# Accounting for Cross Country Differences in Intergenerational Earnings Persistence: The Impact of Taxation and Public Education Expenditure ${ }^{1}$ 

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#### Abstract

A growing body of empirical literature has documented that Western economies exhibits substantial differences in the degree of intergenerational earnings persistence between fathers' and sons'. Earnings persistence is relatively low in Northern Europe, and relatively high in the US, Britain, and Southern Europe. In this paper I first document that there is a strong negative correlation between earnings persistence and tax progressivity, and earnings persistence and public expenditure on tertiary education. I then develop an intergenerational life cycle model of human capital accumulation and earnings, which features taxation, public education expenditure, and borrowing constraints as determinants of earnings persistence. I calibrate the model to US data, and use it to quantify how earnings persistence in the US changes as I introduce policies from Denmark, the country with the highest and most progressive taxes, and greatest public expenditure on tertiary education in my sample. I find that the Danish policies reduce earnings persistence by reducing parental incentives for investing in human capital, and thereby creating a weaker relationship between the financial resources of the parent and the earnings of the child. Quantitatively, taxation is most important. Introducing a Danish tax policy in the US reduces the intergenerational elasticity of earnings by 0.12 , or about $40 \%$ of the difference between the US and the Scandinavian countries, which have the lowest earnings persistence among the countries in my sample. I also find that intragenerational borrowing constraints have very limited impact on earnings persistence.


[^0]
## 1 Introduction

In recent years, several empirical studies have been concerned with estimating and comparing the intergenerational persistence of earnings between fathers and sons in Western economies. The main finding of this literature is that intergenerational persistence is relatively high in the US, Britain, and Southern Europe, and relatively low in Northern Europe, and in Canada. Table 1 below displays the results from a meta study of intergenerational earnings persistence across countries by Heinz Corak (2006) ${ }^{3}$, supplemented with two recent studies from Italy and Spain ${ }^{4}$. The next question follows naturally: What are the reasons for these differences? Western economies differ greatly with respect to public expenditure on education, and with respect to tax schemes. Does the cross country variation in public institutions explain the variation in earnings persistence? Understanding why earnings mobility differs across countries is interesting, even if only for positive reasons. However, the question of whether economic fate is predetermined or whether it is influenced by public institutions may also have important policy implications. For instance if the pattern we observe is due to poor parents in some countries being borrowing constrained from investing optimally in their children's human capital, it may call for policy intervention.

Several explanations that could contribute to the observed cross country pattern in intergenerational earnings persistence have been proposed in the economic literature but there is little quantitative work in the area. There are no previous papers studying the impact of cross country differences in policies on earnings persistence. I start by documenting that there is a strong negative correlation between earnings persistence and tax progressivity, and earnings persistence and public expenditure on tertiary education. I then provide a rich quantitative intergenerational life cycle model of human capital accumulation and earnings. The model determinants of earnings persistence include taxation (or more generally returns to human capital investments), public education expenditure, borrowing constraints, partially inheritable abilities, inter vivos transfers from parent to child, and idiosyncratic wage shocks. I calibrate the model to US data, and decompose the contributions of the different model elements to earnings

[^1]persistence by shutting them down and reintroducing them in the model one by one. Next I study how earnings persistence in the US changes as I introduce policies from Denmark. Denmark is the country in my sample with the highest and most progressive taxes and greatest expenditure on tertiary education. I find that taxation and public education expenditure have a significant impact on earnings persistence and are likely contributors to the cross country patterns which empirical researchers have found. The impact of taxation is quantitatively greater. Introducing a Danish tax system in the US, reduces the intergenerational elasticity of earnings by 0.12 , or about $40 \%$ of the difference between the US and the Scandinavian countries, which have the lowest earnings persistence among the countries in my sample. I also experiment by tightening and loosening the intragenerational borrowing constraints in the model and conclude that they have very little impact on earnings persistence.

## Table 1: Intergenerational Earnings Elasticity Across Countries

| Country | Estimated Earnings Elasticity |
| :--- | :--- |
| Denmark | 0.15 |
| Norway | 0.17 |
| Finland | 0.18 |
| Canada | 0.19 |
| Sweden | 0.27 |
| Germany | 0.32 |
| Spain** | 0.40 |
| France | 0.41 |
| Italy* | 0.43 |
| USA | 0.47 |
| UK | 0.50 |

This table displays the results from a meta study by Heinz Corak (2006). *Taken from Piraino (2007), and adjusted using a formula from Corak (2006). **Taken from Pla (2009) ${ }^{4}$.

## Determinants of Earnings Persistence

In classical human capital theory, it is usually assumed that the earnings of individuals depend on their level of human capital and on market luck, or random shocks. Two factors go into human capital formation. One is a fixed endowment, imperfectly inherited from parents to children, and the other is investments in human capital, which can be made both by the parents and by the

[^2]government, see Becker and Tomes (1979) and (1986), Solon (2004). Endowments here refer to everything from genetically inherited ability to knowledge acquired from the parents, family culture, and the social connections of the parents. In my model below I will refer to the family endowment as ability. The narrowest definition of human capital investments is investments in education, but many authors use broader definitions. It is also commonly assumed that parents care about their children's utility, and that utility depends only on consumption of goods that cannot be considered as investments in human capital, see Becker and Tomes (1986). This way, the only reason to invest in children's human capital is to increase their future consumption through higher earnings. If there are diminishing returns to investments, there will be an optimal level of investment for each child.

From this theory, several explanations for cross country differences in earnings persistence emerge. One possibility is that the inheritability of family endowments is stronger in some countries. There could be many underlying reasons for this. The degree of assortative mating does, for instance, differ across countries. In some countries, couples are more similar in the aspects of education and family background, and since almost all research studies the correlation between fathers and sons, this will cause the sons to be more similar to their fathers. Indeed, there seem to be somewhat higher correlation in spousal education in the US and Italy than in Northern Europe but Britain, which has relatively high earnings persistence, has a relatively low correlation in spousal education ${ }^{5}$.

Han and Mulligan (2001) point out that it is not necessarily only the inheritability of family endowments that matters but also the variance. As they increase the variance of the family endowments in their model, earnings persistence increases. If there is greater variance of family endowments in the US and Britain, perhaps because those countries are more racially and culturally diverse, then this theory could be used to explain higher earnings persistence. However, it is not an obvious result or theoretical implication that larger variance of family endowments should lead to larger and not smaller persistence. This is something that comes out of their specific model for specific parameter values.

[^3]Another possibility is that countries just differ in the returns to human capital or the cost of acquiring it. In standard intergenerational models of earnings formation, earnings persistence increases with the returns to human capital investments, see for instance Restuccia and Urrutia (2004). Depending on modeling choices, there are several channels through which this may work, but to mention a common one: Optimal human capital investments are usually increasing in parental financial resources, as altruistic parents face a tradeoff between their own consumption today and their children's future consumption. If human capital investments become more efficient, then for a given inequality of investments in children of high and low earners the inequality of earnings outcomes will increase. This results in higher intergenerational earnings persistence. In Section 3 below, I illustrate this mechanism with a simple model. Tax codes are also plausible explanations for the cross country differences in earnings persistence, as they effect the incentives to invest in human capital. If taxes are progressive, it will have the effect that human capital investments become less attractive particularly for someone with high ability. This will shrink the dispersion of human capital investments and cause smaller earnings persistence. In Section 2, I document a negative correlation between tax progressivity and earnings persistence.

If there are diminishing returns to human capital investments, and investments made by parents and the government are substitutes, then a parent's incentive to invest will be falling as the government invests more. As the government invests more, the difference between how much is invested in rich and poor children becomes smaller and earnings persistence will fall. Western economies differ with respect to public education expenditure. As I document in Section 2, the countries with low earnings persistence tend to spend more on public investments in education relative to GDP per capita. The difference is particularly large when it comes to spending on tertiary education.

One potential cause of earnings persistence which has received much attention in literature is credit constraints. As mentioned above, in models where it is not possible for parents to borrow against children's future earnings, there will be a direct relationship between parents' and children's earnings, even if the parents are not credit-constrained with respect to their own resources. A stronger relationship may, however, occur if low earners with high
ability/endowment children are credit-constrained from investing in their children's human capital. One potential source of cross country differences in earnings persistence is the degree of credit market completeness. I don't have any good measure of credit market completeness across countries but if the government heavily subsidizes education, it should reduce the number of credit-constrained parents. In my structural model below, I do, however, find that increasing or decreasing borrowing limits for parents or in college have very little quantitative impact on earnings persistence in the US.

## Empirical Literature

The most commonly used measure of earnings persistence is the coefficient, often denoted $\beta$, from the regression of the logarithm of son's earnings on the logarithm of father's earnings and a constant, also called the intergenerational elasticity of earnings:

$$
\begin{equation*}
\log \left(y_{\text {son }}\right)=\alpha+\beta \log \left(y_{\text {father }}\right)+\varepsilon \tag{1}
\end{equation*}
$$

The relevant measure of earnings is lifetime or permanent earnings but as this is rarely available the best a researcher can do is often to average several years of earnings and controlling for age when earnings was observed. What $\beta$ tells us, in a purely statistical sense, is how many percent of a father's earnings advantage, relative to the mean in his generation, that is on average transferred to the son. $A \beta$ of 0 would represent the case when the earnings of fathers and sons are completely unrelated, while a $\beta$ of 1 would represent the case when the earnings advantage of the father is perfectly transferred to the son. Hypothetically, one can also imagine $\beta$ smaller than 0 or greater than 1 . In practice, however, empirical studies have found $\beta$ between 0 and 1 , which also means that earnings tend to revert to the mean over generations.

The statistical literature, which estimates and compares the intergenerational elasticity of earnings for different countries, is by now quite large. Blanden (forthcoming) provides a thorough discussion. There are some difficulties related to methodology and data, which makes it harder to compare different studies (see Appendix A1). It is, however, clear that there are substantial differences between countries. Corak (2006) provides a meta study based on previous empirical studies of earnings persistence in different countries and current knowledge of data and
methodological issues. Table 1 reproduces the main findings of his study, supplemented with two recent studies from Italy and Spain.

## Quantitative Literature

In addition to the empirical work, there is also a theoretical literature, pioneered by Becker and Tomes, which gives us a framework for understanding the factors that may affect the correlation of children's and parents' earnings. The quantitative/structural literature, which takes models to data is, however, very sparse. I will briefly mention the two papers, that are closest in spirit to the work I am undertaking:

Han and Mulligan (2001) develop a very simple 2 period / 2 generation model where parents care about their children and have the opportunity to invest in their human capital and to give them monetary bequests. They calibrate their model to fit characteristics of the US economy, including the intergenerational elasticity of earnings, $\beta$, which they take to be 0.4 . They then study how $\beta$ changes as they eliminate intergenerational borrowing constraints and increase the variance of shocks to ability. The authors conclude that eliminating borrowing constraints reduces $\beta$ by about 0.1 , but also find that $\beta$ increases as the heterogeneity of ability increases. In their model, family endowments are transmitted as an $\operatorname{AR}(1)$-process where the shocks have zero mean. As they increase the variance of the shocks, earnings persistence increases.

Restuccia and Urrutia (2004) develop a model with infinite dynasties where agents live for 4 periods, 2 as children and 2 as adults. Parents decide how much to invest in children's elementary education and whether to send them to college. There is also a government that imposes taxes, runs a balanced budget, and invests the tax revenues in education. The focus of the paper is to determine whether investments in early or college education is quantitatively more important for earnings persistence. They find that early education matters more, and that government investments in early education has a much greater impact than government investments in college education.

My paper is the first to study the impact of cross country differences in policies on $\beta$. It turns out that across countries there is greater variation in tertiary education than in early education
spending. Tertiary education spending therefore seems like a more likely explanation for cross country differences in $\beta$. My paper also offers a richer more realistic model, combining some elements that are present in each of the above papers. In Section 5 I discuss the different model elements in detail and why they are important in a study of earnings persistence.

The remainder of the paper is organized as follows: In section 2, I document the correlation between $\beta$ and tax progressivity and between $\beta$ and spending on tertiary education. Section 3 studies the impact of taxation and public investment in education on $\beta$ in a simple analytical model. Section 4 presents the quantitative model. In section 5 I discuss and justify some of the modeling choices. Section 6 discusses data and calibration. Section 7 decompose the contributions to earnings persistence from the different model elements. Section 8 presents results from policy experiments. Section 9 concludes.

## 2 Correlations between Earnings Persistence and Tax Progressivity and Earnings Persistence and Public Spending on Tertiary Education

It is difficult to give a summarize the tax system in a country just by one number. A commonly used measure of tax progressivity is so-called progressivity wedges, see Guvenen et. al. (2009):

$$
\begin{equation*}
P W\left(y_{1}, y_{2}\right)=1-\frac{1-\tau\left(y_{2}\right)}{1-\tau\left(y_{1}\right)} \tag{2}
\end{equation*}
$$

This measure says something about how fast the tax rate increases as earnings increases from $\mathrm{y}_{1}$ to $\mathrm{y}_{2}$. If there is a flat tax, then the progressivity wedge would be zero for all levels of $\mathrm{y}_{1}$ and $\mathrm{y}_{2}$. For each country in Table 1, I use labor income tax data from the OECD tax database to fit a tax function, see Appendix A2 for a detailed description. I then construct progressivity wedges using the average earnings, AE, in each country for $\mathrm{y}_{1}$ and four times average earnings for $\mathrm{y}_{2}$. In Figure 1, I plot earnings persistence on the $y$-axis against this measure of tax progressivity on the $x$-axis. The correlation between the two quantities is -0.81 and the regression coefficient is highly significant when earnings persistence is regressed on the progressivity wedges. A strong correlation between two variables need of course not imply that one has a causal effect on the

Figure 1: Correlation Between Tax Progressivity and Earnings Persistence


Earnings persistence from Table 1. The tax data is an average of the years 2001-2005, taken from the OECD Tax and Benefit Calculator and the OECD Tax Database. The regression coefficient is significant at the $1 \%$ level.

Figure 2: Correlation Between Public Expenditure on Tertiary Education and Earnings Persistence


Earnings persistence from Table 1. The education spending data is an average of the years 1999-2005, taken from the UNESCO institute for statistics. The regression coefficient is significant at the $1 \%$ level.
other. However, this empirical observation motivates a further investigation of the impact of taxes on earnings persistence in a structural model with careful modeling of the tax systems. In Figure 2, I plot the correlation between earnings persistence and public expenditure per student in tertiary education as a fraction of GDP per capita. The correlation between the two variables is -0.84 , and the regression coefficient is highly significant when earnings persistence is regressed on education expenditure.

## 3 Gaining Intuition: The Impact of Taxation and Public Education Expenditure on Intergenerational Earnings Persistence in a Simple Model

To obtain an understanding of how taxation and public education expenditure affect earnings persistence it may be helpful to start with a simple model. The model is a slight modification of Solon (2004), where I have changed the wage function and the process for inheritance of abilities to similar to the quantitative model of Section 4. Assume that there is a continuum of infinitely lived single individual dynasties. Each individual lives for two periods, one as a child and one as an adult. Parents decide how much to consume and how much to invest in their children's human capital, while children do not make any economic decisions. A parent's utility is a function of today's consumption, $c_{t}$, and his child's future earnings, $y_{t+1}$ :

$$
\begin{equation*}
U_{t}\left(c_{t}, y_{t+1}\right)=\log \left(c_{t}\right)+\alpha \log \left(y_{t+1}\right) \tag{3}
\end{equation*}
$$

The parameter $\alpha$ measures how altruistic parents are with respect to their children. The earnings of the child is determined by his level of human capital. Human capital is a function of investments made by the parents, $I_{t}$, investments made by the government, $I_{g}$, and of the ability or family endowment of the child, $A_{t}$ :

$$
\begin{gather*}
y_{t+1}=\gamma h_{t+1}  \tag{4}\\
h_{t+1}=A_{t+1}\left(I_{t}+I_{g}\right)^{\psi} \tag{5}
\end{gather*}
$$

Abilities are imperfectly transmitted from parent to child. I assume them to be log-normally distributed, and follow an $\operatorname{AR}(1)$-process:

$$
\begin{equation*}
\log A_{t+1}=\theta \log A_{t}+v, \quad v \sim N\left(0, \sigma_{v}^{2}\right) \tag{6}
\end{equation*}
$$

Assuming that labor income is taxed at rate $\tau$, the utility maximization problem of a parent can now be written as:

$$
\begin{gather*}
\max _{c_{t}, I_{t} \geq 0} \log \left(c_{t}\right)+\alpha \log \left(y_{t+1}\right), \quad \text { s.t.: }  \tag{7}\\
c_{t}+I_{t}=y_{t}(1-\tau) \\
y_{t+1}=\gamma A_{t+1}\left(I_{t}+I_{g}\right)^{\psi}
\end{gather*}
$$

Substituting for $c_{t}$, and $y_{t+1}$ :, gives a maximization problem in $I_{t}$ :

$$
\begin{equation*}
\max _{0 \leq I_{t}<y_{t}} \log \left(y_{t}(1-\tau)-I_{t}\right)+\alpha \psi \log \left(I_{t}+I_{g}\right)+\alpha \log \left(A_{t+1}\right)+\alpha \log (\gamma) \tag{8}
\end{equation*}
$$

The first order condition is:

$$
\begin{gather*}
\frac{-1}{y_{t}(1-\tau)-I_{t}}+\frac{\alpha \psi}{I_{t}+I_{g}} \leq 0  \tag{9}\\
\frac{-1}{y_{t}(1-\tau)-I_{t}}+\frac{\alpha \psi}{I_{t}+I_{g}}=0 \quad \text { if } \quad I_{t}>0
\end{gather*}
$$

Rearranging this expression we get the following solution for $I_{t}$ :

$$
I_{t}=\left\{\begin{array}{c}
\frac{\alpha \psi}{1+\alpha \psi} y_{t}(1-\tau)-\frac{1}{1+\alpha \psi} I_{g}, \quad \text { if } \quad y_{t}>\frac{I_{g}}{\alpha \psi(1-\tau)}  \tag{10}\\
0, \quad \text { else }
\end{array}\right.
$$

As long as there is an interior solution, $I_{t}$ is decreasing in the tax rate, $\tau$, decreasing in government investment, $I_{g}$, increasing with the altruism parameter, $\alpha$, and increasing in the human capital production function parameter, $\psi$. Substituting for $I_{t}$ in (5) and taking the log of
(4), we get an equation relating the $\log$ of the earnings of children to the earnings of their parents:

$$
\log \left(y_{t+1}\right)=\left\{\begin{array}{c}
\psi \log \left(y_{t}(1-\tau)+I_{g}\right)+\log \left(\theta A_{t}+v\right)+\log (\gamma)  \tag{11}\\
+\psi \log \left(\frac{\alpha \psi}{1+\alpha \psi}\right), \quad \text { if } \quad y_{t}>\frac{I_{g}}{\alpha \psi(1-\tau)} \\
\psi \log \left(I_{g}\right)+\log \left(\theta A_{t}+v\right)+\log (\gamma)+\psi \log \left(\frac{\alpha \psi}{1+\alpha \psi}\right), \quad \text { else }
\end{array}\right.
$$

## Proposition I

$$
\begin{gather*}
\text { If } y_{i t}>\frac{I_{g}}{\alpha \psi(1-\tau)} \text { and } I_{g}>0 \text { then }  \tag{12}\\
\frac{\partial^{2} \log \left(y_{t+1}\right)}{\partial \log \left(y_{t}\right) \partial \tau}<0, \quad \frac{\partial^{2} \log \left(y_{t+1}\right)}{\partial \log \left(y_{t}\right) \partial I_{g}}<0, \quad \frac{\partial^{2} \log \left(y_{t+1}\right)}{\partial \log \left(y_{t}\right) \partial \psi}>0
\end{gather*}
$$

Proof: See Appendix A3

Proposition I states that as long as both the parental investment and the government investment are positive, the impact of the parent's earnings on the child's earnings become smaller when there is higher taxation, more government investment, or human capital production is more efficient. In the case of the tax, this happens because a smaller share of the parent's earnings can be devoted to investing in human capital when the tax is higher. The government investment, which is equal for all children, then accounts for a larger share of the total human capital investment, and a change in the log of parental earnings will have a smaller impact on the log of the child's earnings. However if government investments were zero, then the flat tax could be separated out as a constant term. When the government investment increases, it has the same effect as when the tax increases. The relative importance of parental earnings is decreasing both because $I_{g}$ is bigger, and because an increase in $I_{g}$ crowds out parental investments. The impact of parental earnings child's earnings is increasing in the human capital production function parameter, $\psi$. This is simply because an increase in $\psi$ increases the effect of parental
investments. The equation usually estimated by empirical researchers studying intergenerational earnings persistence is:

$$
\begin{equation*}
\log \left(y_{i t+1}\right)=\alpha+\beta \log \left(y_{i t}\right)+\varepsilon_{i t+1} \tag{13}
\end{equation*}
$$

Where $i$ denotes the family or dynasty. If we assume $y_{i t}>\frac{I_{g}}{\alpha \psi(1-\tau)} \forall i$, then all parents will invest a positive amount in their child's human capital and we only have to consider the first part of equation (11). Let us also assume that the economy is in steady state, i.e. the distributions of $\log \left(y_{i t+1}\right)$ and $\log \left(y_{i t}\right)$ are equivalent, and that $I_{g}>0$. With the purpose of obtaining an analytical solution for the regression coefficient, $\beta$, we can log-linearize the first part of (11) around average earnings, $\bar{y}$, and average ability, $\bar{A}$ :

$$
\begin{gather*}
\log \left(y_{i t+1}\right)=\alpha^{*}+\frac{\psi(1-\tau) \bar{y}}{\overline{\mathrm{y}}(1-\tau)+I_{g}} \log \left(y_{i t}\right)+\log \left(A_{i t+1}\right)  \tag{14}\\
\text { where } \quad \alpha^{*}=\log \left(\frac{\gamma \bar{y}}{\bar{A}}\right)+\psi \log \left(\frac{\alpha \psi}{1+\alpha \psi}\right)+\frac{\psi(1-\tau) \bar{y}}{\overline{\mathrm{y}}(1-\tau)+I_{g}} \log (\bar{y})
\end{gather*}
$$

Equation (14) now resembles the classical linear regression equation in (13), except that the error term, $\log \left(A_{i t+1}\right)$, is correlated with the explanatory variable, $\log \left(y_{i t}\right)$. This is because both $\log \left(y_{i t}\right)$ and $\log \left(A_{i t+1}\right)$ depends on $\log \left(A_{i t+1}\right)$. OLS estimates of the slope will therefore be biased. Equation (14) is a first-order auto-regression where the error term follows the $\operatorname{AR}(1)$ process as in (6). It is shown in Greene (2000, pp. 534-535) that when $\operatorname{var}\left(\log y_{i t+1}\right)=$ $\operatorname{var}\left(\log y_{i t}\right)$ the probability limit of the OLS-estimator for the slope coefficient in this equation is given by the sum of the true slope coefficient and the autoregressive parameter of the error term divided by one plus their product. Using this result we get that in the population regression where (13) is estimated by OLS:

$$
\begin{equation*}
\beta=\frac{(\psi+\theta)(1-\tau) \bar{y}+\theta I_{g}}{(1+\psi \theta)(1-\tau) \bar{y}+I_{g}} \tag{15}
\end{equation*}
$$

## Proposition II

$$
\begin{equation*}
\frac{\partial \beta}{\partial \tau}<0, \quad \frac{\partial \beta}{\partial I_{g}}<0, \quad \frac{\partial \beta}{\partial \psi}>0, \quad \frac{\partial \beta}{\partial \theta}>0 \tag{16}
\end{equation*}
$$

Proof: See Appendix A3

Thus in this simple model, we have seen that an increase in taxation and/or government investment in education reduces earnings persistence by reducing the direct impact of parental earnings on child's earnings (Proposition I). The intuition behind the result is that the relative importance of parental investments compared to government investments decreases. The difference between how much is invested in rich and poor children becomes smaller in percent/log terms as taxes or government investments increases, and this leads to a fall in earnings persistence. $\beta$ is not surprisingly increasing in the correlation of parent's and child's ability, $\theta$. It is also increasing in the human capital production function parameter, $\psi$. It should be noted that the relationship between the market return to human capital, $\gamma$, and $\beta$ generally is sensitive to the specification of the wage function. I have specified a constant return to a unit of human capital, and $\gamma$ does not enter the expression for $\beta$. In the original model of Solon, an exponential return to human capital was specified and $\gamma$ would then be present in the expression for $\beta$.

## 4 Model

## Economic Environment

The economy is populated by single-individual dynasties, where each individual lives for at least 70 years, and at most 100 years. A model period is five years. For the first 4 periods, or 20 years, of his life, an individual is part of the parent's household and does not make any economic decisions. At age 20, a young individual moves out of the parent's house and forms his own household. At age 30, he has a child, and at age 65 he enters retirement. The first decision a young adult must take is whether or not to enroll in college. All working age households, including college students, decide how much to work, consume, and save at a risk free rate. College students also decide how much to invest in human capital production. There is a fixed
time cost of attending college, and college students have to work a low fixed wage, which is independent of their human capital. There is a probability of failing college, depending on the student's ability and prior level of human capital. Households are altruistic and care about their children's utility. Households with a child, aged 5-19, decide how much to invest in the child's human capital. At the moment a child leaves home and begins his own household, the parent has the option of giving him a one-time gift of liquid assets to secure that he gets a good start in life. This is, of course, a simplifying assumption but it greatly reduces the complexity of the model. Empirically, the fact that the child receives a one-time gift at the beginning of his adult life can be motivated by the observation that many parents help their child with paying for college or with buying a first home. Figure 3 below illustrates the life cycle of a household.

Figure 3: Household's Lifecycle


## Wages and Human Capital

Worker productivity in this economy depends on human capital, college completion, labor market experience, and labor market luck. Since there is no unemployment in the model, experience is equal to potential experience and is fully determined by age, and whether a person attended college. Letting $x$ denote the individual's experience level, and $h$ denote his level of human capital, his wage can be written:

$$
\begin{gather*}
w=h \gamma_{0} e^{\gamma_{1}^{j} x+\gamma_{2}^{j} x^{2}+\gamma_{3}^{j} x^{3}+u}  \tag{17}\\
u \sim N\left(0, \sigma_{u}^{2}\right) \tag{18}
\end{gather*}
$$

Where u is an idiosyncratic productivity shock, and $j \in\{0,1\}$ is an indicator for whether college educated. There are different age/experience paths for the wages of college and high school educated workers. The human capital of a person must be built up during his childhood, and during college. How much human capital a person accumulates depends on his ability, $A$, and how much is invested in his human capital in each time period by the parents, $I_{p}$, by the individual himself in college, $I_{s}$, and the government, $I_{g}$.

$$
\begin{array}{ll}
h^{\prime}=h+A\left[h\left(I_{p}+I_{g}\right)\right]^{\psi_{0}}, & \text { before college }  \tag{19}\\
h^{\prime}=h+A\left[h\left(I_{s}+I_{g}\right)\right]^{\psi_{1}}, & \text { in college }
\end{array}
$$

Here $h^{\prime}$ denotes human capital in the next time period. I follow the tradition in the literature on intergenerational earnings persistence, see Becker and Tomes (1979), and (1986), Solon (2004), and think of human capital investments as investments of money or goods. However, while many definitions of what should be considered human capital investments have been suggested, I will think of it as investment in education. The ability or family endowment of the child, is broadly defined to include things that does not have to be bought, like genetics, family culture, motivation, and knowledge acquired from the parents. Abilities are assumed to be log-normally distributed and imperfectly inherited from parent to child according to an $\operatorname{AR}(1)$ process:

$$
\begin{equation*}
\log A_{c}=\theta \log A_{p}+v, \quad v \sim N\left(0, \sigma_{v}^{2}\right) \tag{20}
\end{equation*}
$$

(19) is the same functional form as in Ben-Porath (1967), except that Ben-Porath allowed for different exponentials on the human capital and goods inputs. The same production function has been used in some recent studies involving human capital accumulation, see for instance Huggett et.al. (2007), or Ionescu (2009). These studies do, however, ignore the input of goods in the
production of human capital and focus on the human capital input, which is modeled as the product of previous human capital and time. They are also different in that they focus on human capital accumulation during work-life and/or college. In my model the input of time is kept constant, and human capital accumulation starts at age 5. It is known that that the efficiency of human capital investments varies by age, see Cunha and Heckman (2007), and this is the rationale for specifying different technologies before college and in college. One could have used a different technology at every age but this would complicate the model.

## Preferences

The momentary utility is a function of consumption in adult equivalents, $\frac{c}{e(t)}$, where $e(t)$ varies depending on whether there is a child in the household, and work hours, $n$ :

$$
\begin{equation*}
u(c, n)=\frac{\left(\frac{c}{e(t)}\right)^{1-\sigma}}{1-\sigma}-\chi \frac{n^{1+\eta}}{1+\eta} \tag{21}
\end{equation*}
$$

A household discounts the future by a factor, $\delta$. When the child leaves from home, the parent cares about the child's utility, $U^{c}$, but discount it by, $\alpha$. Thus a household's lifetime utility, $U$, is given by:

$$
\begin{equation*}
U=\sum_{t=1}^{\text {death }} \delta^{t-1} u(c, n)+\delta^{6} \alpha U^{c} \tag{22}
\end{equation*}
$$

## Borrowing for College and Probability of College Completion

Individuals who attend college are allowed to borrow up to an amount, $z$ while in college. I require that they do not retire in debt, and in subsequent periods I let the borrowing constraint, $\varphi(j, t)$, be linearly decreasing between college and retirement. High school graduates are not allowed to borrow:

$$
\begin{equation*}
\varphi(j=1, t)=\max (0, z(9-t)), \quad \varphi(j=0, t)=0 \tag{23}
\end{equation*}
$$

However if someone took up a loan for college and failed, they will also be subject to borrowing constraint for college graduates. The probability of success in college, $\pi(A h)$, is a function of ability and acquired pre-college human capital:

$$
\begin{equation*}
\pi(A h)=1-e^{-\Omega A h} \tag{24}
\end{equation*}
$$

## Recursive Formulation of the Household's Problem

There are 5 different life stages a household can be in, and therefore 5 different household maximization problems. The first decision a young household must take is whether or not to go to college. This is done at age 20, or $t=1$. In both cases he decides how much to consume, $c$, next period's capital, $k^{\prime}$, and how much to work, $n$. If he goes to college, he must also decide how much to invest in human capital, $I_{s}$. The state variables are age, $t$, capital, $k$, his level of human capital, $h$, his ability, $A$, and the productivity shock, $u$. In all time periods experience, $x$, will be equal to the current model period minus 4 for high school educated workers, and equal to the current model period minus 5 for college educated workers. Formally the individual solves the following Bellman problem:

$$
\begin{gather*}
W(k, h, t=1, A, u)=\max \{V(j=0, \cdot), V(j=1, \cdot)\}, \quad \text { where }:  \tag{25}\\
V(0, k, h, t, A, u)=\max _{c, n, k^{\prime}} u(c, n)+\delta E\left[V^{\prime}\left(0, k^{\prime}, h, t^{\prime}, A, u^{\prime}\right)\right], \quad \text { s.t.: } \\
c\left(1+\tau_{c}\right)+k^{\prime}=(1+r) k+w n-\tau(w n) \\
w=h \gamma_{0} e^{\gamma_{1}^{j} x+\gamma_{2}^{j} x^{2}+\gamma_{3}^{j} x^{3}+u, \quad u \sim N\left(0, \sigma_{u}^{2}\right), \quad k^{\prime} \geq 0, \quad c \geq 0, \quad 0 \leq n \leq 1, \quad t^{\prime}=t+1} \\
V(1, k, h, t, A, u)=\max _{c, n, k^{\prime}, I_{s}} u(c, n+\varpi)+\delta \pi(h, A) E\left[V^{\prime}\left(1, k^{\prime}, h^{\prime}, t^{\prime}, A, u^{\prime}\right)\right] \\
\left.+\delta(1-\pi(h, A)) E\left[V^{\prime}\left(0, k^{\prime}, h, t^{\prime}, A, u^{\prime}\right)\right]\right), \quad s . t .: \\
c\left(1+\tau_{c}\right)+k^{\prime}=(1+r) k+w n-\tau(w n) \\
h^{\prime}=h+A\left[h\left(I_{s}+I_{g}\right)\right]^{\psi_{1}}
\end{gather*}
$$

$$
\begin{gathered}
w^{\prime}=h^{\prime} \gamma_{0} e^{\gamma_{1}^{j} x+\gamma_{2}^{j} x^{2}+\gamma_{3}^{j} x^{3}+u}, \quad u \sim N\left(0, \sigma_{u}^{2}\right) \\
I_{s} \geq 0, \quad 0 \leq n \leq 1-\varpi, \quad w=w_{c}, \quad t^{\prime}=t+1, \quad k^{\prime} \geq \varphi(t), \quad c \geq 0
\end{gathered}
$$

$\varpi$ is here the time cost of attending college, $\tau_{c}$ is a flat consumption tax, and $\tau(w n)$ is a nonlinear labor income tax. Also note that while in college, and individual must work at the fixed wage, $w_{c}$, which is independent of his level of human capital. The problem of a working household without child, and at age 30 when no human capital investments are made is:

$$
\begin{gather*}
V(j, k, h, t, A, u)=\max _{c, n, k^{\prime}} u(c, n)+\delta E\left[V^{\prime}\left(j, k^{\prime}, h, t^{\prime}, A, u^{\prime}\right)\right], \quad \text { s.t.: }  \tag{26}\\
c\left(1+\tau_{c}\right)+k^{\prime}=(1+r) k+w(j, t, h, u) n-\tau(w(j, t, h, u) n) \\
k^{\prime} \geq \varphi(j, t), \quad c \geq 0, \quad 0 \leq n \leq 1, \quad t^{\prime}=t+1, \quad \text { for } t=2,3,8,9(\text { age }=25,30,55,60)
\end{gather*}
$$

At age 30, (20) is also a constraint as the ability of the child will be revealed in the next period, and the parent must have an expectation of his child's ability. Between age 35 and 50 the parent must also decide on how much to invest in the child's human capital. He solves:

$$
\begin{gather*}
V\left(j, k, h_{p}, h_{c}, t, A, u\right)=\max _{c, n, k^{\prime} I_{p}} u(c, n)+\delta E\left[V^{\prime}\left(j, k^{\prime}, h_{p}, h_{c}^{\prime}, t^{\prime}, A, u^{\prime}\right)\right], \quad \text { s.t.: }  \tag{27}\\
c\left(1+\tau_{c}\right)+k^{\prime}=(1+r) k+w(j, t, h, u) n-\tau(w(j, t, h, u) n) \\
h_{c}^{\prime}=h_{c}+A\left[h_{c}\left(I_{s}+I_{g}\right)\right]^{\psi_{0}} \\
k^{\prime} \geq \varphi(j, t), \quad c \geq 0, \quad I_{p} \geq 0, \quad 0 \leq n \leq 1, \quad t^{\prime}=t+1, \quad \text { for } 4 \leq t \leq 6(35 \leq \text { age } \leq 50)
\end{gather*}
$$

$h_{p}$ here denotes the human capital of the parent, and $h_{c}$ denotes the human capital of the child. The parent must keep track of both as state variables. $A$ is now the ability of the child. There is no reason for the parent to know his own ability after the ability of the child is revealed. When the parent is at age 50 and the child is at age 20, the child leaves the household and the parent has a one-time opportunity to give him a gift or inter vivos transfer, $b$. The parent's problem is:

$$
\begin{array}{r}
V\left(j, k, h_{p}, h_{c}, t=7, A, u\right)=\max _{c, n, k^{\prime}, b} u(c, n)+\delta E\left[V_{p}\left(j, k^{\prime}, h_{p}, t=8, u_{p}^{\prime}\right)\right]  \tag{28}\\
+\alpha E\left[V_{c}\left(b, h_{c}, t=1, A, u_{c}\right)\right], \quad \text { s.t.: } \\
c\left(1+\tau_{c}\right)+k^{\prime}=(1+r) k+w(j, t, h, u) n-\tau(w(j, t, h, u) n) \\
b \geq 0 \quad k^{\prime} \geq \varphi(j, t), \quad c \geq 0, \quad I_{p} \geq 0, \quad 0 \leq n \leq 1, \quad t^{\prime}=t+1
\end{array}
$$

$\alpha$ here controls the parent's degree of altruism. I assume that the parent do not observe the child's idiosyncratic shock before the size of the gift is decided. He must therefore take the expectation of the child's value function with respect to the idiosyncratic shock. A household in retirement simply solves:

$$
\begin{gather*}
V(k, t)=\max _{c \geq 0, k^{\prime} \geq 0} u(c, n=0)+\delta \Gamma(t) V^{\prime}\left(k^{\prime}, t^{\prime}\right), \quad \text { s.t. }:  \tag{29}\\
c\left(1+\tau_{c}\right)+k^{\prime}=(1+r) k+T \\
\text { for } 10 \leq t \leq 16(65 \leq \text { age } \leq 95)
\end{gather*}
$$

T is here a constant amount of social security, and $\Gamma(\mathrm{t})$ is an age dependent probability of survival to the next period.

## 5 Discussion of Modeling Choices

## Life Cycle Model with College Decision

Using a life cycle model with college decision allows us to study government expenditure on different levels of education. We can separate the effects of spending on primary, secondary, and tertiary education. The cross country variation in education expenditure is largest for tertiary education. Another argument for using a life cycle model is that when studying the impact of parents' earnings on the earnings of children, we are interested in the financial resources available to parents at the time when there are children in the household. There is a literature documenting that even after controlling for parents' lifetime income, the income of the parents during the childhood years matters for the children's income, see Cunha and Heckman (2007) for a survey.

## Physical Capital, Inter Vivos Transfers, and Human Capital

I will argue that in a realistic quantitative model, developed to study intergenerational earnings persistence, it is important to have financial assets and a mechanism for transfers from parent to child, in addition to human capital. The existence of physical capital in the model affects how much is invested in a child's human capital in various ways. In a model without financial assets, parents will divide their resources between their own consumption today and their children's future consumption, or equivalently their children's human capital. This may create a too strong correlation between the earnings of the parent and the child's human capital, as the optimal investment in the child will always be increasing in the earnings of the parent. If there is physical capital and diminishing returns to human capital investments, there will be a point where the return on capital is strictly higher than the return on human capital, and this will put a cap on human capital investments. Children with low ability but rich parents will earn a lot more in a world with no financial assets, because the only way to help them is to invest in their human capital. With physical capital, their parents will rather give them some financial assets. Furthermore since there is uncertainty in the model, parents will like to accumulate some physical capital to insure against negative shocks, even when the expected return on human capital investments is higher than the returns on physical capital. This will take resources away from human capital investments.

A popular explanation both for earnings persistence, see for instance Han and Mulligan (2001), and college enrollment in the literature is the existence of borrowing constraints. In the literature on intergenerational persistence, the focus has sometimes been on intergenerational borrowing constraints, however, I do not find borrowing towards children's future earnings to be very realistic. Below, I study the impact of intragenerational borrowing constraints on earnings persistence. To do so, it is, however, crucial that the model has financial assets.

## Labor Supply

That agents in the model are able to choose their work hours affects the returns to human capital investments and will be important for the shape of the optimal investment policy as a function of physical capital. In Figure 4 below, I illustrate this point by plotting the optimal investment in human capital for an individual in college.

Figure 4: Human Capital Investment for a Model College Student


As can be seen from the figure, the optimal investment peaks at some point and start sloping downwards. This is because, as the agent becomes wealthier, he will enjoy more leisure in the future and the returns to investing in human capital is falling. Some families accumulate a lot of physical capital but the fact that they enjoy leisure and can control their labor supply will affect the shape of their optimal human capital investments.

Labor supply is also potentially important for college enrollment and for the importance of borrowing constraints with respect to human capital investments, see Garriga and Keightley (2007), Keane and Wolpin (2001). If a poor person cannot borrow to invest in his child, he may choose to compensate by working a bit more. Equivalently if a college student cannot borrow, he may choose to take on a part time job. Having labor choice in the model reduces the importance of borrowing constraints. If a college student has no other way of raising money than borrowing, then borrowing constraints are more likely to be important.

## 6 Calibration

Many of the parameters can be obtained without solving the model. I calibrate 27 model parameters to their empirical counterparts. The remaining 11 parameters are estimated using an
exactly identified simulated method of moments approach. Tables 2 and 3 summarize the parameters calibrated outside and inside the model. The main source of data for the for the estimated parameters, 6 out of the 11 data moments, is employed males from the PSID (19992005). I use employed males because most of the literature on intergenerational earnings persistence is based on the relationship between father and son, and the analysis is carried out on working individuals. In addition there is no unemployment in my model. I use the years 19992005 because these are the years when I also have data on education spending and taxes. Below I describe the data used in the calibration of each parameter as well as the estimation approach.

## Risk Free Interest Rate

Given the partial equilibrium nature of the model, I take the risk free rate as fixed and calibrate it using data. I set the risk free rate equal to the average of 3-month $t$-bill rates minus inflation over the period from 1947-2008 based on data from the Federal Reserve Bank of St. Louis ${ }^{6}$.

## Preferences

The momentary utility function is the standard CRRA utility function in (21), with consumption measured in adult equivalents, $\frac{c}{e(t)}$. I use the so called "OECD-modified" adult equivalence scale and set $e(t)=1.3$ when there is a child in the household, and $e(t)=1.0$ when there is not. Consistent with a survey of the empirical literature in Browning et. al. (1999), I set the coefficient of relative risk aversion, $\sigma$, equal to 2 , and the inverse of the Frisch elasticity of labor supply, $\eta$, equal to 3 . The elasticity of substitution between consumption and labor, $\chi$, the time discount factor, $\delta$, and the altruism parameter, $\alpha$, are among the estimated parameters. The corresponding data moments are average hours worked for employed males, 25-64, asset holdings of employed males 50-54, and asset holdings of employed males 25-29 in the PSID (1999-2005). Consistent with the American Time Use Survey (2003), I assume that the day has 15 hours not needed for personal care and normalize hours so that working 15 hours per day is equivalent to a labor supply of 1 in the model.

[^4]Table 2: Parameters Calibrated Outside of the Model

| Parameter | Value | Description | Target |
| :---: | :---: | :---: | :---: |
| $r$ | 0.011 | Risk free interest rate (annual) | 3-month t-bill rates minus inflation (1947-2008) |
| $\sigma$ | 2 | $u(c, n)=\frac{\left(\frac{c}{e(t)}\right)^{1-\sigma}}{\begin{array}{c} 1-\sigma \\ n^{1+\eta} \end{array}}$ | Consistent with survey in Browning et. al. (1999) |
| $\eta$ | 3 | $-\chi \frac{}{1+\eta}$ |  |
| $e(t)$ | 1.0 or 1.3 |  | OECD-modified equivalence scale. |
| $\gamma_{1}^{0}$ | 0.221 | $w=h \gamma_{0} e^{\gamma_{1}^{j} x+\gamma_{2}^{j} x^{2}+\gamma_{3}^{j} x^{3}+u}$ | PSID (1968-2005) |
| $\gamma_{2}^{0}$ | -0.029 |  |  |
| $\gamma_{3}^{0}$ | 0.001 |  |  |
| $\gamma_{1}^{1}$ | 0.295 |  |  |
| $\gamma_{2}^{1}$ | -0.052 |  |  |
| $\gamma_{3}^{1}$ | 0.003 |  |  |
| $\tau_{1}$ | -0.573 | $\tau(w n)=w n\left(\tau_{1}\left(\frac{w n}{A E}\right)^{0.2}\right.$ | OECD tax data (01-05) |
| $\tau_{2}$ | 1.706 | $+\tau_{2}\left(\frac{w n}{A E}\right)^{0.4}+\tau_{3}\left(\frac{w n}{A E}\right)^{0.6}$ |  |
| $\tau_{3}$ | -1.096 | $\left.+\tau_{1}\left(\frac{W n}{A E}\right)^{0.8}\right)$ |  |
| $\tau_{4}$ | 0.221 |  |  |
| $\tau_{c}$ | 0.084 | Consumption tax | Vertex Inc. (2002) |
| $\varpi$ | 0.110 | Time spent studying in college | American time use survey |
| $w_{c}$ | \$11.14/h | Wage rate in college (2005 dollars) | CPS (1999-2005) |
| $I_{g}(t)$ | Primary: \$4522 <br> Secondary: $\$ 5295$ <br> Tertiary: \$10672 | Public spending per student (annual 2005 dollars) | UNESCO (1999-2005) |
| z | \$24856 | College borrowing limit | Lochner (2008) |
| T | \$13094 | Old age social security | Social Security <br> Administration (1999-2005) |
| $\Gamma(\mathrm{t})$ | Varies | Death probabilities | NCHS (1991-2001) |

## Wages

I calibrate the life cycle profile of wages exogenously, using the entire PSID from 1968-2005. I regress wages on model potential experience and control for the year of observation. I estimate

Table 3: Parameters Calibrated Endogenously

| Parameter | Value | Description | Data Moment |
| :---: | :---: | :---: | :---: |
| $\gamma_{0}$ | 0.372 | $w=h \gamma_{0} e^{\gamma_{1}^{j} x+\gamma_{2}^{j} x^{2}+\gamma_{3}^{j} x^{3}+u}$ | Mean wages of skilled workers |
| $h_{0}$ | 0.467 | Starting level of human capital | Mean wages of unskilled workers |
| $\psi_{0}$ | 0.300 | $h^{\prime}=h+A(h I)^{\psi_{0}}$, before college | Human capital investment in elementary school |
| $\psi_{1}$ | 0.881 | $h^{\prime}=h+A(h l)^{\psi_{1}}$, in college | Human capital investment in college |
| $\sigma_{u}$ | 0.398 | $u \sim N\left(0, \sigma_{u}^{2}\right)$ | Variance of $\log$ of wages |
| $\theta$ | 0.332 | $\log A_{c}=\theta \log A_{p}+v, \quad v \sim N\left(0, \sigma_{v}^{2}\right)$ | The intergenerational elasticity of earnings |
| $\sigma_{v}$ | 0.259 | - " | College enrollment |
| $\Omega$ | -0.427 | $\pi(A h)=1-e^{-\Omega A h}$, Prob. of passing college. | College failure rate |
| $\alpha$ | 0.302 | Parental altruism | Mean assets of people aged 25-29 |
| $\chi$ | 171.2 | $U(c, n)=\frac{\left(\frac{c}{e(t)}\right)^{1-\sigma}}{1-\sigma}-\chi \frac{n^{1+\eta}}{1+\eta}$ | Mean hours worked |
| $\delta$ | 1.016 | Discount factor | Mean assets of people aged 50-54 |

different experience paths for college graduates and non-college graduates. For the data moments used in the structural estimation, I only use the years 1999-2005. I take the average wage of college graduates, the average wage of high school graduates, and the variance of log wages as the corresponding data moments to estimate the following parameters: The market return to human capital, $\gamma_{0}$, the starting level of human capital, $h_{0}$, and the standard deviation of the idiosyncratic earnings shock, $\sigma_{u}$. In the PSID, individuals are observed only every second year from 1999-2005, while they are observed every year until 1997. To get an estimate of the variance of 5-yearly wages in the time period from 1999-2005, I assume that the ratio between the variance of 5-yearly, and 1-yearly wages in this time period is the same as it was in the period 1991-1997.

## Production of Human Capital / Investment in Education

The corresponding data moments to the parameters of the human capital production function, $\psi_{0}$, and $\psi_{1}$, is private spending on elementary and college education. In addition I must know public spending per student at each level of education, $I_{g}(t)$. I follow Restuccia and Urrutia (2004) and think of education spending by local governments in primary and secondary
education as private spending, while I take state and federal education spending as public spending. The rationale behind this is that local government spending is financed by local taxes, and that parents when they choose which neighborhood to live in, choose the level of local government education spending. Public schools receive both local and state/federal funding, and schools in wealthier neighborhoods have larger budgets due to more local funding, see also Fernandez and Rogerson $(1996,1998)$. In one way counting all local government spending as parental investment in education, may be a strong assumption that lead to a high level of private education spending relative to public spending. On the other hand, defining education spending as the only form of monetary investment that parents make in human capital is very conservative. To construct the relevant calibration targets for each level of education under the above assumption, I use data on public expenditure per student as fraction of GDP per capita from the UNESCO institute for statistics (1999-2005), and data on private expenditure as a fraction of total expendiure, as well as local government's share of public expenditure from the OECD (1999-2005).

## Correlation of Ability / Intergenerational Persistence of Earnings

The intergenerational correlation of ability, $\theta$, obviously has an impact on the intergenerational persistence of earnings, and I use that as the calibration target for this parameter. I obtain the value of 0.47 for the intergenerational earnings persistence from a meta study by Corak (2006). This also happens to be the same value as found by Grawe (2004), the latest study, using data from the PSID.

## Time Spent Studying in College, College Enrolment, Failure, and Borrowing

To calibrate the fixed time cost of attending college, $\varpi$, I use data from the American Time use Study (2004-2008). College students spend on average 3.3 hours per day on educational activities on week days. I assume that they attend 2 13-week semesters per year and that they also study 3.3 hour per day on weekends. While this may be a bit optimistsic, many students also attend summer school. I use college enrollment as the data target for the standard deviation of abilities, $\sigma_{v}$, and the college failure rate as the target for the parameter $\Omega$, which determines the probability of failing college. I compute these targets from the fraction of males with college degrees in the PSID (1999-2005), and data on college survival probability from the OECD
(2000, 2004). I get the college borrowing limit from Lochner and Monge-Narajano (2008). This is the borrowing limit for the federal loan program called "Stafford Loans", which is what most students are eligible for. There is another loan program called "Perkins loans", which can provide further loans to the students with greatest financial need but in practice few students make use of this program. Below I study the effect of relaxing the borrowing constraint.

## Taxes

The labor income tax schedule is a polynomial function of an individual's earnings relative to the average earnings, AE :

$$
\begin{equation*}
\tau(w n)=w n\left(\tau_{1}\left(\frac{w n}{A E}\right)^{0.2}+\tau_{2}\left(\frac{w n}{A E}\right)^{0.4}+\tau_{3}\left(\frac{w n}{A E}\right)^{0.6}+\tau_{4}\left(\frac{w n}{A E}\right)^{0.8}\right) \tag{30}
\end{equation*}
$$

As described in more detail in appendix (A2) I fit this polynomial to labor income tax data from the OECD tax database (2001-2005). This data is constructed by the OECD based on tax laws from different countries. It is well suited for cross country comparisons, see also see Guvenen et. al. (2009). Coming up with an accurate estimate of consumption taxes in the US is complicated by the fact that there are local county-level taxes in addition to state taxes. Vertex Inc. (a consulting company) estimated that the average consumption tax in the US was $8.4 \%$ in 2002. I use that number. For simplicity, I abstract from capital taxes. I do this because different types of capital is taxed differently, and this also differs across countries. Households do for instance have about half of their wealth in their homes which may or may not be taxed. In the US, interest income is taxed as labor income, while dividends and capital gains are subject to capital gains tax. The return on capital is, however, set very conservatively in the calibration. It is set equal to the returns on risk free bonds, which was $1.1 \%$ over the past 60 years.

## Death Probabilities and Social Security

I assume that all retirees receive the same constant social security benefit. I obtain the average benefit for males from the Annual Statistical Supplement to the Social Security Bulletin (19992005). The probability that a retiree will survive to the next period, I obtain from the National Center for Health Statistics (1991-2001).

Table 4: Calibration Statistics

| Statistic | Data | Model |
| :--- | :--- | :--- |
| Mean hours worked | 0.417 | 0.417 |
| Mean wages of workers without college degrees | 1.000 | 1.002 |
| Mean wages of workers with college degrees | 1.757 | 1.757 |
| Std. dev. of lnwages | 0.570 | 0.571 |
| Investment in elementary school | 0.038 | 0.037 |
| Investment in college | 0.121 | 0.120 |
| Fraction of workers enrolling in college | 0.588 | 0.590 |
| Fraction failing out of college | 0.400 | 0.399 |
| Intergenerational earnings elasticity | 0.470 | 0.470 |
| Mean assets of people aged 25-29 | 0.092 | 0.092 |
| Mean assets of people aged 50-54 | 0.525 | 0.525 |

## Estimation Method

11 model parameters are calibrated using an exactly identified simulated method of moments approach. I minimize the squared percentage deviation of simulated model statistics from the 11 data moments in Table 4. Let $\Sigma=\left\{\gamma_{0}, h_{0}, \psi_{0}, \psi_{1}, \sigma_{u}, \theta, \sigma_{v}, \Omega, \alpha, \chi, \beta\right\}$ and let $g(\Sigma)=\left(g_{1}(\Sigma), \ldots, g_{11}(\Sigma)\right)^{\prime}$ denote the vector where $g_{i}(\Sigma)=\frac{\bar{m}-\widehat{m}(\Sigma)}{\bar{m}}$ is the percentage difference between empirical moments and simulated moments. Then:

$$
\begin{equation*}
\widehat{\Sigma}=\min _{\Sigma} g(\Sigma)^{\prime} g(\Sigma) \tag{31}
\end{equation*}
$$

Table 3 summarizes the estimated parameter values. As can be seen from Table 4, I get close to match all the moments exactly. Because five of the empirical moments have unknown variance, it is not possible to compute any standard errors in this exercise. I set the intergenerational persistence of earnings equal to 0.47 based on the meta study by Corak (2006). The moments on investment in early and college education is based on aggregate data from UNESCO Institute for Statistics.

## 7 Decomposing Earnings persistence

There are 4 main model elements that govern earnings persistence: the process by which abilities are inherited, the variance of idiosyncratic productivity shocks, inter vivos transfers from parent to child, and investments in human capital. Human capital investments are made by parents
(individuals in college) and the government. Parental/individual investments and inter vivos transfers will be affected by the size of the government investment, returns to human capital investments, taxation, and borrowing constraints. To quantify how the different model elements affect earnings persistence, I shut them down, and reintroduce them in the model one by one. We cannot set human capital investments to zero because everyone would get zero wage, so we will keep government investments constant, in absolute terms, and set parental investments to zero, inter vivos transfers to zero, the correlation of abilities to zero, and the variance of the idiosyncratic shock to zero. Then we will start reintroducing these elements in the model, see Table 5 below. I also keep the variance of the shocks to the log of abilities, $\sigma_{v}$, constant in this exercise.

The main conclusion from Table 5 is that both parental / individual investments and correlation of abilities give significant positive contributions to earnings elasticity. Earnings elasticity falls to approximately zero when all 4 model elements are left out. The reason it is not exactly zero is that I approximate the continuous AR(1)-processes for abilities and idiosyncratic shocks by finite state Markov processes, as proposed by Tauchen (1986), when simulating the model ${ }^{7}$. This leads to slight inaccuracies, which become smaller as one increase the number of states. Introducing correlated abilities leads to an earnings elasticity of 0.266 . One might have expected it to be equal to the correlation of the $\log$ of abilities, 0.332 , but there is a nonlinear relationship between ability and earnings. Having parental / individual investments alone in the model gives an earnings elasticity of 0.159 . However, in this exercise I have kept government investments constant in absolute terms, in the policy experiments below I keep government investment constant relative to average earnings in the economy. Leaving out the other model elements makes the society poorer and therefore the government investments are now relatively larger than in the benchmark economy and crowds out parental / individual investments.

The effect of introducing inter vivos transfers or idiosyncratic shocks in the model is not always monotone. In some instances, having inter vivos transfers leads to lower earnings elasticity because richer children get larger transfers which cause them to reduce hours worked. In other

[^5]| Earnings persistence | Correlated abilities | Idiosyncratic shocks | Parental investments | Inter vivos transfers |
| :---: | :---: | :---: | :---: | :---: |
| 0.002 |  |  |  |  |
| 0.266 | X |  |  |  |
| 0.002 |  | X |  |  |
| 0.159 |  |  | X |  |
| -0.009 |  |  |  | X |
| 0.191 | X | X |  |  |
| 0.396 | X |  | X |  |
| 0.268 | X |  |  | X |
| 0.217 |  | X | X |  |
| -0.025 |  | X |  | X |
| 0.195 |  |  | X | X |
| 0.422 | X | X | X |  |
| 0.184 | X | X |  | X |
| 0.496 | X |  | X | X |
| 0.214 |  | X | X | X |
| 0.470 | X | X | X | X |

instances the transfers may lead to higher earnings persistence because they disproportionally induce children with rich parents to attend college. When all model elements are present, leaving out idiosyncratic wage shocks leads to higher earnings persistence because they are random and not correlated across generations. However, introducing shocks that are log-normally distributed around zero also has the effect of making the society richer, and causing parents to invest more in human capital. This may lead to higher earnings elasticity.

## 8 Policy Experiments

In this section, I study the contribution of country policies to differences in intergenerational earnings persistence. I also study the impact of relaxing and tightening the borrowing constraints. When I perform the policy experiments I keep public education expenditure and taxes as a function of average earnings in the economy. In this way if the society become richer or poorer because of a policy change, education expenditure and taxes will adjust accordingly.

## The Impact of Taxation and Public Education Expenditure

Out of the countries in Table 1, Denmark has the highest and most progressive taxes and they spend the most on tertiary education (see Figures 1 and 2). Denmark is also the country with the lowest earnings persistence. I therefore study how earnings persistence in my model economy, which is calibrated to US data, changes as I introduce Danish policies. I will think of the change in earnings persistence due to the introduction of Danish policies as being in the upper range of how much of cross country differences that can be explained by policies, as the effect of introducing policies from any other country should be smaller. Tables 7 and 8 display how selected model statistics change in the policy experiments.

As can be seen from Table 7, introducing a Danish public education expenditure scheme lowers the intergenerational earnings elasticity by 3.6 percentage points, to 0.434 . This is related to increased public expenditure reducing the incentives for parental / individual expenditure on education. Total private education expenditure has actually increased in absolute terms but this is because society has became richer, and average earnings is up by about $17 \%$. Private education expenditure's share of total education expenditure falls from $53 \%$ to $42 \%$, and the correlation between the log of total human capital investments, private plus public, and parental earnings smaller.

Table 6: Public Education Expenditure per Student as \% of GDP Per Capita

| Education level | US | Denmark |
| :--- | :--- | :--- |
| Primary | 11.1 | 9.6 |
| Secondary | 13.0 | 19.5 |
| Tertiary | 26.3 | 67.1 |

Based on data from UNESCO (1999-2005) and OECD (1999-2005)

Secondary and tertiary private education spending is down with Danish public expenditure, while private spending on elementary education is up. This is because the Danish public investments are very large for tertiary and secondary education, see table 6 , and at about the same level as in the US for elementary education. Therefore parents move their investments from late to early education. Not surprisingly, greatly increasing public expenditure in tertiary
education increases college enrollment. The correlation between college completion and parental earnings decreases.

Introducing a Danish tax system reduces the intergenerational earnings elasticity by 12 percentage points to 0.35 , or about $40 \%$ of the difference between the US and the Scandinavian countries, see Table 1. The higher and more progressive taxes greatly reduces the incentives for private investment in education and the correlation between parental income and how much is invested in a child falls. This leads to lower earnings persistence. We observe that high / progressive taxes also lead to lower college enrollment and less inequality.

## Table 7: Policy Experiments

| Statistic | Bench- <br> mark | Data | Dansih <br> taxes | Danish <br> educ. <br> subsidies | Danish <br> subsidies <br> + taxes |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Mean hours worked | 0.417 | 0.417 | 0.446 | 0.408 | 0.440 |
| Mean wages, workers w. o. college | 1.002 | 1.000 | 0.909 | 1.031 | 0.926 |
| Mean wages, workers with college | 1.757 | 1.757 | 1.280 | 2.067 | 1.540 |
| Std. dev. of log wages | 0.571 | 0.570 | 0.499 | 0.632 | 0.550 |
| Investment in elementary school | 0.037 | 0.038 | 0.009 | 0.053 | 0.015 |
| Investment in college | 0.120 | 0.121 | 0.015 | 0.111 | 0.015 |
| Fraction enrolling in college | 0.590 | 0.588 | 0.511 | 0.890 | 0.832 |
| Fraction failing out of college | 0.399 | 0.400 | 0.438 | 0.426 | 0.459 |
| Intergen. earnings elasticity | $\mathbf{0 . 4 7 0}$ | $\mathbf{0 . 4 7 0}$ | $\mathbf{0 . 3 5 0}$ | $\mathbf{0 . 4 3 4}$ | $\mathbf{0 . 3 5 1}$ |
| Mean assets of people aged 25-29 | 0.092 | 0.092 | 0.029 | 0.104 | 0.020 |
| Mean assets of people aged 50-54 | 0.525 | 0.525 | 0.350 | 0.659 | 0.285 |

Introducing both Danish public education expenditure and taxation at the same time actually increases earnings persistence by 0.1 percentage points to 0.351 . There are two competing effects here. On one hand the society has became richer and therefore people invest more in human capital, in addition to the government investing more. When total human capital investments increase, human capital becomes more important for the log of earnings relative to the idiosyncratic shocks. On the other hand the government investments have increased relative to parental investments, and therefore the correlation between total investments and parental earnings and college enrollment and parental earnings falls.

Table 8: Selected Statistics From Policy Experiments (annual 2005-\$)

| Statistic | Bench- <br> mark | Dansih <br> taxes | Danish <br> educ. <br> subsidies | Danish <br> subsidies + <br> taxes |
| :--- | :--- | :--- | :--- | :--- |
| Human capital investment age 5-9 | $\$ 3998$ | $\$ 868$ | $\$ 5710$ | $\$ 1547$ |
| Human capital investment age 10-14 | $\$ 5127$ | $\$ 1310$ | $\$ 7165$ | $\$ 2144$ |
| Human capital investment age 14-19 | $\$ 5752$ | $\$ 1492$ | $\$ 5055$ | $\$ 864$ |
| Human capital investment in college | $\$ 14692$ | $\$ 1780$ | $\$ 13513$ | $\$ 1881$ |
| Average human capital inv. (all ages) | $\$ 5016$ | $\$ 1041$ | $\$ 6200$ | $\$ 1288$ |
| Gift from parent to child | $\$ 78714$ | $\$ 10333$ | $\$ 128269$ | 21193 |
| tax per worker - educ. expenditure | 0.300 | 0.454 | 0.329 | 0.499 |
| benchmark average earnings |  | $\$ 6111$ | $\$ 53489$ | $\$ 71474$ |
| Average Earnings | 0.525 | 0.215 | 0.417 | $\$ 60883$ |
| $I_{\text {private }} / I_{\text {total }}$ | 0.7418 | 0.6253 | 0.6099 | 0.156 |
| Corr $\left(\log \left(I_{\text {private }}+I_{g}\right), \log \left(y_{\text {parent }}\right)\right)$ | 0.1939 | 0.1367 | 0.1572 | 0.1412 |
| Corr $\left(\right.$ college, $\left(\log \left(y_{\text {parent }}\right)\right)$ |  |  |  |  |

We conclude that tax- and education spending policies significantly impact earnings persistence. Whether having low earnings persistence in the society is good or bad is naturally a different question. More progressive taxation as a stand-alone policy is reducing human capital accumulation and leading to a poorer society, while increased public education expenditure have the opposite effect. Higher taxes may, however, be needed to finance education expenditure. When I introduced Danish education spending, the net change in tax revenues was actually positive. However, the society became richer, and the government only increased its spending on education. I did for instance let the social security payments stay at their old level. Yet another issue is of course general equilibrium effects. I will leave the study of optimal policies to future research.

## The Impact of Borrowing Constraints

The importance of borrowing constraints both for intergenerational earnings persistence and college enrollment has received much attention in the literature. I study the effect of tightening and relaxing the college borrowing constraint, as well as relaxing the assumption that borrowing is only allowed in college. Tables 9 and 10 displays the results from these experiments.

Table 9: Policy Experiments

| Statistic | Bench- <br> mark | Data | 0X BC | 2X BC | 2X BC w. <br> 0. college |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Mean hours worked | 0.417 | 0.417 | 0.417 | 0.416 | 0.417 |
| Mean wages, workers w. o. college | 1.002 | 1.000 | 1.004 | 1.006 | 1.002 |
| Mean wages, workers with college | 1.757 | 1.757 | 1.784 | 1.856 | 1.873 |
| Std. dev. of log wages | 0.571 | 0.570 | 0.563 | 0.586 | 0.585 |
| Investment in elementary school | 0.037 | 0.038 | 0.037 | 0.039 | 0.039 |
| Investment in college | 0.120 | 0.121 | 0.122 | 0.138 | 0.142 |
| Fraction enrolling in college | 0.590 | 0.588 | 0.483 | 0.619 | 0.590 |
| Fraction failing out of college | 0.399 | 0.400 | 0.367 | 0.400 | 0.391 |
| Intergen. earnings elasticity | $\mathbf{0 . 4 7 0}$ | $\mathbf{0 . 4 7 0}$ | $\mathbf{0 . 4 7 4}$ | $\mathbf{0 . 4 6 6}$ | $\mathbf{0 . 4 6 7}$ |
| Mean assets of people aged 25-29 | 0.092 | 0.092 | 0.112 | 0.080 | 0.079 |
| Mean assets of people aged 50-54 | 0.525 | 0.525 | 0.530 | 0.538 | 0.534 |

Columns 3 and 4 displays the result when setting the college borrowing constraint to 0 and doubling the college borrowing constraint. Column 5 displays the results when also people that do not attend college are allowed to borrow up to twice the original college borrowing constraint, $\$ 49712$.

As can be seen from Table 9, relatively large changes to the borrowing constraint have relatively little impact on intergenerational earnings persistence Completely eliminating borrowing for college reduces college enrollment by $18 \%$, and college completion by $14 \%$, however, it is those that have the least to gain from college who drops out. Average earnings in the economy only

## Table 10: Selected Statistics From Policy Experiments

| Statistic | Benchmark | 0X BC | 2X BC | 2X BC w. <br> o. college |
| :--- | :--- | :--- | :--- | :--- |
| Human capital investment age 5-9 | $\$ 3998$ | $\$ 3966$ | $\$ 4167$ | $\$ 4151$ |
| Human capital investment age 10-14 | $\$ 5127$ | $\$ 5063$ | $\$ 5348$ | $\$ 5316$ |
| Human capital investment age 14-19 | $\$ 5752$ | $\$ 5651$ | $\$ 6015$ | $\$ 5978$ |
| Human capital investment in college | $\$ 14692$ | $\$ 14914$ | $\$ 16874$ | $\$ 17300$ |
| Gift from parent to child | $\$ 78714$ | $\$ 77168$ | $\$ 86709$ | $\$ 85770$ |
| Average Earnings | $\$ 61111$ | $\$ 60068$ | $\$ 63143$ | $\$ 62914$ |

fall by $1.7 \%$, and intergenerational persistence only rises by 0.4 percentage points. Letting people borrow more has little impact both on persistence and on college enrollment.

## 9 Conclusion

In this paper I develop an intergenerational life-cycle model of human capital accumulation and earnings, which features taxation, public education expenditure, borrowing constraints, partially inheritable abilities, inter vivos transfers from parent to child, and idiosyncratic wage shocks as determinants of intergenerational earnings persistence. I calibrate the model to US data, and use it to quantify how earnings persistence in the US changes as I introduce policies from Denmark. I find that taxation and public education expenditure have a significant impact on earnings persistence and are likely contributors to the cross country patterns which empirical researchers have found. Taxation is quantitatively more important. As I introduce a Danish tax system in the US, intergenerational earnings elasticity falls by 0.12 , or about $40 \%$ of the difference between the US and the Scandinavian countries, which have the lowest earnings persistence among the countries in my sample. I also find that intragenerational borrowing constraints have very limited impact on earnings persistence.

Future research in this area may include the study of optimal education expenditure and tax policies within an intergenerational general equilibrium framework. An extension is also to explicitly model the supply of educational services. In this paper I have assumed that the technology for human capital production stays the same as the demand for education changes.

## 10 Appendices

## A1 Discussion of Difficulties with Comparing Different Studies of Earnings Persistence

There are some difficulties related to comparing different studies of intergenerational earnings persistence. Solon (1992) and Blanden (forthcoming) provides more in depth discussions of some of the methodological issues. One problem in the estimation of (1) is the measure of earnings. Ideally the measure of earnings used in (1) should be permanent or lifetime earnings. Since this is rarely available, the econometrician will either use earnings observed in a single year or preferably take the average of several years of earnings. This will generally be an inaccurate measure of permanent earnings. It is easy to show that an inaccurate measure of father's earnings in (1) will lead the estimate of $\beta$ to be biased downwards. A first step towards reducing this measurement error is controlling for age in (1), and this is done in pretty much every study. However, if more years of earnings is averaged the measurement error is reduced, and this is a source of discrepancies between different studies. Another obvious source of discrepancies between studies is the quality of data. If the sample is too homogeneous, i.e. the variance of earnings is too small, as is typical for unrepresentative data samples, the problem with measurement error is compounded, see Solon (1992).

A possible solution to the problem with measurement error in father's earnings is the use of instrumental variables. The instruments must be uncorrelated with the measurement error, and in addition uncorrelated to the son's earnings. The problem with the instrumental variable approach is that most variables that are related to father's earnings may also have an independent impact on son's earnings. Solon (1992) shows that in this case the estimate of $\beta$ will be biased upwards. The instrumental variables approach is none the less becoming more popular in the literature.

Finally, the age at which father's and son's earnings are observed may have a substantial impact on the estimates of $\beta$, see Haider and Solon (2006), Grawe (2003). Controlling for age in the regression is not solving this problem as high and low earners have different life-cycle earnings profiles. Often the earnings of young sons are regressed on the earnings of old fathers, which is found to cause a downward bias in the estimate of $\beta$. Haider and Solon (2006) find that the years around 40 will be the best proxies for lifetime earnings.

Corak (2006) provides a cross country meta study of intergenerational earnings persistence that tries to take into account how many years of father's earnings that was used as a measure for permanent earning, whether an IV approach was used, and the age of the father at the time of observation. Table 1 displays the results from this study supplemented with earnings persistence from Italy and Spain, which I take from Piraino (2007) and Pla (2009). The number for Italy I adjust using a formula provided in Corak (2006). I cannot do the same for Spain, because I do not know the average age of the fathers in that study. Given the many problems with comparing different studies of intergenerational earnings persistence, it is clear that Table 1 should be interpreted as a stylized fact.

## A2 Fitting Tax Functions Based on Data from the OECD

For every country in Table 1, I fit the polynomial in (30). I use this functional form because it generally gives me a very good fit, $\mathrm{R}^{2}$ above $99.9 \%$, and because I get functions that are strictly increasing and well behaved on a relatively wide range of labor income. I use labor income tax data from the OECD Tax-Benefit Calculator ${ }^{8}$ and the OECD Tax Database ${ }^{9}$. This data is constructed by the OECD based on tax laws from different countries. The OECD Tax-Benefit Calculator gives the gross- and net-, after taxes and benefits, labor income at every percentage of average labor income on a range between $50 \%$ and $200 \%$ of average labor income, by year and family type starting in 2001. I use the data at every 5 th percentage point for single individuals without children, and take an average of the years 2001-2005. The OECD Tax Database provides the top marginal tax rate in each country and the starting point for this tax rate. To get the tax at earnings above $200 \%$ of average labor income, I use this information, and compute the tax at every multiple of 0.5 times average earnings between 2.5 and 15 times average earnings. For most countries the top marginal tax rate kicks in before $200 \%$ of average labor income but in the US, for instance, the top marginal tax rate starts at about nine times average earnings. I then assume that the marginal tax rate increases linearly between 2 times average earnings and the point where the top marginal tax rate becomes effective. Since I only have tax data starting at $50 \%$ of average earnings, I add a random positive point of close to zero tax for close to zero earnings, to get my tax functions well behaved for very small earnings.

[^6]This, however, has almost no impact on the fit with the real data points. The alternative would have have been to require all people to work enough to make a certain amount of income. I fit the tax functions by running OLS regressions. Table 11 displays the country tax functions, while Figure 5 plots the tax functions for the US and Denmark.

## Table 11: Country Tax Functions

| Country | $\tau_{1}$ | $\tau_{2}$ | $\tau_{3}$ | $\tau_{4}$ | $\mathrm{R}^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Denmark | -1.242825 | 3.603493 | -2.456365 | 0.5239973 | 0.9997 |
| Norway | -0.6488133 | 1.818972 | -1.023706 | 0.1670745 | 0.9999 |
| Finland | -0.71829 | 1.895892 | -1.004558 | 0.1465101 | 0.9996 |
| Canada | -0.3056732 | 0.8059581 | -0.2546371 | -0.0145851 | 0.9997 |
| Sweden | -0.6629891 | 1.966373 | -1.183786 | 0.2152142 | 0.9997 |
| Germany | -1.329006 | 4.017692 | -2.947534 | 0.6809511 | 0.9998 |
| Spain | -0.2001187 | 0.3728243 | 0.1407691 | -0.1200151 | 0.9994 |
| France | -0.5460613 | 1.651868 | -1.011427 | 0.1903222 | 0.9998 |
| Italy | -0.7060691 | 1.782236 | -0.9431628 | 0.137171 | 0.9989 |
| USA | -0.5730303 | 1.705866 | -1.096482 | 0.2207298 | 0.9998 |
| UK | -0.5907906 | 1.778369 | -1.163281 | 0.2362276 | 0.9998 |

Figure 5: Labor Income Tax Functions for the US and Denmark


## A3 Proof of Propositions I and II

## Proposition I

Since $\log \left(y_{t}\right)$ is a monotonic transformation of $y_{t}$, it will be sufficient to take the derivatives of the top part of (11) with respect to $y_{t}$. We have:

$$
\begin{gather*}
\frac{\partial \log \left(y_{t+1}\right)}{\partial y_{t}}=\frac{\psi(1-\tau)}{y_{t}(1-\tau)+I_{g}}>0  \tag{32}\\
\frac{\partial^{2} \log \left(y_{t+1}\right)}{\partial y_{t} \partial \tau}=\frac{-\psi I_{g}}{\left(y_{t}(1-\tau)+I_{g}\right)^{2}}<0  \tag{33}\\
\frac{\partial^{2} \log \left(y_{t+1}\right)}{\partial y_{t} \partial I_{g}}=\frac{-\psi(1-\tau)}{\left(y_{t}(1-\tau)+I_{g}\right)^{2}}<0  \tag{34}\\
\frac{\partial^{2} \log \left(y_{t+1}\right)}{\partial y_{t} \partial \psi}=\frac{y_{t}(1-\tau)^{2}+I_{g}(1-\tau)}{\left(y_{t}(1-\tau)+I_{g}\right)^{2}}>0 \tag{35}
\end{gather*}
$$

Proposition II
Differentiating (16), we obtain:

$$
\begin{gather*}
\frac{\partial \beta}{\partial \tau}=\frac{-\psi\left(1-\theta^{2}\right) \overline{\mathrm{y}} I_{g}}{\left((1+\psi \theta)(1-\tau) \overline{\mathrm{y}}+I_{g}\right)^{2}}<0  \tag{36}\\
\frac{\partial \beta}{\partial I_{g}}=\frac{-\psi\left(1-\theta^{2}\right)(1-\tau) \overline{\mathrm{y}}}{\left((1+\psi \theta)(1-\tau) \overline{\mathrm{y}}+I_{g}\right)^{2}}<0  \tag{37}\\
\frac{\partial \beta}{\partial \psi}=\frac{\left(1-\theta^{2}\right)(1-\tau)^{2} \overline{\mathrm{y}}^{2}+\left(1+\psi \theta-\theta^{2}\right)(1-\tau) \overline{\mathrm{y}} I_{g}}{\left((1+\psi \theta)(1-\tau) \overline{\mathrm{y}}+I_{g}\right)^{2}}>0  \tag{38}\\
\frac{\partial \beta}{\partial \theta}=\frac{\left(1-\psi^{2}\right)(1-\tau)^{2} \overline{\mathrm{y}}^{2}+2(1-\tau) \overline{\mathrm{y}} I_{g}+I_{g}^{2}}{\left((1+\psi \theta)(1-\tau) \overline{\mathrm{y}}+I_{g}\right)^{2}}>0 \tag{39}
\end{gather*}
$$

## A4 Computational Details

## Computation of Optimal policies

I put boundaries on the capital and human capital space and pick a grid in each dimension. I pick 40 grid points in $K=\left[0, k^{\max }\right]$ and 16 grid points in $H=\left[0, h^{\max }\right]$. The grid points for capital is taken to be the scaled zeros of a 40th order Chebyshev polynomial while the grid points for human capital are taken to be the scaled zeros of a 16th order Chebyshev polynomial. Following the method outlined by Tauchen (1986), I approximate the processes for the idiosyncratic productivity shock, $u$, and ability, $A$, as finite state Markov processes. I use 7 equally spaced states for u in $U=\left[-2 \sigma_{u}, 2 \sigma_{u}\right]$, and 13 equally spaced states for A in $\bar{A}=\left[-3 \sigma_{A}, 3 \sigma_{A}\right]$. Let $J=\{0,1\}$ be the state space for whether college educated. The maximum size of the state space occurs in periods 5-7, or age 40-50, when there are 6 state variables apart from time. The state space is then $J \times K \times H \times H \times \overline{\mathrm{A}} \times U$, or $1,863,680$ grid points. I compute the household's optimal policies for each grid point in each time period by iterating backwards. I start from age 100 , the last period of life. In that period, next period's value function is 0 , and the optimal policy is to consume as much as possible. Knowing the value function at age 100, I can compute optimal policies and value functions for age 95 , and so on. Reaching age 50 , when th child leaves home, I need to know both the parent's value function at age 55 and the child's value function at age 20 to compute the optimal policies. The first time around, I use an educated guess for the child's value function at age 20 . When I reach age 20 , I get a new $V(a g e=20, \cdot)$ and start over again from age 50 . I continue this iteration until $V$ converges.
To solve for the optimal policies in each time period, I use the routine called LCONF from the IMSL Fortran library. It is based on M. Powell's method for solving linearly constrained optimization problems, see IMSL documentation for details. To interpolate the value function outside of the grid I use Chebyshev collocation, see Judd (1998), Heer and Maussner (2004). When there is a child in the household and the parent is investing in the child's human capital, next period's value function must be interpolated in the $K \times H$-space. The value function is then represented as a polynomial with $40 \times 16=640$ coefficients. At one point in time, when the agent chooses whether or not to attend college, I am taking the max of two value functions. When these two value functions overlap, the value function considered by the parent, before the child makes the college decision, will generally not be concave. However, what the parent needs to consider is the expectation of the value function over the idiosyncratic shock. It turns out that
the expectation of the value function is concave, although there is no theoretical guarantee for it. To be absolutely sure that I am finding a global max, I am multi starting the solver from points that are far apart.

## Simulation

Knowing today's state, the policy functions, and drawing shocks, $u$ and $v$, I can find next period's state. I make 200000 draws from a random initial distribution of 20 year olds, and run the simulation for 200 generations (enough to reach a stationary distribution). In the simulation, the policy functions must be interpolated on the $K \times H \times H$-space as both the child's and the parent's human capital may be outside the of the grid. I use linear interpolation.

## Hardware and software

I use Intel Fortran, version 11.1 and a computer with a 2.93 GHz Core-i7 processor. To speed up the computation I use OpenMP to parallelize the code on the 8 threads.

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[^1]:    ${ }^{3}$ See also Blanden (forthcoming) for an extensive summary of the empirical literature.
    ${ }^{4}$ There are many difficulties with comparing different studies of earnings persistence, see Appendix A1. Table 1 is to be interpreted as a stylized fact.

[^2]:    ${ }^{4} \mathrm{Pla}$ (2009) estimates one earnings elasticity using sons aged 30-40, and one earnings elasticity using sons aged 4050. Table 1 displays the average of the two.

[^3]:    ${ }^{5}$ See Fernandez et. al. (2005)

[^4]:    ${ }^{6}$ Series TB3MS and GDPDEF

[^5]:    ${ }^{7}$ See Appendix A4 for details on computation

[^6]:    ${ }^{8}$ Available at: www.oecd.org/document/18/0,3343,en_2649_34637_39717906_1_1_1_1,00.html
    ${ }^{9}$ Available at: www.oecd.org/document/60/0,3343,en_2649_34533_1942460_1_1_1_1,00\&\&en-USS_01DBC.html

