Demand Shocks as Productivity Shocks

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Introduction

• We build a model where demand shocks alone (today preference shocks) look like productivity shocks despite not having any.

- In the model the measured Solow residual behaves like that in the data.
- We lay out the theory which is simple and it is built as a growth model.
- We build a business cycle model with only demand shocks, we map it to the U.S. economy. It looks exactly like a standard RBC model.
- We compare quantitatively a standard RBC model with ours. We do as good or better.
- In our economy firms do well because customers show up.

The context

- In a standard business cycle model, the production function requires that either productivity or inputs change output (The only inputs are capital and labor).

$$Y = z\ F(K, N)$$

- So either productivity shocks (z) move or inputs (i.e. labor) move.

- Decreasing returns to scale require that labor productivity and wages drop if labor increases.

- This does not really happen in the data, the residual z is strongly correlated with output. Hence there have to be TFP shocks.

- We have been looking for them for thirty years with limited success.

The logic

- We postulate that in order to transform produced goods into used goods, both consumers and investors must pose efforts.
- Such efforts are not accounted for in NIPA.
- The economy cannot operate at full capacity.
- Operationally, this works as a search friction in the goods market. Increases in search effort imply increased measured productivity.
- Competitive search allows for minimal arbitrariety in the determination of prices (there are no multiple equilibria).
- Preference shocks are a stand in for a variety of demand shocks (credit restrictions, animal spirits, terms of trade shocks) to be developed in future work.

Alternatives in the literature

1. We do not measure inputs properly.
   - We use them more intensively in expansions. We work capital and labor much harder. (Basu and Fernald (1997), Licandro and Puch (2000)).
   - There is labor hoarding as labor have to be adjusted with one period delay (Burnside, Eichenbaum, and Rebelo (1990)). Alternatively, monopolistic firms with putty clay technology face uncertain idiosyncratic demand (Fagnart, Licandro, and Portier (1999)).
2. We do not aggregate properly.
   - Production functions are not perfect aggregates from plants (Hansen and Prescott (2005), Cooley, Hansen, and Prescott (1995)).
3. Distant cousins are Faig and Jerez (2005), Lagos (2006), and Alessandria (2005) also have frictions affecting TFP.

The plan

1. We describe the logic in a simple environment where output is productivity, the Lucas tree.
2. We move on to a growth model suitable for business cycle analysis.
3. We will discuss (very briefly and depending on the amount of interruptions) the subtle calibration issues that show up.
4. We estimate preference shocks from measured Solow residuals.
5. We estimate a model with demand shocks and compare it (favorably) with the estimates yielded by a standard model.
6. We conclude with a sales pitch, discussing natural extensions to the notions of demand shocks that are a lot more palatable than preference shocks (Bernanke and Gertler (1989) and others).

A Lucas-tree version of the model.

- Continuum of trees, measure $T = 1$. Each yields one fruit per period.

- Search friction: If a shopper finds a tree, then trade at price p; otherwise the fruit rots.

- So Consumption = Productivity and is endogenous.

- Competitive Search: Agents choose where to search.

- A "market" is characterized by a price and a "market tightness"

1. p: Price (numeraire: the value of the tree)
2. Q: Market tightness (avgerage available fruits per shopper).

Matching Technology

- Output equal the measure of matches:

$$Y = D^{\alpha} \; T^{1-\alpha}$$

- D is the measure of shoppers. α is a parameter.

- Recall: market tightness is $Q \equiv \frac{T}{D}$

- Probability that a tree is randomly matched with a shopper (i.e., number of matches per tree):

$$\Psi_T(Q) \equiv \frac{D^{\alpha} \; T^{1-\alpha}}{T} = \left(\frac{D}{T}\right)^{\alpha} = Q^{-\alpha} = \frac{Y}{T} \;\; = Y = C = D^{\alpha},$$

- Output and productivity depend only on how many shoppers.

Preferences:

- Many identical, infinitely lived, households. Utility is

$$E \sum_t \beta^t \, U(c_t, d_t, \theta_t),$$

where c_t is fruit consumption, d_t is the measure of shopping units (a search disutility). θ is a Markovian preference shock

- Focus on the case $U(c, d, \theta) = \theta_c \, u(c) \, - \, \theta_d \, d$.

- Consumption is # shopping units (d) times the probability of a unit finding a fruit (Ψ_D):

$$c = d \cdot \Psi_D(Q) \equiv d \; Q^{1-\alpha}.$$

- Households own s shares of the trees.

- Aggregate state: θ. Individual state: (θ, s).

The Values of agents: Households and Firms

Hhold: $$v(\theta, s) = \max_{c,d,s'} U(c, d, \theta) + \beta\, E\left\{v(\theta', s')|\theta\right\} \quad \text{s.t.}$$

$$p(\theta)\, c + s' = s\,[1 + R(\theta)]$$
$$c = d \cdot \Psi_D[Q(\theta)]$$

Firms: $$1 + R(\theta) = \varsigma(\theta) + E\left\{\frac{1 + R(\theta')}{1 + R(\theta')}\right\} = p(\theta)\, \Psi_T(Q) + 1$$

- Equilibrium objects are 4 (really only 2)
  1. Price of consumption (in terms of units of tree): $p(\theta)$.
  2. Market tightness: $Q(\theta)$.
  3. Consumption: $C(\theta) = \Psi_T[Q(\theta)]$.
  4. Dividends from trees: $R(\theta) = p(\theta)\, \Psi_T[Q(\theta)]$.

Consumption rate of return: $1 + r(\theta') = \frac{p(\theta)\,[1+R(\theta')]}{p(\theta')}$

Digression Standard Lucas-tree model: Eq object is just $p(\theta)$

1. Market tightness: $Q(\theta) = \infty$ or $\Psi_T(\infty) = 1$.
2. Consumption $C(\theta) = \Psi_T[Q(\theta)] = 1$.
3. Dividends from trees: $R(\theta) = p(\theta)\, \Psi_T[Q(\theta)] = p(\theta)$,

$$v(\theta, s) = \max_{c,d,s'} U(c, d, \theta) + \beta\, E\left\{v(\theta', s')|\theta\right\} \quad \text{s.t.}$$

$$p(\theta)\, c + s' = s\, [1 + R(\theta)]$$

$$d = 0$$

$$\text{Firms:} \qquad 1 + R(\theta) = p(\theta) + E\left\{\frac{1 + R(\theta')}{1 + R(\theta')}\right\} = p(\theta) + 1$$

- Equilibrium derives from FOC: $\frac{1}{p(\theta)} U_c(\theta) = \beta\, E\left\{\frac{1+p(\theta')}{p(\theta')}\, U_c(\theta')\right\}$

Back to search: Equil. cond. to determine (p, Q)

1. Euler equation:

$$\frac{\partial U}{\partial c} + \frac{\partial U}{\partial d}\frac{\partial d}{\partial c} \;=\; \theta_c u_c(C(\theta)) - \frac{\theta_d}{\Psi_D(Q)} \;=\; p(\theta)\; M(\theta),$$

where M is expected discounted marginal utility of saving,

$$M(\theta) = E\left\{\frac{[1 + R(\theta')]}{p(\theta')}\beta\left(\theta'_c u_{c'} - \frac{\theta'_d}{\Psi_D(Q')}\right) \mid \theta\right\}$$

2. Need one more equilibrium condition to determine Q.
It comes from competitive search.

Competitive Search in the Market for Goods

- This is the mechanism that determines the additional equilibrium object, market tightness.

- Shoppers choose which market to search in. Those markets are differentiated by p and Q.

- Let $R^* = p\,\Psi_T(Q)$ be the "outside value" for firms of going to the best market to sell their fruit. Shoppers take it as given.

- So shoppers can only open markets where trees get at least R^*:

$$R^* \leq \; p\,\Psi_T(Q)$$

The choice of market by the shopper

- Let θ_d be the (sunk) marginal utility cost of an extra shopper. The rewards for the hhold to send a shopper to a (p, Q) market is

$$\Phi = \max_{p,Q} \quad \{-\theta_d + \Psi_d(Q)\ (\theta_c\ u_c\ -\ p\ M)\} \qquad \text{s.t.}$$

$$R^* \leq\ p\,\Psi_T(Q),$$

where again
$$M(\theta) = E\left\{ \tfrac{[1+R(\theta')]}{p(\theta')} \beta \left(\theta'_c u_{c'} - \tfrac{\theta'_d}{\Psi_D(Q')} \right) \Big| \, \theta \right\}.$$

The FOC is

$$0 = (1-\alpha)\ Q^{-\alpha}\ \ \theta_c\ u_c - M\ p\ Q^{-\alpha}$$

or

$$p\ =\ (1-\alpha)\ \ \frac{\theta_c\ u_c}{M}$$

Summary of Equilibrium

- The two conditions that determine the two equilbrium objects $\{p, Q\}$.

1. The Hhold Euler $U_c - \frac{U_d}{\Psi_D} = p\ M$ or

$$\left(\theta_c u_c[C(\theta)] - \frac{\theta_d}{\Psi_D(Q)}\right) = p(\theta)\ E\left\{\frac{[1+R(\theta')]}{p(\theta')}\ \beta\ \left(\theta'_c u_{c'}[C(\theta')] - \frac{\theta'_d}{\Psi_D(Q')}\right)\Bigg|\ \theta\right\}$$

2. The Search Equilibrium Condition $(1-\alpha)\ U_c = p\ M$ or

$$(1-\alpha)\ \theta_c\ u_c[C(\theta)] = p\ E\left\{\frac{[1+R(\theta')]}{p(\theta')}\ \beta\ \left(\theta'_c u_{c'}[C(\theta')] - \frac{\theta'_d}{\Psi_D(Q')}\right)\Bigg|\ \theta\right\}$$

An example:

- Let $u(c) = \log c$ and θ be i.i.d., with $E\{\theta_c\} = E\{\theta_d\} = 1$ and $\theta_d/\theta_c \leq \alpha$. Then,

$$
\begin{aligned}
D(\theta) &= \alpha \frac{\theta_c}{\theta_d} \\
C(\theta) &= (D(\theta))^{\alpha} = \left(\alpha \frac{\theta_c}{\theta_d}\right)^{\alpha} \\
p(\theta) &= \left(\frac{1}{\beta} - 1\right) \left(\frac{\theta_d}{\alpha}\right)^{\alpha} \theta_c^{1-\alpha} \\
R(\theta) &= \left(\frac{1}{\beta} - 1\right) \theta_c
\end{aligned}
$$

- Note: "TFP" can be defined as $Y = TFP \cdot T = (\alpha\, \theta_c/\theta_d)^{\alpha}$, so TFP is driven by demand shocks

Intuition: express stuff in units of consumption

- Price of the tree in terms of consumption units:

$$\frac{1}{p(\theta)} = \frac{\beta}{1-\beta}\,\frac{C(\theta)}{\theta_c} = \frac{\beta}{1-\beta}\,\frac{1}{\theta_c u_c}.$$

(Lucas model has the same price of the tree in terms of c)

- Dividends in terms of consumption units:

$$\frac{R(\theta)}{p(\theta)} = C(\theta)$$

(... as in the Lucas model)

- The interest rate (in terms of consumption) is

$$1 + r(\theta) = \frac{\theta_d^{\alpha}\ \theta_c^{1-\alpha}}{\beta\ E\left\{\theta_d'^{\alpha}\ \theta_c'^{1-\alpha}\right\}}$$

$\Rightarrow$ $r(\theta)$ is increasing in θ_c, with elasticity $1-\alpha$.

Comparison with the standard Lucas tree model

- Lucas model: Lucas tree model $\alpha \to 0 \;\; \Rightarrow \;\; Y = C, \; D = 0$, and

$$
\begin{aligned}
p(\theta) &= \left(\frac{1}{\beta} - 1\right) \theta_c \\
R(\theta) &= p(\theta)\; Y \\
1 + r(\theta) &= \frac{\theta_c}{\beta \; E\left\{(\theta_c')\right\}} = \frac{\theta_c}{\beta}.
\end{aligned}
$$

- Aggregate consumption is invariant to the demand shock (so the elasticity is zero)

- All the adjustment to θ_c takes place through the prices:
  - The elasticity of $1 + r$ and p to θ_c is unity
  - In search model, the elasticity is $1 - \alpha$

Putting the model to work: the Growth Version

- We put a growth model with capital investement and labor choice with the shopping structure that we have developed.

- Some important changes.

1. There is varying capacity or output potential that we denote F and that is the productive capacity.

2. Both households (when purchasing consumption goods) and firms (when purchasing investment goods) face search frictions.

3. In this model capital and wealth are NOT the same. The locations have intrinsic value. Extensions will have creation of new locations as a form of investment.

4. All this generates subtle calibration issues.

Production

- Measure one of firms–locations with installed capital k (depreciates at rate δ). Goods can be used for consumption or investment and capacity is

$$F(k, n) = \bar{z} \;\; k^{\gamma_k} \; n^{\gamma_n}$$

- New capital has to be purchased that requires shoppers n^k.

- Shoppers and sellers trade in decentralized markets at prices (in terms of shares of the economy's wealth) p^i if investment and p^c if consumption.

- Unmatched capacity rots.

Households

Preferences are

$$E\left\{\sum_t \beta^t\, U(c, n, d, \theta)\right\}, \qquad \theta,\ \text{Markovian}$$

Again consumption requires that it is shopped so

$$c = d\ \ \Psi_d(Q^c)\, F^c$$

- $\Psi_d(Q^c)$ is the probability of matching a consumption firm, Q^c is market tightness in the consumption good market and F^c is output capacity in a consumption location.

- Households own the firms.

A few lemmas alleviate notation

1. The state of the economy is the pair $\{\theta, K\}$.
2. There is only one active market in consumption goods and another in investment goods.
3. Firms that produce consumption and investment choose the same inputs.
4. Consumption and investment firms get the same expected revenue (but not necessarily the same price and market tightness).

Consumption (or invt) firms in a (p^c, Q^c) submarket

$$
\begin{aligned}
\Omega(\theta, K, k) \quad = \quad & \max_{n^y, n^k, k', i} \Psi_T(Q^c)\, F(k, n^y)\, p^c - w(\theta, K)\,(n^c + n^k) \\
& -p^i(\theta, K)\, i + E\left\{ \frac{\Omega(\theta', K', k')}{1 + R(\theta', K')} \middle| \theta \right\}
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \qquad & i = (n^k\, \zeta)\ \ \Psi_d[Q^i(\theta, K)]\ \ F[K, N^y(\theta, K)] \\
& k' = i + (1 - \delta)k \\
& K' = G(\theta, K)
\end{aligned}
$$

with FOC (and RA condition)

$$
N^y(\theta, K) \quad = \quad \left(K^{\gamma_k}\, p^c \frac{\Psi_d(Q^c)}{Q^c} \frac{\overline{z}\, \gamma_n}{w(\theta, K)} \right)^{\frac{1}{1-\gamma_n}} .
$$

$$
E\left\{ \frac{\Omega_3(\theta', K', K')}{1 + R(\theta', K')} \middle| \theta \right\} \quad = \quad \frac{w(\theta, K)}{\zeta\, \Psi_d[Q^i(\theta, K)]\, F^i(K, N)} + p^i(\theta, K)
$$

The household problem

$$v(\theta, K, s) \;=\; \max_{c,d,n,s'} \; U(c,d,n,\theta) + \beta\, E\left\{v(\theta', K', s') | \theta\right\} \qquad \text{s.t.}$$

$$p^c(\theta, K)\; c + s' = s\,[1 + R(\theta, K)] + n\, w(\theta, K)$$

$$c = d\; \Psi_d[Q^c(\theta, K)]\;\; F[K, N^y(\theta, K)]$$

$$K' = G(\theta, K)$$

- Hholds' FOC (and RA)

$$U_c - \frac{U_d}{\Psi_d\; F} \;=\; \beta E\left\{ \frac{p^c\;(1+R')}{p^{c\prime}} \;\left[U_c' - \frac{U_d'}{\Psi_d\; F'}\right] \;|\theta\right\},$$

$$U_c - \frac{U_d}{\Psi_d\; F} \;=\; U_n\; \frac{p^c}{w}.$$

Competitive Search in Markets

- As in the tree economy, competitive search yields price and market tightness (they are different) in both markets.

- We get two additional conditions from the FOC of shoppers given expected revenue for sellers.

- The equilibrium objects are functions of (θ, K) for

$$\left\{ Q^c, Q^i, N^y, N^k, N, p^c, p^i, R, G, T^c \right\}.$$

Recursive Equilibrium

1. *Households and firms* solve their problems (4).
2. *Competitive Search Conditions.* (2).
3. *Representative Agent Conditions*
4. *Equal Profit Condition:* $p^i \, \Psi_T(Q^i) = p^c \, \Psi_T(Q^c)$.
5. *Market Clearing Conditions:*

$$\begin{aligned} N &= N^y + N^k = N, \\ C &= T^c \, \Psi_T(Q^C) \, F(K, N^y). \end{aligned}$$

6. Value of the firms is 1.

An aside: The Equilibrium is not Optimal

- The problem is not competitive search by itself (see Kircher (2010)).

- It seems that it is the unpaid facilitation that work has on other people's search:

$$U_n = -\left(U_c - \frac{U_d}{\Psi_D(Q^c)\,F}\right) \Psi_T(Q^c)\,F_n \qquad \text{EQ Intrat FOC}$$

$$U_n = -\;U_c \qquad \Psi_T(Q^c)\,F_n \qquad \text{SP Intrat FOC}$$

Putting the model to work

- We want a clear version of this model. So separable utility with constant Frisch elasticiy and Cobb-Douglas technology. We will place shocks on preferences and on the investment shopping technology.
  - Preferences

$$\tilde{u}(c, n, d, \theta) = \theta_c \frac{c^{1-\sigma}}{1-\sigma} - \theta_n \chi \frac{n^{1+\psi}}{1+\psi} - \theta_d d$$

  - Production function

$$F(k, n^c) = \overline{z} k^{\gamma_k} (n^c)^{\gamma_n}$$

  - Shocks

$$\log(\theta_t) = \rho_\theta \log(\theta_{t-1}) + v_t, \quad v_t \sim N(0, \Sigma^2)$$

Calibration

- There are 11 parameters.
  - Preferences: $\{\beta, \sigma, \chi, \psi\}$
  - Production Technology: $\{\bar{z}, \gamma_k, \gamma_n, \delta\}$.
  - Matching technologies: $\{A, \alpha, \bar{\zeta}\}$.

- Some moments are Standard.

Rate of return	.04
Coefficient of Risk Aversion	2.
Frisch Elasticity of Labor	.7
Time spent working	.3
Labor Share	.67
Investment to Output Ratio	.20
Physical Capital to Output Ratio	2.75

Calibration

- The other moments are specific to this economy

Share of production workers	.96
Capacity in Consumption Industries	.82
Capacity in Investment Industries	.80
Wealth to Output Ratio	3.33

- This calibration uniquely specifies the model economy.

Let's see what the model delivers

- Two Questions:

1. Can shocks to preferences generate the measured Solow residual? We compute the Solow residual z_t^{SR} in the U.S. data :

$$z_t^{SR} = \log GNP_t - (Average\ \ Capital\ \ Share)\quad \log Capital_t\ - \\ (1 - Average\ \ Capital\ \ Share)\quad \log Labor_t$$

   We then estimate processes for the shocks in our model economy using the measure of the Solow residual in the model economy.

2. If so, how does our economy look in terms of business cycle behavior?

Q. 1: Can pref shocks generate measured Solow residual?

- The main moments of the (linearly detrended) Solow residual in the data (1960.Q1–2006.Q4) are a variance of 2.74, and an autocorrelation of .93.

- We estimate (Maximum Likelihood) a process for θ_d that yields

	Estimate	st. dev.	t-stat
ρ_{θ_d}	0.934	0.025	38.13
σ_{θ_d}	0.081	0.004	19.39

- Which yields a variance of the measured Solow residual of 2.75 and an autocorrelation of 0.93 (and a likelihood of 694.8)

- The answer is **yes!!!**

Question 2. Evaluating the behavior of our economies

- We proceed by (Max-Lik) estimating shock processes for two sets of economies.

1. A standard RBC economy with productivity shocks (and sometimes labor supply shocks too).
2. Our model economy with only preference shocks (and also investment demand shocks).

- We compare the two sets of economies.

A reminder: the U.S. business cycles

Table: Data moments: 1960.Q1–2006.Q4

	Variance	Cor w Y	Auto-cor
z	2.74	0.78	0.93
Y	**2.30**	1.00	0.86
N	**0.95**	0.86	0.91
C	0.66	0.86	0.87
I	13.87	0.92	0.79
p_i/p_c	0.47	-0.23	0.92
S&P 500	33.49	0.33	0.82
cor(C, I)	**0.72**		

NOTES: The Solow residual is linearly filtered.

The variances of HP-filtered series are relative to that of output. The likelighood is also 694.8.

A reminder: Data and the Standard RBC model with TFP shocks (likelihood=694.8)

Table: Data

(a) Data | (b) Standard RBC

HP-filtered	Variance	Cor w Y	Acor	Variance	Cor w Y	A-cor
z	2.74	0.78	0.93	2.74	0.99	0.93
Y	**2.30**	1.00	0.86	**0.86**	1.00	0.71
N	**0.95**	0.86	0.91	**0.07**	0.97	0.71
C	0.66	0.86	0.87	0.05	0.93	0.77
I	13.87	0.92	0.79	17.87	1.00	0.71
S&P 500	33.49	0.33	0.82	-	-	-
cor(C, I)	**0.72**			**0.90**		

- As we know hours do not move much unless we use a large elasticity.

Data, the Standard RBC (TFP shocks), and ours with θ_d

	(c) Data			(d) Standard RBC			(e) Our Model		
	Var	C-w-Y	Acor	Var	C-w-Y	A-cor	Var	Cor-w- Y	A-cor
z	2.74	0.78	0.93	2.74	0.99	0.93	2.75	1.00	0.93
Y	**2.30**	1.00	0.86	**0.86**	1.00	0.71	**0.24**	1.00	0.71
N	**0.95**	0.86	0.91	**0.07**	0.97	0.71	**0.15**	**-1.00**	0.72
C	0.66	0.86	0.87	0.05	0.93	0.77	1.31	1.00	0.71
I	13.87	0.92	0.79	17.87	1.00	0.71	0.18	0.99	0.69
p_i/p_c	0.47	-0.23	0.92	-	-	-	2.63	1.00	0.71
S&P 500	33.49	0.33	0.82	-	-	-	4.18	1.00	0.71
cor(C, I)	**0.72**			**0.90**			**.99**		

NOTES: The Solow residual is linearly filtered. The variances of HP-filtered series are relative to that of output.

- Consumption and investment move well but hours are awful (small volatility and negative correlation.)

- A shock to θ_d is like a positive wealth effect (so more consumption, more investment and more leisure).

As Richard and Andrés wanted, what if Frisch=1.1, and again θ_d

	(f) Data			(g) Standard RBC			(h) Our Model		
	Var	C-w-Y	Acor	Var	C-w-Y	A-cor	Var	Cor-w- Y	A-cor
z	2.74	0.78	0.93	2.74	0.99	0.93	2.74	1.00	0.94
Y	**2.30**	1.00	0.86	**0.98**	1.00	0.71	**0.29**	1.00	0.71
N	**0.95**	0.86	0.91	**0.11**	0.97	0.71	**0.31**	**-1.00**	0.71
C	0.66	0.86	0.87	0.04	0.92	0.77	1.47	1.00	0.71
I	13.87	0.92	0.79	18.27	1.00	0.71	0.02	0.99	0.68
p_i/p_c	0.47	-0.23	0.92	-	-	-	3.14	1.00	0.71
S&P 500	33.49	0.33	0.82	-	-	-	4.18	1.00	.71
cor(C, I)	**0.72**			**0.89**			**.99**		

NOTES: The Solow residual is linearly filtered. The variances of HP-filtered series are relative to that of output.

- Hours move more. Same features.

A different type of preference shock θ_c

- This is a shock to the utility of consumption (alternatively it is a negative shock to all leisures, future consumptions and disutility of shopping).

- The likelyhood is 695.6. The estimates are

	Estimate	s.d	t-stat
ρ_{θ_c}	0.851	0.07	12.97
σ_{θ_c}	0.144	0.01	14.66

- Which yields a variance of the measured Solow residual of 2.71 and an autocorrelation of 0.87.

- Again success in replicating the behavior of the Solow residual.

Data, Standard RBC (TFP shocks), and ours w θ_c shocks

	(i) Data			(j) Standard RBC			(k) Our Model		
	Var	C-w-Y	Acor	Var	C-w-Y	A-cor	Var	Cor-w- Y	A-cor
z	2.74	0.78	0.93	2.74	0.99	0.93	2.71	0.87	0.93
Y	**2.30**	1.00	0.86	**0.86**	1.00	0.71	**5.01**	1.00	0.65
N	**0.95**	0.86	0.91	**0.07**	0.97	0.71	**1.21**	0.97	0.70
C	0.66	0.86	0.87	0.05	0.93	0.77	5.62	1.00	0.65
I	13.87	0.92	0.79	17.87	1.00	0.71	20.16	-1.00	0.64
p_i/p_c	0.47	-0.23	0.92	-	-	-	0.02	0.97	0.70
S&P 500	33.49	0.33	0.82	-	-	-	1.90	-0.87	0.78
cor(C, I)	**0.72**			**0.90**			**-1.00**		

- Hours move plenty and positively correlated but investment is negatively correlated.

- The more expensive labor makes it unattractive to invest.

Data, Standard RBC (TFP shocks), and ours with θ_d, θ_n

	(l) Data			(m) Standard RBC			(n) Our Model		
	Var	C-w-Y	Acor	Var	C-w-Y	A-cor	Var	Cor-w- Y	A-cor
z	2.74	0.78	0.93	2.74	0.99	0.93	2.75	0.74	0.92
Y	**2.30**	1.00	0.86	**0.86**	1.00	0.71	**1.11**	1.00	0.71
N	**0.95**	0.86	0.91	**0.07**	0.97	0.71	**0.61**	**0.76**	0.69
C	0.66	0.86	0.87	0.05	0.93	0.77	0.58	0.95	0.74
I	13.87	0.92	0.79	17.87	1.00	0.71	5.47	0.91	0.68
p_i/p_c	0.47	-0.23	0.92	-	-	-	0.63	0.75	0.70
S&P 500	33.49	0.33	0.82	**0.90**			1.76	0.93	0.70
cor(C, I)	**0.72**						**.73**		

- We targetted $\{z, y\}$ but is pretty good.
- Variance decomposition.
  - Shocks to θ_d account for bw 88% and 99% of the variance of TFP.
  - Shocks to θ_n account for bw 91% and 98% of the variance of Labor.
  - For output is more split. Between 74% and 16% for θ_d.

A very preliminary attempt to do full estimation

- Estimate by CI ML 4 variables (detrended output, labor, Solow residual (TFP) and investment) and four uncorrelated shocks (θ_d, θ_n, z ζ).

Table: Variance Decomposition in percentages: 1960.Q1–2006.Q4

	θ_d	θ_n	z	ζ
TFP	31.8	10.3	**45.7**	12.2
N	0.8	94.8	1.2	3.2
Y	5.1	58.6	21.8	14.5
I	0.0	17.4	16.2	66.4
C	6.2	65.4	14.1	14.3
p_i/p_c	4.4	1.0	0.0	94.6

- Productivity Shocks are now less than half of what moves TFP.
- Still most of the action comes from shocks to labor.
- The demand shocks are a big part of the fluctuations.

Let's reestimate with R & A Frisch's (1.1)

- Estimate by CI ML 4 variables (detrended output, labor, Solow residual (TFP) and investment) and four uncorrelated shocks (θ_d, θ_n, z ζ).

Table: Variance Decomposition in percentages: 1960.Q1–2006.Q4

	θ_d	θ_n	z	ζ
TFP	66.5	9.3	**14.0**	10.2
N	2.8	91.4	0.41	5.3
Y	5.3	55.0	4.48	35.2
I	0.1	17.5	4.68	77.8
C	14.4	69.7	3.12	12.8
p_i/p_c	11.4	1.0	0.0	87.5

- Productivity Shocks are now less than one sixth of what moves TFP.
- Still most of the labor action comes from shocks to labor.
- The demand shocks are again a big part of the fluctuations.

Conclusions

1. We have constructed a model were demand (preference) shocks generate fluctuations that move productivity.

2. This structure is quantitatively powerful. It generates observed movements in productivity.

3. It is very easy to use (dynare code is on the web).

4. We think that it is very promising. It may prove powerful in analyzing things like the effects of stimulus packages.

5. It is important to develop identification strategies to separate technology shocks from demand shocks like those in this paper all of which may show up as changes in the Solow residual.

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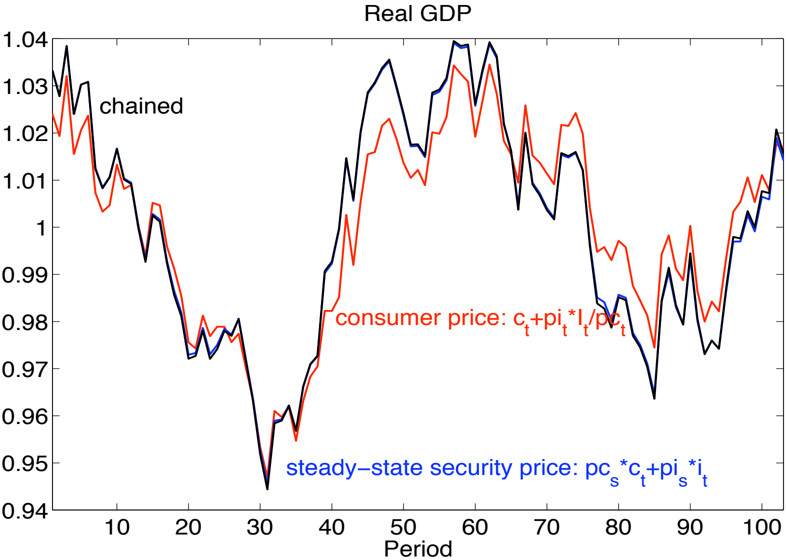
A few more things

- The exact manner in which we measure output does not really matter that much: Ways to measure output

  1. Consumption Goods units. $C_t + \frac{p_t^i}{p_t^c} I_t$.
  2. Base year prices (Old NIPA) $GDP_t = C_t \; p_0^c + I_t \; p_0^i$.
  3. Chained-Indexed prices (New NIPA)

- We use base year prices. It is just easier (in Dynare).


Real GDP
1.04
1.03
1.02
1.01
1
0.99
0.98
0.97
0.96
0.95
0.94
10
20
30
40
50
60
70
80
90
100
Period
chained
consumer price: c_t+pi_t*I_t/pc_t
steady–state security price: pc_s*c_t+pi_s*i_t

Equilibrium

- Decisions: $c(\theta, s)$, $d(\theta, s)$, $s'(\theta, s)$; aggregates $D(\theta)$ $Q(\theta) = \frac{1}{D(\theta)}$, $C(\theta)$, $p(\theta)$ and $R(\theta)$ such that

1. Households solve their problem.
2. Representative Agent Condition:

$$\begin{aligned} D(\theta) &= d(\theta, 1) \\ C(\theta) &= c(\theta, 1) = D(\theta)^{\alpha} \end{aligned}$$

3. Equilibrium in the asset market

$$s'(\theta, 1) = 1$$

4. Equilibrium in the good markets (via competitive search)

$$\begin{aligned} p(\theta) &= (1-\alpha) \frac{\theta_c \ u_c(C(\theta))}{E\left(\frac{[1+R(\theta')]}{p(\theta')}\beta \left(\theta'_c u_{c'} - \theta'_d D'^{\alpha}\right) \mid \theta\right)} \\ R(\theta) &= p(\theta) \ D^{\alpha}. \end{aligned}$$

Table: Standard RBC model with z and θ_c (likelihood=1422.6)

(a) Estimates and Model Behavior

	Estimate	s.d	t-stat
ρ_z	0.938	0.021	45.37
σ_z	0.006	0.000	19.43
ρ_{θ_c}	0.963	0.016	45.37
σ_{θ_c}	0.041	0.009	4.62
cor(z, θ_c)	-0.064	0.079	0.81

Non-filt	Variance	Cor w Y	Auto-cor
z	2.95	0.81	0.94
Y	7.14	1.00	0.95
N	13.04	0.41	0.98

HP-filt	Variance	Cor w Y	Auto-cor
z	0.50	0.80	0.71
Y	1.20	1.00	0.71
N	0.82	0.71	0.72
C	2.82	0.63	0.72
I	28.09	0.14	0.71
cor(C, I)	-0.68		

(b) Variance decomp

Non-filtered	z	θ_c
z	100.00	0.00
Y	68.12	31.88
N	3.53	96.47

Filtered	z	θ_c
z	100.00	0.00
Y	64.24	35.76
N	2.91	97.09

Table: Shock on θ_c only (likelihood=695.6)

	Estimate	s.d	t-stat
ρ_{θ_d}	0.835	0.070	12.005
σ_{θ_d}	0.144	0.010	14.663

Non HP-filtered	Variance	Cor with Y	Auto-cor
z	2.73	0.83	0.93
Y	11.77	1.00	0.79
N	26.38	0.90	0.90
p_i/p_c	0.32	0.90	0.90

HP-filtered series	Variance	Cor with Y	Auto-cor
z	0.10	0.90	0.74
Y	6.71	1.00	0.63
N	1.25	0.96	0.70
C	6.48	1.00	0.63
I	28.19	-1.00	0.63
$p_i I/p_c$	26.96	-1.00	0.63
p_i/p_c	0.02	0.96	0.70
cor$(C, p_i I/p_c)$	-1.00		
cor(C, I)	-1.00		

Table: Shock on θ_d only (likelihood=694.8)

	Estimate	s.d	t-stat
ρ_{θ_d}	0.934	0.025	38.131
σ_{θ_d}	0.081	0.004	19.390
Non HP-filtered	Variance	Cor with Y	Auto-cor
z	2.75	1.00	0.93
Y	2.01	1.00	0.94
N	0.20	-1.00	0.95
p_i/p_c	2.91	1.00	0.93
HP-filtered series	Variance	Cor with Y	Auto-cor
z	1.42	1.00	0.71
Y	0.42	1.00	0.71
N	0.09	-1.00	0.72
C	0.74	1.00	0.71
I	0.11	0.99	0.69
$p_i I/p_c$	2.41	1.00	0.70
p_i/p_c	1.49	1.00	0.71
cor$(C,_i I/p_c)$	1.00		
cor(C, I)	0.99		

Table: Uncorrelated Shocks on θ_d and θ_n (likelihood=1412.3)

(a) Estimates and Model Behavior

	Estimate	s.d	t-stat
ρ_{θ_d}	0.908	0.021	44.17
σ_{θ_d}	0.083	0.004	19.23
ρ_{θ_n}	0.952	0.017	55.78
σ_{θ_n}	0.019	0.001	16.47

Non-filtered	Variance	Cor w Y	Auto-cor
z	2.20	0.37	0.91
Y	4.91	1.00	0.95
N	4.69	0.69	0.93
p_i/p_c	2.27	0.44	0.91

HP-filtered	Variance	Cor w Y	Auto-cor
z	0.72	0.67	0.70
Y	0.87	1.00	0.71
N	1.16	0.55	0.70
C	0.51	0.97	0.73
I	4.33	0.80	0.69
$p_i I/p_c$	5.42	0.96	0.69
p_i/p_c	0.75	0.65	0.70
$\text{cor}(C, I)$	0.64	$\text{cor}(C, \frac{p_i}{p_c} I)$	0.86

(b) Variance decomp

Non-filtered	θ_d	θ_n
z	94.25	5.75
Y	32.93	67.07
N	2.76	97.24

Filtered	θ_d	θ_n
z	99.19	0.81
Y	52.44	47.56
N	2.96	97.04

Table: Correlated Shocks on θ_d and θ_n (likelihood=1433.2)

(a) Estimates and Model Behavior

	Estimate	s.d	t-stat
ρ_{θ_d}	0.902	0.021	42.29
σ_{θ_d}	0.083	0.004	19.20
ρ_{θ_n}	0.961	0.016	60.33
σ_{θ_n}	0.019	0.001	15.76
$cor(\theta_d, \theta_n)$	0.448	0.059	7.66

Non-filtered	Variance	Cor w Y	Auto-cor
z	1.78	0.52	0.89
Y	7.40	1.00	0.95
N	4.85	0.85	0.95
p_i/p_c	1.87	0.60	0.90

HP-filtered	Variance	Cor w Y	Auto-cor
z	0.47	0.81	0.69
Y	1.24	1.00	0.71
N	0.69	0.74	0.70
C	0.50	0.99	0.73
I	3.29	0.91	0.69
$p_i I/p_c$	5.08	0.98	0.69
p_i/p_c	0.48	0.80	0.69
$cor(C, I)$	0.85	$cor(C, p_i I/p_c)$	0.94

(b) Variance decomp

First θ_d, then θ_n

Non-HP filtered	θ_d	θ_n
z	93.09	6.91
Y	57.42	42.58
N	10.51	89.49

HP filtered	θ_d	θ_n
z	99.31	0.69
Y	73.77	26.23
N	8.79	91.21

First θ_n, then θ_d

Non-HP filtered	θ_d	θ_n
z	87.75	12.25
Y	16.63	83.37
N	1.94	98.06

HP filtered	θ_d	θ_n
z	84.01	15.99
Y	29.55	70.45
N	2.65	97.35

Table: Uncorrelated Shocks on θ_d, θ_n, and ζ (likelihood=2081.7)

(a) Estimates and Model Behavior

	Estimate	s.d	t-stat
ρ_{θ_d}	0.923	0.020	46.60
σ_{θ_d}	0.082	0.004	19.23
ρ_{θ_n}	0.979	0.012	80.37
σ_{θ_n}	0.019	0.001	16.28
ρ_{ζ}	0.985	0.008	127.23
σ_{ζ}	0.095	0.005	19.21

Non-filt	Variance	Cor w Y	Auto-cor
z	2.88	0.05	0.93
Y	9.66	1.00	0.97
N	9.56	0.72	0.97
p_i/p_c	21.07	-0.24	0.98
Filt	Variance	Cor w Y	Auto-cor
z	0.74	0.66	0.70
Y	0.82	1.00	0.72
N	1.18	0.53	0.71
C	0.61	0.94	0.73
I	7.96	0.59	0.70
$p_i I/p_c$	5.46	0.89	0.69
p_i/p_c	1.84	0.31	0.71
$cor(C, I)$	0.28	$cor(C, p_i I/p_c)$	0.67

(b) Variance decomp

Non-filt	θ_d	θ_n	ζ
z	82.55	9.22	8.24
Y	18.60	69.05	12.35
N	1.68	96.28	2.03
p_i/p_c	12.02	0.70	87.28

Filt	θ_d	θ_n	ζ
z	98.88	0.80	0.33
Y	52.38	45.20	2.42
N	3.40	93.52	3.07
p_i/p_c	41.57	0.49	57.93

Table: Correlated Shocks on θ_d, θ_n, and ζ (likelihood=2169.6)

(a) Estimates and Model Behavior

	Estimate	s.d	t-stat
ρ_{θ_d}	0.958	0.014	68.42
σ_{θ_d}	0.082	0.004	19.36
ρ_{θ_n}	0.983	0.011	87.20
σ_{θ_n}	0.020	0.001	15.94
ρ_ζ	0.996	0.004	251.25
σ_ζ	0.095	0.005	19.37
$cor(\theta_d, \theta_n)$	0.374	0.064	5.88
$cor(\theta_d, \zeta)$	-0.748	0.032	23.08
$cor(\theta_n, \zeta)$	-0.273	0.071	3.84
Non-filt	Variance	Cor with Y	Auto-cor
z	3.89	0.23	0.95
Y	23.39	1.00	0.98
N	11.13	0.61	0.98
p_i/p_c	57.16	-0.57	1.00
HP-filt	Variance	Cor w Y	Auto-cor
z	0.48	0.80	0.71
Y	1.24	1.00	0.73
N	0.71	0.72	0.71
C	0.51	0.98	0.76
I	5.99	0.91	0.70
$p_i I/p_c$	5.20	0.97	0.70
p_i/p_c	0.32	-0.04	0.73

(b) Variance decomp

Order: θ_d, θ_n, ζ

Non-filt	θ_d	θ_n	ζ
z	83.46	7.60	8.94
Y	61.26	31.97	6.77
N	3.36	94.34	2.30
p_i/p_c	48.98	0.24	50.79

HP filtered	θ_d	θ_n	ζ
z	99.14	0.74	0.12
Y	72.86	26.62	0.52
N	7.17	91.74	1.09
p_i/p_c	4.34	1.46	94.19

$cor(C, p_i I/p_c)$	0.90
$cor(C, I)$	0.82

Table: Partially Correlated Shocks on θ_d, θ_n, and ζ (likelihood=2094.6)

(a) Estimates and Model Behavior

ρ_{θ_d}	0.925	0.021	44.95
σ_{θ_d}	0.082	0.004	19.33
ρ_{θ_n}	0.984	0.011	89.05
σ_{θ_n}	0.019	0.001	15.95
ρ_ζ	0.991	0.007	140.49
σ_ζ	0.095	0.005	19.33
$cor(\theta_d,\theta_n)$	0.365	0.064	5.66

Non-filt	Variance	Cor w Y	Auto-cor
z	2.80	0.09	0.94
Y	14.68	1.00	0.98
N	11.91	0.78	0.98
p_i/p_c	33.76	-0.25	0.99

HP-filt	Variance	Cor w Y	Auto-cor
z	0.53	0.78	0.70
Y	1.10	1.00	0.72
N	0.76	0.68	0.71
C	0.61	0.96	0.73
I	5.64	0.64	0.70
p_iI/p_c	4.66	0.92	0.70
p_i/p_c	1.32	0.40	0.72
$cor(C, p_iI/p_c)$	0.78	$cor(C, I)$	0.42

(b) Variance decomp

Order: θ_d, θ_n, ζ

Non-filt	θ_d	θ_n	ζ
z	75.13	10.97	13.90
Y	35.12	52.27	12.60
N	7.85	89.68	2.46
p_i/p_c	6.67	0.53	92.79

HP filtered	θ_d	θ_n	ζ
z	98.96	0.73	0.31
Y	69.28	29.19	1.53
p_i/p_c	40.04	0.48	59.49
N	3.26	93.76	2.99

Table: High Frisch Elasticity: Shock on θ_d only (likelihood=694.8)

	Estimate	s.d	t-stat
ρ_{θ_d}	0.933	0.025	37.626
σ_{θ_d}	0.058	0.003	19.391
Non HP-filtered	Variance	Cor with Y	Auto-cor
z	2.76	1.00	0.93
Y	1.46	1.00	0.93
N	0.43	-1.00	0.93
p_i/p_c	2.97	1.00	0.93
HP-filtered series	Variance	Cor with Y	Auto-cor
z	1.87	1.00	0.71
Y	0.32	1.00	0.71
N	0.30	-1.00	0.70
C	0.84	1.00	0.71
I	0.01	-0.98	0.67
$p_i I/p_c$	1.77	1.00	0.71
p_i/p_c	2.03	1.00	0.71
cor$(C, p_i I/p_c)$	1.00		
cor(C, I)	-0.98		

Table: High Frisch Elasticity: Shock on θ_c only (likelihood=694.8)

	Estimate	s.d	t-stat
ρ_{θ_c}	0.914	0.050	18.324
σ_{θ_c}	0.102	0.006	16.505
Non HP-filtered	Variance	Cor with Y	Auto-cor
z	2.32	1.00	0.92
Y	54.57	1.00	0.91
N	81.58	1.00	0.92
p_i/p_c	0.24	1.00	0.92
HP-filtered series	Variance	Cor with Y	Auto-cor
z	0.04	1.00	0.70
Y	14.97	1.00	0.70
N	1.45	1.00	0.70
C	1.63	1.00	0.70
I	0.04	-0.98	0.66
$p_i I/p_c$	0.02	-0.96	0.65
p_i/p_c	0.00	1.00	0.70
cor(C, I)	-0.98		
cor$(C, p_i I/p_c)$	-0.96		

Table: High Frisch Elasticity: Correlated Shocks on θ_d, θ_n, and ζ (likelihood=2169.4)

(a) Estimates and Model Behavior

	Estimate	s.d	t-stat
ρ_{θ_d}	0.960	0.014	70.63
σ_{θ_d}	0.059	0.003	19.33
ρ_{θ_n}	0.990	0.009	111.86
σ_{θ_n}	0.013	0.001	13.69
ρ_ζ	0.995	0.005	220.56
σ_ζ	0.067	0.004	19.32
$cor(\theta_d, \theta_n)$	0.403	0.064	6.29
$cor(\theta_d, \zeta)$	-0.744	0.033	22.56
$cor(\theta_n, \zeta)$	-0.219	0.077	2.85
Non-filt	Variance	Cor w Y	Auto-cor
z	4.07	0.16	0.96
Y	23.75	1.00	0.98
N	15.08	0.64	0.98
p_i/p_c	45.71	-0.53	1.00
HP-filt	Variance	Cor w Y	Auto-cor
z	0.48	0.80	0.71
Y	1.21	1.00	0.72
N	0.74	0.70	0.70
C	0.49	0.97	0.75
I	6.64	0.90	0.69
$p_i I/p_c$	5.72	0.96	0.69
p_i/p_c	0.33	-0.05	0.73

(b) Variance decomp

Order: θ_d, θ_n, ζ

Non-filt	θ_d	θ_n	ζ
z	81.07	10.81	8.13
Y	58.48	36.96	4.57
N	1.84	95.12	3.03
p_i/p_c	47.92	0.12	51.96
HP-filt	θ_d	θ_n	ζ
z	98.88	0.92	0.19
Y	73.41	25.20	1.40
N	7.21	88.78	4.01
p_i/p_c	4.02	0.30	95.68

$cor(C, p_i I/p_c)$	0.86
$cor(C, I)$	0.78