing. They also found that interaction effects between reputation (proxied by VC age and number of previous IPOs done by the VC firm) and underpricing are negative, supporting grandstanding.

The real loss in underpricing for the venture capital firm is that it transfers wealth from existing shareholders, including the venture capitalists (who on average own 36% of the firm prior to the IPO and 26.3% immediately after), to new shareholders.

However, there are features in the market that are not answered by the traditional grandstanding model presented by Gompers (1993). First, although empirical papers discuss the smaller equity stakes held by young VC firms, there is no intrinsic reason in the model for this, the explanation being based on different models. The grandstanding model also has nothing to say about the relation between hot issue markets and smaller equity stakes hold by venture capitalists, as observed by Ljungqvist and Wilhelm (2003), and the higher fund-raising obtained by the whole industry after hot IPO markets, as presented by Bouis (2004) and Black and Gilson (1998).

Another important relation observed in the VC market but not explained by Gompers's theory is the relation between technological shocks, IPO market and fundraising by VC firms. As argued by many authors including Jovanovic and Rosseau (2001), technological shocks increase the speed at which firms come to an IPO, these shocks being considered by many as one of the main driving forces behind hot IPO markets. Such shocks also seem to increase the fund-raising by VC firms, as empirically confirmed by Gompers and Lerner (1998) and Bouis (2004).

Behind the impact of technological shocks in VC fund-raising and faster IPOs lies an important feature missed by grandstanding models: demand-side factors in the VC industry. The demand of capital from entrepreneurs in innovative industries is a major determinant of the amount and allocation of funds. According to Gompers and Lerner(1998) and Poterba (1989), demand-factors actually have a determinant effect on VC fund-raising. As Hellman (1998) says:

"At a theoretical level, it is hard to argue that demand considerations are of no importance. And casual observation suggests that in many countries the obstacles to investing in venture capital are relatively minor, yet there is no active venture capital market, suggesting that supply alone cannot be the problem. Instead, it is frequently argued that the lack of venture capital is due first and foremost to the lack of entrepreneurs."

Therefore, there is space to introduce an alternative explanation that can preserve the empirical results derived from the grandstanding hypothesis and still be able to address additional features that are observed in the market. This is my goal in this paper. I will introduce here a model that posits capacity constraints (represented by human capital constraints) associated with the random arrival of new opportunities as the driving forces in this market. In this framework, grandstanding empirical results come from differences in the strength of these constraints: younger VC firms have a smaller number of senior partners with less experience and are thus more human-capital-constrained than well established firms. Once a new project appears, these companies have to exit younger projects on average than older VC companies. As we show in the fifth section, the expected return in an early IPO is smaller and therefore the underpricing is higher.

My view that underpricing is related to a higher fund-raising comes from what underpricing shows us, that there is a higher inflow of good projects in the market or that a good project has been found, enabling the VC fund to make higher profits. Therefore, instead of the reputational story that is presented in the literature, I consider here that the investors are forward-looking, i.e., they see the underpricing as a signal of good news and higher expected returns in the future (the VC firm had to go public with the current projects to release human capital to undertake new projects). Since the young VC firms are more capacity constrained, the effect of underpricing must be higher to them.

Finally, the relation between technological shocks, higher fund-raising and, lower average time until the IPO is related to demand-side explanations; technological shocks, proxied in the empirical literature by number of patents registered and investment in R&D, can change the inflow, distribution, cost, and return of new projects in the market, inducing the VCs to exit earlier from current projects to realize profits and engage in new ventures.

3 Setup

Time is continuous and the horizon is infinite. The economy is populated by a continuum of VCs and entrepreneurs, with measures 1 and m, respectively. VCs and entrepreneurs are infinitely lived and risk neutral, although we will consider the case in which entrepreneurs leave the VC market after their project is terminated. Both types discount future utility at rate $r > 0$.

Each entrepreneur has one project to fund. The initial quality S of the project will be considered to be a draw from a distribution H with support $\left[0, \overline{T}\right]$. The project's quality improve if time is spent running it. In this sense, if a project with initial quality S is run by its entrepreneur, with or without a VC backing it, during an interval of time ΔT, it's quality at the end of the interval will be $T = S + \Delta T$.

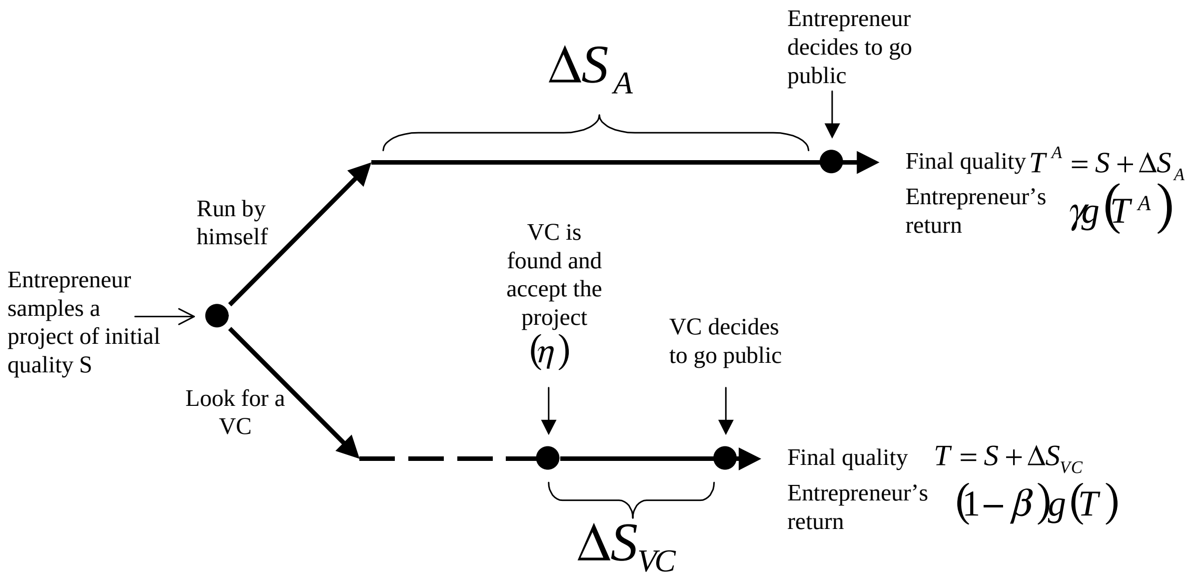
All projects will be considered identical but the initial quality, i.e., all projects have the expected revenue from sale following the deterministic rule $g(T)$ over time, where T is the project's quality at the time of the sale. We assume $g'(\cdot) > 0$ and $\frac{g''(T)}{g'(T)} < r$, for reasons that will be clear later. In the fifth section, we present one way to endogenize $g(T)$ modelling explicitly the IPO process.

Once he draws his project, an entrepreneur must decide if he tries to find a VC or run the project by himself. If he runs by himself, he obtains $\gamma g\left(T^A\right)$, where γ is the discount factor that determines the reduction on the output that can be obtained from the project, given the entrepreneur has worse skills on handling/publicizing the project and T^A is the project's quality at the time of the sale. Therefore, T^A optimal stopping time chosen by the entrepreneur. If the initial quality was S, this means that the entrepreneur spent a time interval of $T^A - S$ running the project.

If he chooses to look for a VC, his project is held constant until he finds a VC (i.e., project's quality is kept constant). If he finds a VC, which happens with probability $\eta > 0$ and the VC accepts his project, they start running it together. Once the project is sold, the entrepreneur will receive $(1-\beta)\, g(T)$, where $(1-\beta) > \gamma$ is the fraction of the sales revenue that belongs to the entrepreneur and T is the project's quality at the IPO. In this case, however, we will assume that the VC determines when the project will be sold[1], which will depend on the outside options that she faces, i.e., the new projects that are randomly offered to her. We suppose each VC cannot handle more than one project.

We summarize this structure in the picture below.

[1]We could consider here that both parts need to agree to keep the partnership, otherwise the project is sold and the partnership dissolved. However, it would make no difference since we show that the VC always have a higher incentive to walk away.



Given these features the only choice made by the entrepreneur is to enter or not to enter in the VC market. Once he entered, he would accept any VC that accepts him, since all VCs are homogeneous after they accepted his project. Therefore, the entrepreneurs decisions can be summarized by the distribution of projects in the market, F, with support possibly on $\left[0, \overline{T}\right]$.

A definition of equilibrium in this economy is given below:

Definition 1 An Equilibrium is this economy is a vector $\left\{T_c^*(\cdot), S^{\blacklozenge}(\cdot), F(T)\right\}$ such that:

- Given $F(T)$; Venture Capitalists are choosing optimally their termination rule $T_c^*(\cdot)$;
- Given $F(T)$ and $T_c^*(\cdot)$, entrepreneurs choose optimally their entrance rule $S^{\blacklozenge}(\cdot)$;
- Given $T_c^*(\cdot)$ and $S^{\blacklozenge}(\cdot)$, the distribution of initial projects' quality in the market is given by $F(T)$.

As we will see later, $S^{\blacklozenge}(\cdot)$ is given by a cut off rule: A project will enter the VC market if its initial quality $S \geq S^{\blacklozenge}$ and choose autarky otherwise.

To obtain some intuition about the model and see the importance of introducing "on-the-project" search, we will start considering a simple case in which a VC only receives offers of new projects after she goes public with the current one. Then, in the next subsection we will relax this assumption.

3.1 No "on-the-project" search

In this section we will consider the case in which only unmatched VCs will be able to enter in a new partnership. This case will be important to give us some intuition for the more complex set ups, while showing why it is important to consider these better structured set ups to understand some of the stylized facts obtained in the empirical literature.

Consider that each unmatched VC finds a new project with probability λ. Then, the optimal termination time is the solution for the following problem:

$$\max_{T} e^{-r(T-S)} \left[\beta g(T) + V^0\right]$$

where S is the initial time/quality, T is the project time/quality at the sale, β is the VC's share in the project while V^0 is the value of having no current project and therefore, the opportunity of looking for a new one.

At the optimal selling time, we must have[2] :

$$g'(T^*) - rg(T^*) = \frac{rV^0}{\beta} \tag{1}$$

Because our constraint on $g(T)$, we know that the LHS is decreasing on T. Since the RHS is a constant on T, we guarantee that there is a unique T^* that satisfies the above equation.

The value V^0 is the value of having a vacancy and therefore, the value of searching for a new project. We can see that, as expected, the higher the value of a vacancy, the lower the waiting time to finish the project. Now, let's calculate V^0 :

$$(1 + rdT) V^0 = \lambda dT \int \max\left\{V\left(\widetilde{S}\right), V^0\right\} dF\left(\widetilde{S}\right) + (1 - \lambda dT) V^0$$

Therefore, the value of being able to look for a new project $\left(V^0\right)$ is given by the expected return of the arrival of a new opportunity with initial quality $\widetilde{S}$, given by a draw from the distribution of initial qualities in the market F .

But note that since you can always get the project and finish it immediately, receiving $\beta g(T)$, where $T \geq 0$, the VC will always accept it. Then, we have, manipulating and taking $dT \to 0$:

$$V^0 = \frac{\lambda}{r + \lambda} \int_0^{\overline{T}} V\left(\widetilde{S}\right) dF\left(\widetilde{S}\right) \tag{2}$$

Therefore, we have that, the value of a project with current quality T is:

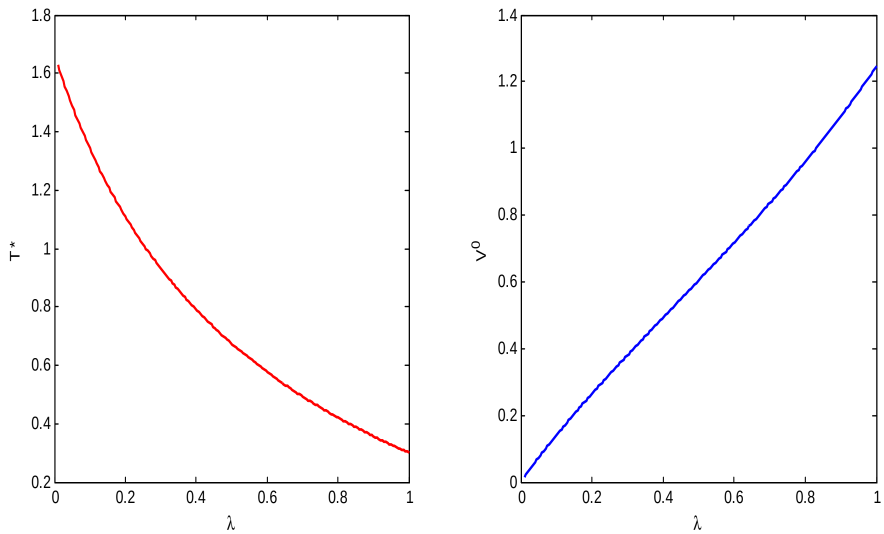
$$V(T) = e^{-r(T^* - T)} \left[\beta g(T^*) + V^0\right] \tag{3}$$

Substituting these expressions in our previous results, we obtain:

[2] We could have corner solutions in which the VC immediately sells the project without compromising the model. However, given our assumptions, no VC would keep a project forever.

$$\frac{g'(T^*)}{g(T^*)} = \frac{(r+\lambda) r}{r + \lambda \left(1 - e^{-rT^*} \int_0^{\overline{T}} e^{r\widetilde{S}} dF\left(\widetilde{S}\right)\right)} \tag{4}$$

The above expression implicitly defines the optimal time to go public, as a function of the distribution of initial qualities in the market and parameters of the model. Once we have T^*, we can easily obtain V^0. From these expressions, we can show that $\frac{dT^*}{d\lambda} < 0$ and $\frac{dV^0}{d\lambda} > 0$, which intuitively means that, if the market has too many projects relatively to the number of VCs, each project would be kept for a shorter period of time, since the value of looking for and starting a new project goes up. The graph below presents an example where we have $g(T) = T^{\frac{1}{2}}$ and uniform distribution.



These results capture the hot issue market behavior, in which we observe a wave of many underpriced IPOs occurring at the same time. My explanation to these behavior is that in these occasions there are too many new opportunities entering in the market, such that keeping a project longer would incur a very high opportunity cost.

To close this model as an equilibrium model, we would need to consider the entrepreneurs' decision of looking for a Venture Capitalist or not. If a entrepreneur decides to run the project by himself, he would solve the following problem:

$$\max_T e^{-r(T-S)} \left[\gamma g(T)\right]$$

with solution:

$$\frac{g'\left(T^A\right)}{g\left(T^A\right)} = r \tag{5}$$

We can clearly see that $T^A > T^*$, since $\frac{r+\lambda}{r+\lambda\left(1-e^{-rT^*}\int_0^{\overline{T}} e^{r\widetilde{S}}dF\left(\widetilde{S}\right)\right)} < 1$. It is easy to see why: In this set up, notice that the Venture Capitalist has two gains whenever she goes public with her current project: First, she realizes the gains, obtaining $\beta g(T)$. But she also can now receive offers from new projects. So, in this framework the vacancy by itself is valuable for a VC.

However, this difference in potential sources of earnings would create a clear distinction for the entrepreneur: He knows that if he looks for a Venture Capitalist, the company will necessarily be sold before its optimal time, while, if he decides to run the company by himself, he would choose the optimal time to sell it, however he would not be able to usufruct all the expertise that a VC has (therefore, obtains $\gamma << 1-\beta$). If a entrepreneur with a project with initial quality S enters the VC market, he has a expected value of:

$$\frac{\eta}{\eta+r}e^{-r(T^*-S)}\left(1-\beta\right)g\left(T^*\right)$$

Therefore, he would choose to look for a VC if and only if:

$$\frac{\eta}{\eta+r}e^{-r(T^*-S)}\left(1-\beta\right)g\left(T^*\right) > e^{-r\left(T^A-S\right)}\gamma g\left(T^A\right)$$

simplifying, we obtain:

$$\frac{\eta}{\eta+r}e^{-rT^*}\left(1-\beta\right)g\left(T^*\right) > e^{-rT^A}\gamma g\left(T^A\right)$$

Therefore, the initial quality S does not affect the decision of looking for a VC or not. This implies that all qualities would join the market or it would simply shut down, with all entrepreneurs going to autarky.

Finally, we can clearly see that this framework cannot address the question why young VCs would go public earlier than tenured ones: Once VCs are only allowed to search for new opportunities if they have a opening, capacity constraints are not binding in this framework. Another point we should emphasize for future references is that the introduction of an initial sunk cost generates an increase in T^* and a decrease in the value of a vacancy.

The proposition below summarizes our findings on this section.

Proposition 1 In an economy with no "on-the-project search":

- All qualities of projects enter the VC market or not;
- All projects would go public at same age/quality;

- Hot issue markets would be occur whenever the market is tight (λ is close to 1);
- There would have no differentiation between the behavior of young and old VC firms.
- If there is a initial sunk cost ($c > 0$), T^* will be increasing in c, while V^0 will be decreasing on it.

Therefore, if we want to study the differences in the initial quality of projects in autarky and in VC partnerships and also to obtain different behavior between young and old VCs we need to modify this model. As we are going to see in the next sections, we are able to obtain results in agreement with the empirical literature by introducing "on-the-project" search.

3.2 General case: "On-the-project" search

Now, let's consider that, with the same probability λ, the matched VC can find a new project with initial quality $\widetilde{S} \sim F\left(\widetilde{S}\right)$. From the previous section, we know that now the VC has two decisions that must be done: First, as before, to terminate the project and open a new vacancy if nobody showed up. Second, to finish a project to engage in a new one just sampled. In this section, I will consider a starting cost, i.e., a sunk cost to start a new project.

Then, the Bellman equation is:

$$\begin{aligned}(1 + rdT) V(T) &= \lambda dT E_{\widetilde{S}} \max\left\{V\left(\widetilde{S}\right) - c + \beta g(T + dT), V(T + dT)\right\} + \\ &\quad (1 - \lambda dT) \max\left\{\beta g(T + dT) + V^0, V(T + dT)\right\}.\end{aligned}$$

Therefore, the expected value of a project of current quality T is given by a expected value that consider two decisions taken in two different states of nature: the case in which the VC finds a new project and needs to decide if she accepts the new project and therefore goes public, obtaining a payoff of $V\left(\widetilde{S}\right) - c + \beta g(T + dT)$, or she keeps the current project a little longer, improving its quality but losing the opportunity of the one just offered, and the state of nature in which no new opportunity is found and the VC must choose between keeping and improving the current project or go for an IPO and opening a vacancy ($\beta g(T + dT) + V^0$).

We know that there is a threshold, given by $T^*_\emptyset$, at which the VC will exit the project even without having found another. It's straightforward to show that:

$$\frac{g'\left(T^*_\emptyset\right)}{g\left(T^*_\emptyset\right)} = r$$

Notice that $T^*_\emptyset$ only depends on g and r and it is higher than the stopping time obtained in the previous section and identical to T^A, the entrepreneur's optimal time to exit a project. The reason is that once “on-the-project” search is as effective as search while holding a vacancy, there is no additional benefit of going public than the realization of the payoff $\beta g(T)$. Therefore, if no new project appears, the VC will hold the project until the cost of postponing consumption is equal to the benefit of holding it and increasing the project's quality and therefore, expected return from sale.

For any $T < T_{\emptyset}^{*}$, we have $V(T + dT) > \beta g(T + dT) + V^{0}$. Then, taking $T \in \left[0, T_{\emptyset}^{*}\right]$ and $dT \to 0$, we obtain:

$$rV(T) = \lambda E_{\widetilde{S}} \max\left\{V\left(\widetilde{S}\right) - c + \beta g(T) - V(T), 0\right\} + \frac{\partial V(T)}{\partial T} \tag{6}$$

Given the nature of this problem, we know that there is a threshold $T_{c}^{*}(T)$ in which the VC is indifferent between keeping the current project of size T or ending it to engage in a new project of size $T^{*}(T)$. This threshold is defined by:

$$V\left(T_{c}^{*}(T)\right) + \beta g(T) - c = V(T) \tag{7}$$

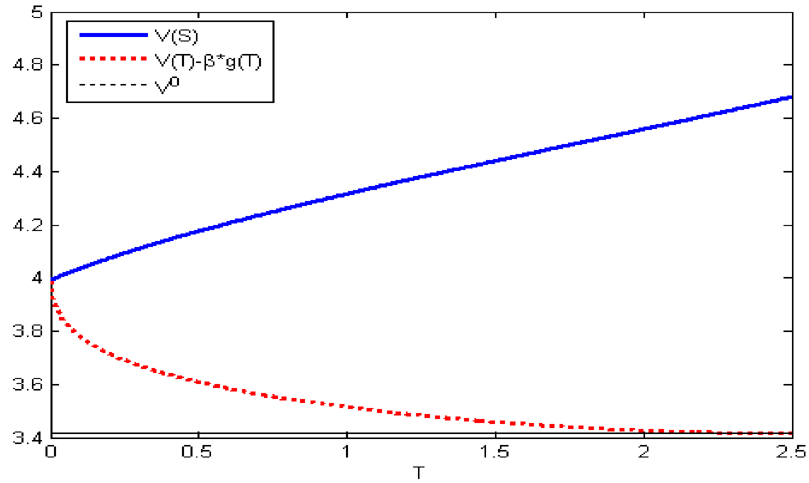
According to this equation, a VC would finish the current project of quality T to enter in a new project with starting quality $T_{c}^{*}(T)$ if the gain from the termination of the current project, $\beta g(T)$ plus the value of the new project minus the initial sunk cost, $V\left(T_{c}^{*}(T)\right) - c$ compensates the loss of the value of the current project $V(T)$. Comparing this to the investment literature, this is simply telling us that the VC will compare Net Present Values.

Based on the above expression, we can show the following results:

Proposition 2 In an economy with "on-the-project search":

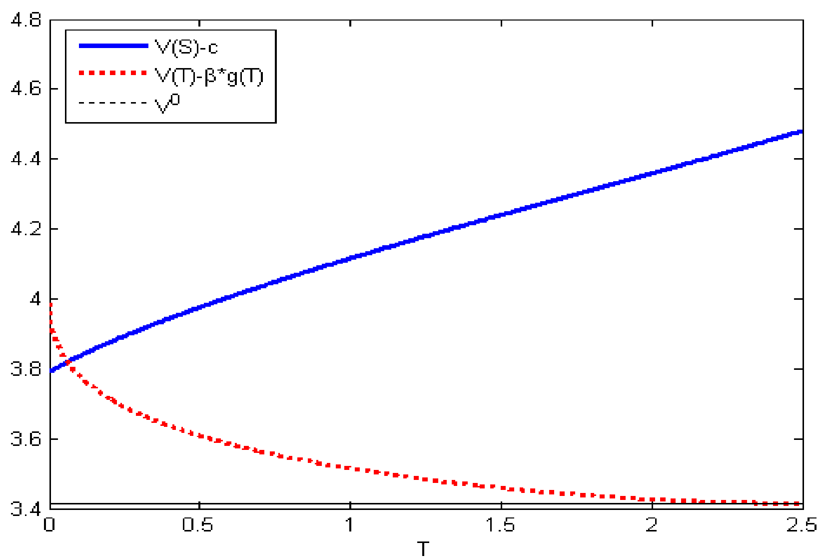
- If there are no sunk cost to enter in a new project, $T^{*}(T) = 0,\ \forall T \in \left[0, T_{\emptyset}^{*}\right]$;
- If sunk cost is positive $(c > 0)$, $T_{c}^{*}(\cdot)$ is strictly decreasing;
- If the model is extended such that a VC can hold two projects at the same time, the average time a VC keeps a given project goes up.
- A higher sunk cost shifts up the cut off rule, i.e., if $c_1 > c_2$, $T_{c_1}^{*}(T) \geq T_{c_2}^{*}(T), \forall T \in \left[0, T_{\emptyset}^{*}\right]$.

we obtain the following graph:



Then, regardless of the size of the current project that the venture capitalist has in her hand, she will finish it and enter in a new project, whenever she has the opportunity to do.

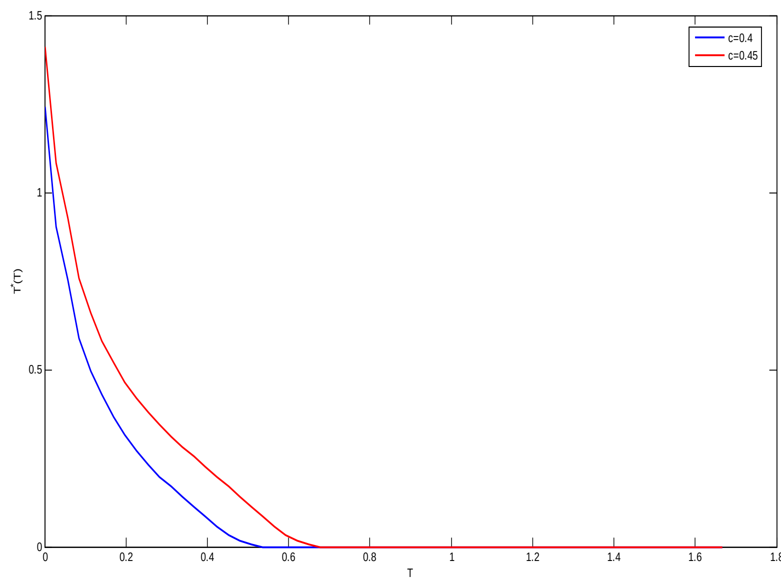
In the case in which we introduce an initial sunk cost, we can clearly see that we would shift $V(S)-c$ downwards relatively to $V(T)-\beta g(T)$. Graphically we have:



Once the basic framework is developed, we can easily modify it to evaluate how VC's behavior changes as we relax or alter some of our assumptions.

As we mention before, we consider that young and mature VC firms are distinct in terms of how binding capacity constraints are. To address this point, in Appendix B we extend the model presented here to the case in which a VC can hold two projects at the same time. In this case, we show that whenever a VC has to go public with a given project to enter in a new venture, she chooses the older/better quality one. Then, in the case in which the entrepreneur still accepts any VC[3], the expected time until an IPO is larger for a mature (two spots) VC firm. Therefore, we obtain the same result obtained by the grandstanding hypothesis literature.

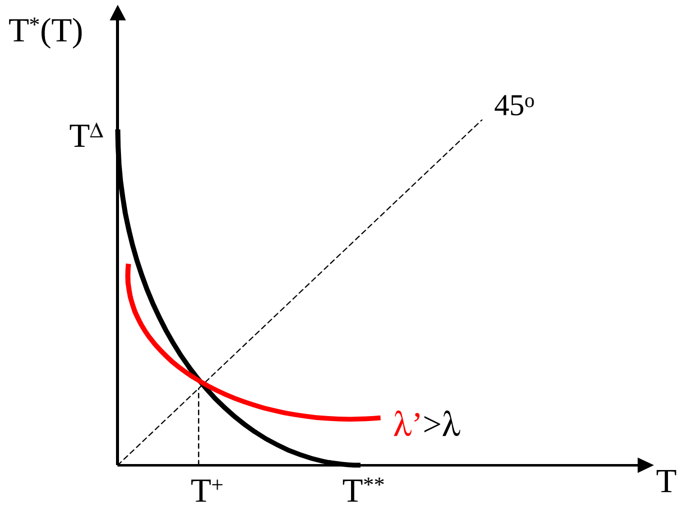
Other questions that we can regard here are changes in the cost of entering in a new project and also the impact of changes in the expected return on a sale, given by changes in the $g(\cdot)$ function. This could be seen as changes in technology that would generate a lower initial investment and a higher expected return, which are both hypothesis considered by the literature when discussing the impact of Information Technology in reducing the time until IPO in the last 30 years, as mentioned by Jovanovic and Rosseau (2001). In our model, we can easily see that an increase in c shifts outwards the cut off rule $T_c^*(T)$, while an increase in the expected return from an IPO would shift inwards the cut off rule, agreeing that a reduction in costs and higher expected return would reduce the expected time until an IPO.



Finally, let's also consider the impact of an increase in λ (arrival rate of new projects). We can show that $T_c^*(T)$ twist counterclockwise around the 45° line when λ increases. This means that as the arrival rate increases, the VC becomes more patient with larger projects and less patient with small ones. This is true because once the arrival rate increases, the risk of being in a large project and having to cut it without having a replacement reduces (the value of a vacancy also goes up, reducing the cost

[3] The presence of young and mature VCs can also change the distribution of projects in the market. However, whenever both VCs are accepted by the same projects, they face the same distribution.

of this worst case scenario). Therefore, VCs can keep these projects growing until a more reasonable offer arrives. Graphically, we have:



Proposition 3 As λ increases, $T_c^*(T)$ counterclockwise around the 45° line.

Therefore, we notice that the result from an increase in the arrival rate can be ambiguous: although the VC receives more offers, and therefore there is a higher probability of going for an IPO earlier, she becomes less picky to finish young projects and more picky to sell old projects. In this case, it is clear the importance looking at the impact of changes in λ on the distribution of projects in the VC market to determine its impact on the expected time until an IPO.

However, in all the extensions we presented here we should worry about potential changes in the distribution of projects in the market, F. For example, notice that as c increases, low quality projects will start being rejected by VCs, which implies that the time until the match increases, while it initially reduces the probability of a going public too soon. So, we need to endogenize F through optimal entry decision in the VC market by entrepreneurs, which is the goal of our next section.

4 Entrepreneurs' Entry Decision

We consider here the simple case in which entrepreneurs leave the market after their project are sold (so any entrepreneur just draws one project) and in which all VCs can handle only one project at a time. Initially, assume that there is no initial sunk cost. $(c = 0)$

The entrepreneur's decision is a binary choice between looking for a VC or going for autarky. If he decides to go to autarky (undertaking the project by himself), he faces the following optimal stopping problem:

$$\max_T e^{-r(T-S)}\left[\gamma g(T)\right]$$

with solution:

$$\frac{g'\left(T^A\right)}{g\left(T^A\right)}=r \tag{8}$$

Notice, as we saw before, that $T_{\emptyset}^{*}=T^{A}$. Therefore, if no new project is offered to the VC while she has a current project, both VC and entrepreneur agree when the project should be finished.

Now, let's consider the case in which the entrepreneur is in a partnership with a VC. He knows that she will terminate the partnership whenever a new project is offered to her. Therefore, his value function at current quality T is:

$$rP(T)=\lambda\left\{(1-\beta)g(T)-P(T)\right\}+\frac{dP(T)}{dT} \tag{9}$$

Then, let's consider the value function of a entrepreneur in the VC's market searching for a partner with a project with initial quality S. Since the entrepreneur would accept any VC and would also be accepted by any of them, we would have:

$$S(S)=\frac{\eta}{(r+\eta)}P(S) \tag{10}$$

where η is the meeting rate for an entrepreneur.

Then, an entrepreneur would enter VC's market after getting a project of initial quality S, if and only if:

$$S(S)\geq A(S)$$

Claim 1 *There exists a $S^{\blacklozenge}$ in which any project larger than $S^{\blacklozenge}$ enters the VC market.*

Therefore, we have that the better projects will enter the market, being terminated earlier than the worse projects that chose autarky, even though those ones give a higher expected value for their entrepreneurs.

Finally, we can see that a measure $m\left[1-H\left(S^{\blacklozenge}\right)\right]$ of entrepreneurs will be in the market and the distribution F will be given by: $\frac{H(S)}{1-H(S^{\blacklozenge})}$, with support on $\left[S^{\blacklozenge},\overline{T}\right]$. Since the acceptance/termination rule does not depend on the characteristics of the projects, this distribution is the same as the steady state distribution.

Let's now find the distribution of projects on partnerships. Define $G(T)$ the distribution of VCs with projects that are smaller or equal to T and u_{VC} the measure of unmatched VCs. Then, the evolution of $G(T)$ over time is given by:

$$\begin{aligned}\dot{G}(T) &= -(1-\lambda dT)\{G(T)-G(T-dT)\} \\ &\quad -\lambda dT[1-F(T)]G(T)+\lambda dTF(T)[u_{VC}+(1-G(T))]\end{aligned}$$

Since in steady state, $\dot{G}(T)=0$, after some manipulations and taking $dT\to 0$, and solving the differential equation (remember that $G(0)=0$), we have:

$$G(T)=(1+u_{VC})\left\{F(T)-\int_0^T e^{-\lambda(T-s)}f(s)\,ds\right\} \tag{11}$$

Note that this distribution puts more weight on higher values of T than $F(T)$. The reason is that once the project is in a partnership it grows over time, while it's constant when unmatched.

Now, we need to find the measure of VCs unmatched. If $\overline{T}>\frac{1}{r}$, then, we have projects that will be terminated immediately after they are accepted. Therefore, there will be an inflow of brokers to the unmatched pool, more specifically, all the VCs that are matched and find an outside offer that is in the range $\left[\frac{1}{r},\overline{T}\right]$ will accept it, terminate the project and open a vacancy. Then, the steady state measure of unmatched VCs is given by:

$$u_{VC}=\frac{\left[1-F\left(\frac{1}{r}\right)\right]+\frac{1}{\lambda}\left(G'(T)|_{T=\frac{1}{r}}\right)}{1+\frac{1}{\lambda}\left(G'(T)|_{T=\frac{1}{r}}\right)} \tag{12}$$

if $\left(G'(T)|_{T=\frac{1}{r}}\right)=0$, we obtain that $u_{VC}=1-F\left(\frac{1}{r}\right)$ in this case.

However, if $\overline{T}\leq\frac{1}{r}$, considering that F has no mass points, there is no outflow to the unmatched pool but the one from VCs with no outside offers and current projects that hit $T=\frac{1}{r}$. Then, we have:

$$u_{VC}=\frac{\frac{1}{\lambda}G'(T)|_{T=\frac{1}{r}}}{1+\frac{1}{\lambda}G'(T)|_{T=\frac{1}{r}}} \tag{13}$$

if $\left(G'(T)|_{T=\frac{1}{r}}\right)=0$, we obtain that $u_{VC}=0$ in this case.

Now, let's consider the case in which we have the initial sunk cost. Then, we have:

$$rP(T)=\lambda\left[1-F\left(T_c^*(T)\right)\right]\{(1-\beta)g(T)-P(T)\}+\frac{dP(T)}{dT}$$

Claim 2 *There exists a $T^{\blacklozenge}(c)$ in which any project larger than $T^{\blacklozenge}(c)$ enters the VC market.*

Therefore, the existence of a threshold of quality in projects entering the VC market is robust to the introduction of an initial sunk cost. However now the acceptance/termination rule depends on the size/quality of the project (better developed projects have a higher probability of being accepted and terminated than less developed ones) which would induce a steady state distribution of projects in the market that has a higher weight on smaller projects than the one observed in the inflow of projects. A reduction in c has a ambiguous effect on the distribution: it increases the acceptance probability of a given project, reducing the costly time of looking for a VC, but also reduces the value of a partnership, since increases the probability of an early termination. Further assumptions are necessary to determine which effect is stronger.

In the next section, we look more carefully to the IPO process, aiming to show one reasonable way of endogenizing the expected return on sale that we took as exogenous up to now.

5 Microfoundations on IPO procedure: Endogeneizing $g(T)$

In this section, we are going to present one way in which we can endogenize the return from the exit in a venture investment. Since the most successful exits are through initial public offers (IPOs), endogenizing $g(T)$ necessarily involves a discussion about IPOs and their main players and features.

The main feature that we observe in IPOs is the presence of underpricing: According to Jenkinson and Ljungqvist (2001), the first day return that investors experience is positive in virtually every country, and typically averages more than 15 per cent in industrialized countries and around 60 per cent in emerging markets. These returns are viewed as anomalies, since in efficient markets, firms shouldn't leave "money on the table" and competition between investors in the market would necessarily exhaust all possible gains from private information, as presented by Grossman (1976) in a Rational Expectations Equilibrium Model.

Many theories were developed to address this issue, all with limited success. The traditional explanations are based on asymmetric information: in Rock (1986), there is asymmetric information between potential buyers, which generates a lemon's problem for the uninformed buyer, while in Benveniste and Spindt (1989), the seller that is actually trying to extract information from institutional investors about the value of the company being sold. The main problem with these theories is that they take the sale's procedure as given: Rock (1986) takes as given firm commitment offers, avoiding the transmission of information from informed to uninformed buyers through price changes , while Benveniste and Spindt (1989) assume the existence of a pre-market in which only regular investors participate under the assumption that "cost of conducting an all-inclusive pre-market is prohibitive".

Another theoretical explanation for underpricing presented by the literature is signaling. In this case, the seller knows the quality of the firm being sold, while underpricing would be a way to signal better quality. This explanation, although theoretically elegant suffers from empirical flaws (some assumptions used in these models are wrong and the data don't corroborate their results) and it also seems susceptible of collusion between the seller and some buyers or even the seller can "create" false investors, as discussed lately in the auction literature.

The explanation we are going to present here is related to the one defended by Jovanovic and Szentes (2007) and empirically discussed by Loughram and Ritter (2004). In this paper, Jovanovic and Szentes (2007) claim that underprice is created by an agreement between institutional investors and underwriters, in which investment bankers allocate underpriced stocks to institutional investors in the hope of winning their future investment banking business, a practice known as 'spinning'. This practice is well-documented, being the most famous case the $100 million fine that Credit Suisse First Boston received because of these activities.

The reason why firm's original owners accept this comes from the disclosure of information that IPO reveals to the market.

In our argument, we introduce a step further, considering the competition between institutional investor for underpriced shares and how this impacts the amount of information released and therefore, the size of the first day return.

Given this effect of competition, we consider how the increase in competition is related with time. We claim that one important role performed by Venture Capitalists and underwriters is to publicize the firm to be marketed. The more institutional investors that get information about the firm, higher the expected competition for its shares and therefore, lower underprice.

One simple way that we can see this effort on publicizing an IPO firm and its impact on competition is looking at the length of the road show, that we will proxy looking at the number of days in registration. The Road Shows are tours taken by IPO firms' top managers and investment bankers to visit groups of invited institutional investors, publicizing the firm and also to elicit bids from investors. Although these bids don't have legal tender, there is a strong presumption that investors should be prepared to honour their bids.

Looking at the data for more than 1500 IPOs in the period between 1984 and 2004 (we used data from the SDC Platinum), we can show a negative correlation between the number of days in registration and the first day return. This result is robust to different specifications of our multivariate linear regressions or even multivariate fractional polynomial models. Although the lack of data with respect to the number of bidders doesn't allow us look for deeper empirical relations, we imagine that this is a clear indication that timing and its impact on building up competition is an important factor to understand the size of the underpricing. In the next section, we give details on our empirical results.

We clearly agree that we are presenting one of many ways in which we could endogenize the expected return. However, we believe that our explanation not only adds in matching some empirical evidence that was not considered before, but it also gives a clear theoretical foundation for some hypothesis presented by the empirical literature on firm managers behavior. In addition, our specifications are not in disagreement with other conjectures, as the idea that Venture Capitalists not only publicize the project but also increase its quality (as we see below, all results are kept if we imagine that the real value of the venture ω is an increasing function of time spent in the partnership).

5.1 Basic Framework

In this basic framework, we model IPOs as first price auctions in which only institutional investors participate. We initially consider that there are N potential buyers participating in this auction. Later, we show ways to endogenize N and therefore evaluate the expected return on exiting as time passes.

We assume that the value of the company ω is known by institutional investors and/or it can be credible communicated by the underwriter. We also assume that the underwriter and the venture capitalist knows ω but other players in the market don't . However, from our claim about the agreement between underwriter and investors, the investor that wins the auction obliges himself to a future contract with the underwriter. Let's consider that this future contract between institutional investor and investment banker generates a private cost to the investor that is seen here as an i.i.d. draw ε from a distribution Z with support on $[0,\omega]$ [4]. Therefore, investor i's gain in winning the auction is given by $\omega - \varepsilon_i - p$ when he bids p and this is the highest bid. Therefore, we have the following payoff function:

$$\Pi_i = \begin{cases} \omega - \varepsilon_i - p_i & \text{if} \quad p_i > \max_{j \neq i} p_j \\ 0 & \text{if} \quad p_i < \max_{j \neq i} p_j \end{cases}$$

Then, the problem of investor i is:

$$\max_{p \geq 0} \left(\omega - \varepsilon_i - p\right) \left[1 - Z\left(\omega - P^{-1}\left(p\right)\right)\right]^{N-1}$$

Then, solving the symmetric equilibrium case, we obtain:

$$P\left(\omega - \varepsilon_i\right) = \omega - \varepsilon_i - \int_{\varepsilon_i}^{\omega} \left[\frac{1 - Z\left(y\right)}{1 - Z\left(\varepsilon_i\right)}\right]^{N-1} dy \tag{14}$$

Then, the expected revenue is:

$$R\left(\omega, N\right) = \omega - N \int_0^{\omega} \left[1 - Z\left(\varepsilon\right)\right]^{N-1} F\left(\varepsilon\right) d\varepsilon - \int_0^{\omega} \left[1 - Z\left(\varepsilon\right)\right]^{N} d\varepsilon \tag{15}$$

[4] The support being between 0 and ω is just a simplifying assumption that can be dropped without qualitative changes in the results.

Claim 3 *Expected Return is increasing in N.*

Claim 4 *Expected payment converges to ω as $N \to \infty$.*

Now consider that the winner paid a price p. How much would the market pay for this company in the next day?

Remember that p is given by:

$$p = \omega - \varepsilon_w - \int_{\varepsilon_w}^{\omega} \left\{ \frac{1 - Z(y)}{1 - Z(\varepsilon_w)} \right\}^{N-1} dy.$$

where ε_w is the winner's private cost of sealing the agreement with the underwriter. Considering the market agents are risk neutral, we are looking for:

$$E[\omega \,| p \text{ is the winner}]$$

Then, we can above the expression above:

$$\omega = p + \varepsilon_w + \int_{\varepsilon_w}^{\omega} \left\{ \frac{1 - Z(y)}{1 - Z(\varepsilon_w)} \right\}^{N-1} dy$$

Since the agent won the auction, ε_w is the minimum between N. Therefore, taking the expected value of the last two terms on RHS, we have:

$$\widehat{\omega} = p + N \int_0^{\widehat{\omega}} [1 - Z(y)]^{N-1} dy - (N-1) \int_0^{\widehat{\omega}} [1 - Z(y)]^N dy$$

It is easy to show that the RHS of the above expression is constant in $\widehat{\omega}$. Since the LHS is increasing, we can show that it crosses once.

Let's show an example with a Uniform distribution

Example 5 $\varepsilon_i \sim U[0, \omega]$. *Then, we have:*

$$\widehat{\omega} = p + \widehat{\omega} - \frac{N-1}{N+1} \widehat{\omega}$$

Therefore:

$$\widehat{\omega} = \frac{N+1}{N-1} p.$$

We notice that as N increases $\widehat{\omega}$ converges to p. Therefore, as N increases, p becomes a better signal of ω.

In this example, the expected first day return is given by:

$$\begin{aligned}\widehat{\omega} - p &= \frac{N+1}{N-1}p - p \\ fdr &= \frac{2}{N-1}p\end{aligned}$$

substituting p, we have:

$$\begin{aligned}\widehat{fdr} &= \left(\frac{2}{N-1}\right) \times \left(\frac{N-1}{N+1}\right)\omega \\ &= \frac{2}{N+1}\omega.\end{aligned}$$

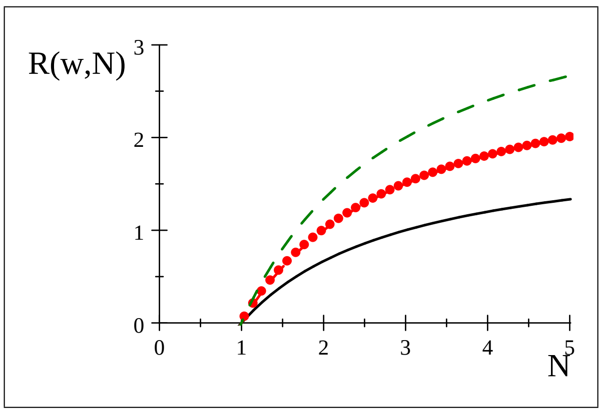
Therefore, the higher ω, higher the expected first day return.

This result is in agreement with the intuition presented by Ritter (1998) to why pre-IPO shareholders don't get upset when they see a large underprice: "Bad news that a lot of money was left on the table arrived at the same time that the good news of high market price", i.e., since more valuable companies (High ω) usually have a higher first day return, a large underprice in a crowded IPO implies a high value to the shares kept by the pre-IPO shareholders.

It also gives us an indication about the relationship between hot issue markets and technological shocks. Considering a technological shock as a jump in ω, this would imply an increase in the expected underprice, as advocate by some authors.

Finishing this example, it's easy to show that $R(\omega, N) = \left(\frac{N-1}{N+1}\right)\omega$ is concave in (ω, N). In this way, even if we consider that ω increases through time given VC's activity (changing managers, restructuring production, etc...), we still obtain the same results and the concave shape necessary for our previous results about optimal selling time.

Graphically, we have:



Where $\omega = 2$,3 and 4 in the black (solid), red (dot-dash) and green (dash) lines, respectively.

Concavity in general is not necessarily granted, especially given that we have a jump in revenue from the entrance of the second bidder in the auction[5]. However, it is easy to show that there is a cutoff number of bidders N^* such that for any $N \geq N^*$, $R(\omega, N)$ is concave in N :

$$N^* = \frac{\int_0^\omega [1 - Z(\varepsilon)]^{N^*-2} Z(\varepsilon)^2 d\varepsilon}{\int_0^\omega [1 - Z(\varepsilon)]^{N^*-2} Z(\varepsilon)^3 d\varepsilon}.$$

Up to now, we considered the number of bidders in a given IPO as constant. However, our intuition from the length of the road show and importance of good marketing skills by underwriters and VCs are related with an inflow of institutional investors that get to know the IPO firm and then decide to participate or not in the IPO. We will model this considering the arrival of potential buyers as a Poisson Process with average μ. Therefore, the number of investors that observed their valuations in an interval of length T is a random variable with Poisson distribution with parameter μT. Therefore the probability that an auction realized after a waiting time of length T has N bidders is $p_N(T) = \frac{e^{-\mu T}(\mu T)^N}{N!}$. Notice that in this case we consider that all investors that where contacted and draw their ε will wait for the auction. Even though this seems a strong assumption it doesn't affect qualitatively the results. A simple generalization would be considering that investors could sample other opportunities and leave. This generalization would have very similar results to the basic case since the investors that are more probable to leave are the ones that sampled high opportunity costs, while the ones with low costs, which are the ones important to obtain the expected revenue, stay waiting for the auction with high probability.

Therefore, the expected return of an auction realized after waiting T is given by:

$$\sum_{N=2}^{\infty} p_N(T) R(\omega, N).$$

Manipulating it, we obtain:

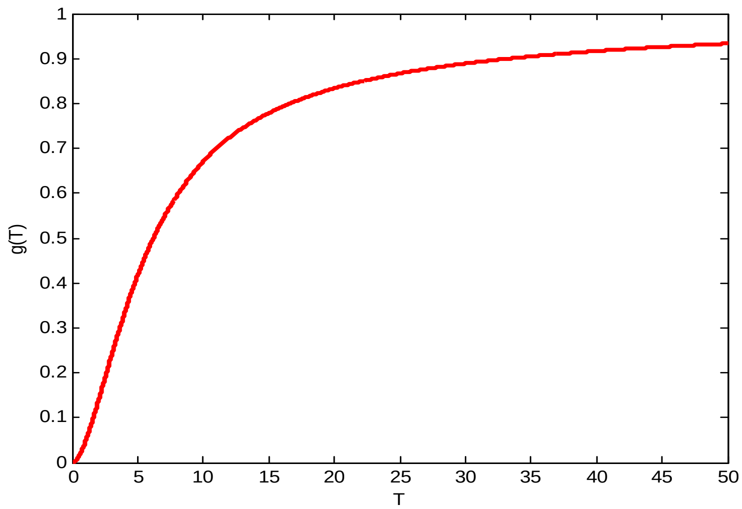
$$g(T) = \omega - \int_0^\omega [1 + \mu T Z(\varepsilon)] e^{-\mu T Z(\varepsilon)} d\varepsilon$$

It is easy to see that $g'(\cdot) > 0$ and $g''(T) < 0$, $\forall T \geq T^C$ given by:

$$T^C = \frac{\int_0^\omega \mu Z(\varepsilon) e^{-\mu T^C Z(\varepsilon)} d\varepsilon}{\int_0^\omega (\mu Z(\varepsilon))^2 e^{-\mu T^C Z(\varepsilon)} d\varepsilon}$$

Plotting $g(T)$, we have:

[5] The introduction of reserve prices and some additional assumptions can mitigate this problem.



The initial convexity comes from the large impact on auction's expected revenue generated by the entry of a second potential buyer, since it introduces competition and raises the price from zero to a positive value. The introduction of reserve prices and assuming log concavity of the distribution $Z(\cdot)$ would help us to avoid the initial non-concavity in $g(T)$. Finally, we can see that our previous discussions on technological shocks in $g(T)$ could be modeled as jumps in ω.

6 Empirical Evidence

We will present now some evidence about the impact of days in registration and the age of the firm on first day return and therefore, underpricing. The number of days in registration is considered here as a proxy for the length of the road show, indicating the effort of sale and/or the expected number of potential bidders.

The source of our data is SDC Platinum, from which we look at US data on IPOs from 1984-2004, focusing on Common Stocks. We include as control variables the number of days in registration, the age at which the first investment was made (difference between year of foundation and year at the first investment), number of investment rounds, book value per share, book value before offer, market indexes at the IPO date and firm's age at the IPO. We also include in our analysis dummy variables that control for: sector in which the firm is, year in which the IPO was realized, market in which the IPO was realized.

Results from regression analysis with robust errors are presented in the table below. It show that Days in Registration have an impact on First Day Return that is always significative (consider $\alpha \leq 5\%$). The size varies between $[-0.11, -0.04]$, while age at the IPO has a negative and significant impact on first day return (size varies but it is usually big: around -0.4). These results are in agreement with our theory on IPOs: The higher the time until the IPO, the lower the amount of money left on the table for investors that buy at the initial public offer[6].

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SEE TABLE 3

Another empirical analysis that we present here comes from a nonlinear analysis using a Multivariable fractional polynomial model. The obtained results are presented below:

SEE TABLE 4

The adjustments in the variables are obtained after 3 iterations in the fractional polynomial fitting algorithm. Dummy variables are included in the estimation, although their results are omitted here. As we can see, again there is a negative impact of days in registration and age at the IPO in the first day return, showing that our intuition that the longer the venture capitalist keeps the firm/project, the lower is the amount of money left on the table for institutional investors.

As we mentioned before, these results are only indications in favor of our theory, showing that the correlations obtained in the data are in agreement to our results. Unfortunately, a deeper empirical analysis, with the estimation of more structured models is not possible since most data on the IPO process is not public, not being disclosed by investment banks for further analysis.

7 Conclusion

In this paper, we present a new theory about underpricing and VC-backed companies where the key feature is VC's capacity constraints, i.e., Venture Capital firms can only handle a limited number of projects.

We show that this theory can match not only the empirical evidence that VC firms take younger companies public and that younger VC firms take even younger firms public than their more mature counterparts, but it also presents as a nice framework to address additional features in the market, as the impact of technological shocks and hot issues markets in underpricing, being our preliminary results in agreement with what was encountered by the empirical literature.

[6] As additional results we have that: Dummy for 1999 is the only year dummy consistently significant. It has a positive impact on first day return. Sector dummies have the expected signals from a asymmetric information claim (positive for high tech, negative for manufacture and health) but they are not significant in many cases. All other variables are usually not statistically significant at 5%

Finally, our model also presents endogenizes the IPO-underpricing mechanism in a way that matches our initial empirical evidence between time to IPO -measure by firm's age at the IPO and days in registration - and underpricing.

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9 Appendix A

Claim A.1: $T^*_\emptyset$ is given by:

$$\frac{g'\left(T^*_\emptyset\right)}{g\left(T^*_\emptyset\right)} = r.$$

Proof: Notice that the VC must be indifferent between keeping the project or finishing it and opening a vacancy, i.e.:

$$\beta g\left(T^*_\emptyset\right) + \frac{1}{1+rdT}\left\{\lambda dT E_{\tilde{T}} V\left(\widetilde{T}\right) + (1-\lambda dT)\, V^0\right\} = \frac{1}{1+rdT}\left\{\begin{array}{c} \lambda dT\left[\beta g\left(T^*_\emptyset + dT\right) + E_{\tilde{T}} V\left(\widetilde{T}\right)\right] \\ +(1-\lambda dT)\left[\beta g\left(T^*_\emptyset + dT\right) + V^0\right] \end{array}\right\}$$

Simplifying:

$$\begin{aligned} g\left(T^*_\emptyset\right) &= \frac{1}{1+rdT} g\left(T^*_\emptyset + dT\right) \\ (1+rdT)\, g\left(T^*_\emptyset\right) &= g\left(T^*_\emptyset + dT\right) \\ rdTg\left(T^*_\emptyset\right) &= g\left(T^*_\emptyset + dT\right) - g\left(T^*_\emptyset\right) \end{aligned}$$

Dividing by dT and taking $dT \to 0$, we have:

$$\frac{g'\left(T^*_\emptyset\right)}{g\left(T^*_\emptyset\right)} = r$$

■

Proof of Proposition 2

To obtain the proof of this propostion, we need to show a few intermediary steps:

Lemma $V(T)$ is strictly increasing.

Proof: By now, assume that V is increasing in T. Then, we know that any project with starting age $T \geq T^*(T)$ will induce the termination of the current project. Therefore, we have that for $T < T^*(T)$:

$$rV(T) = \lambda \int_{T^*(T)}^{T_n^*} \left[V\left(\widetilde{S}\right) + \beta g(T) - V(T) \right] dF\left(\widetilde{S}\right) + \frac{dV(T)}{dT} \tag{16}$$

Rearranging and using (6), we have:

$$rV(T) = \frac{dV(T)}{dT} + \lambda \int_{T^*(T)}^{T_\emptyset^*} \left[1 - F\left(\widetilde{S}\right) \right] \frac{dV\left(\widetilde{T}\right)}{d\widetilde{T}} d\widetilde{S} \tag{17}$$

If we solve this ODE, we obtain:

$$V(T) = e^{-r\left(T_\emptyset^* - T\right)} \left[\beta g\left(T_\emptyset^*\right) + V_0 \right] + \lambda \int_T^{T_\emptyset^*} e^{-r(s-T)} \int_{T^*(s)}^{T_\emptyset^*} \frac{dV\left(\widetilde{S}\right)}{d\widetilde{T}} \left[1 - F\left(\widetilde{S}\right) \right] d\widetilde{S} ds$$

Observe that the first term on RHS is exactly the same that we had in the case without on the project search (although the optimal time will not be the same, as we will see later). The second term on the RHS is the gain the VC has because of on the project search, i.e., the growth of the current project while he searches for a new one.

Taking the derivative of (8) and (6) with respect to T and manipulating, we get:

$$\frac{d^2V(T)}{dT^2} = \{r + \lambda [1 - F(T^*(T))]\} \frac{dV(T)}{dT} - \lambda [1 - F(T^*(T))] \beta g'(T)$$

Note that this is a first order ODE in $\frac{dV(T)}{dT}$.

For notational reasons, let's define:

$$\Psi(S) = r + \lambda [1 - F(T^*(S))]$$

Then, solving the ODE, we obtain:

$$\frac{dV(T)}{dT} = e^{\int_{T_0}^T \Psi(s)ds} x_0 - \lambda \int_{T_0}^T e^{\int_s^T \Psi(z)dz} \beta g'(s) [1 - F(T^*(s))] ds$$

where T_0 is an initial condition. Using $T_\emptyset^*$ as a terminal condition and manipulating, we obtain:

$$\frac{dV(T)}{dT} = e^{-\int_T^{T_\emptyset^*} \Psi(s)ds} \beta \left\{ g'\left(T_\emptyset^*\right) + \lambda \int_T^{T_\emptyset^*} e^{\int_s^{T_\emptyset^*} \Psi(z)dz} g'(s) [1 - F(T^*(s))] ds \right\} > 0$$

■

Claim A.2: For any $c > 0$, $\frac{dV(T)}{dT} > 0$.

Proof: Doing integration by parts and rearranging, using , we obtain:

$$rV(T) = \lambda \int_{T_c^*(T)}^{\overline{T}} [1 - F(s)] \frac{dV(s)}{ds} ds + \frac{dV(T)}{dT}$$

taking the derivative, we have:

$$r\frac{dV(T)}{dT} = -\lambda [1 - F(T_c^*(T))] \left.\frac{dV(T)}{dT}\right|_{T=T_c^*(T)} \frac{dT_c^*(T)}{dT} + \frac{d^2V(T)}{dT^2}.$$

but, from (20), we know that:

$$\left.\frac{dV(T)}{dT}\right|_{T=T_c^*(T)} \frac{dT_c^*(T)}{dT} = -\beta g'(T) + \frac{dV(T)}{dT}$$

therefore:

$$\frac{d^2V(T)}{dT^2} = (r + \lambda [1 - F(T_c^*(T))]) \frac{dV(T)}{dT} - \lambda [1 - F(T_c^*(T))] \beta g'(T)$$

For notational sake, define:

$$\Psi_c(s) = r + \lambda [1 - F(T_c^*(s))]$$

then, solving the ODE and using $T_\emptyset^*$ as the terminal condition, we obtain:

$$\frac{dV(T)}{dT} = e^{-\int_T^{T_\emptyset^*} \Psi_c(s)ds} \beta \left\{ g'\left(T_\emptyset^*\right) + \lambda \int_T^{T_\emptyset^*} e^{\int_s^{T_\emptyset^*} \Psi_c(z)dz} g'(s) [1 - F(T^*(s))] ds \right\} > 0.$$

■

Claim $\frac{dV(T)}{dT} \leq \beta g'(T)$.

Proof: From previous calculations, we obtained:

$$\frac{dV(T)}{dT} = e^{-\int_T^{T_\emptyset^*} r+\lambda[1-F(T_c^*(s))]ds} \beta g'\left(T_\emptyset^*\right) + \int_T^{T_\emptyset^*} \lambda [1 - F(T_c^*(s))] e^{-\int_T^s r+\lambda[1-F(T_c^*(z))]dz} \beta g'(s) ds$$

Using integration by parts, we have:

$$\frac{dV(T)}{dT} = \beta g'(T) + \int_T^{T_\emptyset^*} e^{-\int_T^s r+\lambda[1-F(T^*(z))]dz} \beta \left[g''(s) - rg'(s)\right] ds$$

Since $g''(T) - rg'(T) < 0$, $\forall T \in \left[0, T_\emptyset^*\right]$, we must have $\frac{dV(T)}{dT} < \beta g'(T)$

■

The remaining steps to prove proposition 2 can be seen graphically in the main text. ■

Proof of Proposition 3:

Proof: From previous calculations, we obtained:

$$\frac{dV(T)}{dT} = e^{-\int_T^{T_\emptyset^*} r+\lambda[1-F(T_c^*(s))]ds}\beta g'\left(T_\emptyset^*\right) + \int_T^{T_\emptyset^*} \lambda\left[1-F\left(T_c^*(s)\right)\right] e^{-\int_T^s r+\lambda[1-F(T_c^*(z))]dz}\beta g'(s)\, ds$$

Using integration by parts, we have:

$$\frac{dV(T)}{dT} = \beta g'(T) + \int_T^{T_\emptyset^*} e^{-\int_T^s r+\lambda[1-F(T^*(z))]dz}\beta\left[g''(s) - rg'(s)\right] ds$$

Then, taking the derivative with respect to λ, we have:

$$\frac{d^2V(T)}{d\lambda dT} = \int_T^{T_\emptyset^*}\left\{\begin{array}{c} e^{-\int_T^s r+\lambda[1-F(T^*(z))]dz}\beta\left[rg'(s) - g''(s)\right] \\ * \int_T^s \left[1-F\left(T^*(z)\right)\right] - \lambda f\left(T^*(z)\right)\frac{dT^*(z,\lambda)}{d\lambda} \end{array}\right\} dzds$$

Now, let's take the equation that defines $T_c^*(T)$:

$$V\left(T_c^*(T)\right) + \beta g(T) - c = V(T)$$

deriving with respect to λ, we have:

$$\frac{dV\left(T_c^*(T)\right)}{dT_c^*(T)}\frac{dT_c^*(T)}{d\lambda} + \frac{dV\left(T_c^*(T)\right)}{d\lambda} = \frac{dV(T)}{d\lambda}$$

Therefore, we have:

$$\frac{dT_c^*(T)}{d\lambda} = \frac{\frac{dV(T)}{d\lambda} - \frac{dV(T_c^*(T))}{d\lambda}}{\frac{dV(T_c^*(T))}{dT_c^*(T)}}$$

Since $\frac{dV(T_c^*(T))}{dT_c^*(T)} > 0$, the numerator defines the sign of this derivative. We know that $V\left(T^+\right) = V\left(T_c^*\left(T^+\right)\right)$, therefore $\left.\frac{dT_c^*(T)}{d\lambda}\right|_{T=T^+} = 0$. Notice that de numerator will depend on $\frac{d^2V(T)}{d\lambda dT}$. Let's consider the following cases:

$\frac{d^2V(T)}{d\lambda dT} > 0$: Then $\frac{dV(T)}{d\lambda}$ increases with T. Since $\forall T > T^+$, $T_c^*(T) > T$, we have that this hypothesis implies $\frac{dT_c^*(T)}{d\lambda} > 0,\ \forall T > T^+$ and $\frac{dT_c^*(T)}{d\lambda} < 0,\ \forall T < T^+$.

$\frac{d^2V(T)}{d\lambda dT} < 0$: Then $\frac{dV(T)}{d\lambda}$ decreases with T. Since $\forall T > T^+$, $T_c^*(T) > T$, we have that this hypothesis implies $\frac{dT_c^*(T)}{d\lambda} > 0,\ \forall T < T^+$ and $\frac{dT_c^*(T)}{d\lambda} < 0,\ \forall T > T^+$.

Let's start considering the case in which $\frac{dT_c^*(T)}{d\lambda} < 0,\ \forall T > T^+$. Then, we have that :

$$\frac{d^2V(T)}{d\lambda dT} = \left\{\begin{array}{c} \int_T^{T_\emptyset^*} e^{-\int_T^s r+\lambda[1-F(T^*(z))]dz}\beta\left[rg'(s) - g''(s)\right]\int_T^s \left[1-F\left(T^*(z)\right)\right] \\ -\lambda f\left(T^*(z)\right)\frac{dT^*(z,\lambda)}{d\lambda}dzds \end{array}\right\} > 0$$

which contradicts the assumption that is needed to actually obtain $\frac{dT_c^*(T)}{d\lambda} < 0$. Therefore, we cannot have $\frac{dT_c^*(T)}{d\lambda} < 0,\ \forall T > T^+$. Following a simple induction argument, we can show that we cannot have $\frac{dT_c^*(T)}{d\lambda} < 0$ for any $T > T^+$.

However, to obtain $\frac{dT_c^*(T)}{d\lambda} > 0$, for $T > T^+$, we must have $\frac{d^2V(T)}{d\lambda dT}$ switching signals at $T = T^+$, which will be generically not true.■

Proof of Claim 5:

Proof: First of all, remember that the value of a partnership for the entrepreneur with a project of size T is given by

$$rP(T) = \lambda\{(1-\beta)g(T) - P(T)\} + \frac{dP(T)}{dT} \tag{18}$$

solving the ODE and remembering that:

$$P\left(T_\emptyset^*\right) = (1-\beta)\,g\left(T_\emptyset^*\right)$$

we have:

$$P(T) = (1-\beta)\,g\left(T_\emptyset^*\right)e^{-(r+\lambda)\left(T_\emptyset^*-T\right)} + \lambda\int_T^{T_\emptyset^*}(1-\beta)\,g(s)\,e^{-(r+\lambda)(s-T)}ds$$

solving the integral and rearranging,

$$P(T) = \frac{(1-\beta)}{r+\lambda}\left\{\lambda\left[g(T) + \int_T^{T_\emptyset^*} g'(s)\,e^{-(r+\lambda)(s-T)}ds\right] + rg\left(T_\emptyset^*\right)e^{-(r+\lambda)\left(T_\emptyset^*-T\right)}\right\}$$

Finally, let's consider the value function of a entrepreneur that is in the VC's market searching for a partner. Since the entrepreneur would accept any VC and would also be accepted by any of them, we would have:

$$S(S) = \frac{\eta}{(r+\eta)}P(S) \tag{19}$$

Then, substituting $P(S)$, we have:

$$S(S) = \frac{\eta(1-\beta)}{(r+\lambda)(r+\eta)}\left\{\lambda\left[g(t) + \int_S^{T_\emptyset^*} g'(x)\,e^{-(r+\lambda)(s-t)}dx\right] + rg\left(T_\emptyset^*\right)e^{-(r+\lambda)\left(T_\emptyset^*-S\right)}\right\} \tag{20}$$

Then, an seller would enter the broker's market after getting a project of initial quality S, if and only if:

$$S(S) \geq A(S)$$

i.e.,

$$\frac{\eta(1-\beta)}{(r+\lambda)(r+\eta)}\left\{g\left(T_{\emptyset}^{*}\right)e^{-(r+\lambda)\left(T_{\emptyset}^{*}-T\right)}+\lambda\int_{S}^{T_{\emptyset}^{*}}g(x)\,e^{-(r+\lambda)(s-T)}dx\right\}\geq e^{-r\left(T_{\emptyset}^{*}-t\right)}\gamma g\left(T_{\emptyset}^{*}\right)$$

Then, rearranging the above expression, we have:

$$e^{\lambda T}\left\{g\left(T_{\emptyset}^{*}\right)e^{-(r+\lambda)T_{\emptyset}^{*}}+\lambda\int_{S}^{T_{\emptyset}^{*}}g(x)\,e^{-(r+\lambda)s}dx\right\}\geq\frac{(r+\eta)\gamma}{\eta(1-\beta)}g\left(T_{\emptyset}^{*}\right)e^{-rT_{\emptyset}^{*}}\qquad(\bigstar)$$

Take the derivative of each term:

$$\frac{d}{dS}\left\{e^{\lambda T}g\left(T_{\emptyset}^{*}\right)e^{-(r+\lambda)T_{\emptyset}^{*}}\right\}=\lambda e^{\lambda S}g\left(T_{\emptyset}^{*}\right)e^{-(r+\lambda)T_{\emptyset}^{*}}>0.$$

$$\begin{aligned}\frac{d}{dS}\left\{\lambda e^{\lambda S}\int_{S}^{T_{\emptyset}^{*}}g(x)\,e^{-(r+\lambda)x}dx\right\} &= \lambda^{2}e^{\lambda S}\int_{S}^{T_{\emptyset}^{*}}g(x)\,e^{-(r+\lambda)x}dx-\lambda e^{-rS}g(S)\\ &= \lambda\left\{-e^{-rS}g(S)+\lambda e^{\lambda T}\int_{S}^{T_{\emptyset}^{*}}g(x)\,e^{-(r+\lambda)x}dx\right\}\end{aligned}$$

Note:

$$\lambda e^{\lambda S}\int_{S}^{T_{\emptyset}^{*}}g(x)\,e^{-(r+\lambda)x}dx=_{I.P.}\frac{\lambda e^{\lambda S}}{r+\lambda}\left\{\begin{array}{c}-g\left(T_{\emptyset}^{*}\right)e^{-(r+\lambda)T_{\emptyset}^{*}}+\\ g(S)\,e^{-(r+\lambda)S}+\int_{S}^{T_{\emptyset}^{*}}g'(x)\,e^{-(r+\lambda)x}dx\end{array}\right\}$$

Now, assume that $T_{\emptyset}^{*}>0$[7], this implies that $g'(S)-rg(S)\geq 0$ (being $=0$ iff $S=T_{\emptyset}^{*}$). Then, $g'(S)\geq rg(S)$. Using this, we have:

$$\lambda e^{\lambda S}\int_{S}^{T_{\emptyset}^{*}}g(x)\,e^{-(r+\lambda)x}dx\geq\frac{\lambda e^{\lambda S}}{r+\lambda}\left\{\begin{array}{c}-g\left(T_{\emptyset}^{*}\right)e^{-(r+\lambda)T_{\emptyset}^{*}}+\\ g(S)\,e^{-(r+\lambda)S}+r\int_{S}^{T_{\emptyset}^{*}}g(x)\,e^{-(r+\lambda)x}dx\end{array}\right\}$$

rearranging:

$$\lambda\int_{S}^{T_{\emptyset}^{*}}g(x)\,e^{-(r+\lambda)x}dx\geq -g\left(T_{\emptyset}^{*}\right)e^{-(r+\lambda)T_{\emptyset}^{*}}+g(S)\,e^{-(r+\lambda)S}$$

Substituting this back, we have:

$$\frac{d}{dS}\left\{\lambda e^{S}\int_{S}^{T_{\emptyset}^{*}}g(x)\,e^{-(r+\lambda)x}dx\right\}\geq\lambda\left\{-e^{-rS}g(S)+g(S)\,e^{-rS}-g\left(T_{\emptyset}^{*}\right)e^{-(r+\lambda)T_{\emptyset}^{*}}e^{\lambda S}\right\}$$

rearranging:

[7] Note that we already assumed this when we wrote $P(S)$

$$\frac{d}{dS}\left\{\lambda e^{\lambda T}\int_{T}^{T_{\emptyset}^{*}} g\left(s\right)e^{-(r+\lambda)s}ds\right\}\geq -\lambda e^{\lambda T}g\left(T\right)e^{-(r+\lambda)T}$$

Then, putting everything together, we have:

$$\frac{dLHS}{dS}\geq \lambda e^{\lambda S}g\left(T_{\emptyset}^{*}\right)e^{-(r+\lambda)T_{\emptyset}^{*}}-\lambda e^{\lambda S}g\left(S\right)e^{-(r+\lambda)S}=0.$$

■

Proof of Claim 6:

Proof: Solving the ODE and using the terminal condition, we have:

$$P\left(T\right)=\left(1-\beta\right)g\left(T\right)+\int_{T}^{T_{\emptyset}^{*}}\left(1-\beta\right)\left[g'\left(s\right)-rg\left(s\right)\right]e^{-\int_{T}^{s}\Psi(z)dz}.ds$$

Then:

$$\frac{dP\left(T\right)}{dT}=\left\{r\left(1-\beta\right)g\left(T\right)+\Psi\left(T\right)\int_{T}^{T_{\emptyset}^{*}}\left(1-\beta\right)\left[g'\left(s\right)-rg\left(s\right)\right]e^{-\int_{T}^{s}\Psi(z)dz}.ds\right\}>0$$

Since $g'\left(s\right)-rg\left(s\right),\ \forall s\in\left[0,T_{\emptyset}^{*}\right]$. Then:

$$S\left(T\right)=\frac{\eta\left[1-F\left(T^{*-1}\left(T\right)\right)\right]}{r+\eta\left[1-F\left(T^{*-1}\left(T\right)\right)\right]}P\left(T\right)$$

As we know, a entrepreneur enters the VC market if:

$$S\left(T\right)\geq A\left(T\right)$$

which means:

$$\frac{\eta\left[1-F\left(T^{*-1}\left(T\right)\right)\right]}{r+\eta\left[1-F\left(T^{*-1}\left(T\right)\right)\right]}e^{-rT}P\left(T\right)\geq e^{-rT_{\emptyset}^{*}}\gamma g\left(T_{\emptyset}^{*}\right)$$

Then:

$$\frac{dLHS}{dT}=\left\{\begin{array}{c}-\frac{r\eta f\left(T^{*-1}(T)\right)\frac{dT^{*-1}(T)}{dT}}{\left\{r+\eta\left[1-F\left(T^{*-1}(T)\right)\right]\right\}^{2}}e^{-rT}P\left(T\right)\\+\frac{\eta\left[1-F\left(T^{*-1}(T)\right)\right]}{r+\eta\left[1-F\left(T^{*-1}(T)\right)\right]}e^{-rT}\left[-rP\left(T\right)+\frac{dP(T)}{dT}\right]\end{array}\right\}$$

notice that:

$$\frac{dP\left(T\right)}{dT}-rP\left(T\right)=\left[\lambda\left(1-F\left(T^{*}\left(T\right)\right)\right)\right]\int_{T}^{T_{\emptyset}^{*}}\left(1-\beta\right)\left[g'\left(s\right)-rg\left(s\right)\right]e^{-\int_{T}^{s}\Psi(z)dz}.ds>0$$

Since $\frac{dT^{*-1}(T)}{dT} < 0$, we have that $\frac{dLHS}{dT} \geq 0$. Therefore, there exists a threshold $T^{\blacklozenge}(c)$ such that if $T \geq T^{\blacklozenge}(c)$ the entrepreneur enters the VC market.

Proof of Claim 7:

Proof: Using induction. We can easily show that it increases from $N = 1$ to $N = 2$. Now consider a given $N \geq 2$. For $N + 1$, we have:

$$\omega - (N+1)\int_0^{\omega} [1 - Z(\varepsilon)]^N Z(\varepsilon)\, d\varepsilon - \int_0^{\omega} [1 - Z(\varepsilon)]^{N+1} d\varepsilon$$

while for N we already saw:

$$\omega - N\int_0^{\omega} [1 - Z(\varepsilon)]^{N-1} Z(\varepsilon)\, d\varepsilon - \int_0^{\omega} [1 - Z(\varepsilon)]^{N} d\varepsilon$$

Therefore, the change in expected return as we increased the number of bidders in 1 is:

$$\begin{aligned} & N\int_0^{\omega} [1 - Z(\varepsilon)]^{N-1} Z(\varepsilon)^2\, d\varepsilon + \int_0^{\omega} [1 - Z(\varepsilon)]^{N+1} d\varepsilon - \int_0^{\omega} [1 - Z(\varepsilon)]^{N+1} d\varepsilon \\ = \; & N\int_0^{\omega} [1 - Z(\varepsilon)]^{N-1} Z(\varepsilon)^2\, d\varepsilon > 0 \end{aligned}$$

■

Proof of Claim 8:

Proof:

$$\lim_{N\to\infty} \omega - N\int_0^{\omega} [1 - Z(\varepsilon)]^{N-1} Z(\varepsilon)\, d\varepsilon - \int_0^{\omega} [1 - Z(\varepsilon)]^{N} d\varepsilon$$

notice that:

$$\lim_{N\to\infty} \int_0^{\omega} [1 - Z(\varepsilon)]^{N} d\varepsilon = 0$$

Consider that there exists $z'(\varepsilon)$ and $z(\cdot) > 0$, then we have:

$$\begin{aligned} -N\int_0^{\omega} [1 - Z(\varepsilon)]^{N-1} Z(\varepsilon)\, d\varepsilon &= \int_0^{\omega} N[1 - Z(\varepsilon)]^{N-1} (-z(\varepsilon)) \frac{Z(\varepsilon)}{z(\varepsilon)} d\varepsilon \\ &= -\int_0^{\omega} [1 - Z(\varepsilon)]^{N} \left[1 - \frac{Z(\varepsilon)\, z'(\varepsilon)}{z(\varepsilon)^2}\right] d\varepsilon \end{aligned}$$

But then, it is easy to see that this expression goes to zero as $N \to \infty$. Therefore, expected value converges to ω.■

10 Appendix B

In this appendix, we will consider the case in which the Venture Capital can handle two projects at the same time. In this case, her Bellman function is given by:

$$\begin{aligned}(1+rdT)\,V\left(T_1,T_2\right) &= \lambda dT E_{\widetilde{T}} \max\left\{\begin{array}{c} V\left(\widetilde{T},T_2+dT\right)+\beta g\left(T_1+dT\right)-c, \\ V\left(T_1+dT,\widetilde{T}\right)+\beta g\left(T_2+dT\right)-c, \\ V\left(T_1+dT,T_2+dT\right) \end{array}\right\} \\ &\quad +\left(1-\lambda dT\right)\max\left\{\begin{array}{c} V\left(\emptyset,T_2+dT\right)+\beta g\left(T_1+dT\right), \\ V\left(T_1+dT,\emptyset\right)+\beta g\left(T_2+dT\right), \\ V\left(\emptyset,\emptyset\right)+\beta g\left(T_1+dT\right)+\beta g\left(T_2+dT\right), \\ V\left(T_1+dT,T_2+dT\right) \end{array}\right\}\end{aligned}$$

Then, considering $\max\{T_1,T_2\}<T_{\emptyset}^{*}$, we have:

$$\begin{aligned}(1+rdT)\,V\left(T_1,T_2\right) &= \lambda dT E_{\widetilde{T}} \max\left\{\begin{array}{c} V\left(\widetilde{T},T_2+dT\right)+\beta g\left(T_1+dT\right)-c, \\ V\left(T_1+dT,\widetilde{T}\right)+\beta g\left(T_2+dT\right)-c, \\ V\left(T_1+dT,T_2+dT\right) \end{array}\right\} \\ &\quad +\left(1-\lambda dT\right)V\left(T_1+dT,T_2+dT\right)\end{aligned}$$

Rearranging:

$$\begin{aligned}rdTV\left(T_1,T_2\right) &= \lambda dT E_{\widetilde{T}} \max\left\{\begin{array}{c} \left[\begin{array}{c} V\left(\widetilde{T},T_2+dT\right)+\beta g\left(T_1+dT\right) \\ -c-V\left(T_1+dT,T_2+dT\right) \end{array}\right], \\ \left[\begin{array}{c} V\left(T_1+dT,\widetilde{T}\right)+\beta g\left(T_2+dT\right) \\ -c-V\left(T_1+dT,T_2+dT\right) \end{array}\right], \\ 0 \end{array}\right\} \\ &\quad +V\left(T_1+dT,T_2+dT\right)-V\left(T_1,T_2\right)\end{aligned}$$

Then, dividing by dT and taking $dT\to 0$, we have:

$$rV(T_1,T_2) = \lambda E_{\widetilde{T}} \max \left\{ \begin{array}{c} \left[\begin{array}{c} V\left(\widetilde{T},T_2\right) + \beta g(T_1) \\ -c - V(T_1,T_2) \end{array} \right], \\ \left[\begin{array}{c} V\left(T_1,\widetilde{T}\right) + \beta g(T_2) \\ -c - V(T_1,T_2) \end{array} \right], \\ 0 \end{array} \right\}$$
$$+ \left[\frac{\partial V}{\partial T_1}(T_1,T_2) + \frac{\partial V}{\partial T_2}(T_1,T_2) \right]$$

Then, looking at the cut off rule, let's analyse the conditions in which we have:

$$V\left(\widetilde{T},T_2\right) + \beta g(T_1) - c - V(T_1,T_2) \geq V\left(T_1,\widetilde{T}\right) + \beta g(T_2) - c - V(T_1,T_2)$$

Simplifying, we have:

$$V\left(\widetilde{T},T_2\right) + \beta g(T_1) \geq V\left(T_1,\widetilde{T}\right) + \beta g(T_2)$$

First of all, consider a simmetry condition:

$$V\left(\widetilde{T},T_2\right) = V\left(T_2,\widetilde{T}\right)$$

Then, rearranging,we have:

$$[\beta g(T_1) - \beta g(T_2)] - \left[V\left(T_1,\widetilde{T}\right) - V\left(T_2,\widetilde{T}\right)\right] \geq 0$$

Therefore, since both functions are increasing (as we showed before), we must have the increase in $\beta g(\cdot)$ higher than the increase in $V\left(\cdot,\widetilde{T}\right)$ as T increases. Taking $T_1 = T_2 + dT$, dividing by dT and taking $dT \to 0$, we have:

$$\beta g'(T_2) - \frac{\partial V}{\partial T_1}\left(T_2,\widetilde{T}\right) \geq 0$$

which is exactly what we showed previously. Therefore, if the VC finishes one project to undertake a new one, she chooses the bigger one. Then, we have a decision about cutting a project or not: Imagining again that T_1 is the bigger one,

$$V\left(\widetilde{T},T_2\right)+\beta g\left(T_1\right)-c\geq V\left(T_1,T_2\right)$$

We will again have a $T_c^*\left(T_1\right)$, in which the only contribution of T_2 is that $T_2\leq T_1$.

Let's consider now calculations related to the expected time until an IPO. First of all, notice that in the usual case in which a Venture Capital firm can only handle one project, the survival rate of a project of size/quality T is given by:

$$S\left(t\left|T\right.\right)=\frac{e^{-\lambda\int_0^t\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T}\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}},\quad \text{for } t\in\left[0,T_\emptyset^*-T\right]$$

then the hazard rate conditional on T is:

$$\hbar\left(t\left|T\right.\right)=-\frac{S'\left(t\left|T\right.\right)}{S\left(t\left|T\right.\right)}=\lambda\left[1-F\left(T^*\left(T+t\right)\right)\right].$$

and the lifetime distribution, conditional on T is:

$$K\left(t\left|T\right.\right)=1-S\left(t\left|T\right.\right)=1-\frac{e^{-\lambda\int_0^t\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T}\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}}$$

which implies:

$$k\left(t\left|T\right.\right)=\lambda\left[1-F\left(T^*\left(T+t\right)\right)\right]\frac{e^{-\lambda\int_0^t\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T}\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}}$$

Then the expected lifetime of a partnership given a starting size/quality T is:

$$\frac{\int_0^{T_\emptyset^*-T}e^{-\lambda\int_0^t\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}dt-\left(T_\emptyset^*-T\right)e^{-\lambda\int_0^{T_\emptyset^*-T}\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T}\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}}$$

Then the expected lifetime of a partnership is given by:

$$\int_0^{T_\emptyset^*}\frac{\int_0^{T_\emptyset^*-T}e^{-\lambda\int_0^t\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}dt-\left(T_\emptyset^*-T\right)e^{-\lambda\int_0^{T_\emptyset^*-T}\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}}{1-e^{-\lambda\int_0^{T_\emptyset^*-T}\left[1-F\left(T^*\left(T+s\right)\right)\right]ds}}dF\left(T\right).$$

Now consider the case in which a VC can handle two projects. Whenever she has a free spot, she will prefer fulfilling that spot instead of going public with the current project to accomodate the new project than cutting a current project. So she will cut projects when they are mature or if she finds a new one and have both vacancies fulfilled. Considering this second case, the Survival rate for a project with size quality T is given by:

$$S(t|T) = \frac{e^{-\lambda \int_0^t G(T+s)[1-F(T^*(T+s))]ds}}{1-e^{-\lambda \int_0^{T^*_\emptyset - T} G(T+s)[1-F(T^*(T+s))]ds}}, \quad \text{for } t \in \left[0, T^*_\emptyset - T\right]$$

which is bigger than the previous one, since $G(\cdot) \leq 1$. It's routine to show that in this case the expected lifetime of a partnership is given by:

$$\int_0^{T^*_\emptyset} \frac{\int_0^{T^*_\emptyset - T} e^{-\lambda \int_0^t G(T+s)[1-F(T^*(T+s))]ds} dt - \left(T^*_\emptyset - T\right) e^{-\lambda \int_0^{T^*_\emptyset - T} G(T+s)[1-F(T^*(T+s))]ds}}{1-e^{-\lambda \int_0^{T^*_\emptyset - T} G(T+s)[1-F(T^*(T+s))]ds}} dF(T).$$

which is bigger than the previous one.

TABLE 1: Characteristics of VC backed and non-VC backed IPOs

	VC- backed IPOs		Non-VC backed IPOs		
	Mean	*N*	*Mean*	*N*	*T-stat*
Age	7.0	1159	14.7	1446	12.13
Book Value	0.76	1961	6.63	3253	11.28
Revenue	19.9	1732	52.1	2759	3.16
EPS (% pos.)	49.7	806	76.5	1358	12.71
Total Assets	104.4	1719	543.2	2628	3.97
Net proceeds	40.5	2286	58.3	3782	5.44
Underwriter Rank	7.80	2383	6.79	4030	21.02
Gross spread	7.09	2285	7.36	3781	1.76

Source: Lee and Wahal (2004)

TABLE 2: Comparison of characteristics for IPOs backed by young and old VC firms

	VC firms <6 yr. at IPO	VC firms ≥6 yr. at IPO	p-value test of no difference
Avg. time from IPO date to next follow-on fund in months	16.0 [12.0]	24.2 [24.0]	0.001 [0.002]
Avg. size of next follow-on fund (1997 $mil)	87.9 [68.0]	136.6 [113.4]	0.018 [0.024]
Avg. age of VC-backed company at IPO date in months	55.1 [42.0]	79.6 [64.0]	0.000 [0.000]
Avg. duration of board representation for lead VC in months	24.5 [20.0]	38.8 [28.0]	0.001 [0.000]
Avg. underpricing at the IPO date	0.136 [0.067]	0.073 [0.027]	0.001 [0.036]
Avg. offer size (1997 $mil)	18.3 [13.0]	24.7 [19.1]	0.013 [0.000]
Avg. Carter and Manaster underwriter rank	6.26 [6.50]	7.43 [8.00]	0.000 [0.000]
Avg. number of previous IPOs	1 [0]	6 [4]	0.000 [0.000]
Avg. fraction of equity held by all VCs prior to IPO	0.321 [0.287]	0.377 [0.371]	0.025 [0.024]
Avg. fraction of equity held by lead VC after IPO	0.122 [0.100]	0.139 [0.120]	0.098 [0.031]
Avg. market value of lead VC's equity after IPO (1997 $mil)	9.5 [4.3]	14.7 [8.7]	0.033 [0.000]
Avg. aftermarket std. deviation	0.034 [0.030]	0.030 [0.028]	0.080 [0.324]
Number of Observations	99	240	

Note: Sample is 433 VC-backed companies that went public between January 1, 1978 and December 31, 1987. Medians are in brackets. Significance tests in the third column are p-values of t-tests for differences in averages and p-values of two-sample Wilconson rank-sum tests for differences in medians in brackets.
Source: Gompers and Lerner (2001)

TABLE 3: Multivariate Regression with Robust Errors

No. obs.: 1550
F(26, 1523) = 9.64
Prob > F = 0.000
R^2 = 0.239
Root MSE = 60.18

fdr	Coef.	Std. Error	t	P>|t|	[95% Conf. Interval]	
Reg	-0.09982	0.02185	-4.57	0.000	-0.142670	-0.056962
Age	-0.48988	0.11376	-4.31	0.000	-0.713018	-0.266745
AMEXdummy	-7.01536	9.55909	-0.73	0.463	-25.76573	11.73501
NASDQdummy	0.56059	2.79067	0.20	0.841	-4.913374	6.034551
NYSEdummy	-7.36085	4.33795	-1.70	0.090	-15.86984	1.148125
Djod	-0.02486	0.01441	-1.73	0.085	-0.053122	0.003401
Dummy1999	57.81929	20.6124	2.81	0.005	17.38751	98.25106
Health	-11.10043	3.07635	-3.61	0.000	-17.13477	-5.066087
High tech	3.380294	2.82414	1.20	0.232	-2.159312	8.919901
Spod	0.196895	0.11041	1.54	0.125	-0.046873	0.386251
_cons	90.97264	44.9188	2.03	0.043	2.863437	179.0818

Where: reg ≡ days in registration; age ≡ firm age at IPO; djod ≡ dow jones index at offer date; health ≡ dummy for firm in health sector; high tech ≡ dummy for firm in tech sector; spod ≡ S & P 500 at offer date.

TABLE 4: Multivariable Fractional Polynomial Model

Source	SS	df	MS
Model	1830577.8	28	65377.8
Residual	5419252.1	1521	3562.95
Total	7249829.9	1549	4680.33

No. obs.: 1550
F(28,1521) =18.35
Prob > F = 0.000
R^2 = 0.2525
Root MSE = 59.69

fdr	Coef.	Std. Error	t	P>|t|	[95% Conf. Interval]	
lreg	-10.6595	2.183401	-4.88	0.000	-14.9423	-6.3767
Lage1	-0.50835	0.187997	-2.70	0.007	-0.87711	-0.1396
Ldjod_1	-3.47512	0.844651	-4.11	0.000	-5.13193	-1.8183
Ldjod_2	1.26426	0.313333	4.03	0.000	0.64965	1.8789
High tech	2.71745	3.927244	0.69	0.489	-4.98594	10.421
Health	-10.5799	8.051582	-1.31	0.189	-26.3733	5.2135
lspod_1	435.539	92.85873	4.69	0.000	253.394	617.68
lspod_2	-657.809	141.3594	-4.65	0.000	-935.089	-380.53
NASDQdummy	-0.15150	5.607602	-0.03	0.978	-11.1509	10.8479
NYSEdummy	-7.50743	9.710198	-0.77	0.440	-26.5542	11.5393
_cons	107.522	29.09031	3.70	0.000	50.4606	164.583

Where: lreg_1 = ln(reg/100) + 0.3457942617, lnage_1 = age – 7.612903226, Ldjod_1 = $(djod/100)^3$ – 218.8132734, Ldjod_2 = $(djod/100)^3$ln(djod/100) – 393.004593; lspod_1 = $(spod/100)^3$ – 0.4248286209; lspod_2 = $(spod/100)^3$ ln(spod/100) – 0.1212275993.