Efficient Auditing and Enforcement in Dynamic Contracts (Job Market Paper)*

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Abstract

Private information and limited enforcement are two frictions that impede the provision of first best insurance against income risk. To mitigate these frictions, societies make costly investments into technologies such as auditing and court systems. The implicit assumption through most of the literature is that either or both of these technologies is either costless or infinitely costly. I consider a model of efficient insurance in which at each point in time the principal can choose a level of enforceability that inhibits an agent's ability to renege on the contract and a level of auditing that inhibits his ability to conceal income. The dynamics of the optimal contract imply an endogenous lower bound on the lifetime utility of an agent, strictly positive auditing at all points in the contract and positive enforcement only when the agent's utility is sufficiently low. Furthermore, the two technologies operate as complements and substitutes at alternative points in the state space, uncovering dynamics that differ dramatically from the case in which the technologies are studied separately.

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1 Introduction

Limited commitment and private information are two sources commonly identified as impediments to an agent's ability to insure himself against idiosyncratic income risk. However, models of efficient insurance with limited commitment generally ignore all considerations of private information and implicitly assume both that enforcement is prohibitively costly and that information is completely costless. Similarly, models of efficient insurance under private information often abstract away from the issue of enforcement, assuming that it is costless to compel agents to remain in the contract.

In this paper, I consider the provision of efficient insurance under both limited commitment and private information when the insurer has access to enforcement and auditing technologies that,

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respectively, reduce the returns from walking away from the contract and concealing income. While this is not the first paper in the literature to consider an insurance problem with both information and enforcement frictions, it is among the first that I am aware of to allow for an insurer's access to one or more technologies designed to mitigate these frictions in a dynamic environment.[1] I show that the implications on an agent's consumption, along with use of these technologies, critically depend on the presence of both frictions and the relative costs of auditing and enforcement. Therefore, analyses that ignore one dimension of the problem may provide misleading conclusions on the nature of the problem.

Consumption seems to react to idiosyncratic income shocks in a variety of settings.[2] To account for this observation, both normative and positive models have been adapted to include limited commitment and information frictions. In environments with limited commitment, an agent is assumed to have access to an "outside option," typically taken to be his value of consuming his contemporaneous income and autarky consumption thereafter. Should the contract ever be sufficiently undesirable, the agent may then choose to break the contract and receive the value of this option. A first best contract that requires the transfer of income from high income types to low income types may then be impossible to implement as an agent receiving a large income may prefer a high consumption today and the outside option to paying the transfer. Meanwhile, in environments with private information, low income agents are indistinguishable from high income agents. Thus, first best insurance is impossible to implement as all agents would choose to mimic the agent who is due the largest net transfer; any insurance plan that transfers resources from high income to low income agents must also provide high income agents the appropriate incentives to reveal their information.

Most applications of insurance problems not only are likely to entail some level of both these frictions, but also endogenous choices on the part of insurers that affect the extent to which one would expect these frictions to impede first best insurance. For example, implementation of efficient unemployment insurance entails decisions on the monitoring of agents' income claims, job offers, or search effort to mitigate these sources of information frictions. Moreover, efficient unemployment insurance also relies on a choice of investment in a court system, law enforcement, and even prisons to compel employed agents with high income to pay taxes. This paper's analysis can then be thought of as one of optimal unemployment insurance. The analysis of this paper could also be cast in the literature on managerial compensation, in which private information and limited commitment are thought to play an important role. Jensen (1986) hypothesizes that firm structure is the result of the optimal balancing of agency and information frictions between institutional investors and management. The ability of debt and equity holders to internally audit management and hire lawyers to sue opportunistic managers for fraud will then have implications on corporate financial structure. Finally, this paper applies to the informal risk sharing literature[3] to the extent that small

[1]One exception is Krasa and Villamil (2000) who study a three period model with private information and enforcement, allowing enforcement to be endogenous.

[2]See, for example, Cochrane (1991) for evidence from US consumer data, Townsend (1994) for evidence in village India.

[3]See Townsend (1994) and Ligon, et al (2002).

villages suffer from these frictions and have the capacity to mitigate them.

I study three dynamic principal agent problems in which the principal is risk neutral and the agent is risk averse. The first is one of limited commitment (costless information) and an enforcement technology and is referred to as the *Enforcement Model*, the second is one of private information (costless enforcement) and an auditing technology and is referred to as the *Auditing Model*, and the third features both sets of frictions and technologies and is referred to as the *General Model*. From the first two models, I show how the principal optimally uses enforcement and auditing, respectively, to insure the agent. In the *General Model*, I show how the implications of a joint model differ dramatically from the joint implications of the separate models.

The *Enforcement Model* is a version of the limited commitment and endogenous enforcement model studied in Koeppl (2007) and the results from this section are used to contrast and provide insight into the role of enforcement in the *General Model*. As in environments with exogenous enforcement, such as Thomas and Worrall (1988), the limited commitment in the *Enforcement Model* implies an increasing level of consumption and long run consumption constant and equal to an amount at which the highest income type would not choose to renege. Efficient insurance that requires the transfer of income from high to low income states must also provide an agent who realizes high income with an incentive to not renege. Such a transfer can be implemented by increasing future consumption across future shocks for an agent with high income. Higher consumption across future income states then diminishes the agent's future incentives to renege and, eventually, first best insurance is possible. With the addition of endogenous enforcement, enforcement is decreasing as the agent's promised utility increased. Enforcement alters these dynamics by slowing the growth of consumption to allow for smoother inter-temporal consumption. However, in the long run insurance is first best and the principal uses no enforcement. Notice that this long run result relies heavily on the costless information assumption, as long run consumption patterns are identical for high and low income types. Such a model should then be viewed skeptically if it is believe that there is any level of information friction.

Models of efficient insurance with private information[4] deliver a conflicting prediction on consumption. In these models, providing an agent with high income realization the incentives to reveal his information is "cheap" at lower levels of consumption and utility. The reason for this is that when marginal utility of consumption is high, agents with high income and agents with low income evaluate quite differently small differences in transfers. This eases the ability of the principal to separate income realizations and yields the well known immiseration result in the long run in which agents are driven down to their lower bound of utility (or toward $-\infty$ if utility is unbounded below) almost surely. This extreme prediction of models with private information is a direct consequence of the implicit costless enforcement assumption; given any capacity to renege an agent would surely undertake such an option to avoid the consequences of the implied contract.

In the *Auditing Model* I show that while the auditing technology slows this process, it does not

[4]See, for example, Green (1987), Atkeson and Lucas (1992), and Thomas and Worrall (1990)

prevent immiseration and thus, also relies heavily on costless enforcement. Furthermore, I show that as the agent is immiserised, the principal's use of auditing also vanishes. A novel feature of the *Auditing Model* is its specification of the auditing technology, which allows for more complete analytic results than have been obtained in other dynamic contracting environments with private information and some form of auditing. For example, Wang (2005) studies a dynamic version of costly state verification á la Townsend (1979). However, the complex nature of his problem hampers his ability to make statements on, for example, the qualitative response of the agent's future utility to income shocks. The auditing technology I consider is such that the level of investment into the technology reduces the agent's return to misreporting directly, but does not itself reveal information. This greatly simplifies the information structure of the problem and allows for more analytic results.

The dependence of both the *Enforcement* and *Auditing* models on costless information and enforcement, respectively, reveals the tension that exists in a model where both technologies are assumed to be costly. In the *General Model*, I study such a model. Past models that consider environments with both sources of frictions include Atkeson and Lucas (1995), who study unemployment insurance when job offers are unobservable and there is an exogenous lower bound on wealth. Phelan (1995) similarly studies an environment of limited commitment and private information in which competitive insurance contracts determine the lower bound on an agent's utility. However, neither paper allows a principal any recourse against these frictions, thus assuming that both information and enforcement are infinitely costly.

Studying auditing and enforcement together reveals that the two technologies interact in important ways that cannot be seen when studying the frictions and technologies separately. For example, the opposing forces on wealth of limited commitment and private information give rise to an endogenous lower bound. To contrast this result to those aforementioned papers with both frictions, their lower bounds are assumed by the (sometimes implicit) assumption of infinitely costly enforcement.[5] I do not rely on infinitely costly enforcement to establish this lower bound and show that such a bound exists whenever enforcement costs are convex, independent of its costs relative to auditing and consumption.

Lastly, I show that auditing and enforcement technologies act alternatively as both complements and substitutes at alternative points in the state space. Consider the case when wealth is at a level such that the high income agent is indifferent between remaining in the contract and reneging. Additional auditing allows the principal to better smooth the agent's consumption between high and low states, reducing the principal's consumption cost. However, this smoothing can be done only when accompanied by additional enforcement cost, as the high income agent must be kept from reneging. Thus, the two technologies are *complements*. On the other hand, when wealth is sufficiently low, it is the agent with a low income realization who is indifferent between reneging and remaining in the contract. In this case, an increase in auditing that increases the low agent's utility and smooths utility across states is accompanied by a decrease in the necessary enforcement.

[5] The lower bound in Phelan paper is still endogenous via competitive insurance markets.

Thus, the two technologies are *substitutes* in this region.

The remainder of the paper is organized as follows. Section 2 lays out the general problem and shows how the problem can be written recursively, which is used as the basis for the rest of the paper. Sections 3 and 4 examine the *Enforcement* (costless information) and *Auditing* (costless enforcement) models, respectively, while Section 5 analyzes the *General Model.* Section 6 provides s numerical example of the Enforcement, Auditing, and General models. Section 7 concludes.

2 Model

There are three models considered in this paper. The first is a model of efficient insurance with limited commitment and an enforcement technology, the second is a model with private information and an auditing technology, and the last model has both frictions and both the enforcement and auditing technologies. The enforcement technology makes breaking the contract costly to the agent. The agent could walk away from the contract at any time, consume that period's income and his autarky consumption at any point thereafter.[6] However, with an investment of $\gamma \in \Gamma = \mathbf{R}_+$ into enforcement, his utility from leaving the contract is shifted down by γ units of utility. We could think of this cost as being a disutility of court proceedings to break a contract or even more extreme measures such as debtor prisons. The first model studied is one of costless information and limited commitment with endogenous enforcement and will be called the *Enforcement Model.* This environment is similar to the contracting problem studied by Koeppl (2007), only that the principal is assumed to be risk neutral in this paper and the enforcement penalties on the agent differ slightly.

The second model considers an environment of private information in which the principal has access to an auditing technology. The nature of the auditing technology makes hiding income (or consumption) costly to the agent. Namely, for each unit of output that he tries to hide, α will be lost in an effort to avoid detection, where $\alpha \in A = [0, 1]$. For simplicity, I assume that should the agent choose to lie about his output (which he will, of course, not do in the optimal allocation) he will also choose to hide his income. The cost to hiding income can be interpreted as an agent having to consume less desirable goods (for example, the agent may have to buy a smaller house or less fancy car to remain inconspicuous). Thus, the purpose of auditing solely serves the function of reducing the incentives of the agent to lie. This model is one of costless enforcement and private information and will be referred to as the *Auditing Model.* It is worthwhile to note that the auditing technology in this paper differs from the common "costly state verification" auditing model. In the model here, auditing does not change the principal's information on the agent's true income, but rather, decreases the agent's return to misreporting. The third model combines the first two models, so that there are both enforcement and information frictions and the principal has access to auditing and enforcement to mitigate each friction.

[6] I could consider the agent as having an option to recontract with another principal should he choose to renege on the contract. Under the enforcement environment in this paper, this set up would be identical to one in which the agent's outside option is the value of this alternative contract rather than autarky. See Krueger and Uhlig (2006).

A risk neutral principal looks to provide insurance to a risk averse agent who is subject to a risky income stream. Each period, an agent's income shock $\theta_t \in \Theta = \{\theta_1, .., \theta_S\}$ is realized with θ_t i.i.d. over time with $\theta_s < \theta_{s+1}$. Generic elements in Θ are denoted by θ_s and occur with probability p_s where $\sum_s p_s = 1$. I assume that the principal and the agent share a common discount factor $\beta \in (0, 1)$. In the *General Model*, there is an information friction and an enforcement friction. The enforcement friction implies that the agent can renege on the contract at any point in time with the consequence of being restricted from future insurance. In the absence of enforcement, I take the future value of the walk away option, U_0, to be the autarky value: $U_0 = \sum_s u(\theta_s)p_s$.

Before each period's realization of income the principal can invest in two technologies to ameliorate these frictions: an auditing technology α and an enforcement technology γ at costs of $f(\alpha)$ and $g(\gamma)$, respectively. The choice of investment is observed by the agent and the investment is effective only for one period. [7]

After observing his private income, θ_t, and the principal's choice of auditing and enforcement, the agent reports income to the principal. Based on the history, the principal provides transfers τ_t to the agent each period, so that his consumption is $\tau_t + \theta_t$ and his contemporaneous utility given this consumption is $(1 - \beta)u(\tau_t + \theta_t)$. It is assumed that the agent is an expected utility maximizer and that utility is separable across periods. Thus, the agent's t-period consumption when his true income is θ_s under a falsely reported income, $\theta_{\tilde{s}}$, is $\tau_{\tilde{s}} + \theta_s - \alpha(\theta_s - \theta_{\tilde{s}})$. For ease of notation, define $x_s \equiv \tau_s + \theta_s$, so that the agent's consumption under truth telling is x_s and his consumption under a false report, $\theta_{\tilde{s}}$, is $x_{\tilde{s}} + (1 - \alpha)(\theta_s - \theta_{\tilde{s}})$. Let the period t public information of reports, truthful consumption level (which has a one-to-one mapping with transfers), auditing levels and enforcement levels be denoted by $h_t = (\tilde{\theta}_t, x_t, \alpha_t, \gamma_t)$ with private information (θ_t, h_t). Denote the t-period public history $h^t = (h_0, .., h_t) \in H^t$, where H^t is the set of t-period public histories. Denote the set of private histories by $\tilde{H}^t$

A *plan*, $\nu = (\nu_0, \nu_1, ..,)$, where $\nu_t = (\nu_t^1, \nu_t^2)$ such that $\nu_t^1 : H^t \to A \times \Gamma$ and $\nu_t^2 = H^t \times A \times \Gamma \times \Theta \to \mathbf{R}$, is a sequence of duple mappings of the principal's enforcement and auditing choices given public histories and the principal's choice of transfers given the public history, this period income report, and this period's auditing and enforcement choices. The set of plans is denoted by N. An agent's *strategy* is the mapping of private histories and this period's auditing, enforcement, and income realizations into income reports.[8] Let $\sigma_t : \tilde{H}^{t-1} \times A \times \Gamma \times \Theta \to \Theta$ be agent's the t-period announcement. Denote $\Sigma = \{(\sigma_0, \sigma_1, ..)\}_{t=0}^{\infty}$ as the set of strategies such that histories are consistent with strategies. I make one additional and non-standard restriction on allowable strategies; namely, given an income θ_t, the agent cannot report any income $\theta > \theta_t$. The thought behind this assumption is that it costless to validate that the agent does not have a high income by virtue of his inability

[7]The fact that the auditing choice is made and observed before the agent's income is realized implies that the principal would never have an ex-post incentive to reverse his auditing choice. In the costly state verification literature, the auditing technology allows the principal to decide, conditional on the agent's announced income, to observe the true state at some cost. However, as the optimal contract is written as one in which the agent never misreports, the principal has no ex-post incentive to monitor the agent given the income announcement. See Khalil (1997).

[8]Note that I ignore the agent's decision to walk away. See the discussion in the following paragraph.

to show resources when asked. This assumption is also useful in simplifying the constraints in the recursive formulation of the *General Model*.

If at any point in the contract it is optimal to let the agent walk away then there also must exist an optimal contract in which the agent chooses to not walk away and the principal replicates the autarky consumption profile by setting transfers equal to 0 every period. This has no cost to the principal relative to the agent reneging, and can therefore be incorporated into any contract. Furthermore, a version of the Revelation Principle applies. Thus, the principal's problem is to choose the plan that maximizes resources such that the agent never reneges and that the agent has no incentive to lie. Denote the probability of a history given a plan ν and a strategy σ by $\pi(\tilde{h}^t|\sigma,\nu)$, where the conditioning variables are dropped when there is no confusion.

Given a plan ν and a strategy σ, an agent's utility at time t' given history $\tilde{h}^{t'}$ is

$$U_{t'}(\sigma|\nu,\tilde{h}^{t'-1}) = \sum_{t=t'}^{\infty}(1-\beta)\beta^{t-t'} \sum_{\tilde{h}^t\in\tilde{H}^t|\tilde{h}^{t'}} u\left(x(h^{t-1},\alpha_t,\gamma_t,\tilde{\theta}_t) + (1-\alpha_t(h^{t-1}))(\theta_t(\tilde{h}^t) - \sigma(\tilde{h}^{t-1},\alpha_t,\gamma_t,\theta_t))\right)\pi(\tilde{h}^t|\tilde{h}^{t'-1})$$

where x,α denote the consumption component and auditing components of plan ν, $\tilde{\theta}_t$ is the agent's t period announcement given the private history, and the notation $\tilde{h}^t \in \tilde{H}^t|\tilde{h}^{t'}$ denotes the set of histories that are consistent with the t'-period private history $h^{t'}$ and h^t denotes the public history consistent with private history $\tilde{h}^t$. Denote the truth-telling strategy σ^* as the strategy such that for all histories and periods $\sigma^*_t(\tilde{h}^t) = \theta_t$. Abusing notation slightly, let $U_{t'}(\sigma|\nu,\tilde{h}^{t'-1}),\theta_s$ be the agent's t'-period valuation of the contract under a plan ν and strategy σ conditional on the t period income realization θ_s.

Given an initial reservation value of autarky, the principal's problem, $(P1)$, can be written as the maximum net present value of resources such that truth-telling is incentive compatible and the agent never chooses to renege:

$$\begin{aligned}
(P1) \quad \tilde{V} \quad &= \quad \max_{\nu\in N}\sum_t \beta^t \sum_{\tilde{h}^t}[(1-\beta)(\theta_t(\tilde{h}^t) - x(h^{t-1},\alpha_t,\gamma_t,\tilde{\theta}_t)) - f(\alpha(h^t)) - g(\gamma(h^t))]\pi(\tilde{h}^t|\sigma,\nu) \\
s.t.(PK) \quad & \quad U(\sigma^*|\nu) = U_0 \\
\forall\, \tilde{\sigma}\in\Sigma; t',\ h^{t'}(IC) \quad & \quad U_{t'+1}(\sigma^*|\nu,\tilde{h}^{t'}) \geq U_{t'+1}(\tilde{\sigma}|\nu,\tilde{h}^{t'}) \\
\forall\, s,\ t',\ h^{t'-1}(E) \quad & \quad (1-\beta)u(x(h^{t-1},\nu_{t',1}(h^{t'-1}),\theta_s)) + \beta U_{t'+1}(\sigma^*|\nu,\tilde{h}^{t'}) \geq (1-\beta)u(\theta_s) + \beta U_0 - \gamma(h^{t'-1})
\end{aligned}$$

A dynamic model of costless information and limited commitment is one where $f(\alpha) = 0$ for all α. Similarly, a dynamic model of asymmetric information and costless enforcement is one in which $g(\gamma) = 0$ for all γ. These models are considered separately, respectively, in Sections 3 and 4. The *General Model* is analyzed in Section 5.

Before analyzing the separate models, I first state a proposition following Spear and Srivastava (1987) that allows us to write the problem recursively using the agent's expected utility of the continuation contract, ω as a state variable. The interpretation of the state variable is that this represents the agent's wealth (or indebtedness). I use the recursive model as the starting point for the Enforcement, Auditing, and General models in subsequent sections.

Proposition 1 *The problem* $(P1)$ *can be written recursively as:*

$$\begin{aligned}
V(\omega) &= \max_{\alpha,\gamma,\{x_s,\omega'_s\}_{s\in S}} \sum_s [(1-\beta)(\theta_s - x_s) - \beta V(\omega'_s)]p_s - f(\alpha) - g(\gamma) \\
s.t.\ (IC) &\quad (1-\beta)u(x_s) + \beta\omega'_s \geq (1-\beta)u(x_{\tilde{s}} + \alpha(\theta_s - \theta_{\tilde{s}})) + \beta\omega'_{\tilde{s}},\ \forall\ s\ \forall\ s > \tilde{s} \in S \\
(E) &\quad (1-\beta)u(x_s) + \beta\omega'_s \geq (1-\beta)u(\theta_s) + \beta U_0 - \gamma,\ \forall\ s \in S \\
(PK) &\quad \sum_s [(1-\beta)u(x_s) + \beta\omega'_s]p_s = \omega
\end{aligned}$$

All omitted proofs appear in the appendix.

Notice that the expected transfer to the agent each period is $\mathbf{E}[\theta_s - x_s]$. As $\mathbf{E}[\theta_s]$ is constant, I drop it hereafter when writing the principal's maximization problem.

The following assumptions that will be used throughout the remainder of the paper:

Assumption 1 *i)* $u : [\underline{c}, \infty) \to (-\infty, \bar{u}]$ *is unbounded below with* $u', -u'' > 0$
ii) u''/u' *is non-decreasing.*
iii) $\lim_{x\to 0} u(x) = -\infty$ *and* $\lim_{x\to+\infty} = \bar{u}$
iv) $f', f'', g', g'' > 0$.
v) $\lim_{\alpha\to 0} f'(\alpha) = 0$, $\lim_{\alpha\to 1} f'(\alpha) = +\infty$, $\lim_{\gamma\to 0} g'(\gamma) = 0$, $\lim_{\gamma\to+\infty} g'(\gamma) = +\infty$

Assumptions i, iii, iv, and v are fairly common assumptions on the regularity properties of the cost functions and the utility function. Note that utility is unbounded below and bounded above and consumption is unbounded above and may or may not be bounded from below. Assumption ii states that the agent's absolute risk aversion is non-increasing and is a sufficient condition for the concavity of the value function in the dynamic programming problem when the information friction is non-trivial.

3 Enforcement Model

In this section I consider a special case of the *General Model* in which information is free ($f(\alpha) = 0$ for all $\alpha \in [0, 1]$). Namely, the principal's problem is:

$$V(U_0) = \max_{\gamma,\{x_s,\omega'_s\}_{s\in S}} \sum_s [-(1-\beta)x_s + \beta V(\omega'_s)] - g(\gamma)$$

subject to (E) for all $s \in S$ and (PK). Concavity and differentiability of the value function are established for the *General Model* in Section 5. The proof here follows accordingly and is omitted.

Letting λ be the multiplier on the promise keeping constraints and $\xi_s p_s$ be the multipliers on enforcement constraints, the first order conditions and envelope condition for this problem are:

$$
\begin{aligned}
u'(x_s)(\lambda + \xi_s) - 1 &= 0 \\
V'(\omega'_s) + \lambda + \xi_s &= 0 \\
g'(\gamma) - \sum_s \xi_s p_s &= 0 \\
V'(\omega'_s) &= -\lambda'_s
\end{aligned}
$$

An immediate conclusion from this model is that the highest income agent determines the level of enforcement. Namely, if (E) binds from some state s, then it also binds for all $\tilde{s} > s$. This result is common in the limited commitment literature, but does not hold in the model considered in Section 5.

Lemma 1 *If $\xi_s = 0$ then $\xi_{\tilde{s}} = 0$ for all $\tilde{s} < s$.*

Proof. Suppose not. Then $(1-\beta)u(x_s)+\beta\omega'_s > (1-\beta)u(\theta_s)+\beta U_0-\gamma$ and $(1-\beta)u(x_{\tilde{s}})+\beta\omega'_{\tilde{s}} = (1-\beta)u(\theta_{\tilde{s}}) + \beta U_0 - \gamma$ where $\theta_{\tilde{s}} < \theta_s$ so that the RHS of the former is greater than that of the latter. This implies that either (i) $x_s > x_{\tilde{s}}$ or (ii) $\omega'_s > \omega'_{\tilde{s}}$. For whichever of (i) or (ii) holds, we can increase the variable (say x) for $\tilde{s}$ and decrease it for s in such a way that γ is unchanged and PK holds. The concavity of u and the value function imply that this yields strictly more resources to the principal. ■

The previous lemma is what would be expected from results in the limited enforcement literature. Namely, for any level of promised utility, an agent with the highest income realization has the largest incentive to walk away from the contract. For low income realizations, an agent does not have an incentive to renege and the principal can perfectly smooth consumption across such realizations. Thus, for each level of incoming promise there is some threshold level of income below which consumption and future promises are smoothed across states and above which the principal receives a net payment from the agent, $\theta_s - x_s > 0$. An agent with a high income realization is then deterred from reneging via increased consumption in future periods.

The dynamics of the agent's promised utility are simple in this environment and can easily be seen from the first order conditions. If the agent receives an income θ_s for which the enforcement constraint does not bind, then $\xi_s = 0$ and $\lambda'_s = \lambda$, implying that the agent's wealth remains constant. Likewise, consumption is equalized over states in which the enforcement constraint does not bind and is increased when income shocks are sufficiently high. At the moment when the constraint binds, promised utility increases and the process repeats itself. Thus, the continuation promise is weakly increasing over time, eventually reaching a maximum value of $(1-\beta)u(\theta_S)+\beta U_0$ when the highest income state is realized.

In models of efficient insurance and limited commitment, the set of continuation promises that can be reached with positive probability from any state ω (with the exception of that amount) are a subset of $\mathcal{U} \equiv \{(1-\beta)u(\theta_s)+\beta U_0\}_{s\in S}$. The reason for this is that the outside option of an agent who receives an income realization of θ_s is equal to $(1-\beta)u(\theta_s)+\beta U_0$ independent of his promised utility upon entering the period. As the optimal mix of consumption and future promises to delivering this level of utility does not depend on history, the set of possible values for ω is always a subset of $\mathcal{U}$. However, in a model with endogenous enforcement, the enforcement variable and, consequently, the value of the agent's outside option, depend on the contractual state variable ω. Thus, in a model with endogenous enforcement, the continuation promise of an agent receiving income θ_s is history dependent.

In particular, as the continuation promise increases and the autarky option is less attractive to the agent, then enforcement also decreases. This is established in the following proposition and is similar to the result achieved in Koeppl (2007). He studies a case in which two risk averse agents split a fixed amount of resources and finds that enforcement increasing in inequality. Alternatively phrased, his result shows that enforcement decreases as the wealth of the poorest agent increases. A parallel result is achieved here.

Proposition 2 *γ is strictly decreasing in ω when $\gamma > 0$.*

Given the dynamics of the continuation promise, the dynamics of enforcement are easily determined. Namely, the use of the enforcement technology is constant over time so long as the enforcement constraint does not bind and decreases when sufficiently high income shocks cause the enforcement constraint to bind and increase future ω. One consequence of this is that in the long run, enforcement goes to zero almost surely. Therefore, the enforcement technology is eventually obsolete.

Proposition 3 *$\gamma_t \to 0$ almost surely.*

Note also that the enforcement technology implies that the principal could implement *any* level of ω, in particular those below U_0. In the *Enforcement Model* where there are no information frictions, the agent's continuation promise is increasing and implementing $\omega < U_0$ is off the equilibrium path. However, this will not be the case in later sections. Furthermore, costly enforcement implies that V is increasing for ω sufficiently small. This can easily be seen as $\gamma \geq U_0 - \omega$ from the enforcement constraints.

4 Auditing Model

In this section I consider a model in which the agent has private information on his income shocks and a planner has access to an auditing technology; enforcement is costless ($g(\gamma)=0$ for all γ).

The efficient contracting problem with only auditing is as follows:

$$
\begin{aligned}
V(\omega) &= \max_{\alpha,x,\omega'} \sum_s [-(1-\beta)x_s - \beta V(\omega'_s)]p_s - f(\alpha) \\
s.t.\ (IC) \quad & u(x_s) + \beta\omega'_s \geq u(x_{\tilde{s}} + \alpha(\theta_s - \theta_{\tilde{s}})) + \omega'_{\tilde{s}} \qquad \forall \tilde{s} < s \\
(PK) \quad & \sum_s [u(x_s) + \beta\omega'_s]p_s = \omega
\end{aligned}
$$

Concavity and differentiability of the value function follow a similar argument to the one presented in Proposition 9. The proof is therefore skipped.[9]

The problem can be simplified further by showing that it is enough to consider only local incentive compatibility constraints and that these constraints bind. That is, if for each type s the agent does not have the incentive to lie downward to $s-1$, then all of the incentive compatibility constraints are satisfied. To do this, I first establish that higher income realizations receive higher consumption and lower transfers than lower income realizations in the optimal allocation. This is a straightforward result, as one would expect insurance schemes to generally give a larger transfer payment (or demand a smaller fee) to an agent who realizes low income than to one who realizes high income. The second lemma establishes that it is enough to only consider "local" lies. Define $\Delta_s \equiv \theta_s - \theta_{s-1}$.

Lemma 2 *For all s, $x_{s-1} \leq x_s \leq x_{s-1} + \Delta_s(1-\alpha)$.*

Throughout the remainder of the paper let $\tilde{u}(x, \alpha, \theta_s - \theta_{\tilde{s}}) = u(x + (1-\alpha)(\theta_s - \theta_{\tilde{s}}))$ denote an agent's contemporaneous utility from reporting an income $\theta_{\tilde{s}}$ when his true income is θ_s.

Lemma 3 *Local constraints are enough. Let $C_{\tilde{s},s} = (1-\beta)[u(x_{\tilde{s}}) - \tilde{u}(x_s, \alpha, \theta_{\tilde{s}} - \theta_s)] + \beta[\omega'_{\tilde{s}} - \omega'_s]$. Then, $C_{s+1,s} \geq 0$ and $C_{s,s-1} \geq 0$ for all s imply that $C_{k,s} \geq 0$.*

Furthermore, it is also the case that all of the local downward constraints bind. Fixing any level of auditing, if the downward constraint were to ever not bind, then the principal could increase consumption to the agent when a low state is realized and decrease consumption when a high state is realized without affecting any of the other IC constraints. From Lemma 2, the marginal consumption under low realizations is greater than under high realizations so that such a scheme would increase the principal's utility.

Lemma 4 *Local constraints bind so that $\alpha > 0$ for all ω.*

[9] The one major difference is the domain of V, which is unbounded below here. To adapt the argument for this environment it is sufficient to note that the value function defined here lies between the functions F_1 and F_2 defined in the proof. Furthermore, absent enforcement, the difference between these functions is bounded for all ω, established in the proof of the proposition and the fact that the slope of the value function tends toward 0 as $\omega \to -\infty$. See Thomas and Worrall (1990).

Auditing is useful to the principal in that it allows for better smoothing of agent's consumption across states. In a world with costless information, if marginal utility is higher in low states than in high states, then a risk neutral planner can increase his utility while holding the agent's utility constant by decreasing consumption in the high state and increasing consumption in the low state. With costly auditing, this requires an increase in auditing costs to keep an agent with high income from lying. The following proposition derives an expression for the optimal auditing when $S = 2$, with $\Delta = \theta_H - \theta_L > 0$ so that the marginal cost of auditing is equal to the principal's benefit from consumption smoothing. A similar, but more cumbersome, expression is easily derived for the case where $S > 2$. The left hand side of the expression can be interpreted as the marginal auditing cost of smoothing consumption, while the right hand side is the marginal benefit to the principal of smoothing the agent's consumption.

Proposition 4 *The optimal allocation satisfies* $\frac{f'(\alpha)}{\Delta}\left[1+\frac{u'(x_L)p_L}{\tilde{u}'(x_L,\alpha,\Delta)p_H}\right] = p_L\left[\frac{u'(x_L)}{u'(x_H)} - 1\right]$

Proof. Consider the following scheme. Increase α by ϵ/Δ, increase x_L by η_L and decrease x_H by η_H. The cost of this scheme is $-\frac{f'(\alpha)}{\Delta} - \eta_L p_L + \eta_H p_H$. Now, we must show that we can choose values such that IC and PK are satisfied, while decreasing resource costs. Choose $\eta_H = \eta_L \frac{u'(x_L)p_L}{u'(x_H)p_H}$ so that the PK constraint holds. Under this scheme, the LHS of the IC constraint is decreased by $\eta_L \frac{u'(x_L)p_L}{p_H}$ while the RHS is decreased by $(\epsilon - \eta_L)u'(x_L,\alpha,\Delta)$. Choosing $\epsilon = \eta_L\left[1+\frac{u'(x_L)p_L}{u'(x_L,\alpha,\Delta)p_H}\right]$, the total cost of the scheme is then

$$\eta_L\left[-\frac{f'(\alpha)}{\Delta}\left(1+\frac{u'(x_L)p_L}{u'(x_L,\alpha,\Delta)p_H}\right) - p_L + \frac{u'(x_L)p_L}{u'(x_H)}\right]$$

which must be less than or equal to zero in an optimal allocation. The opposite perturbation implies that the expression must be greater than or equal to zero. ■

Although auditing is positive for all ω, it vanishes as ω tends toward $-\infty$ and, under an additional assumption on the utility function (which is satisfied by CRRA utility functions), auditing also vanishes as ω approaches its upper bound $\bar{u}$. Auditing tends toward zero in the extremes for different reasons. As ω becomes small, providing incentives for truth-telling to the agent is cheap. The reason for this is that as the marginal utility from consumption increases, small consumption differences across states are enough to induce truth telling. Meanwhile, the agent's high marginal utility implies that the value function is flat as ω diverges to $-\infty$. As the incentive compatibility constraint is inexpensive to satisfy via variations in current and future consumption, the auditing technology becomes unnecessary. In the case of ω large, the agent is already being promised high consumption. The fixed differences between income levels combined with the concavity of the utility function guarantees that the marginal value to the agent from lying is small. Thus, less auditing is necessary to deter him from doing so.

Proposition 5 $\alpha \to 0$ *as* $\omega \to -\infty$

Proposition 6 *If* $\lim_{x \to +\infty} u'(x)/u'(x+\Delta) = 1$ *for* $\Delta = \theta_S - \theta_1$ *(which is true under CRRA with a coefficient greater than 1), then* $\alpha \to 0$ *as* $\omega \to \bar{u}$*.*

Consequently, α reaches a maximum in the interior of $(-\infty, \bar{u}]$ and vanishes in the limits. This is in contrast to the *Enforcement Model* in which enforcement costs are high when the promises are low and are decreasing in ω. One could extend these insights by thinking of this model as the "component planner" problem[10] in a model of efficient insurance with a continuum of agents and a period by period resource constraint equal to the expected value of income. The implications in a more general model with a resource constraint would be that inequality requires large enforcement costs to overcome the friction arising from the agent's limited commitment. However, the *Auditing Model* implies that costs of auditing are *lower* when there are large inequalities and agents are toward the utility extremes and higher when there is more equality.

Perhaps surprisingly, the long run dynamics of the *Auditing Model* resemble that of a model in which information is prohibitively costly. That is, despite the presence of an auditing technology designed to mitigate the information friction, the principal's incentive to front-load consumption and push ω toward where incentives are "cheap" dominates and in the long run the auditing technology becomes obsolete.

Proposition 7 $\omega \to -\infty$ *almost surely.*

Proof. First, it is straightforward to show that λ_t is a martingale by summing over states the first order conditions on ω'_s. Furthermore, as V is decreasing it must be the case that λ is bounded below by 0. Then, by the Martingale Convergence Theorem, λ_t converges almost surely. Suppose λ_t converges to any interior value. This is a contradiction as for every $\omega \in \Omega$ it is the case that $\omega'_L < \omega < \omega'_H$. This implies that $\omega \to -\infty$. ■

The robustness of the immiseration result to auditing can be understood in relation to the relative costs of auditing when consumption is front-loaded and back-loaded. An agent with low income or high income realizations values continuation contracts identically, but values current transfers differently. Therefore, the more front-loaded the contract facing an agent, the easier the principal can induce truth-telling by manipulating this difference. Consider an optimal insurance plan and imagine reallocating consumption for the low income realization from the future period into the current period. That is, increase x_L and decrease ω'_L such that the agent's overall utility (and his utility from the low state in particular) is unchanged. What effect does this have on auditing? Denote by ϵ_x and ϵ_ω the increase and decrease, respectively of current consumption and future utility. Then $(1-\beta)\epsilon_x u'(x_L) - \beta\epsilon_\omega = 0$ by assumption. It is clear that such a reallocation of future consumption to current consumption relaxes the IC constraint as the change in the right hand side of the constraint is $(1-\beta)\epsilon_x \tilde{u}'(x_L, \alpha, \Delta) - \beta\epsilon_\omega < 0$ while the left hand side of the IC constraint is unchanged. It follows that front-loading consumption to agent reduces auditing costs.

[10]See Atkeson and Lucas (1992)

Thus, as in standard models of insurance with private information, the principal continues to have an incentive to front-load payments to the agent.

5 General Model

From the *Enforcement* and *Auditing* models one is led to ask, to what extent do the conclusions of these models rely on the assumptions of costless information and costless enforcement, respectively? Looking at the *Auditing Model*, the principal relies heavily on cheap truth-telling incentives for the agent at low levels of ω. However, we know from the *Enforcement Model* that this is precisely when the principal chooses the largest levels of enforcement. Conversely, in the *Enforcement Model* the principal back-loads the agent's consumption, with long run utility equal to the walk-away option when income is high and there is no enforcement. From the *Auditing Model* we know that it is in this interior region where the information friction binds the most tightly.

To answer this question we turn to the *General Model* of costly auditing and enforcement. The conclusions from this section demonstrate that these technologies interact in a non-uniform manner. Thus, in environments with both limited commitment and private information where technologies ameliorating each friction are costly, examining the problems separately may provide misleading conclusions.

In this section I take $\Theta = \{\theta_L, \theta_H\}$ with $\Delta = \theta_H - \theta_L > 0$. The principal solves:

$$
\begin{aligned}
V(\omega) \quad &= \quad \max_{\alpha,x,\omega'} \sum_{s=L,H} [-(1-\beta)x_s - \beta V(\omega'_s]p_s - f(\alpha) \\
s.t.\ (IC) \quad & \quad (1-\beta)u(x_H) + \beta\omega'_H \geq (1-\beta)u(x_L + (1-\alpha)\Delta) + \beta\omega'_L \\
(E) \quad & \quad (1-\beta)u(x_s) + \beta\omega'_s \geq (1-\beta)u(\theta_s) + \beta U_0 - \gamma \\
(PK) \quad & \quad \sum_{s=L,H} [(1-\beta)u(x_s) + \beta\omega'_s]p_s = \omega
\end{aligned}
$$

First I establish the regularity properties of the value function. The assumption of non-increasing absolute risk aversion plays an important role in establishing concavity.

Proposition 8 *V is concave.*

The following proposition establishes the differentiability of the value function over a bounded interval of ω. Setting the lower bound sufficiently low, I use differentiability to establish the properties of the value function. It is later shown in Proposition 10 that bounding utility from below is without loss of generality, as low enough values of ω are never reached.

Proposition 9 *For any $\underline{\omega} < \bar{u}$, V is differentiably continuous on $[\underline{\omega}, \bar{u})$.*

Denote the Lagrange multipliers for the IC, E, and PK constraints as η_s, ξ_s, and λ, respectively. Then, the first order conditions and the envelope condition are:

$$\begin{array}{ll} x_H: & p_H = (p_H\lambda + \eta + p_H\xi_H)u'(x_H) \\ x_L: & p_L = (p_L\lambda - \eta + p_L\xi_H)u'(x_H) \\ \omega'_H: & p_H V'(\omega'_H) = p_H\lambda + \eta + p_H\xi_H \\ \omega'_L: & p_L V'(\omega'_L) = p_L\lambda - \eta + p_L\xi_L \\ \alpha: & f' = \tilde{u}'(x_L, \alpha, \Delta)\eta \\ \gamma: & g' = \sum_s p_s\xi_s \\ EC: & V'(\omega) = \lambda \end{array}$$

Summing up the constraint on ω' yields $\lambda + g' = \mathbf{E}[\lambda']$. Therefore λ is a sub-martingale with $\mathbf{E}[\lambda'] > \lambda$ when at least one of the enforcement constraints binds and $\mathbf{E}[\lambda'] = \lambda$ otherwise. There are two opposing forces at work. The first force is from the enforcement constraint. Because keeping agents at a suppressed level of utility is costly, the optimal allocation features an upward push on utility when ω is sufficiently low and is seen in the g' term. When continuation promises are higher and enforcement for both types is unnecessary, the contract resembles that in the *Auditing Model*. In this region, the marginal cost of delivering utility to the planner today is equal to the expected marginal cost of delivering utility to the agent tomorrow. The concavity of the value function then suggests that the agent's expected promise is less than his current promise.

As in the case of the *Auditing Model*, the high income agent never receives a larger transfer $(x_s - \theta_s)$ than the low income agent. This is naturally the case absent the enforcement constraints given that smoothing consumption across states and holding the agent's utility constant reduces the principal's consumption cost. The following lemma establishes that this continues to hold in the *General Model*.

Lemma 5 $x_H \leq x_L + (1-\alpha)\Delta$.

In the *Auditing Model*, the value function is downward sloping because the principal could always decrease the agent's utility by decreasing the agent's consumption for the lowest income realization without affecting any of the incentive compatibility constraints. In the *Enforcement Model*, the value function was downward sloping over the set of utilities reached with positive probability. The reason for this is the same as in the *Auditing Model*, noting that starting at U_0, the lowest enforcement constraint never binds. In the model with both auditing and enforcement, this is not necessarily the case. The information friction forces utility below U_0 when the low state is realized (proven in Corollary 2), thereby forcing an increase in future enforcement costs. Therefore, one cannot exclude the possibility that the agent reaches some level of ω for which the value function is upward sloping. Such a level of utility would not be Pareto efficient ex-post, but it may be optimal ex-ante in providing the appropriate truth-telling incentives. The following establishes that the value function is upward sloping for some ω, following from the costly enforcement necessary to induce such a level of utility.[11]

Lemma 6 *There is some ω^* such that $V'(\omega) > 0$ for all $\omega < \omega^*$.*

Notice also that the first order conditions on x_H and ω'_H immediately imply that $V'(\omega'_H) < 0$. This implies that the principal will choose to increase the continuation promise to the decreasing portion of the value function whenever the high state is realized.

I now turn to the binding patterns of the enforcement constraints and the incentive compatibility constraints. Studying these binding patterns is useful in determining the nature of the frictions at alternative points in the state space and, consequently, the principal's choices of enforcement and auditing.

Lemma 7 *For ω sufficient small, EL binds.*

It is shown further, in Lemma 12 in the Appendix, that if EL binds for some ω then it also binds for all $\hat{\omega} < \omega$. This is useful in establishing the relationship between enforcement and auditing later in the paper.

The following lemma establishes that the auditing technology is used at all nodes in the optimal dynamic contract. This is in contrast to the enforcement technology which is used only when the agent's entitlements are sufficiently low. Unlike the case with only auditing, the fact that the marginal cost of auditing at zero is zero does not guarantee that positive auditing is optimal. The reason for this is that interstate smoothing of consumption may still be costly via the need through increased enforcement. That is, decreasing high income consumption and increasing low income consumption may require an increase in enforcement costs when EH is already binding. The following lemma shows that some positive auditing is always optimal.

Lemma 8 *The incentive compatibility constraint binds for all ω.*

Having established that auditing is positive for all ω, I now turn to the interaction of the low enforcement constraint with the incentive compatibility constraint. This interaction provides the following lemma, which is unique to this environment. It states that as continuation promises decrease, eventually only the low state enforcement constraint and the incentive compatibility constraint bind. The reason for this is that satisfying the IC constraint when promises are low enough guarantees that the high state enforcement constraint is also satisfied. To understand this, imagine that x_L is small. The IC constraint ensures that the utility in the high state is at least $u(x_L + \Delta(1-\alpha)) - u(x_L) \approx u'(x_L)(1-\alpha)\Delta$ greater than it is if the low state is realized. Meanwhile, if both EL and EH bind, then the utility differential is exactly $u(\theta_H) - u(\theta_L)$. In other words, the gap between the agent's valuation of the outside option is constant across x_L, whereas the implied gap between the high and low income agents' utilities is decreasing in x_L. As x_L decreases, the IC constraint will then imply that EH holds.

[11] I am not currently able to conclude whether such an ω is necessarily reached with positive probability.

Lemma 9 *Only EL and IC bind as ω sufficiently small.*

As the cost of enforcement increases with lower wealth, the principal has higher incentives to back-load consumption to the agent and forego future enforcement costs. Consequently, for sufficiently low levels of entitlement the principal increases the agent's wealth independent of the income shock. This fact implies that ω'_L crosses ω from above and establishes an endogenous lower bound on entitlements.

Proposition 10 *$\omega'_L \geq \omega$ for some ω. There exists some $\underline{\omega} < U_0$ such that for all periods and histories, $\omega \geq \underline{\omega}$.*

Having established the binding patterns in when ω is small, I show that at the initial utility level, U_0, only the enforcement constraint for the high type binds with the incentive compatibility constraint. This is useful in showing that U_0 falls in the interior of set $[\underline{\omega}, \bar{u})$ and also allows for the demonstration of other properties of the value function and optimal contract at this point.

I now examine the binding of the constraints at the autarky (initial) level of utility. Under the assumption that the marginal cost of enforcement and auditing vanish at zero, the principal finds it optimal to smooth the agent's consumption across states.

Proposition 11 *At $\omega = U_0$, only the high type enforcement constraint and IC bind.*

Corollary 1 *V is downward sloping at U_0.*

The following corollary follows directly from the previous proposition and first order conditions on continuation promises.

Corollary 2 *At U_0 continuation utilities are such that $\omega'_L < U_0 < \omega'_H$.*

Figure 1 summarizes the binding patterns of the enforcement and incentive compatibility constraints over the state space. When ω is high, the agent does not have incentives to renege on the contract and the principal uses the auditing technology as a means of mitigating the information friction. For levels of ω around the autarky level, smoothing the agent's consumption across the high and low income states increases the agent's incentives to renege in the high income state and, consequently, high income enforcement constraint binds. For lower levels of ω the benefit to smoothing the agent's consumption across states diminishes and the tension between the incentive compatibility constraint and the enforcement constraints increase. Thus, as ω decreases the low enforcement constraint begins to bind as well. Finally, for ω sufficient low, the tension between the constraints is such that the high enforcement constraint ceases to bind.

The theory of the maximum guarantees that the multipliers and decision variables are continuous in ω. For the region below U_0 it is not possible that only IC binds, as the non-binding of both enforcement constraints (combined with the implied zero enforcement) contradict the PK constraint. However, we know that as ω decreases only the EL and IC constraints bind. Therefore, there must

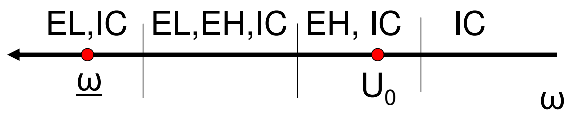

Figure 1: Binding Constraints for Values of ω

be a region in which EL, EH, and IC each bind. Within this region, α and γ are both decreasing in ω. Additionally, it must be the case that $\underline{\omega}$ is contained in a region where EL binds. This follows from the fact that $p_L\xi_L - \eta = 0$ at $\underline{\omega}$ so that EL must bind at the point where the low continuation promise is equal to the current continuation promise.

An interesting implication of the model is that there is a positive probability that the agent will reach a node at which he consumes less than θ_L, the low level of income. That is, after some histories an agent receiving low income is a payor rather than recipient of transfers as part of the efficient insurance plan. While this appears as contrary to the notion of insurance, it is necessary to guarantee the appropriate incentives arising from the information friction.[12] More specifically, low continuation promises are used to induce truth-telling, but are also costly to the principal in terms of enforcement. The latter effect pushes the principal to compensate the agent with more consumption in future periods, reducing future enforcement costs. When ω is sufficiently small and enforcement costs sufficiently high, this effect dominates.

Lemma 10 *For some $\omega > \underline{\omega}$, reached with positive probability, $x_L < \theta_L$.*

Proof. It is enough to show that $x_L < \theta_L$ at $\underline{\omega}$ given continuity of consumption in the state variable. $\underline{\omega}$, by definition, is such that $\omega'_L = \underline{\omega}$ so that $\xi_L = \eta > 0$ and EL binds. The enforcement constraints and promise keeping imply that $U_0 - \gamma \leq \underline{\omega}$. Meanwhile, EL implies that $(1-\beta)u(x_L) + \beta\underline{\omega} = (1-\beta)u(\theta_L) + \beta U_0 - \gamma$. Together, these imply $(1-\beta)(u(x_L) - u(\theta_L)) = \beta(U_0 - \underline{\omega}) - \gamma < 0$. ■

5.1 Optimal use of Enforcement and Auditing Technologies

This section examines the use of the enforcement and auditing technologies in the optimal contract. While the qualitative use of enforcement in the optimal contract is similar to that when there was no information friction, the presence of the enforcement friction qualitatively changes the use of auditing. The next proposition is akin to that in the *Enforcement Model*.

[12] A similar result is obtained in Atkeson (1991)

Proposition 12 *γ is decreasing in ω.*

However, efficient auditing differs dramatically in the presence of the enforcement constraint. The reason for this is that the principal uses auditing not only to smooth the agent's consumption across states, but also to diminish the costs of enforcement that arise at lower levels of wealth. Unlike standard models of enforcement, enforcement is determined in for some states of ω by the low income realization. Therefore, by increasing auditing and smoothing the agent's consumption across states, the principal can relax the level of enforcement. Because this incentive for auditing increases with lower levels of promised utility, auditing is decreasing for levels of wealth in which EL binds.

Proposition 13 *α is decreasing in ω whenever EL binds.*

When EL does not bind, auditing is qualitatively similar to the *Auditing Model*. As in Proposition 6, auditing vanishes in the upper limit when the assumption of the proposition is satisfied. When EL does not bind, but enforcement is still positive, the comparative statics of α are unclear for the same reason they are unclear in the *Auditing Model*. Namely, as consumption increases marginal utility from lying is decreased. However, manipulating future promises to the agent to induce truth-telling is also more costly to the principal. The balance of these two forces determines the direction of auditing as ω increases.

Propositions 12 and 13 together imply that an agent near the lower utility bound $\underline{\omega}$ is costly to the principal in terms of auditing and enforcement. While the consumption cost to the principal of an agent with a low promised utility is low, the investment required to suppress the agent at these levels of utility are high. Thus, as the agent's promised utility decreases in this region, an increasingly large percentage of the principal's costs stem from the need to satisfy the enforcement and incentive compatibility constraints rather than the direct costs of providing the agent with consumption. This is examined further in the numerical example.

Auditing and Enforcement as Complements and Substitutes

As shown above, the space of utility promises can be divided into sections, the first in which EL is binding, the second in which EH and EL bind, the third in which EH binds, and the last in which no enforcement constraint binds. When EL is binding, the principal must pay a cost of enforcement that is determined by the utility conditional on receiving the low income shock. The intuition for this is as follows. Suppose that EL binds and there is an increase in auditing. The effect of the increase in auditing is that consumption may be better smoothed across the high and low income state. This implies that *less* enforcement cost is needed to prevent an agent with a low income realization from reneging. That is, more interstate smoothing of consumption then has the consequence of reducing the necessary enforcement costs, implying that auditing and enforcement are *substitutes* for low levels of utility promise. This is evidenced by the expression at the optimal allocation[13]

$$
\begin{aligned}
f'\left[1+\frac{p_L u'(x_L)}{p_H u'(x_L+(1-\alpha)\Delta}\right] &= p_L\left[\frac{u'(x_L)}{u'(x_H)}-1+g'\right] \\
&> p_L\left[\frac{u'(x_L)}{u'(x_H)}-1\right]
\end{aligned}
$$

The LHS of the equation is the cost of auditing, while the RHS of the inequality is the benefit to the principal of smoothing the agent's consumption across states. Supposing that enforcement were increased so that the EL constraint was slack, the inequality implies that the principal would find it optimal to decrease auditing and un-smooth consumption across states.

On the other hand, as EH binds and EL is slack, interstate smoothing of consumption becomes *costly* to the principal in the form of higher enforcement. In this region, the auditing and enforcement technologies are *complements*. Suppose that the principal were to increase auditing in the region in which EH binds and EL is slack. This increase in auditing allows the principal to smooth the agent's consumption across states, lowering the total consumption cost to the principal, only if it is accompanied by an increase in enforcement as well. If we look at the region in which EH and IC bind (but not EL), the optimal allocation satisfies:

$$
\begin{aligned}
f'\left[1+\frac{p_L u'(x_L)}{p_H u'(x_L+(1-\alpha)\Delta}\right] &= p_L\left[\frac{u'(x_L)}{u'(x_H)}-1-\frac{g'u'(x_L)}{p_H u'(x_H)}\right] \\
&< p_L\left[\frac{u'(x_L)}{u'(x_H)}-1\right]
\end{aligned}
$$

Supposing now that enforcement were increased so that the EH constraint were slack, the inequality implies that the cost of auditing in this case is less than the benefit to the principal from smoothing the agent's consumption across states. Therefore, an increase in enforcement in this case is accompanied with an increase in auditing as well.

The argument above, together with a threshold value of ω for which EL binds (see Lemma 12 in the Appendix) imply that auditing and enforcement are substitutes below this threshold and complements above it.

Unfortunately, there is no immediate proof for the existence of an ergodic distribution. While it must be the case that $\omega_t \in [\underline{\omega}, \bar{u})$, there is no uniform bound on the probability that ω reaches any particular value. In fact, as the set of histories is countable, there are only a countable number of values of ω that can be reached with positive probability. As such, for there to exist an ergodic distribution it would have to be the case that from some node ω^* there is a non-zero probability that the contract returns to exactly this value. However, the forces of the model suggest that the

[13] To derive this expression, one can use first order conditions. Alternatively, imagine an allocation in which x_L is raised and x_H is decreased. So that PK still holds. When only EL and IC bound initially, this scheme requires an increase of α and decrease of γ to maintain that IC and EL continue to bind. This scheme cannot yield any more resources to the principal than the optimal scheme. We can perform the opposite perturbation as well which then implies the expression.

process ω_t is mean reverting. This follows from the downward pressure on ω' when promises are high and the upward pressure on ω' when promises are low.

6 Numerical Example

In this section I explore the properties of auditing and enforcement through a numerical example.

Let $\Theta = \{\theta_L = 1, \theta_H = 5\}$ with $p_L = p_H = 0.5$ and $\beta = 0.9$. I take $u = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma = 2$. These imply that $U_0 = -0.6$. For the cost functions let $g(\gamma) = \gamma^2$ and $f(\alpha) = \frac{\alpha^2}{250(1-\alpha)^4}$, $\alpha \in [0,1]$. Note that $f'(0) = g'(0) = 0$, $g', g'', f', f'' > 0$ and $\lim_{\alpha \to 1} f'(\alpha) = \lim_{\gamma \to +\infty} g(\gamma) = +\infty$.

First, consider the *Enforcement Model*. Figure 2 shows the decreasing relationship of enforcement, γ, in continuation promises, ω, as is consistent with Proposition 2. As ω_t is weakly increasing over time, ω is shown only for those values greater than the initial promise U_0. Note that some of those ω plotted will not be reached with positive probability, see the left panel of Figure 3.

The left panel of Figure 3 shows the continuation promises ω'_H and ω'_L as a function of state ω as well as the 45^0 line. The right panel of Figure 3 shows a sample path of ω_t for a draw of income shocks $\{\theta_t\}_{t=1}^{500}$. For the remainder of the section, all sample paths will use this same draw. As there are only two shocks, the enforcement constraint binds only if $\theta_t = \theta_H$, so that $\omega'_L = \omega$ and $\omega'_H \geq \omega$ with strict equality if and only if $\omega < (1-\beta)u(\theta_H) + \beta U_0 = -0.56$. Contrast this to the case in which enforcement is prohibitively costly. In that case, the first draw of θ_H would push ω immediately to -0.56. Because this valuation is at least as great as the high income type's outside option, first best insurance is implementable thereafter. When enforcement is endogenous, the growth of ω is slowed by the fact that enforcement keeps the high income agent's outside option below -0.56. Thus, wealth approaches -0.56 more slowly than in the model with no enforcement.

Next, consider the *Auditing Model*. Figure 4 shows the relationship of auditing, α, to continuation promises, ω. Note that auditing is vanishing in the limits consistent with Propositions 5 and 6. In this and other numerical examples, auditing is single peaked in wealth, but I am unable to generally prove that this is the case.

The left panel of Figure 5 shows the continuation promises ω'_H and ω'_L as a function of state ω as well as the 45^0 line. Auditing is positive for all interior ω and the incentive compatibility constraint, combined with Lemma 5 implies that $\omega'_L < \omega < \omega'_H$. The right panel of Figure 5 shows a sample path of ω_t for the same sample draw of $\{\theta_t\}_{t=1}^{500}$ when auditing costs are specified as above. Note the downward drift of ω_t corresponding to the immiseration result.

Finally, I turn to the *General Model*. As determined in Proposition 12, enforcement is decreasing in ω and is show in Figure 8. Note that it behaves qualitatively similarly to that in the *Enforcement Model* depicted in Figure 2. Auditing in the *General Model*, on the other hand, differs qualitatively from that in the *Auditing Model*. At lower values of ω, auditing is sharply decreasing in ω. In this region, the IC constraint combined with the PK constraint interact so as to force the low income agent's enforcement constraint to bind. The principal then uses auditing as a substitute for enforcement, as it allows for greater interstate income smoothing. At the point where EL ceases to bind, auditing behaves more similarly to that in the *Auditing Model*. Namely, it is single peaked in this region and vanishes in the upper limit. One conclusion from this numerical example is that low income agents are institutionally very costly, in the sense that greater resources are required both greater enforcement and auditing investment relative to other agents. As x_L and x_H are also smaller when ω decreases, a decreasingly small percentage of the principal costs are used to deliver consumption to the agent.

As discussed in Section 5, there exists an endogenous lower bound $\underline{\omega}$ on the continuation promises of the agent. This is consistent with the left panel of Figure 7, which shows the continuation promises from high and low income shocks. When $\omega = \underline{\omega}$, continuation promises are qualitatively like that in the *Enforcement Model*, namely: promises respond positively to high income shocks, but are unchanged by low income shocks. The implication of this is that consumption is heavily back-loaded in this region. As promised utility increases and enforcement decreases, the dynamics more closely resemble that of the *Auditing Model*; high (low) income shocks propel increases (decreases) in the utility promise and, as enforcement vanishes, the utility promise begins to experience a downward drift. In this region, the consumption is predominantly front-loaded.

Given the draw of income shocks as above, the right panel of Figure 7 depicts the sample path ω_t. Unlike the other models, ω exhibits mean reversion as a consequence of the forces described in the previous paragraph. The relative costs of auditing and enforcement then play a role in establishing both the mean and volatility of this process.

Finally, consider the components of the principal's costs in the *General Model*. First, there is the principal's cost of delivering consumption to the agent $\mathbf{E}[x_s]$; second, there are the *institutional* costs of auditing and enforcement $f(\alpha) + g(\gamma)$ that are used in providing the appropriate incentives for the agent, but do not directly provide the agent with utility. At the point where EL binds, Propositions 12 and 13 establish that the *institutional* costs increase as ω decreases. Meanwhile, as ω decreases the principal also delivers fewer consumption resources to the agent. Thus, the fraction of that period's costs devoted to consumption, $\mathbf{E}[x_s] / (\mathbf{E}[x_s] + f(\alpha) + g(\gamma))$ shrinks as ω declines.

7 Concluding Remarks

In this paper, I evaluated a problem of efficient insurance under limited commitment and private information frictions when the principal has access to auditing and enforcement technologies. I showed that long run use of auditing depends on the presence of the limited commitment friction and, likewise, long run usage of enforcement depends on the information friction. Furthermore, the combination of frictions implies a lower bound on the wealth of the agent with the lower bound determined endogenously by the interaction of frictions and technologies. At the lower bound the contract resembles that of the *Enforcement Model* and the incentives to back-load the agent's consumption dominate. At higher levels of wealth, the contract resembles the *Auditing Model* and the contract is front-loaded. Finally, I show alternatively how the auditing technology acts as a complement to enforcement to smooth consumption at higher levels of wealth, but is used instead to substitute enforcement at low levels of wealth.

By examining the extent to which agents are able to insure themselves against idiosyncratic risk, future work could use the analysis of this paper to determine the relative costs of mitigating information and enforcement frictions. In particular, the cheaper is enforcement relative to auditing, the more front-loaded one should expect consumption. On the other hand, as enforcement is more expensive relative to auditing, the more one should expect consumption to resemble that of the *Enforcement Model* and contracts should be increasingly back-loaded. The framework provided in this paper may then allow future work to tease out these costs.

8 Appendix

Proposition 1: **Proof.** Consider an optimal allocation under $(P1)$. Now, suppose that the principal could reoptimize after some history $h^{t'}$. Clearly, the principal can be no worse off at that node given the option to change plans. Then, it is sufficient to show that the original plan does as well as when given the chance to reoptimize after the history $h^{t'}$. Consider some alternative plan, ν' that is equal to ν along every path, but switches to the reoptimized plan after $h^{t'}$.

By the assumption of reoptimization, the profile ν' satisfies enforcement constraints for all period after t' and incentive compatibility constraints after t'. Furthermore, after history $h^{t'}$ it must guarantee the agent a promised utility of $U_{t'}(\sigma|\nu, h^{t'})$.

Because all future nodes of ν' other than $h^{t'}$ are identical to ν and ν' satisfies the constraints for all periods after $h^{t'}$, it is only possible that ν' violates some constraint for $k < t'$ along the nodes consistent with history $h^{t'}$. Rewrite the agent's utility at some time $k < t'$ and some realized income θ_s as:

$$(1-\beta)u(x_s) + \sum_{t=k+1}^{t'} \sum_{\tilde{h}^t|\tilde{h}^k} [\beta^{t-k}(1-\beta)u(x_t(\cdot)) + \beta^{t'-k}U_{t'}(\sigma|\nu', h^t)]\pi(\tilde{h}^t)$$

It immediately follows that ν' necessarily satisfies promise keeping and enforcement as the LHS of each of those conditions is unchanged moving to plan ν' from ν.

Suppose that ν' violated the IC constraint. Then there is some σ, some $k < t'$ such that $U_k(\sigma|\nu', h^k) > U_k(\sigma^*|\nu', h^k)$. In turn, this implies that:

$$\sum_{t=0}^{k} \sum_{h^t} (1-\beta)\beta^t u(x(\cdot) + \beta^k U_k(\sigma|\nu', h^{t'}) > \sum_{t=0}^{k} \sum_{h^t} (1-\beta)\beta^t u(x(\cdot) + \beta^k U_k(\sigma^*|\nu', h^{t'})$$

Because the plans are identical up until time t', this can be rewritten as:

$$\sum_{t=0}^{t'}\sum_{h^t}(1-\beta)\beta^t u(x(h^t)) + \beta^{t'}U_{t'}(\sigma|\nu', h^{t'}) > \sum_{t=0}^{t'}\sum_{h^t}(1-\beta)\beta^t u(x(h^t)) + \beta^{t'}U_{t'}(\sigma^*|\nu', h^{t'})$$

However, by assumption we know that the continuation utilities at t' are identical between ν and ν' so the conditioning on ν' in equation 1 can be replaced with ν. This implies that the original plan was not incentive compatible yielding a contradiction.

■

Proposition 2: Proof. First note that the concavity of u and convexity of g imply that V is strictly concave (only weak concavity is proven for the general model in Proposition 8). Suppose that the statement did not hold so that for some $\hat{\hat{\omega}} < \tilde{\omega}$ it was the case that $\hat{\hat{\gamma}} \leq \tilde{\gamma}$. As γ is decreasing for ω sufficiently large (it must be 0 in the region for which first best insurance is possible), there is some $\hat{\omega} > \tilde{\omega}$ for which $\hat{\gamma} = \hat{\hat{\gamma}}$. Then, note that for any state s at which the enforcement constraint binds it is the case that $(1-\beta)u(\hat{x}_s) + (1-\beta)\hat{\omega}'_s = (1-\beta)u(\hat{\hat{x}}_s) + (1-\beta)\hat{\hat{\omega}}'_s$ and by the concavity of V and u it must be that for any such state $\hat{x}_s = \hat{\hat{x}}_s$ and $\hat{\omega}'_s = \hat{\hat{\omega}}'_s$. Let $\underline{s}$ be the lowest state for which both $\hat{\xi}_s > 0$ and $\hat{\hat{\xi}}_s > 0$. Consequently, $\sum_{s=\underline{s}}^S \hat{\lambda} + \hat{\xi}_s = \sum_{s=\underline{s}}^S \hat{\lambda} + \hat{\xi}_s$. As $\hat{\lambda} > \hat{\hat{\lambda}}$ it must be the case that $\sum_{s=\underline{s}}^S \hat{\xi}_s < \sum_{s=\underline{s}}^S \hat{\hat{\xi}}_s = g'(\hat{\gamma})$ so that there is some state $s' < \underline{s}$ for which

$$\begin{aligned}(1-\beta)u(\theta_{s'}) + \beta U_0 - \hat{\gamma} &= (1-\beta)u(\hat{x}_{s'}) + \beta\hat{\omega}'_{s'} \\ &< (1-\beta)u(\hat{\hat{x}}_{s'}) + \beta\hat{\hat{\omega}}'_{s'}\end{aligned}$$

Additionally, by the fact that $\hat{\omega} > \hat{\hat{\omega}}$ there must also exist some state $s'' < s'$ where: $(1-\beta)u(\hat{x}_{s''}) + \beta\hat{\omega}'_{s''} > (1-\beta)u(\hat{\hat{x}}_{s''}) + \beta\hat{\hat{\omega}}'_{s''}$. As utility is constant across states of the double-hat profile for which enforcement constraint does not bind this implies that $(1-\beta)u(\hat{x}_{s''}) + \beta\hat{\omega}'_{s''} > (1-\beta)u(\hat{x}_{s'}) + \beta\hat{\omega}'_{s'}$ yielding a contradiction. ■

Proposition 3: Proof. Note that first order conditions imply that $\mathbf{E}[\lambda'] = \lambda + g'$ so that λ_t is a sub-martingale. Furthermore, when $\omega \geq u(\theta_S) + \beta U_0 = \bar{\omega}$, first best insurance is possible without any enforcement costs so that no enforcement constraints bind and $\omega'_s = \omega$ for all s. For $\omega > \bar{\omega}$ it is the case that the cost of providing the agent additional utility is simply $\lambda = u^{-1}(\omega)$ and as λ'_s is continuous in ω from the Theory of the Maximum, $\max_{\omega,s}\lambda'_s(\omega)$ is bounded on $\omega \in [U_0, \bar{\omega}]$. Therefore, λ_t is a submartingale that is bounded from above and converges almost surely by the Martingale Convergence Theorem. If ever an enforcement constraint binds, it must be that $\lambda'_s > \lambda$, so it must be that $g' = 0$ almost surely. ■

Lemma 2: Proof. Suppose that there are also upward constraints (specified by the C function next defined). It will then be shown that the upward constraints do not bind. Let $C_{s,s-1} = u(x_s) + \beta\omega'_s - u(x_{\tilde{s}} + (1-\alpha)(\theta_s - \theta_{\tilde{s}}))$. Then, consider $C_{s,s-1} + C_{s-1,s} = u(x_s) - u(x_s - \Delta_s(1-$

$\alpha)) + u(x_{s-1}) - u(x_{s-1} + \Delta_s(1-\alpha)) \geq 0$. The concavity of u guarantees that this expression is true only if $x_s \geq x_{s-1}$.

Let $\Delta_s = \theta_s - \theta_{s-1}$. For the second part of the statement, suppose that $x_s > x_{s-1} + \Delta_s(1-\alpha)$. Rearrange terms in the expression above to get,

$$\{u(x_s) - u(x_{s-1} + \Delta_s(1-\alpha))\} - \{u(x_s - \Delta_s(1-\alpha)) - u(x_{s-1})\} \geq 0$$

The LHS of the expression is decreasing in x_s and is equal to zero when x_s is equal to the upper bound specified. ■

Lemma 11 *Assuming V is concave and that the problem features upward constraints in addition to downward constraints, (i) Downward constraints always bind and (ii) the upward constraints are not binding.*

Proof. (i) To show that $C_{s,s-1} = 0$. Suppose to the contrary that $C_{s,s-1} > 0$, for some s. Then $\omega'_s > \omega'_{s-1}$ by Lemma 2. Consider changing $(x_i, \omega'_i)_{i \in S}$ as follows: keep ω'_1 fixed and if the downward constraint is slack, reduce ω'_s so that the downward constraint binds. Do the same for $s = 2, 3..$ until all downward constraints bind. Add a constant to each ω'_s so as to satisfy promise keeping. Each $\omega'_s - \omega'_{s-1}$ has been reduced increasing the principal's objective. The new contract offers the borrower the same utility and satisfies upward constraints because $x_s \leq x_{s-1} + \Delta_s(1-\alpha)$ still holds and this combined with the downward constraints binding implies that upward constraints also hold.

(ii) Suppose we ignore the constraint $C_{s-1,s}$. If $x_s \leq x_{s-1} + \Delta_s(1-\alpha)$ then by (i) the upward incentive constraint is automatically satisfied. So suppose that the solution has $x_s \geq x_{s-1} + \Delta_s(1-\alpha)$. Then $\omega'_s < \omega'_{s-1}$ and $C_{s-1,s} < 0$. But then replacing $x_{s-1} - (1-\alpha)\theta_{s-1}$ by $x_s - (1-\alpha)\theta_s$ and ω'_{s-1} by ω'_s cannot decrease the principal's objective and cannot violate incentive compatibility. However, by the concavity of the agent's utility function, this increases his utility, yielding a contradiction.

■

Lemma 3: Proof. Consider the case of $k > s$. Note that

$$C_{k,s} = \sum_{j=s}^{k} C_{j+1,j} + \left[\sum_{i=s+1}^{k-1} u(x_i + (1-\alpha)\Delta_{i+1}) - u(x_i)\right] - [u(x_s + (\theta_k - \theta_s)(1-\alpha))u(x_s + \Delta_{s+1}(1-\alpha))]$$

Lemma 2 implies that for $i > s$,

$$u(x_i + (1-\alpha)\Delta_{i-1}) - u(x_i) \geq u(x_s + (\theta_{i-1} - \theta_s)) - u(x_s + (\theta_{i-2} - \theta_s))$$

and the first term in brackets is thus larger than the second term in brackets. ■

Proposition 5: Proof. At the lower limit: First order conditions establish that $\lambda = E[\lambda'_s]$.

We know that λ tends toward 0 and given that $\lambda'_L \geq 0$, it must be that λ'_H vanishes as well. Then, from the first order condition $u'(x_H)\lambda'_H = 1$ it must be that $1/u'(x_H)$ vanishes. Then, dividing numerator and denominator by $u'(x_H)$ in Equation 1 it must be that the numerator tends toward zero. Given INADA conditions on f, α must be bounded strictly below 1 and the denominator of Equation 1 is bounded away from 0. ■

Proposition 6: Proof. At the upper bound: From earlier propositions we have that

$$f' = p_L \frac{\frac{u'(x_L)}{u'(x_H)} - 1}{1 + \frac{p_L u'(x_L)}{p_H u'(x_L + (1-\alpha)\Delta)}} < p_L \frac{\frac{u'(x_L)}{u'(x_L + (1-\alpha)\Delta)} - 1}{1 + \frac{p_L u'(x_L)}{p_H u'(x_L + (1-\alpha)\Delta))}} \tag{1}$$

Notice that the denominator is bounded away from zero. As for the numerator, $\lim_{x\to\infty} \frac{u'(x)}{u'(x+(1-\alpha)\Delta)} \leq \lim_{x\to\infty} \frac{u'(x)}{u'(x+\Delta)} = 1$, proving that auditing vanishes at the upper limit. ■

Lemma 4: Proof. Suppose not. Then in the optimal allocation it is the case that $u(x_H) + \beta\omega'_H = u(x_L + \Delta) + \beta\omega'_L$ so that either $x_H > x_L$ or $\omega_H > \omega_L$. Suppose the former (the same logic applies with the latter). Imagine increasing α by ϵ_α, decreasing x_H by ϵ_H and increasing x_L by ϵ_L so that $p_H u'(x_H)\epsilon_H = p_L u'(x_L)\epsilon_L$ and $(\epsilon_\alpha - \epsilon_L)u'(x_L + \Delta) = u'(x_H)\epsilon_H$. Then if IC and PK held in the original allocation they continue to hold. The additional cost of this plan is $\epsilon_\alpha f'(0) - \epsilon_H \left(p_H - \frac{p_H u'(x_H)}{u'(x_L)}\right) < 0$ yielding a contradiction. ■

Proposition 8: Proof.

Consider the operator:

$$T\hat{V}(\omega) = \max_{\alpha,\gamma,x,\omega'} \sum_s p \left[-(1-\beta)x_s + \beta\hat{V}(\omega'_s)\right] - f(\alpha) - g(\gamma) \tag{2}$$

subject to the IC, E, and PK constraints. We will show that V is its fixed point. Consider the space of functions

$$F = \{\hat{F} \in \mathcal{C} : [\underline{\omega}, \bar{u}) \to \mathbf{R} | F_1(\omega) \leq F(\omega) \leq F_2(\omega), \omega \in [\underline{\omega}, \bar{u})\}$$

where $\mathcal{C}$ is the set of continuous functions defined on the appropriate domain. First, we show that F is a complete metric space in the supremum metric and T is a contraction mapping on F.

Using the sup norm, we will construct F_2 and F_1 so that the distance between these two functions bounded, which implies that F is a complete metric space. The upper bound function F_2 is the first best allocation, in which $F_2(\omega) = u^{-1}(\omega) \equiv C(\omega)$. The lower bound function F_1 grants each agent his income plus/minus a constant y and pays the necessary enforcement cost associated with this. To guarantee that this satisfies PK it must be that $\sum_s [u(y + \theta_s)]p_s = \omega$ and that the enforcement cost is

$$\bar{\gamma}(\omega) = \max_s \left\{(1-\beta)u(\theta_s) + \beta U_0 - [(1-\beta)u(y + \theta_s) + \beta\omega]\right\}.$$

As ω is bounded from below, the difference between these two functions is bounded. To show this, note that $\sum_{s\in S} u(y+\theta_s) = C$ for all ω so that $y+\theta_1 \leq C$ and for all s, $y+\theta_s+\theta_1 \leq y+\theta_N+\theta_1 \leq C+\theta_N$. Then, the difference in cost to the principal (ignoring enforcement) between the two plans is $\sum_s [y+\theta_s - C] \leq \theta_N - \theta_1$. Given that γ is bounded above from the plan associated with F_1, difference between F_1 and F_2 must be bounded. By construction, $T(V)$ lies in F and, in addition, Bellman's sufficient conditions are satisfied. Therefore, T is a contraction mapping on a complete metric space, so that there exists a unique fixed point V.

Next, we show that T maps concave functions into concave functions. As V is the fixed point of the contraction mapping T, and the set of concave functions is closed, V is then also concave. Consider any $\hat{\omega} < \hat{\hat{\omega}}$ with corresponding contracts $\{\{\hat{x_s}, \hat{\omega'_s}\}_{s\in S}, \hat{\alpha}, \hat{\gamma}\}$ and $\{\{\hat{\hat{x}}_s, \hat{\hat{\omega'_s}}\}_{s\in S}, \hat{\hat{\alpha}}, \hat{\hat{\gamma}}\}$. Consider some $\omega = (1-\delta)\hat{\omega} + \delta\hat{\hat{\omega}}$, $\delta \in [0,1]$. Assume that V is concave and consider the contraction mapping in Equation 2. We want to show that $TV(\omega) \geq (1-\delta)TV(\hat{\omega}) + \delta TV(\hat{\hat{\omega}})$.

Construct the following contract $\{\{x^*_s, \omega'_s\}_{s\in S}, \alpha^*, \gamma^*\}$ such that:

$$
\begin{aligned}
f(\alpha^*) &= (1-\delta)f(\hat{\alpha}) + \delta f(\hat{\hat{\alpha}}) \\
u(x^*_s) &= (1-\delta)u(\hat{x}_s) + \delta u(\hat{\hat{x}}_s) \\
\omega^{*\prime}_s &= (1-\delta)\hat{\omega}' + \delta\hat{\hat{\omega}}'_s \\
\gamma^* &= (1-\delta)\hat{\gamma} + \delta\hat{\hat{\gamma}}
\end{aligned}
$$

and note that by convexity of f, concavity of u, V, it is the case that $\alpha^* \geq (1-\delta)\hat{\alpha} + \hat{\hat{\alpha}}$, $x^*_s \leq (1-\delta)\hat{x}_s + \hat{\hat{x}}_s$ and that $V(\omega^{*\prime}_s) \geq (1-\delta)V(\hat{\omega}'_s) + \delta V(\hat{\hat{\omega}}'_s)$ so that this yields at least as must resources to the principal as the convex combination. Clearly, this scheme satisfies promise keeping and enforcement given that the plans at $\hat{\omega}$ and $\hat{\hat{\omega}}$ satisfied those constraints. Then, we must show that the starred plan satisfies incentive compatibility. There are two cases, one in which the α and x move in the same direction, in which case concavity follows from standard arguments. In the case where the α and x move in opposite directions $u(x^* + (1-\alpha^*))$ need not be less than the convex combination of $u(\hat{x} + (1-\hat{\alpha}))$ and $u(\hat{\hat{x}} + (1-\hat{\hat{\alpha}}))$, complicating the proof.

Case 1: Suppose that α and x move in the same direction: $(\hat{\alpha} - \hat{\hat{\alpha}})(\hat{x} - \hat{\hat{x}}) \geq 0$.

We must show that $C^* = (1-\beta)u(x^*_H) + \beta\omega^{*\prime}_H - [(1-\beta)u(x^*_L + (1-\alpha^*)\Delta) + \beta\omega^{*\prime}_L]$. Consider $(1-\delta)\hat{C}_{H,L} + \delta\hat{\hat{C}}_{H,L} \geq 0$, where the C terms represent the incentive constraints for the appropriate contracts. Note that

$$
C^* = (1-\delta)\hat{C}_{H,L} + \delta\hat{\hat{C}}_{H,L} + (1-\beta)[(1-\delta)u(\hat{x} + (1-\hat{\alpha})\Delta) + \delta u(\hat{\hat{x}} + (1-\hat{\hat{\alpha}})\Delta) - u(x^*_L + (1-\alpha^*)\Delta)]
$$

so given that the IC constraints hold for the hat contracts by assumption, it is enough to show that $(1-\delta)u(\hat{x} + (1-\hat{\alpha})\Delta) + \delta u(\hat{\hat{x}} + (1-\hat{\hat{\alpha}})\Delta) - u(x^*_L + (1-\alpha^*)\Delta) \geq 0$. Replace all of the α terms with $(1-\delta)\hat{\alpha} + (1-\delta)\hat{\hat{\alpha}}$. This will decrease the sum of the first two terms as more weight is put on the term with the lower marginal utility (by the assumption of Case 1) and the second term is more negative as α^* is greater than this term. Then, the case is proven by the assumption u''/u'

increasing.

Case 2: Suppose that α and x move in opposite directions: $(\hat{\alpha} - \hat{\hat{\alpha}})(\hat{x} - \hat{\hat{x}}) < 0$.

By way of contradiction, suppose that TV is not concave and assume that $TV(\omega)$ lies below secant connecting $TV(\hat{\omega})$ and $TV(\hat{\hat{\omega}})$ for δ small.[14] Then, it must be the case that the starred profile violates incentive compatibility as δ increases above 0 (otherwise the starred profile would be better than the convex combination). The following is true (using the inverse function theorem) of the starred profile at $\delta = 0$:

$$\begin{aligned}\frac{\partial \alpha^*}{\partial \delta} &= \frac{f(\hat{\hat{\alpha}}) - f(\hat{\alpha})}{f'(\hat{\alpha})} \\ \frac{\partial x^*}{\partial \delta} &= \frac{u(\hat{\hat{x}}) - u(\hat{x})}{u'(\hat{x})}\end{aligned}$$

So that at $\delta = 0$,

$$\frac{\partial C^*}{\partial \delta} = -u'(\hat{x} + (1-\hat{\alpha})\Delta)) \left[\frac{u(\hat{\hat{x}}) - u(\hat{x})}{u'(\hat{x})} - \frac{f(\hat{\hat{\alpha}}) - f(\hat{\alpha})}{f'(\hat{\alpha})} \right] + u(\hat{\hat{x}} + (1-\hat{\hat{\alpha}})\Delta) - u(\hat{x} + (1-\hat{\alpha})\Delta)$$

Suppose that $\hat{x} < \hat{\hat{x}}$ so that $\hat{\alpha} > \hat{\hat{\alpha}}$, an identical argument will hold for the opposite case. Then,

$$\begin{aligned}\frac{\partial C^*}{\partial \delta} &\geq -u'(\hat{x} + (1-\hat{\alpha})\Delta)) \left[\frac{u(\hat{\hat{x}}) - u(\hat{x})}{u'(\hat{x})} \right] + u(\hat{\hat{x}} + (1-\hat{\alpha})\Delta) - u(\hat{x} + (1-\hat{\alpha})\Delta) \geq 0 \\ &\Leftrightarrow -\frac{u(\hat{\hat{x}}) - u(\hat{x})}{u'(\hat{x})} + \frac{u(\hat{\hat{x}} + (1-\hat{\hat{\alpha}})\Delta) - u(\hat{x} + (1-\hat{\hat{\alpha}})\Delta)}{u'(\hat{x} + (1-\hat{\alpha})\Delta))} \geq 0\end{aligned}$$

The following claim then concludes the proof.

Claim: For any $\Delta > 0$, $z(x, \Delta) = \frac{u(x+\Delta) - u(x)}{u'(x)}$ is increasing in x by Assumption ii. Proof: $z_x(x, \Delta) = \frac{[u'(x+\Delta) - u'(x)]u'(x) - u''(x)[u(x+\Delta) - u(x)]}{(u'(x))^2} \geq 0$ if and only if $[u'(x+\Delta) - u'(x)]u'(x) - u''(x)[u(x+\Delta) - u(x)] > 0$, which is increasing in Δ by Assumption ii. The claim is proven by noting $\lim_{\Delta \to 0} z_x(x, \Delta) = 0$. ■

Proposition 9: Proof.

To show that V is differentiably continuous we adapt the argument of Thomas and Worrall (1990). Consider a neighborhood of ω around any $\tilde{\omega}$, and consider a contract that satisfies IC, E, and PK in which α and ω' are held constant at their values implied by $\tilde{\omega}$ so that only x and γ are varied to yield ω in which IC continues to bind. There is a unique way to do this construction and the cost of doing this is differentiably continuous and yields a utility to the principal less than or equal to to $V(\omega)$ with equality holding at $\tilde{\omega}$. That V is continuously differentiable then follows from

[14] If this is not true, then define $\hat{\omega}$ as the ω at which this is true.

Lemma 1 of Benveniste and Scheinkman (1979). ■

Lemma 5: Proof. Suppose not. It then must be the case that IC binds. Note that IC not binding implies $\omega_L', \omega_H' > \omega$. Consider the cases (i) $x_L \geq \theta_L$ and (ii) $x_L < \theta_L$. For case (i) note that PK and $\omega_L', \omega_H' > \omega$ imply that $\omega > u(x_H) + u(x_L) > U_0$ so that no constraints bind and $x_L = x_H$ yielding a contradiction. Case (ii) note that $x_L < \theta_L$ implies that if the IC and EL constraints holds, then so too must the EH constraint as the high type's utility is at least $u(x_L + \Delta) - u(x_L)$ greater than the low type's utility. Then, the principal could be better off by raising x_L and decreasing x_H, fixing the agent's utility, with no additional enforcement or auditing costs. Thus, IC must bind.

Given that IC binds, if $x_L + (1-\alpha)\Delta < x_H$ then it must be that $\omega_L' \geq \omega_H'$. First order conditions imply that $u'(x_H)\lambda_H' = 1$ and $u'(x_L)\lambda_L' + \eta(u'(x_L) - u'(x_L, \alpha, \Delta))/p_L = 1$. Since $u'(x_H) < u'(x_L)$ and $\lambda_H' \leq \lambda_L'$, this is a contradiction. ■

Lemma 6: Proof. By the concavity of V we need only establish that there exists some ω such that $V'(\omega) > 0$. Note that the enforcement constraints combined with the promise keeping constraint imply that $U_0 - \gamma < \omega$ so that γ increases indefinitely as ω decreases. Now, $V(U_0) \geq E[\theta]$ as setting $\gamma = \alpha = 0$ and $x_s = \theta_s$ satisfies all conditions. Therefore, $V'(\omega) > 0$ for some $\omega < U_0$. ■

Lemma 7: Proof. First order conditions imply that $(1-\beta)u'(x_L)(p_L\lambda + p_L\xi_L) - \eta\tilde{u}(x_L, \alpha, \Delta) > 0$. As $\lambda < 0$ for ω sufficient small, it must be that $\xi_L > 0$ for all such ω. ■

Lemma 8: Proof. We show for each possibility of enforcement constraints binding that IC must bind as well:

i) Neither EL nor EH bind. It is immediate that IC must bind, otherwise this would imply implementation of the efficient allocation.

ii) EH and EL bind. This implies that the difference between the high type utility and low type utility is $(1-\beta)(u(\theta_H) - u(\theta_L))$. If IC does not bind, this implies that the difference between the high type utility and low type utility is greater than $u(x_L + \Delta) - u(x_L)$ which implies that $x_L > \theta_L$. Both EL and EH binding, but IC not binding implies that $\eta = 0$ so that $\omega_L' \geq \omega$ by the concavity of V. The PK constraint then implies that $(1-\beta)[p_H u(x_L + \Delta) + p_L u(x_L)] + \beta\omega_L' < \omega$ so that $p_H u(x_L + \Delta) + p_L u(x_L) < \omega$. However, by $x_L > \theta_L$ this implies that $\omega > U_0$ which implies that $\gamma = 0$, contradicting the binding of EL and EH.

iii) EL only binding. This implies that $\omega_L' > \omega = \omega_H'$. The IC constraint then implies that $u(x_H) > u(x_L + \Delta)$ contradicting Lemma 2.

iv) EH only binding. This implies that $(1-\beta)u(\theta_H) + \beta U_0 - \gamma > (1-\beta)u(x_L + \Delta) + \beta\omega_L' > (1-\beta)u(\theta_L) + \beta U_0 - \gamma$. This in turn implies that $u(\theta_H) - u(\theta_L) > u(x_L + \Delta) - u(x_L)$ so that $x_L > \theta_L$. Furthermore, $x_L + \Delta$ is an upper bound on x_H and FOC (along with IC, EL not binding) imply that $\omega_L' = \omega$. Together, these imply that $\omega < U_0$. The PK constraint and IC imply further that $(1-\beta)[p_H u(x_L + \Delta) + p_L u(x_L)] + \beta\omega < \omega$ implying that $U_0 < p_H u(x_L + \Delta) + p_L u(x_L) < \omega$ yielding a contradiction, where the first inequality follows from $x_L > \theta_L$.

■

Lemma 9: Proof. Suppose not, so that EH also binds. Both enforcement constraints binding imply $\tilde{u}(x_L, \alpha, \Delta) - u(x_L)) = u(\theta_H) - u(\theta_L)$ so that $\alpha = \phi(x_L) \equiv \frac{1}{\Delta}\left[x_L + 1 - u^{-1}(u(x_L) + u(\theta_H) - u(\theta_L))\right]$. From first order conditions $\eta = \frac{f'}{\Delta \tilde{u}(x_L, \alpha, \Delta)}$. Using the inverse function theorem and L'Hopital's rule:

$$\lim_{x_L \to 0} \eta = \lim_{x_L \to 0} \frac{f''(\phi)\left[1 - u'(x_L)\frac{1}{u'(x_L)}\right]}{\tilde{u}''(x_L, \alpha, \Delta)(1 - \phi')} = 0$$

As η vanishes, from first order conditions $\lambda'_L + \frac{1}{u'(x_L)}\eta(u'(x_L) - \tilde{u}(x_L, \alpha, \Delta) = \frac{p_L}{u'(x_L)}$ and $\lambda'_H = \frac{p_H}{u'(x_L)}$. Therefore, λ'_s tends toward zero for $s = L, H$ and continuation promises are equal. By the IC constraint binding this implies that it cannot be the case that $\tilde{u}(x_L, \alpha, \Delta) - u(x_L) = u(\theta_H) - u(\theta_L)$ yielding a contradiction. ■

Lemma 12 *If EL binds for ω, then it also binds for all $\hat{\omega} < \omega$.*

Proof. First, note that at the greatest ω for which EL binds it is the case that $\omega < U_0$ and so it must also be the case that EH binds as well (this follows the proof from Lemma 11). Furthermore, if EL does not bind at $\hat{\hat{\omega}}$ then it must be that $V'(\hat{\hat{\omega}}) < 0$. Finally, there must also be some $\hat{\omega} \in [\hat{\hat{\omega}}, \omega)$ both EL and EH hold with equality and $\xi_L = 0$. Then, $\omega - \hat{\omega} = u(\theta_H) - u(\theta_L)$. Furthermore $\alpha = \phi(x_L$ where $\tilde{u}(x_L, \phi, \Delta) - u(x_L) = u(\theta_H) - u(\theta_L)$. Then, as x_L decreases it is straightforward to show that $\eta = \frac{f'}{\Delta \tilde{u}(x_L, \phi(x_L), \Delta)}$ increases. Furthermore it must be that $\hat{\gamma} - \gamma = \omega - \hat{\omega}$. As $\hat{\eta} > \eta$ and $p_H \hat{\xi} = g'(\hat{\gamma}) > g'(\gamma) = p_H \xi_H + p_L \xi_L$ it must be that $\hat{\lambda}'_H - \hat{\lambda}'_L > \lambda'_H - \lambda'_L$. However, by concavity of the value function it must be that the constant difference between the high and low utilities implies that $\lambda'_H - \lambda'_L$ decreases as promised utility decreases. ■

Proposition 10: Proof.

For some ω only EL and IC bind. First order conditions on continuation promises yield

$$p_L \lambda'_L + p_H \lambda'_H = \lambda + g'. \tag{3}$$

EH not binding implies $\xi_L = g'$ so that $u'(x_L)p_L(\lambda + g') - \eta u'(x_L + (1 - \alpha)\Delta) = p_L$ implying that $\lambda + g' > 0$. From the first order conditions on ω'_H and x_H we have that $u'(x_H)\lambda'_H = p_H$ so that $\lambda'_H > 0$ for all ω. Moreover, as ω decreases it is the case that λ'_H tends toward 0. Thus, from the Equation 3 it cannot be the case that λ'_L decreases indefinitely with λ as ω decreases.

The lower bound is then established by the fact Theory of the Maximum, which guarantees that the multipliers are continuous in ω. Finally, the fact that $\underline{\omega} < U_0$ follows from Proposition 11. ■

Proposition 11: Proof. To show this I show that other possible binding combinations are not possible. i) Suppose neither EL nor EH were to bind. This immediately implies that $\gamma = 0$ and $\sum_s \left[(1 - \beta)u(x_s) + \beta \omega'_s\right] > U_0$ yielding a contradiction.

ii) Suppose only EL were to bind. Imagine that the planner did not have to pay $g(\gamma)$ for the first period only, so that the objective function was $\sum_s[-(1-\beta)x_s + \beta V(\omega'_s)]p_s - f(\alpha)$ subject to PK, E, IC. Call this problem (P'). The solution to this revised problem looks like a problem with private information in which income is shared across states ($\theta_L < x_L < x_H < \theta_H$), utility is shared across states $u(\theta_L) + \beta U_0 < u(x_L) + \beta\omega'_L < u(x_H) + \beta\omega'_H < u(\theta_H) + \beta U_0$ and the following relation holds: $u'(x_H)\lambda'_H = p_H$. Call the original problem (P) and the solution to this problem $\mathcal{C}_1$ and the maximum V^*. Clearly, the planner could implement this allocation in the original problem (in which the principal pays $g(\gamma)$ by choosing $\mathcal{C}_1$ and setting $\gamma_1 = u(\theta_H) + \beta\omega'_H - u(x_H) - \beta\omega'_H$. Call the principal's utility from this plan V_1 and note that $V^* = V_1 + g(\gamma_1)$. Now, suppose that EL bound in the optimal allocation, $\mathcal{C}_2$ and the maximum principal utility was V_2. Note that the fact that $\mathcal{C}_2$ was not chosen in the revised problem implies that $V^* > V_2 + g(\gamma_2)$, where γ_2 is the first period enforcement technology under the optimal allocation. Abusing notation, note that $u'(x_H)\lambda'_H = p_H$ holds in this case as well. Since $V_2 > V_1$ by assumption, it must be that $\gamma_2 < \gamma_1$, which will lead to the contradiction. Note that $\gamma_1 = (1-\beta)[u(\theta_H) - u(x_{H,1})] + \beta(U_0 - \omega'_{H,1})$ and $\gamma_2 > (1-\beta)[u(\theta_H) - u(x_{H,2})] + \beta(U_0 - \omega'_{H,2})$, where subscript H, i infers the first period variable implied by $\mathcal{C}_i$. $\gamma_2 < \gamma_1$ together with $u'(x_H)\lambda'_H = p_H$ and the concavity of the value function imply that $x_{H,1} < x_{H,2}$. Consider a perturbation of the optimal allocation of the (P') problem in which $x_{L,1}$ is increased, $x_{H,1}$ is decreased and α_1 is increased and vice versa. Optimality requires that:

$$\frac{p_L}{p_H u'(x_{H,1})} = \frac{p_L}{u'(x_{L,1})} + \frac{f'}{\Delta}\left[\frac{p_L u'(x_{L,1}) + u'(x_{L,1} + (1-\alpha)\Delta)}{p_H u'(x_{L,1}) u'(x_{L,1} + (1-\alpha_1)\Delta)}\right] \tag{4}$$

Note that a similar perturbation implies the following for problem (P):

$$g' + \frac{p_L}{p_H u'(x_{H,2})} = \frac{p_L}{u'(x_{L,2})} + \frac{f'}{\Delta}\left[\frac{p_L u'(x_{L,2}) + u'(x_{L,2} + (1-\alpha)\Delta)}{p_H u'(x_{L,2}) u'(x_{L,1} + (1-\alpha_2)\Delta)}\right] \tag{5}$$

If EL binds, then IC, PK imply that $U_0 = u(x_{L,2}) + \beta U_0 + p_H[u(x_{L,2} + (1-\alpha)\Delta) - u(x_{L,2})]$ where the second term must be greater than $u(\theta_H) - u(\theta_L)$, which is possible only if $x_{L,2} \leq \theta_L$. Finally, note that $\alpha_1 > \alpha_2$ follows from the IC constraint and the fact that $x_{H,1} < x_{H,2}$, $x_{L,1} > x_{L,2}$ and that $u(x_{H,1}) + \beta\omega'_{H,1} < u(x_{H,2}) + \beta\omega'_{H,2}$. Looking toward equations 4 and 5 this yields a contradiction, as the LHS of equation 5 is greater than that of 4 and the RHS of 5 is less than that of equation 4, yielding a contradiction. The latter point requires that the term with f' is decreasing in α, which is easily shown.

iii) Both EL and EH bind. IC must bind, otherwise we can perturb the allocation to smooth utility across states at no cost $f'(0) = g'(0) = 0$. All constraints binding imply that $u(x_L + (1-\alpha)\Delta) - u(x_L) = u(\theta_H) - u(\theta_L)$ implying that $x_L \leq \theta_L$ from the curvature of u. Because $\gamma = 0$, first order conditions imply that $\omega'_L < U_0$. Thus, $(1-\beta)u(x_L) + \beta\omega'_L < (1-\beta)u(\theta_L) + \beta U_0$ yielding a contradiction.

iv) If EH only binds, the IC must bind. If EH binds then $(1-\beta)u(x_L) + \beta\omega'_L > (1-\beta)u(\theta_L) +$

$\beta U_0 - \gamma$. The IC constraint and the concavity of u imply that if $\alpha = 0$ so IC does not bind, the high type's utility is at least $u(\theta_H) - u(\theta_L)$ higher than the low type, yielding a contradiction. ■

Corollary 1: Proof. If only EH binds then consider reducing the agent's utility by some small amount ϵ. The principal can guarantee $U_0 - \epsilon$ level of utility by replicating the allocation at U_0 and reducing x_L by $\epsilon/u'(x_L)$. Because EL is not binding, for ϵ sufficiently small, the EL constrain will still hold. Furthermore, reducing x_L loosens the IC constraint, while having no impact on the EH constraint. Thus, the principal is strictly better off at $U_0 - \epsilon$. ■

Lemma 12: Proof. Claim: if EH and EL both bind for any ω then for any $\hat{\omega} < \omega$ it is the case that the corresponding levels of enforcement satisfy $\hat{\gamma} > \omega$. Suppose not so $\hat{\gamma} \leq \gamma$. Then, at ω it is the case that

$$\begin{aligned}&(1-\beta)u(x_s) + \beta\omega'_s = (1-\beta)u(\theta_s + \beta U_0 - \gamma \\ &\leq (1-\beta)u(\theta_s) + \beta U_0 - \gamma \leq (1-\beta)u(\hat{x}_s) + \beta\hat{\omega}'_s\end{aligned}$$

Summing over states this implies that $\omega \leq \hat{\omega}$ yielding a contradiction.

Given the claim, we need only prove that γ is strictly decreasing when only EH and IC bind. When EL does not bind, then $p_H\xi_H = g'$. Suppose that γ is not strictly decreasing over the region in which only EH and IC bind. Then, continuity implies that there must be some $\hat{\omega} < \hat{\hat{\omega}}$ at which $\hat{\gamma} = \hat{\hat{\gamma}}$. As only EH binds it must be the case that $(1-\beta)u'(\hat{x}_H) + \beta\hat{\omega}'_H = (1-\beta)u'(\hat{x}_H) + \beta\hat{\omega}'_H$ and by the concavity of the value function and u, this implies that $\hat{x}_H = \hat{\hat{x}}_H$ and $\hat{\omega}'_H = \hat{\hat{\omega}}'_H$. By IC binding it is also the case that $(1-\beta)\tilde{u}(\hat{\hat{x}}_L, \hat{\hat{\alpha}}, \Delta) + \beta\hat{\hat{\omega}}'_L = (1-\beta)\tilde{u}(\hat{x}_L, \hat{\alpha}, \Delta) + \beta\hat{\omega}'_L$ and by PK it must be that $p_L[(1-\beta)(u(\hat{x}_L) - u(\hat{\hat{x}}_L) + \beta(\hat{\omega}'_L - \hat{\hat{\omega}}'_L)] = \hat{\hat{\omega}} - \hat{\omega}$. Together these imply that $\hat{x}_L - \hat{\hat{x}}_L < \hat{\hat{\alpha}} - \hat{\alpha} > 0$. Given optimality at $\hat{\omega}$, this implies the principal could be strictly better off by reducing $\hat{\hat{\alpha}}$ slightly and adjusting consumption and wealth accordingly. ■

Proposition 13: Proof. Suppose that EL and EH bind. Then $u(x_L + (1-\alpha)\Delta) - u(x_L) = u(\theta_H) - u(\theta_L)$. By the concavity of u, it is clear that α decreases as x_L increases.

Now, consider some $\hat{\omega}$ at which only EL binds and consider $\hat{\hat{\omega}} < \hat{\omega}$ sufficiently close to $\hat{\omega}$ such that only EL binds at $\hat{\hat{\omega}}$. Suppose, by way of contradiction, that $\hat{\hat{\alpha}} \leq \hat{\alpha}$. Using EL and IC, it is the case that $(1-\beta)u(\theta_L) + \beta U_0 - \hat{\gamma} + (1-\beta)p_H[u(\hat{x_L} - (1-\hat{\alpha})\Delta) - u(\hat{x_L})] = \hat{\omega}$ and similarly for the double hat contract. Then:

$$\hat{\hat{\gamma}} - \hat{\gamma} + \quad p_H(1-\beta)\left[(u(\hat{x_L} - (1-\hat{\alpha})\Delta) - u(\hat{x_L})) - \left(u(\hat{\hat{x}}_L - (1-\hat{\hat{\alpha}})\Delta) - u(\hat{\hat{x}}_L)\right)\right] = \hat{\omega} - \hat{\hat{\omega}}$$

Notice that it must be the case that $\hat{\hat{\gamma}} - \hat{\gamma} - \hat{\gamma} \leq \hat{\omega} - \hat{\hat{\omega}}$ as the principal can do at least as well at $\hat{\hat{\omega}}$ by taking the allocation at $\hat{\omega}$, reducing $\hat{x}_L$ so that utility in the low state is reduced by $\hat{\omega} - \hat{\hat{\omega}}$ and increasing enforcement by $\hat{\omega} - \hat{\hat{\omega}}$. Therefore, $\left[(\tilde{u}(\hat{x}_L, \hat{\alpha}, \Delta) - u(\hat{x}_L)) - \left(\tilde{u}(\hat{\hat{x}}_L, \hat{\hat{\alpha}}, \Delta) - u(\hat{\hat{x}}_L)\right)\right] > 0$ which implies that $\hat{\alpha} \geq \hat{\hat{\alpha}}$ so long as $\hat{x}_L \geq \hat{\hat{x}}_L$, which is proven in Lemma 13. ■

Lemma 13 *x_L is increasing in ω.*

Lemma 13: Proof. Consider $\hat{\hat{\omega}} < \hat{\omega}$ with corresponding double hat and single hat allocations. Consider an allocation with the single hat contract and varying only x_L, γ, and α. Let $\hat{\hat{x}}_L^*, \hat{\hat{\gamma}}^*$ and $\hat{\hat{\alpha}}^*$ be so that the agent's utility is $\hat{\hat{\omega}}$. Let $\hat{x}_L^*, \hat{\gamma}^*$ and $\hat{\alpha}^*$ likewise denote the components of the allocation solving the principal's problem for $\hat{\omega}$ when the other variables are as specified in the double hat contract. Note that to satisfy PK, $\hat{\hat{x}}_L \leq \hat{x}_L^*$ and $\hat{\hat{x}}_L^* \leq \hat{x}_L$. Optimality of the hat and double hat contracts imply:

$$\begin{aligned}
-p_L\hat{\hat{x}}_L - p_H\hat{\hat{x}}_H + p_H V(\hat{\hat{\omega}}'_H) + p_L V(\hat{\hat{\omega}}'_L) - f(\hat{\hat{\alpha}}) - g(\hat{\hat{\gamma}}) \quad &\geq \\
&-p_L\hat{\hat{x}}_L^* - p_H\hat{x}_H + p_H V(\hat{\omega}'_H) + p_L V(\hat{\omega}'_L) - f(\hat{\hat{\alpha}}^*) - g(\hat{\hat{\gamma}}^*) \\
-p_L\hat{x}_L - p_H\hat{x}_H + p_H V(\hat{\omega}'_H) + p_L V(\hat{\omega}'_L) f(\hat{\alpha}) - g(\hat{\gamma}) \quad &\geq \\
&-p_L\hat{x}_L^* - p_H\hat{\hat{x}}_H + p_H V(\hat{\hat{\omega}}'_H) + p_L V(\hat{\hat{\omega}}'_L) - f(\hat{\alpha}^*) - g(\hat{\gamma}^*) \\
\Rightarrow p_L(\hat{\hat{x}}_L^* - \hat{\hat{x}}_L) + f(\hat{\hat{\alpha}}^*) - f(\hat{\hat{\alpha}}) + g(\hat{\hat{\gamma}}^*) - g(\hat{\hat{\gamma}}) \quad &\geq \quad p_L(\hat{x}_L - \hat{x}_L^*) + f(\hat{\alpha}) - f(\hat{\alpha}^*) + g(\hat{\gamma}) \geq -g(\hat{\gamma}^*)
\end{aligned}$$

As the starred and unstarred profiles differ only on x_L, α, and γ, then from the PK constraints

$$u(\hat{\hat{x}}_L^*) - u(\hat{\hat{x}}) = u(\hat{x}) - u(\hat{x}_L^*) \tag{6}$$

Suppose it were the case that $\hat{\hat{x}} > \hat{x}$, then it must be that

$$\hat{\hat{x}}_L^* - \hat{\hat{x}} > \hat{x}_L^* - \hat{x} \tag{7}$$

by the concavity of u. Similar calculations from the binding of EL constraints imply that $u(\hat{x}_L) - u(\hat{\hat{x}}_L^*) = \hat{\hat{\gamma}}^* - \hat{\gamma}$ and $u(\hat{\hat{x}}_L) - u(\hat{x}_L^*) = \hat{\gamma}^* - \hat{\hat{\gamma}}$. Summing together an combining with (6) implies that

$$\hat{\gamma} - \hat{\gamma}^* = \hat{\hat{\gamma}} - \hat{\hat{\gamma}}^* \tag{8}$$

Given that utility in the high state is equalized across the $\hat{\cdot}$ and $\hat{\hat{\cdot}}^*$ profiles, $\hat{\hat{x}} > \hat{x}$ implies that $\hat{\omega}'_L > \hat{\omega}'_L$. Furthermore, from the IC constraints it must be that $(1-\beta)\tilde{u}(\hat{x}_L, \hat{\alpha}, \Delta) + \beta\hat{\omega}'_L = (1-\beta)\tilde{u}(\hat{\hat{x}}_L^*, \hat{\hat{\alpha}}^*, \Delta) + \beta\hat{\hat{\omega}}'_L$ implying that $\hat{\alpha} >\geq \hat{\hat{\alpha}}^*$ from $\hat{x}_L \geq \hat{\hat{x}}_L^*$. A similar calculation shows $\hat{\hat{x}}_L \geq \hat{x}_L^*$. These two facts, combined with (8) and (7) contradict (6). ■

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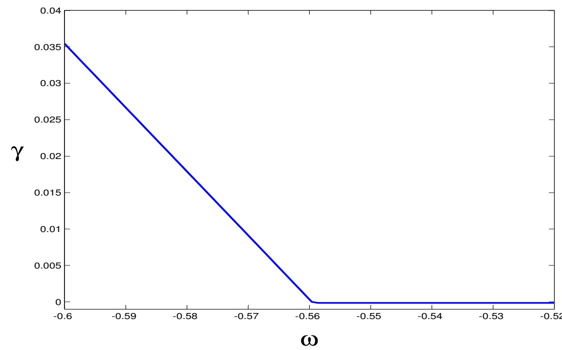

Figure 2: Enforcement as a Function of ω

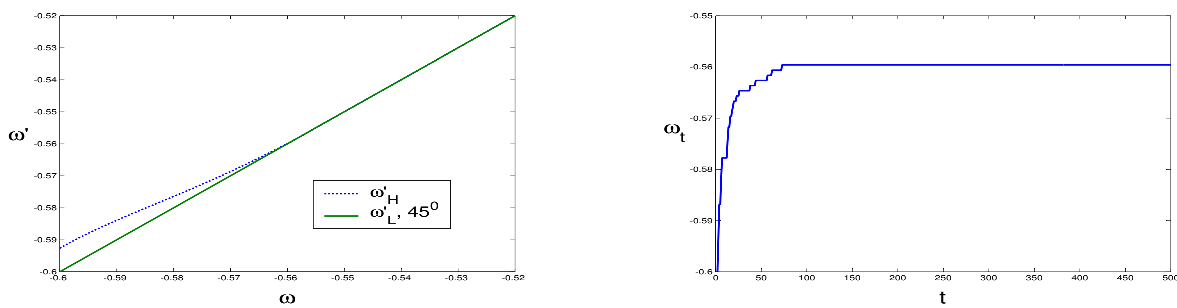

Figure 3: Left Panel: Continuation Promises, Right Panel: Sample Path ω

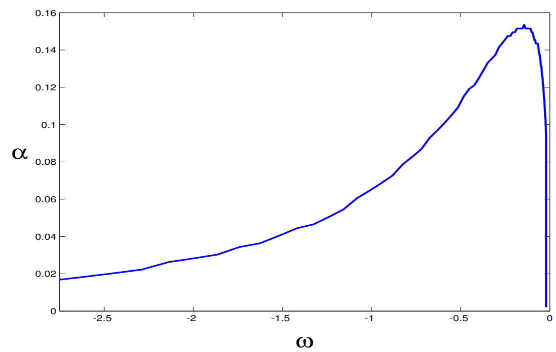

Figure 4: Auditing as a Function of ω

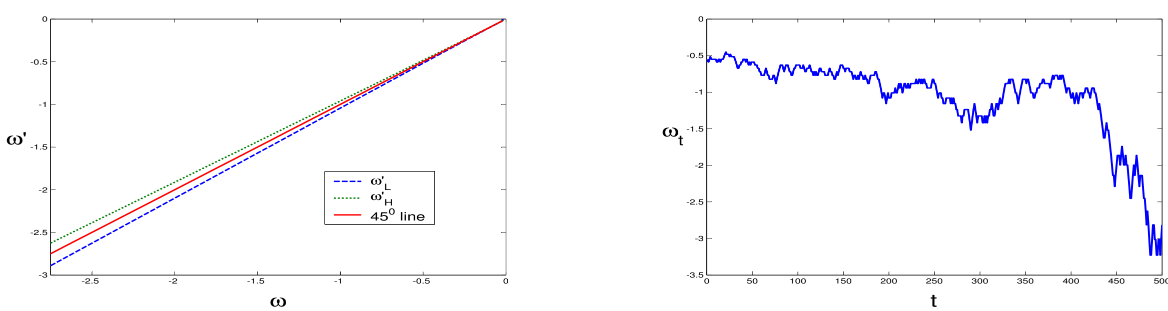

Figure 5: Left Panel: Continuation Promises, Right Panel: Sample Path ω

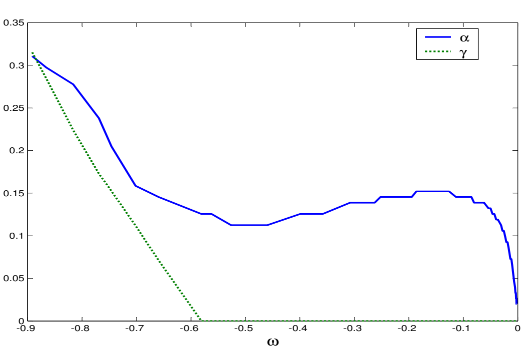

Figure 6: Auditing and Enforcement as a Function of ω

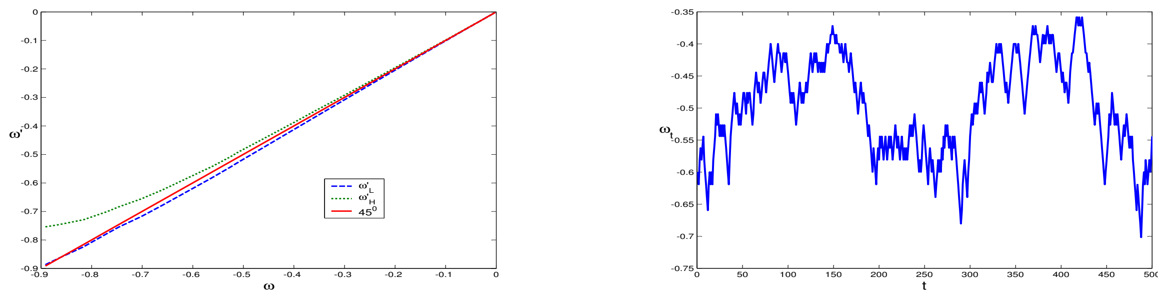

Figure 7: Left Panel: Continuation Promises, Right Panel: Sample Path ω

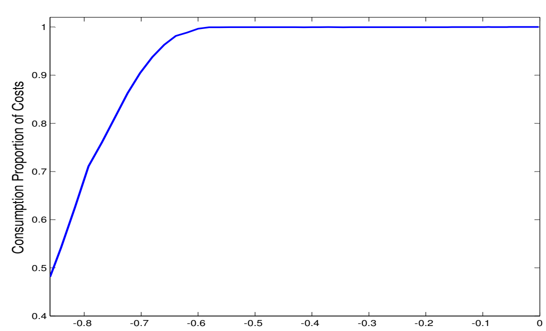

Figure 8: Proportion of Costs Devoted to Consumption