International Portfolios: An Incomplete Markets General Equilibrium Approach

Serhiy Stepanchuk
University of Pennsylvania

Viktor Tsyrennikov
Cornell University

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Abstract

We build a two-country two-good model of international portfolio choice and current account adjustment. We calibrate the model so that it is consistent with the home equity bias and consumption-real exchange rate disconnect. Financial markets are incomplete and trade three assets: domestic and foreign stocks and an international bond. First, we show that if the bond is denominated in domestic good, than domestic economy in the long run can accumulate a sizable negative net foreign asset position. This result suggests that the international role of the U.S. dollar cannot be ignored. Consistent with the data, we also find that the negative NFA position is achieved by accumulating debt, while the portfolio share invested by domestic investors into stocks increases. Second, we show that the net foreign asset (NFA) position could also deteriorate if the volatility in the domestic economy decreases relative to the rest of the world, as happened after 1984. This provides another explanation why the U.S. NFA position has been declining.

1 Introduction

A recent wave of financial integration that started in mid-80's has led to a surge in international asset trade, and a build-up of large cross-border gross asset positions[1]. Simultaneously, there has been an emergence of the so-called "global imbalances". One of the most-often cited developments here that came to the forefront of policy discussion has been a large and persistent deterioration of the net international investment position of the largest world economy, the United States. This has revived the interest in the analysis and economic modeling of the countries' international portfolios. It has been argued that the structure of the countries' portfolios is of first-order importance in the analysis of external adjustment. For instance, it has been observed that the changes in the portfolio valuations play a substantial role in the cyclical adjustment of the countries' net foreign asset positions[2]. Obstfeld (2004) states that "appropriate concepts of external balance adjustment cannot be defined without reference to the structure of national portfolios".

This lead to a development of two different strands of literature. One is concerned with introducing the international portfolio choice into the open economy macroeconomics models and developing methods for solving them. Notable recent contributions here are Tille & Wincoop (2007), Devereux & Sutherland (2006), Evans & Hnatovska (2008) and Pavlova & Rigobon (2010). The other strand attempts to uncover the driving forces behind the deterioration of the U.S. net foreign asset position[3]. We contribute to both of these literatures.

Figure 1, using the data from Lane & Milesi-Ferretti (2007) dataset, shows the sharp increase in U.S. gross positions in two major asset categories –

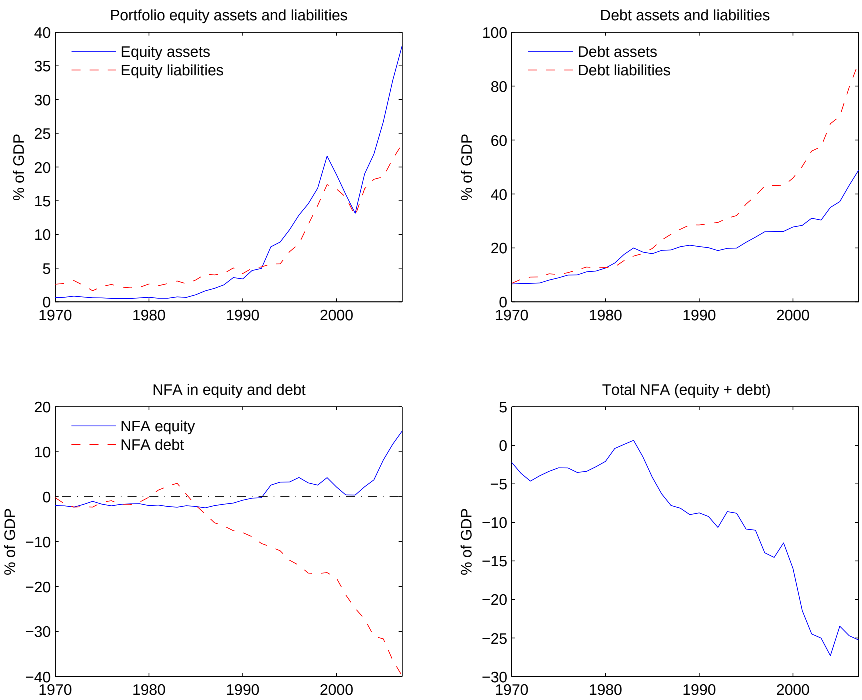
[1]See Gourinchas & Rey (2005), Lane & Milesi-Ferretti (2005) and Lane & Milesi-Ferretti (2007) for a detailed account of these developments.

[2]Gourinchas & Rey (2007) find that stabilizing valuation effects constitute 27% of cyclical adjustment in U.S.'s net foreign asset position.

[3]See Caballero, Farhi & Gourinchas (2008), Mendoza, Quadrini & Rios-Rull (2008) and Fogli & Perri (2006)

portfolio debt and equity instruments, since mid-80's. It also shows the country's net position in these two asset categories.

Figure 1: US external positions in debt and portfolio equity



It is clear from the picture that the deterioration in the overall U.S. net foreign asset position has been driven by a growth in debt obligations, while the net position in equity has actually improved. Gourinchas & Rey (2005) conclude that "as financial globalization accelerated its pace, the U.S. transformed itself from a world banker into a world venture capitalist, investing greater amounts into high yield assets such as equity and FDI", while "its liabilities have remained dominated by bank loans, trade credit and debt, i.e. low yield safe assets". A similar observation is made by Obstfeld (2004), who states that for the United States, "the striking change since the early 1980s is the sharp growth in foreign portfolio equity holdings", while on the liabilities' side, "the most dramatic percentage increase has been in the share of U.S. bonds held by foreigners"[4].

To explain the behavior of the net international investment position of the U.S. and its portfolio composition, we develop a two-country general equilibrium model with many assets, incomplete financial markets and portfolio choice. We consider the effect of the following two features that make our two economies asymmetric:

1. *Exorbitant priviledge.* Figure 2, that uses the data from Lane & Milesi-Ferretti (2007) and Lane & Shambaugh (2009) datasets, shows that international debt markets are dominated by the assets denominated in only a few "global" currencies [5]. U.S. dollar played a dominant role here until recently, when the introduction of euro has lead to the increased share of euro-denominated internationally traded debt assets.

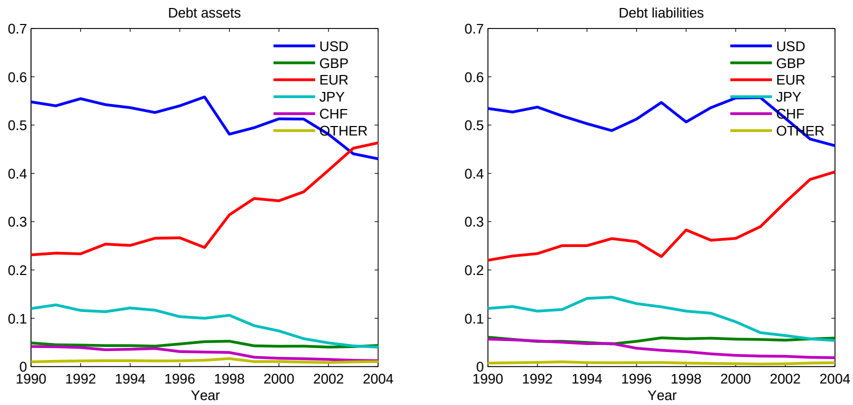


Figure 2: Currency composition of internationally-traded debt assets and liabilities

Figure 13 in the appendix also shows that the U.S., as the issuer of the global currency, has been able to issue most of its debt in its own

[4]See also Higgins, Tille & Klitgaard (2007), Tille (2005), Mendoza et al. (2008) and Obsfled & Rogoff (2005).

[5]See Eichengreen & Hausmann (2005) for further evidence of this point.

currency, while until the introduction of the Euro, only about 20% of the internationally-traded debt issued by all other countries has been denominated in the local currency of the issuring country[6].

Eichengreen, Hausmann & Panizza (2002) find that the country size is the only robust determinate of the country's ability to issue debt in own currency, while they find the effects of various measures of economic and financial development, the soundness of the country's monetary and fiscal policy, the degree of openness statistically and economically insignificant. They conclude that the internationally traded debt "is concentrated in a very few currencies for reasons largely beyond the control of the excluded countries", the finding that they call "the original sin"[7].

We assume that the international debt market structure is exogenously fixed, and we attempt to model the role of the U.S. dollar as the leading global currency by assuming that there is only one internationally-traded bond that pays off in the good of one of the two countries.

2. *Great Moderation.* Fogli & Perri (2006) document that after 1985, the U.S. experienced a larger fall in business cycle volatility than its partners, and argue that it can be one of the causes for the deterioration in the U.S. net foreign asset position. Since their model has only one internationally-traded asset – riskless bond, they don't obtain any implications for the portfolio structure.

We show that both of these features produce a sizable negative net foreign asset position for the home country, the U.S., in our model, and also

[6]Even after the introduction of the Euro, this number remains under 50%; most of this increase comes from the expansion of Euro-denominated debt, and a large part of it is traded between the Euro zone countries.

[7]Hasan (2010) develops a model where he shows that the debt issued in the currency of a larger country will have lower equilibrium interest rate. If this effect is large enough, one can imagine that no one would be willing to borrow in high-interest debt instruments issued in the currencies of smaller countries

deliver the portfolio structure that resembles the one observed in the U.S., with a short position in debt and increased share of wealth invested in risky assets like equity. The "exorbitant" privilege works through the correlation structure of the bond's payoff and home country non-traded income, making the bond whose payoffs are denominated in units of home country goods an undesirable hedging instrument for domestic investors. The main driving force in the "great moderation" experiment is the precautionary savings channel which makes domestic investors endogenously less risk-averse than their foreign counterparts.

We also test whether a lower borrowing capacity abroad can contribute to generating the negative net foreign asset position in the home country. We find only some moderate effects here.

In terms of the methodological approach, we build on the numerical method developed in Kubler & Schmedders (2003). This allows us to obtain a numerical solution to our model that is globally accurate, while the perturbation methods developed in Tille & Wincoop (2007) and Devereux & Sutherland (2006) are designed to offer an accurate solution only locally around some fixed point in a state space. Our numerical approach also allows us to analyze the case with the countries that are exogenously asymmetric. It is not clear how one could do this with the perturbation methods, since they require one to make an arbitrary guess in terms of the steady state wealth distribution between the countries, making it hard to generate large and persistent deviations of the net foreign asset position from zero.

The structure of our paper is as follows. In section 2, we set up the model and reformulate it recursively, so that it can be analyzed using our numerical algorithm. In section 3, we describe the algorithm, and test its performance in a special case of our model, where we can describe solution analytically. In section 5, we obtain and discuss the results from our calibrated model. Section 6 concludes.

2 Model

2.1 Economic Environment

Our model consists of two economies that we will call "Home" (representing U.S.), and "Foreign" (representing the rest of the world, or RoW). We will assume that each of these two economies is populated by a continuum of infinitely-lived consumers who share the same preferences within the country. Each period, both countries are endowed with some quantity of their own perishable good. The consumers in both countries like to consume both of these goods, but have a relative preference towards the consumption of their local good ("home bias in consumption"), which we will model by assigning a higher weight to the local goods in the utility function. To differentiate between the domestic and foreign consumers, we will denote all foreign consumers' choice variables with an asterisk.

Time is discrete, $t = 0, \ldots, \infty$. Each period t, one of finitely many possible states of the world, $z_t \in Z = \{z_1, \ldots, z_n\}$, realizes. The state of the world characterizes all relevant uncertainty. In particular, the available quantity of both consumption goods in each period is a time-invariant function of the state of the world, $e_h : Z \to \mathbb{R}_{++}$ and $e_f : Z \to \mathbb{R}_{++}$. We will assume that only some part of the output in each country is "capitalized" – it comes in the form of the dividends from the Lucas trees that represent the stock indices in the two countries. The rest of the output is non-traded. We will often refer to the non-traded part of the output as "wages", but in our calibration, it will also include the profits of the companies that are not publicly traded through the stock market. We will allow the share of non-traded income in total output to be stochastic, so that the division of the output between dividends and wages is also determined by the realization of the state of the world: $e_h(z_t) = d_h(z_t) + w_h(z_t)$ and $e_f(z_t) = d_f(z_t) + w_f(z_t)$[8].

We assume that the exogenous shock that determines the state of the

[8] In particular, we will not assume that non-traded income is a constant fraction of the output. The reason for this will become apparent from our discussion in section 3.2.

world follows a first-order linear Markov process, with the probability transition matrix Π, so that $\Pi(z_{t+1}|z_t)$ is the probability that z_{t+1} realizes next period, given that this period, the state of the world is z_t.

Consumers in both countries maximize their expected utility. For domestic consumers, their utility function is:

$$U(c) = E\Big[\sum_{t=0}^{\infty} \beta^t u(g(c_{ht}, c_{ft}))\Big|\mathcal{I}_0\Big], \tag{2.1}$$

The instantaneous utility function is a composition of the Armington aggregator over the two consumption goods, $g(c_h, c_f) = (s_h c_h^{\rho} + (1 - s_h) c_f^{\rho})^{1/\rho}$ (where parameter ρ controls the elasticity of substitution between the goods, and we will assume that $s_h > 0.5$ to model the "home bias in consumption"), and standard CRRA utility function, $u(g) = \frac{g^{1-\sigma}}{1-\sigma}$.

Foreign consumers maximize a similar expected utility function, with c^*_{ht}, c^*_{ft} and s^*_h replacing c_{ht}, c_{ft} and s_h.

Financial Markets. In a model with endowment economies and consumers who have standard expected utility preferences, assuming complete financial markets has a number of unrealistic implications. First, Judd, Kubler & Schmedders (2000) demonstrate that this typically implies that consumers choose constant financial portfolios that do not change over time, with no trade in assets in any period beyond period 0. This in turn implies that the only source of the changes in a country's net foreign asset position would be the valuation effects (changes in assets' prices), with the traditional measure of the current account (which excludes the asset valuation changes) being always zero. Second, Backus, Kehoe & Kydland (1992) show that it also has counterfactual implications for cross-country consumption correlations. We will assume that financial markets are incomplete, with the number of internationally-traded assets (with linearly independent payoffs) being smaller than the number of the states of the world. In fact, we will assume that only the following 3 assets are traded internationally – two Lucas trees representing the stock indices in the two countries, and a single one-period internationally traded bond. The stocks represent the claims to the future stream of dividends (paid in the two countries' consumption goods, as explained above), and are traded at an ex-dividend prices q_h and q_f. The bond pays off in some fixed combination of the two consumption goods, $r_b = \alpha p_h + (1-\alpha)p_f$, where p_h and p_f are the two consumption goods' prices, and is traded at price q_b.

Taste shocks and spot traders. Since we assume that all internationally-traded assets in our model are "real" assets (meaning that they pay off in some bundles of the two consumption goods), one can reasonably expect that their rates of return, and thus the resulting portfolio choices predicted by the model, will be heavily influenced by the behavior of the relative consumption goods' prices, or real exchange rates. Thus, it is important for us to obtain realistic predictions for the behavior of relative goods' prices. Another reason why the behavior of relative prices is important for us is that we are interested in analyzing the behavior of the net foreign asset position in the U.S., and the valuation changes in U.S. NFA position caused by exchange-rate changes are comparable in magnitude to the size of the financial flows[9]. The salient feature of the data here is the so-called Backus-Smith puzzle (or the "relative consumption/RER anomaly"). Backus & Smith (1993) demonstrated that a model with complete financial markets and only output shocks predicts that the growth rate of relative consumptions between the two countries should be perfectly correlated with the growth in relative consumption goods' prices (or real exchange rate), while in the data this correlation for most countries in close to zero. To address this observation, we add the following two features to our model. First, similar to Stockman & Tesar (1995), Pavlova & Rigobon (2007) and Heathcote & Perri (2009), we introduce taste shocks (as a simple reduced-form way to model the demand-side shocks). Second, similar to Kollmann (2009), we introduce some positive measure μ of consumers in both countries who do not participate in the international financial markets,

[9]According to the BEA data, the average size of the annual changes in the U.S. NFA position caused by exchange rate fluctuations over 1989-2009 period was 0.32 of the changes caused by the financial flows.

and only trade in spot markets for consumption goods (we will call them "spot traders", and we will refer to the regular consumers who trade in both spot markets for consumption goods and financial asset markets as "active traders"). We will show in section 5.3 that both of these features increase the volatility of relative prices and reduce the correlation between the relative prices and relative consumption growth rates.

Budget constraints. Let θ_h and θ_f denote the domestic active traders' positions in home and foreign stocks, and let b denote their bond position. Let z^t denote a finite history of shocks up to date t, and let I_h denote domestic active traders' "cash-in-hand" – the market value of their non-traded income and their financial portfolio (including the dividends):

$$I_h(z^t) \equiv p_h(z^t)(1-\mu)w_h(z^t) + r_b(z^t)b(z^{t-1}) + r_h(z^t)\theta_h(z^{t-1}) + r_f(z^t)\theta_f(z^{t-1}),$$

where r_b, r_h and r_f are the returns from the bond, the home and the foreign stock respectively:

$$\begin{aligned} r_b &\equiv \alpha p_h + (1-\alpha)p_f, \\ r_h &\equiv q_h + p_h d_h, \\ r_f &\equiv q_f + p_f d_f, \end{aligned}$$

and μ is the share of the spot traders in the country's total population.

Domestic active traders maximize utility subject to the sequence of budget constraints that have the following form:

$$p_h(z^t)c_h(z^t) + p_f(z^t)c_f(z^t) + q_h(z^t)\theta_h(z^t) + q_f(z^t)\theta_f(z^t) + q_b(z^t)b(z^t) = I_h(z^t), \tag{2.2}$$

Spot traders each period receive their share of non-traded income, μw_h. They are precluded from participating in financial markets, and thus face a sequence of static budget constraints:

$$p_h(z^t)c_h^{st}(z^t) + p_f(z^t)c_f^{st}(z^t) = p_h(z^t)\mu w_h(z^t)$$

In addition to the budget constraints, we will assume that the active traders face short-selling constraints on stocks, $\theta_h \geqslant 0$, $\theta_f \geqslant 0$, $\theta_h^* \geqslant 0$ and $\theta_f^* \geqslant 0$[10], and borrowing limits that we describe next.

Borrowing limit. We will assume that the amount that the active traders in both countries can borrow using the internationally-traded bond is proportional to the lowest possible realization of the value of their wages next period. Namely, we will require that:

$$\min_{z^{t+1}} \Big(k(1-\mu) w_h(z^{t+1}) p_h(z^{t+1}) + b(z^t) r_b(z^{t+1}) \Big) \geq 0, \quad \forall z^t \tag{2.3}$$

for some $k > 0$, and similarly for the foreign active traders. Intuitively, for $k = 1$ this borrowing limit requires the consumer to be able to completely repay his debt in all possible states of the world next period, using his non-traded income only. This form of the borrowing limit will be particularly convenient for our numerical algorithm. However, it is also quite general, since we are not restricting k to be equal to 1. This gives us a lot of flexibility – larger values of k increase the amount of debt that the consumer is allowed to take on. In our baseline model in section 5.1 we will attempt to choose k to make our borrowing limits generous enough, so that our results are not driven by potentially binding borrowing constraints. However, in section 5.4 we will take a different approach − we will allow the borrowing limits to be different in two countries, and will investigate the impact of the tighter constraint in one of the countries on our results.

Competitive equilibrium. For any initial realization of the exogenous state z_0 and the initial distribution of asset holdings, we can define a competitive equilibrium in a standard manner, as a sequence of prices $\mathcal{P} = \{p_h(z^t), p_f(z^t), q_h(z^t), q_f(z^t), q_b(z^t), \forall z^t\}$, consumption allocation for active traders $\mathcal{C} = \{c_h(z^t), c_f(z^t), c_h^*(z^t), c^*(z^t), \forall z^t\}$ and spot traders $\mathcal{C}^{st} = \{c_h^{st}(z^t), c_f^{st}(z^t), c_h^{*st}(z^t), c_f^{*st}(z^t), \forall z^t\}$, and portfolio choices $\mathcal{A} = \{\theta_h(z^t), \theta_f(z^t), \theta_h^*(z^t), \theta_f^*(z^t), \forall z^t\}$ such that:

[10]The short-selling constraints can be easily generalize to $\theta_h \geqslant -\bar{t}$, $\theta_f \geqslant -\bar{t}$, $\theta_h^* \geqslant -\bar{t}$ and $\theta_f^* \geqslant -\bar{t}$ for some $\bar{t} \in \mathbb{R}_+$.

a) Given the prices, the allocations solve the optimization problem for every consumer.

b) For all z^t, markets clear:

$$\begin{aligned} c_h^*(z^t) + c_h(z^t) + c_h^{\mathrm{st}*}(z^t) + c_h^{\mathrm{st}}(z^t) &= e_h(z^t), \\ c_f^*(z^t) + c_f(z^t) + c_f^{\mathrm{st}*}(z^t) + c_f^{\mathrm{st}}(z^t) &= e_f(z^t), \\ \theta_h(z^t) + \theta_h^*(z^t) &= 1, \\ \theta_f(z^t) + \theta_f^*(z^t) &= 1, \\ b(z^t) + b^*(z^t) &= 0. \end{aligned}$$

We add the following price normalization: $p_h(z^t) + p_f(z^t) = 1$ for all z^t.

2.2 Wealth-recursive equilibria

Since we are interested in solving our model economy numerically, we will concentrate on equilibria that can be represented in a recursive form, as a map from some state space into all current endogenous variables (the policy function), and a transition function that describes the evolution of the state variable(s) over time[11].

The choice of the state space has both theoretical and practical consequences. On the one hand, the current state must be a sufficient statistic for the future evolution of the system. On the other hand, a high-dimensional state-space can lead to insurmountable computational difficulties – the so-called "curse of dimensionality". The description of the budget sets of the active traders (the only agents who solve dynamic problem in our model) in the previous section suggests that the distribution of "cash-in-hand", or wealth between these agents is a natural candidate to be the only endogenous state

[11] Duffie, Geanakoplos, Mas-Colell & McLennan (1994) call this type of equilibria "dynamically simple". They argue that it is reasonable to concentrate on these equilibria, since equilibria that do not display some minimal regularity through time will require implausibly high degree of coordination between the agents. Krueger & Kubler (2008) provide an overview of this type of equilibria and their applications in macroeconomics.

variable (in addition to exogenous shock) in our model. Following Kubler & Schmedders (2002), we will call this type of equilibria "wealth-recursive". Using wealth shares (as opposed to the beginning-of-period portfolios) as the only state variable offers a practical advantage of reducing the dimensionality of the numerical problem that we need to solve.

Wealth share as a state variable. For our numerical algorithm, it will be important to have a compact state space. We assume that the exogenous shocks come from some finite set. We will define our endogenous continuous state variable, the wealth share, so that given our portfolio constraints, it will lie in a unit interval. To achieve this, first let us redefine the active traders' "cash-in-hand" as:

$$\tilde{I}_h(z^t) = k(1-\mu)w_h(z_t)p_h(z^t) + r_b(z^t)b(z^{t-1}) + r_h(z^t)\theta_h(z^{t-1}) + r_f(z^t)\theta_f(z^{t-1}),$$
$$\tilde{I}_f(z^t) = k(1-\mu)w_f(z_t)p_f(z^t) + r_b(z^t)b^*(z^{t-1}) + r_h(z^t)\theta_h^*(z^{t-1}) + r_f(z^t)\theta_f^*(z^{t-1}).$$

The total "cash-in-hand", or wealth of the active traders in the two countries, is:

$$\tilde{I}(z^t) = \tilde{I}_h(z^t) + \tilde{I}_f(z^t) = k(1-\mu)(w_h(z_t)p_h(z^t) + w_f(z_t)p_f(z^t)) + r_h(z^t) + r_f(z^t),$$

which follows from the asset market clearing conditions, $b+b^* = 0$, $\theta_h+\theta_h^* = 1$, $\theta_f+\theta_f^* = 1$. Note that with strictly positive prices, the total wealth is always strictly positive.

As the next step, let as define the wealth shares of domestic and foreign active traders as:

$$\omega_h(z^t) = \frac{\tilde{I}_h(z^t)}{\tilde{I}(z^t)}, \quad \omega_f(z^t) = \frac{\tilde{I}_f(z^t)}{\tilde{I}(z^t)}. \tag{2.5}$$

Note that $\omega_h(z^t)+\omega_f(z^t) = 1$ for all z^t by construction. The following lemma is very useful for our computations.

Lemma 2.1. *Given the short-sale constraints on equity positions and the borrowing limit defined by 2.3, the wealth shares of domestic and foreign* *active traders remain in the unit interval,* $\omega_h(z^t) \in [0,1]$*,* $\omega_f(z^t) \in [0,1]$ *for all* z^t*.*

Proof. If the short-sale constraints on equity positions, and the borrowing limit 2.3 is always satisfied for both domestic and foreign active traders, we obtain that $\omega_h(z^t) \geqslant 0$ and $\omega_f(z^t) \geqslant 0$ for all z^t. Since $\omega_h(z^t) + \omega_f(z^t) = 1$, the desired result follows immediately. □

We can now rewrite the budget constraints of the domestic active traders as:

$$p_h(z^t)c_h + p_f(z^t)c_f + q_h(z^t)\theta'_h + q_f(z^t)\theta'_f + q_b(z^t)b' = \omega_h(z^t)\tilde{I}(z^t) + (1-k)(1-\mu)w_h(z_t)p_h(z^t)$$

Intuitively, the right-hand side of this equation, which determines the resources available to the consumer in node z^t, depends only on the current realization of the exogenous shock z_t, equilibrium prices (which together determine the total wealth, $\tilde{I}(z^t)$) and the consumer's wealth share, $\omega_h(z^t)$, and not on the consumer's positions in each of the assets separately. We thus can expect that (ω_h, z) could serve as a sufficient statistic for for the whole past history z^t, so that we can use them as our state variables.

More formally, let us define the wealth-recursive Markov equilibrium. Let $\Delta = [0,1]$, and let Φ be the set of all possible realizations of all endogenous variables in our model[12]. Let us define the "expectations correspondence":

$$g : \Big(Z \times \Delta \times \Phi\Big) \rightrightarrows \Big(\Delta \times \Phi\Big)^{|Z|}$$

Given the current values of all endogenous variables, it specifies all next-period values of endogenous variables that are consistent with the market-clearing, and first-order static and dynamic optimality conditions for all consumers in our model. It is described by the system of non-linear equations and inequalities specified in the appendix. Note that in addition to the

[12] Formally, $\Phi = \{\phi = (c_h, c_f, c^*_h, c^*_f, \theta_h, \theta_f, \theta^*_h, \theta^*_f, b, b^*, p_h, p_f, q_h, q_f, q_b) \in \mathbb{R}^4_+ \times \mathbb{R}^6 \times \mathbb{R}^3_+\}$.

market-clearing and first-order optimality equations, this system contains the equation that implicitly defines the evolution of the wealth share. Our numerical algorithm will attempt to approximate the policy and pricing functions by solving this system on some grid over $Z \times \Delta$.

We follow Kubler & Schmedders (2003) and define the generalized wealth-recursive Markov equilibrium as consisting of the nonempty-valued "policy correspondence" P, and the transition function F,

$$P : Z \times \Delta \rightrightarrows \Phi, \quad F : \text{graph}(P) \to \Big(\Delta \times \Phi\Big)^{|Z|}$$

such that for all $(z, \omega_h, \phi) \in \text{graph}(P)$ and all $z \in Z$:

$$F(z, \omega_h, \phi) \in g(z, \omega_h, \phi), \quad \text{and } (z, F_z(z, \omega_h, \phi)) \in \text{graph}(P)$$

Unfortunately, there are no known conditions that would guarantee that the policy correspondence P is single-valued[13]. Our numerical approach can be interpreted as an attempt to find a single-valued section from P and approximate it with some continuous functions.

2.3 Two-stage budgeting

To interpret some of our results in section 5, it will turn out to be useful to consider a slight reformulation of the optimization problem of the active traders in our model that uses the so-called "two-stage budgeting" procedure. It conceptually separates the overall optimization problem into the static and the dynamic parts, and will allow us to obtain an intuitive expression for the consumers' stochastic discount factor, which we will later use to interpret our portfolio results.

The dynamic part of the optimization problem deals with the reallocation of the consumer's income across time and states of the world. The active traders in our model achieve this by participating in the financial markets. In the static part, the consumer decides on how to spend the income available

[13] Kubler & Schmedders (2002) provide a counterexample.

to him in each date-event node on the consumption goods, by participating in the spot markets[14]

Let's start with the static problem. Suppose that the domestic consumer has some amount of income $\tilde{c}_h$ that he can spend on consumption in some date-event node[15]. To decide how to spend this income on the two consumption goods, he solves:

$$\max \left(s_h c_h^\rho + s_f c_f^\rho\right)^{1/\rho}$$
$$\text{s.t. } p_h c_h + p_f c_f = \tilde{c}_h.$$

The solution to this problem is:

$$\left(c_h^{\max}, c_f^{\max}\right) = \left(\frac{\tilde{c}_h}{p_h + p_f\big((s_f p_h)/(s_h p_f)\big)^{1/(1-\rho)}}, \frac{\tilde{c}_h}{p_f + p_h\big((s_h p_f)/(s_f p_h)\big)^{1/(1-\rho)}}\right).$$

This produces the indirect utility from income:

$$v_h(\tilde{c}_h|p_h, p_f) = g(c_h^{\max}, c_f^{\max}) = \tilde{c}_h/\pi_h(p_h, p_f),$$

where $\pi(p_h, p_f) = \left(s_h^{1/(1-\rho)} p_h^{\rho/(\rho-1)} + s_f^{1/(1-\rho)} p_f^{\rho/(\rho-1)}\right)^{1-1/\rho}$ is the domestic consumption-based price aggregator (domestic CPI). We can substitute it into 2.1, and rewrite the dynamic problem as:

$$\max E\Big[\sum_{t=0}^{\infty} \beta^t \frac{(\tilde{c}_h/\pi_h)^{1-\sigma}}{1-\sigma}\Big|\mathcal{I}_0\Big], \tag{2.6}$$

subject to the sequence of budget constraints:

$$\tilde{c}_h(z^t) + q_h(z^t)\theta_h(z^t) + q_f(z^t)\theta_f(z^t) + q_b(z^t)b(z^t) = I_h(z^t)$$

[14]Note that this division is useful for us only conceptually. Practically, we cannot separately find the static equilibria in spot markets for goods, and dynamic equilibria in asset markets, since there will be a feedback between the goods' prices and the distribution of the consumers' wealth. In section 3.2, we will consider a special case of our model, where one can in fact find the two equilibria separately.

[15]Note that $\tilde{c}_h$ is simply some amount of units of account, not "money".

As is always the case in the competitive equilibrium, the consumer treats all the prices (and thus the price index π_h) as given. This reformulation of his optimization problem makes it clear that the consumer, through trade in financial markets, attempts to achieve two goals. First, he has standard concerns about smoothing his consumption expenditures $\tilde{c}_h$ over time and states of the world. This is similar to the case with only one consumption good – in our case, $\tilde{c}_h$ plays the role of the single consumption good. However, in a setup with several consumption goods and homothetic preferences, he also wants to hedge against the fluctuations in the prices of the consumption goods, which are conveniently summarized by the changes in the appropriate CPI (π_h for the domestic consumer and π_f for the foreign consunmer). The changes in the CPI act as "preference shocks", increasing the marginal utility from income when π_h is high, and decreasing it when π_h is low[16].

This can be seen more clearly when we derive the following expression for the stochastic discount factor used by the consumer to value the assets. Ignoring the Lagrange multipliers from possibly binding portfolio constraints, the Euler equation for domestic consumer and asset j can be written as:

$$q_j(z^t) = E\Big[\beta \frac{\pi_h(z_{t+1})^{\sigma-1}\tilde{c}_h(z_{t+1})^{-\sigma}}{\pi_h(z_t)^{\sigma-1}\tilde{c}_h(z_t)^{-\sigma}} r_j(z_{t+1})|z_t\Big] \tag{2.7}$$

Thus, we obtain the following expression for the stochastic discount factor of domestic consumer:

$$SDF_h(z_{t+1}) = C \cdot \Big(\pi_h(z_{t+1})^{\sigma-1}\tilde{c}_h(z_{t+1})^{-\sigma}\Big)$$

where $C = \beta/\big(\pi_h(z_t)^{\sigma-1}\tilde{c}_h(z_t)^{-\sigma}\big)$ is a known constant in node z^t. The consumer's willingness to hold a particular asset in his portfolio will be determined by the covariance of the asset's payoffs with the consumer's stochastic discount factor. Note that before the consumer chooses his portfolio in the current period, his next-period income available for consumption consists only of his non-traded income, $\tilde{c}_h = w_h p_h$. Thus, the hedging value of the

[16] Assuming $\sigma > 1$, which is a standard assumption in international business cycles literature.

asset will depend on the covariance of the asset's payoffs with the consumer's non-traded income (where a negative covariance makes the asset more desirable), and with the appropriate consumption-based price index (where a positive covariance makes the asset more desirable).

We will use this intuition when we will analyze our portfolio results.

3 Computing the equilibrium

3.1 The algorithm

To solve the model numerically, we use the time-iteration collocation algorithm similar to the one described in Kubler & Schmedders (2003). We use the projection method[17] that is designed to provide a solution which is globally accurate on the whole state space (as opposed to the local perturbation methods that are designed to provide a good approximation only around some given point in a state space). We provide a more detailed description of the algorithm in the appendix, while here we only outline its main features. We project the policy and pricing functions into the space of piecewise polynomials (splines). We start with some initial guess, and update the polynomial coefficients by solving the system of non-linear equations that describes the "expectations correspondence" (described in details in the appendix) in each point of some predetermined grid over the state space, $\mathcal{Z} \times \Delta$, and iterate until convergence. Portfolio constraints can lead to non-differentiable policy functions, and the location of the kinks are not known a priori. To deal with this complication, we use several hundred grid points over $[0, 1]$. Another complication is that the portfolio constraints introduce inequalities (through Kuhn-Tucker complementarity conditions) into the system of temporal equilibrium conditions. We deal with this by using the "Garcia-Zangwill" trick (described in detail in Garcia & Zangwill (1981)). It essentially transforms the Kuhn-Tucker inequalities into equalities by an appropriate change of

[17] See Judd (1998), chapter 11.

variables. We provide a brief explanation of how it works in the appendix.

The algorithm was implemented in Fortran 90. The code is available upon request [18].

3.2 A special case with known solution

To check the performance of our algorithm and to obtain the intuition for our home bias in equity results in section 5, we consider the special case of our model where we can characterize the solution in details analytically. We show that our numerical solution produces a very good approximation to the analytic solution in this case.

Consider the model that we described in section 2, with the following modifications (*Model LS*[19].):

1. There are no spot traders, so that all consumers have access to the financial markets.

2. Consumers in both countries have identical preferences towards the two consumption goods. In particular, we will assume that the preference weights for both consumers are fixed at $s_h(z) = s_h^*(z) = 1/2$ for all z[20].

3. The "wages" and the "dividends" are some fixed fractions of the output in both countries, so that $w_h(z) = \nu e_h(z)$ and $w_f(z) = \nu e_f(z)$ for some $\nu \in [0, 1)$ for all z (which implies that $d_h(z) = (1 - \nu)e_h(z)$ and $d_f(z) = (1 - \nu)e_f(z)$).

4. The initial portfolio distribution (or, alternatively, the initial wealth share of the home country, $\omega_h(z_0)$) is such that the short-selling con-

[18] To solve the non-linear system of equilibrium equations, we used two non-linear solvers: HYBRD (faster but less robust) and KNITRO (slower but more robust).

[19] The consumption allocation in the one-good version of this model is known in the general equilibrium literature as the "linear sharing (LS) rule" (see Magill & Quinzii (1996), p. 173)

[20] Intuitively, with this assumption the model behaves as the model with one consumption good.

straints will not be binding at the solution that we will present next (the precise meaning of this assumption will become clear when we present the solution).

The exact values for all other parameters in the model (ρ, σ, β and the specification of the two output processes) are not important for this example, but we will set them equal to the values from our calibrated model in section 4. We will assume that $\nu = 0.1$, which on one hand will ensure that the short-selling constraints are not binding for a large portion of our state space, and on the other hand will ensure that our predicted bond positions do not follow trivially from our borrowing limits.

Lemma 3.1. *The model that satisfies assumptions* "Model LS" *has an equilibrium with the following consumption allocation (which is Pareto optimal):*

$$c_h^{PO}(z) = ke_h(z), \quad c_f^{PO}(z) = ke_f(z),$$

$$c_h^{*PO}(z) = (1-k)e_h(z), \quad c_f^{*PO} = (1-k)e_f(z)$$

where $k = 1/\left(1 + \left[\frac{1-\mu^{po}}{\mu^{po}}\right]^{1/\sigma}\right)$ *and* μ^{po} *is the weight that the planner assigns to the domestic consumer (which depends on* $\omega_h(z_0)$*), and the following time-invariant portfolio allocation:*

$$\theta_h^{PO} = \frac{k-\nu}{1-\nu}, \quad \theta_f^{PO} = \frac{k}{1-\nu}, \quad b^{PO} = 0,$$

$$\theta_h^{PO*} = \frac{k}{1-\nu}, \quad \theta_f^{PO*} = \frac{k-\nu}{1-\nu}, \quad b^{PO*} = 0,$$

Proof. One can easily check that $(c_h^{PO}, c_f^{PO}, c_h^{*PO}, c_f^{*PO})$ is indeed Pareto optimal, by verifying that it satisfies the first-order conditions in the planner's problem. It is also easy to check that the suggested consumption and portfolio allocations satisfy the budget constraints for both consumers at each note z^t. Finally, one can set the relative prices to be equal to the appropriate ratios of marginal utilities of either of the consumers, and check that the first-order conditions in the consumer's optimization problem are satisfied. Note that the Pareto weight μ^{po} depends on the initial distribution of wealth between the consumers (it must be chosen so that the budget constraints are satisfied). We need to assume that the initial wealth distribution is such that the predicted equity positions are non-negative ($\theta_h^{PO} \geqslant 0, \theta_f^{PO} \geqslant 0, \theta_h^{PO*} \geqslant 0, \theta_f^{PO*} \geqslant 0$), so that the short-selling constraints are satisfied. The predicted bond positions are identically zero, so that the borrowing limits are satisfied. □

Note that our portfolio allocation in this case is the same as in Baxter & Jermann (1997), who consider the model with one consumption good and Cobb-Douglas production functions (which produce the result that wages and dividends are constant fractions of output). If $\nu = 0$ (no non-traded income), we get $\theta_h^{PO} = \theta_f^{PO} = k$ and $\theta_h^{PO*} = \theta_f^{PO*} = (1-k)$, so that both consumers should hold only a fixed share in a mutual fund that fully owns both equities. If $\nu > 0$, domestic consumers should reduce their exposure to domestic equity and increase their position in foreign equity. This demonstrates that the model with one consumption good and wages that are proportional to output is unable to produce the home bias in equity portfolios that we observe in the data. In the calibrated version of our model in section 5, we will assume (1) home bias in consumption preferences, and (2) shocks to factor shares, which will combine to deliver the home bias in equity portfolios.

Lemma 3.1 describes portfolio positions for a given Pareto weight μ^{po}. For every initial exogenous state z_0 and every initial wealth share $w_h(z_0)$, we can find μ^{po} numerically using the Negishi algorithm, as described in Judd et al. (2000).

Figure 3 compares the equity and bond positions obtained by our numerical solution with the ones from lemma 3.1. The upper part of the figure shows the two portfolio solutions over the whole state space, and the lower part shows the absolute difference between the two solutions (only the part of the state space where the short-sale constraints do not bind is shown). This figure demonstrates that our numerical solution is very close to the analytic solution.

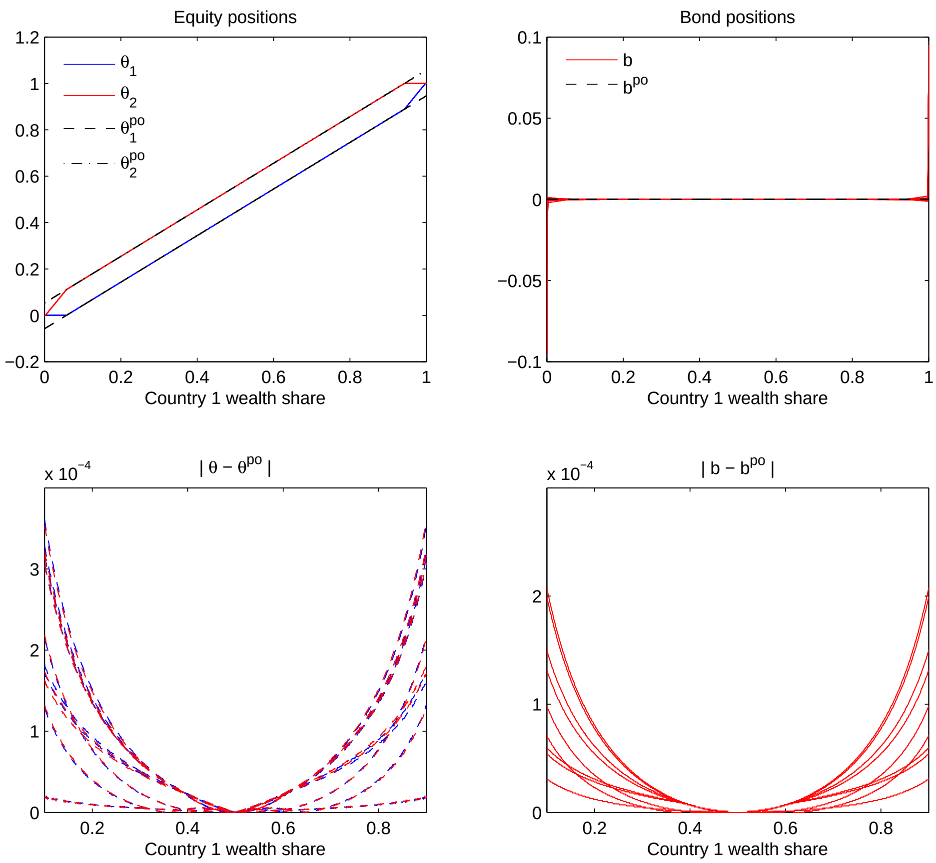


Figure 3: Numerical and analytic portfolios for Model LS

Next, we compare the simulated stationary distributions of our endogenous state variable, ω_h, and simulated portfolio positions (with several million draws of the exogenous shock). We simulate the model assuming that $\omega_h(z_0) = 0.5$. We can expect that ω_h should stay close to 0.5. In fact, from lemma 3.1 it follows that there should be finitely many realized values for ω_h, equal to the number of exogenous states[21].

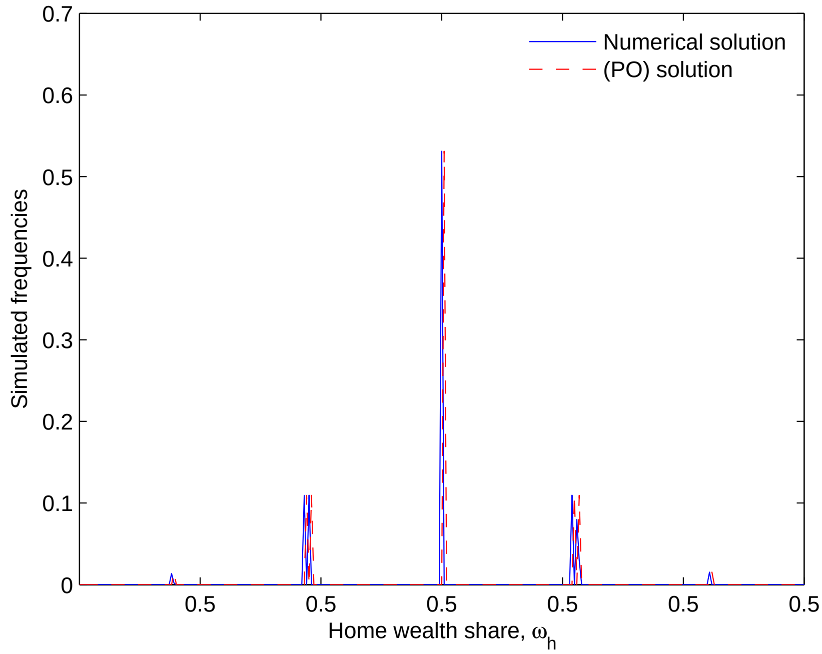


Figure 4: Stationary distributions of endogenous state variable

Figure 4 confirms that the stationary distribution obtained from our numerical solution is very close to the distribution implied by the analytic

[21]From the definition in 2.5, it is clear that ω_h depends on the selected portfolio positions, goods' and assets' prices and wage and dividend realizations. Lemma 3.1 shows that portfolio positions should not change over time. It also shows that the consumption allocations, and thus all the prices will be "strongly stationary" (will depend only on the realization of the exogenous state). It follows that ω_h will also depend only on the realization of the exogenous state.

solution. Table 1 shows that the predicted portfolio positions are also very close to the ones from lemma 3.1.

	Numerical solution	(PO) solution
mean(ω_h)	0.5000	0.5000
mean(θ_h)	0.4444	0.4444
mean(θ_f)	0.5556	0.5556
mean(b)	0.0000	0.0000

Table 1: Simulated portfolio allocation

We conclude that our numerical algorithm provides a very good approximation to both the policy functions, and the implied stationary distribution over ω_h.

We also can check how the numerical solution obtained by the perturbation approach performs in this case. For this purpose, we use the second-order approximation toolkit developed by Stephanie Schmitt-Grohe and Martin Uribe, which was used in Heathcote & Perri (2009). This algorithm, however, cannot be applied to the portfolio choice problem directly, since the non-stochastic "steady state" portfolio allocation is not well-defined. Without the shocks, all assets must have the same returns, making the consumer indifferent between them – hence, there is a continuum of portfolio allocations satisfying the non-stochastic version of the model's first-order conditions, whereas the algorithm requires the point of approximation to be a locally unique solution. To deal with this problem, Heathcote & Perri (2009) propose to start with some initial guess about the portfolio allocation, and introduce small adjustment costs for any deviation from this predetermined point, which makes the guessed portfolio choice a valid approximation point. They then rely on an iterative scheme which first approximates the solution around some portfolio guess, and uses the simulations to update the guess.

Figure 5 shows the results of applying this algorithm using the analytic solution from lemma 3.1 as the stationary portfolio (note that the iterative update scheme, if it works, should converge to this point).

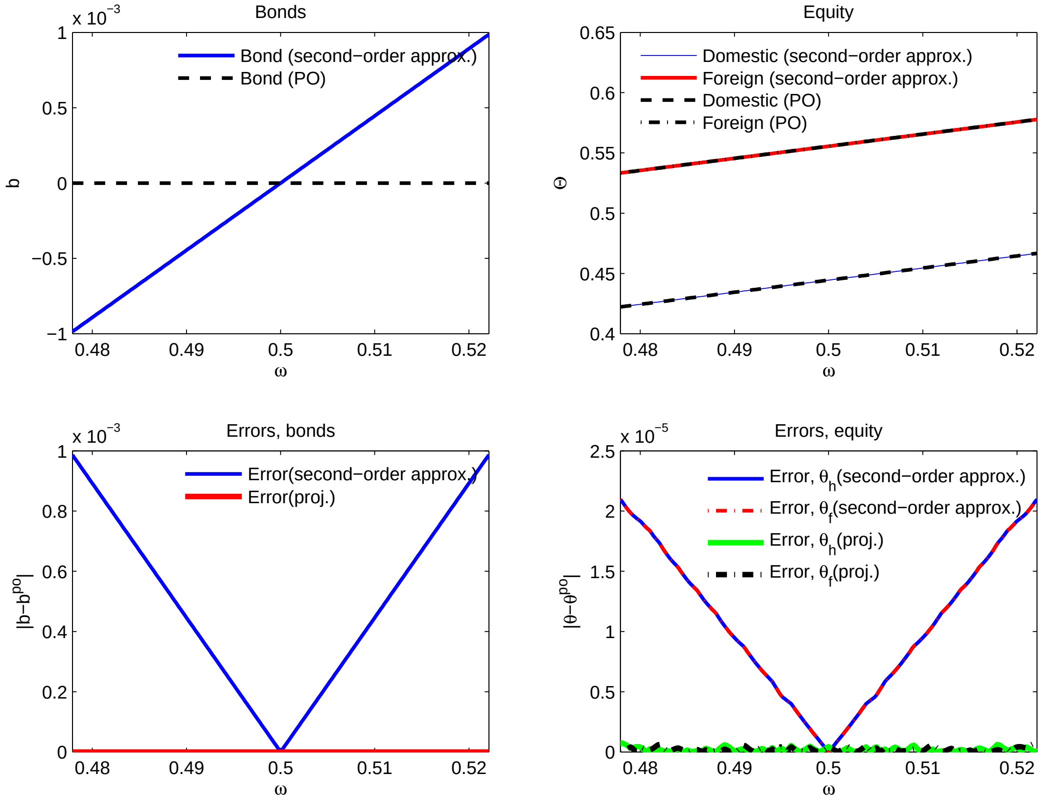


Figure 5: Second-order perturbation solution

First thing to note is that the errors from the projection method that we use are an order of magnitude smaller than those from the second-order approximation. Second, we give the perturbation method an unfair advantage of providing the analytic solution for the portfolios at $\omega_h = 0.5$ from lemma 3.1 as the point of approximation. The iterative updating scheme unfortunately has not worked (always diverging) when we started with alternative portfolio guesses (we have always used the analytic solution for the bond position, and only changed the guess for the equity positions).

4 Calibration

Data sources. To calibrate the joint output process for our two model economies, we use the national income accounts data from the Penn World Table dataset. We include in our sample 42 countries, with the data over the 1985-2007 period[22]. We are interested in this period because it coincides with the period of increased international capital flows, and also the "great moderation" period in the U.S. In the appendix, we show that our empirical finding that the output in the United States in this period is less volatile than in the rest of the world is robust when we use a smaller sample of the OECD countries, with the data from the OECD.Stat database.

We also use Robert Shiller's dataset for the U.S. stock market dividends, and the dataset compiled by Lane and Milesi-Ferretti for the countries' net foreign asset positions and portfolio composition.

Output processes. In our model, we abstract from the government expenditures and investment into the physical capital. Thus, to obtain a measure of a country's output, we use the sum of its consumption expenditures and

[22]Our sample includes: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Colombia, Denmark, Egypt, Finland, France, Germany, Greece, Hong Kong, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Peru, Philippines, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom, United States.

net exports. We filter the series for the natural log of our measure of output using the Hodrick-Prescott filter with the smoothing parameter set to 100 (because we use the data at the annual frequency). We also report the results from using the output growth rates.

To obtain a measure of output volatility for our second model economy that represents the rest of the world (RoW), we compute a weighted average of individual countries' standard deviations, using the individual countries' shares in the total output of our sample of the RoW economies over the 1985-2007 period as weights. Similarly, we obtain a measure of the cross-country output correlation with the U.S. Our use of the the countries' output shares as weights addresses a potential criticism that our finding of the higher output volatility for the RoW is driven by the high volatility of output in smaller countries in our sample. Table 2 summarizes the results.

	HP-filtered series	**Growth rate series**
$\sigma(y_t^{us}), \%$	0.88	0.81
$\sigma(y_t^{row}), \%$	2.16	2.42
$\rho(y_t^{us}, y_t^{row})$	0.17	0.13
$\rho(y_t^{us}, y_{t-1}^{us})$	0.63	0.39
$\rho(y_t^{row}, y_{t-1}^{row})$	0.53	0.26

Table 2: Summary statistics for the output process, PWT data

As table 2 shows, we find that during the 1985-2007 period, output in the U.S. was about twice less volatile as in our RoW sample. Table 8 in the appendix shows that we reach a similar conclusion if we confine ourselves to a smaller sample of OECD countries.

An alternative way to obtain a measure of the volatility of output for the RoW would be first to construct the total output of the RoW as the sum of the outputs of all individual RoW countries in our sample. However, such total output for RoW turns out to be slightly *less* volatile than the output in the U.S. (with $\sigma(y_{hp}^{tot\,row}) = 0.86\%$), despite the fact that our output measure in all individual countries in our sample is more volatile than in the U.S.

(see table 7 in the appendix for the details).[23] However, we believe that this approach would be misleading, since it would implicitly assume that RoW countries can insure against their individual output shocks much better among themselves (in fact, perfectly well) than bilaterally with the U.S. We will make another extreme assumption that the RoW countries cannot hedge their shocks among themselves.

Dividends. Since we lack consistent data for stock dividends for many countries in our sample, we will use the U.S. data to calibrate the dividend process. In particular, we use the data on the dividends accruing to the S&P Composite Stock Market Index from Robert Shiller's dataset. Figure 6 shows the joint behavior of dividends and our measure of output in the U.S. over the last four decades.

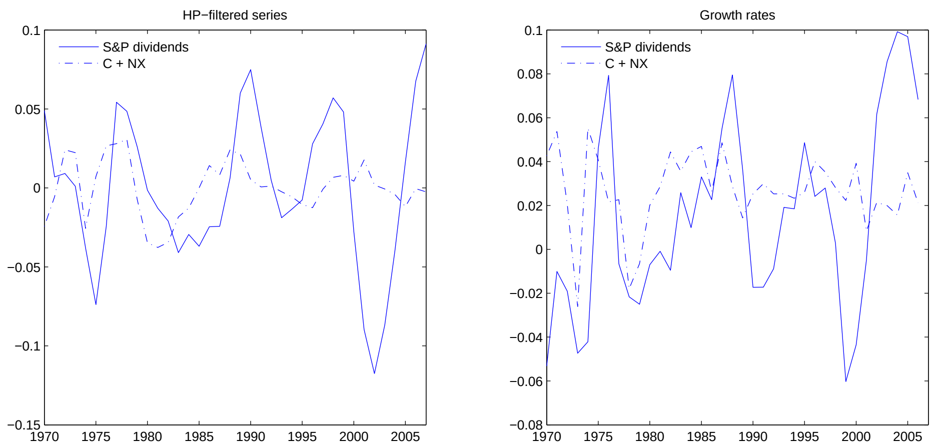


Figure 6: Dividends and output in the U.S.

The above graph illustrates the well-known fact that since mid-80s, there has been a reduction in the volatility of output and consumption in the U.S.,

[23] This suggests that RoW countries might have achieved a significant degree of insurance against their individual output shocks by pooling these shocks amoung themselves, which strengthens the argument that the home equity bias observed in the data appears as a puzzle.

which was not accompanied by a similar reduction in the volatility of many financial variables. In particular, stock dividends remained as (or even more) volatile as before the onset of the "great moderation". As a result, we obtain that in the 1985-2007 period, the dividends were significantly more volatile than our measure of output, and the correlation between the two was very low (in fact, slightly negative). Table 3 summarizes these findings.

	HP-filtered series	**Growth rate series**
$\sigma(d_t)/\sigma(y_t)$	6.25	4.35
$\rho(d_t, y_t)$	-0.02	-0.03

Table 3: Dividends and output in the U.S.

In our calibration, we start by assuming that the output in our two model economies follows a bivariate VAR process, and approximate it with a discrete-state first-order Markov process with 9 states (3 for each country) using the approach described in Knotek & Terry (2008)[24]. In our baseline calibration, we assume that the output shocks in both countries have the same volatility (which we set equal to the the RoW's weighted average, $\sigma(y_t) = 2.16\%$) and the same persistence (we set it to $\rho(y_t, y_{t-1}) = 0.6$). We set the cross-country output correlation to 0.17, the weighted average of cross-country correlations of RoW countries in our sample with the U.S. The discretisation procedure produces the two vectors of the output shock realizations, and the probability transition matrix. Then we choose the realizations of the dividend process to match the following 3 moments: (1) relative volatility of dividends: $\sigma(d_t)/\sigma(y_t) = 6.25$, (2) correlation between dividends and output: $\rho(d_t, y_t) = -0.02$ and (3) average dividend share: $E\left(\frac{d_t}{y_t}\right) = 0.12$.

Taste shocks. It has been noted in the literature [25] that a model with

[24]This approach implements a bivariate version of the Tauchen discretization procedure using Monte Carlo simulation to compute the cell probabilities.

[25]See, for example, Stockman & Tesar (1995), Pavlova & Rigobon (2007) and Heathcote & Perri (2009).

only output shocks performs poorly in terms of predicting the joing behavior of relative consumption and relative prices (or real exchange rates). Even with incomplete financial markets, these models usually deliver the results that are close to those described in Backus & Smith (1993), who showed that with complete financial markets, one should expect that the growth in relative consumptions should be perfectly correlated with the growth in relative prices (or real exchange rate). At the same time, Backus & Smith (1993) demonstrate that in the data, this correlation for most countries is close to zero (this observation is usually refered to as the "Backus-Smith puzzle").

As we mentioned in section 2.1, to improve our predictions in terms of the joint behavior of relative consumption and relative prices, we follow the literature and introduce two features into our model: (1) taste shocks as a simple reduced-form way to model demand-side shocks; (2) some share of consumers that are restricted from participating in the international financial markets (similar to Kollmann (2009)). We introduce the taste shocks by making the consumption goods' weights in the utility function (s_h and s_f for domestic and foreign consumers respectively) stochastic. We assume that the taste shocks are independent from the output shocks, and take one of the two possible realizations, $\bar{s}_i - \epsilon_{s,i}$ and $\bar{s}_i + \epsilon_{s,i}$. We set $\bar{s}_h = \bar{s}_f = 0.87$ to target the average ratio of trade (measured by the sum of imports and exports) to our measure of output, $\frac{0.5(Im+X)}{C+X-Im}$ in the U.S., which was 16.1% during the 1985-2007 period.

We choose $\epsilon_{s,i}$ that controls the volatility of the taste shocks so that the standard deviation of the taste shocks (in percentage terms) is proportional to the standard deviation of output shocks in each country. Neither the magnitude of the taste shocks, nor the share of financially restricted consumers is directly observable in the data. Our strategy here is to choose these two parameters to obtain realistic predictions in terms of the correlation between the relative consumption and relative prices, and the home bias in equity. In our baseline calibration, we choose the taste shocks that are 3 times less volatile than the output shocks, and we follow Kollmann (2009) and set the share of the financially constrained consumers to 0.5. This is consistent with the findings in Hess & Shin (2009), who use the data from the OECD countries and conclude that the fraction of risk-shares is from 20 to 50 % of the population. We will perform sensitivity analysis for both of these parameters.

Other parameters. Since our model period is one year, we set the discount factor to $\beta = 0.961$, which gives us an annual interest rate of 4%. We set the value of σ (the parameter that controls both the relative risk aversion and the intertemporal elasticity of substitution) to 2, which is a common value in the international macro literature.

There is no general agreement on the value of ρ, which controls the elasticity of substitution between our two consumption goods in the model [26]. Static general equilibrium models that study the response in the trade growth following a one-time episodes of trade liberalization usually need high elasticities, in the range between 10 and 15 (which translate into ρ between $\frac{9}{10}$ and $\frac{14}{15}$). International business cycle models usually use much lower elasticities to account for high-frequency fluctuations in trade balances and the terms of trade. We choose $\rho = -0.15$, which lies between two recent estimates by Heathcote & Perri (2002) who get $\rho = -0.11$, and Corsetti, Dedola & Leduc (2008) who find $\rho = -0.18$.

Table 4 summarizes our choice of parameters.

5 Results

5.1 Numerical results

We consider the following 3 versions of our model:

- *Model 1.* Fully symmetric model, where both countries have the same volatility of shocks, and the internationally traded bond pays in equal

[26] See Ruhl (2004) for the review

	Value	Moment/Source
β	0.961	Return on bond = 4%
σ	2.00	Common benchmark value
ρ	-0.15	Heathcote & Perri (2002), Corsetti et al. (2008)
$\bar{s}$	0.87	Trade/Output in US = $0.5(X+M)/(C+NX) = 16.1\%$
$E(d_t/e_t)$	0.12	Corporate Profits/Output Ratio, US
$\sigma(e_t^{us})$	0.01	Volatility of output in US
$\sigma(e_t^{row})$	0.02	Volatility of output in RoW
$\rho(e_t, e_{t-1})$	0.60	Persitence of output in US
$\rho(e_t^{us}, e_t^{row})$	0.17	Cross-country correlation of output

Table 4: Parameter values

combination of both consumption goods, $r_b = 0.5p_h + 0.5p_f$ (*"Symmetric"*).

- *Model 2.* A model where the only asymmetry between the two countries is that the internationally traded bond pays in units of country 1's goods, $r_b = p_h$ (*"Dollar bonds"*).

- *Model 3.* A model where the shocks in country 1 ("home") are twice less volatile than shocks in country 2 ("foreign") (with the symmetric bond payoffs) (*"Great moderation"*).

Figure 7 compares the simulated stationary distributions from Model 2 and Model 3 to the one generated by the fully symmetric Model 1. We use these simulated stationary distributions to obtain all the summary statistics that we report below.

Both asymmetries that we introduce into the model (Model 2 and Model 3) lead to country 1 loosing financial wealth over time. As table 5 shows, this happens through an accumulation of a negative net foreign asset (NFA) position. In Model 2 with asymmetric bond payoffs, country 1 accumulates an average negative NFA position that reaches -24% of the average value of output in country 1. In Model 3, the negative NFA position of country

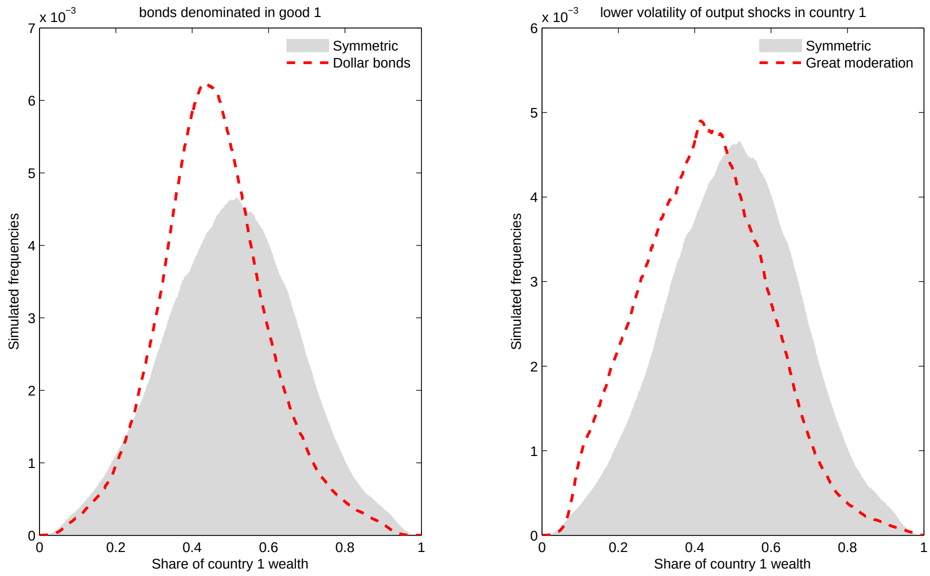


Figure 7: Simulated distributions

is even larger, reaching -39% of the average value of output in country 1. According to the data from Lane – Milesi-Ferretti's dataset, the NFA position in portfolio equity and debt in the U.S. in 2007 reached -38% of $C + NX$.

All three versions of our model deliver home bias in equity. Using the data on U.S. stock market capitalization from the World Bank database, and the U.S. portfolio assets and liabilities from the Lane – Milesi-Ferretti dataset, we can compute the share of the U.S. equity portfolio invested in domestic stocks as $\frac{q_h \theta_h}{q_f \theta_f + q_h \theta_h} = \frac{\text{U.S. Stock Market Cap. - Portfolio Equity Liabilities}}{\text{U.S. Stock Market Cap. - Portfolio Equity Liabilities + Portfolio Equity Assets}}$. We obtain that in 2007, it was 0.76 (for the whole path over 1988-2007 period, see figure 12 in the appendix). This number is close to the one reported in Heathcote & Perri (2009), who find that foreign assets' share in the U.S. portfolio over the 1990-2004 period was around 25%. All three versions of our model closely match this feature of the data.

We also obtain a realistic prediction for the correlation between the relative consumption and relative price indices ("Backus-Smith" correlation).

Table 5: Simulated and data moments				
	Symmetric	Dollar bonds	Moderation	Data
ω_h (*wealth share of country 1*)	0.50	0.46	0.43	n.a.
$NFA_h/(p_h e_h)$	0%	−24%	−39%	−38%
$\rho(\Delta\tilde{c}, -\Delta\tilde{\pi})$ (*Backus-Smith*)	0.05	0.06	0.09	0.07
$q_h\theta_h/(q_h\theta_h + q_f\theta_f)$ (*Home Bias*)	0.77	0.79	0.78	0.76
Domestic portfolio shares:				
$\mu(\theta_h)$	0.75	0.81	0.77	1.01
$\mu(\theta_f)$	0.25	0.25	0.27	0.32
μ(equity)	1.00	1.06	1.04	1.33
$\mu(b)$	0.00	−0.06	−0.04	−0.33

We compute the correlations between the relative consumption and relative CPIs for all countries in our OECD.Stat dataset with the U.S. over the 1985-2007 period, and find that the weighted average (again, using the countries' output shares as weights) is 0.07. All 3 versions of our model come close to delivering the same prediction. In section 5.3, we show that we need both preference shocks and spot traders to achieve this result.

Table 5 also shows portfolio composition in all three models for the domestic representative investor. The asset shares are defined as the value of the asset holdings devided by the value of the total portfolio. For instance, $\mu(\theta_h) = \frac{\theta_h q_h}{\theta_h q_h + \theta_f q_f + b q_b}$. The table shows that in both Model 2 and 3, the deterioration of the NFA position at home is driven by the accumulation of debt obligations by the domestic investor. In both of these asymmetric models, domestic investor's portfolio is more leveraged than in the symmetric model, with a negative position in debt, and the increased holdings of the equity. For comparison, we also construct the U.S. asset shares using the data from the Lane–Milesi-Ferretti dataset, and the World Bank data for the U.S. stock market capitalization. It shows that in reality, the U.S. portfolio is even more leveraged than what our two models predict.

Interestingly, while our Model 3 (reduced volatility of domestic shocks) produces a larger deterioration of the home NFA position, the change in the portfolio composition (in particular, the increased in the total share of equity holdings) is smaller than in Model 2. The reason for this is that with reduced volatility of shocks in country 1, foreign investor has stronger precautionary savings motive, and thus seeks to invest a larger share of his portfolio into a relatively safer assets. At the same time, the reduced volatility of the dividends from country 1's equity makes country 1's stocks a more attractive instrument for foreign investor.

To summarize our findings, we get that:

Result 1. A country that can issue bonds in own currency accumulates on average a sizable negative net foreign asset position. This is driven by a large negative position in bonds, while at the same time the share of financial wealth invested by country 1's investors in equity increases.

Result 2. Similarly, a country with less volatile shocks accumulates a large negative NFA position. The effect for country 1's portfolio composition is similar to our first result, but smaller in magnitude.

Next, we will argue that our first result is driven by the covariance structure of the bonds' payoff and non-traded income, while our second result is driven by the differences in the precautionary motive between the domestic and foreign investors.

5.2 Economic intuition

Recall that[27], ignoring the potential effects of binding portfolio constraints, the pricing formula for asset j for domestic investor can be written as:

$$q_j(z^t) = E\big(SDF_h(z_{t+1})|z^t\big)E\big(r_j(z_{t+1})|z^t\big) + Cov\big(SDF_h(z_{t+1}), r_j(z_{t+1})|z^t\big) \tag{5.1}$$

where $SDF_h(z_{t+1}) = C\cdot\Big(\pi_h(z_{t+1})^{\sigma-1}\tilde{c}_h(z_{t+1})^{-\sigma}\Big)$, and $C = \beta/\big(\pi_h(z_t)^{\sigma-1}\tilde{c}_h(z_t)^{-\sigma}\big)$ is a known constant in node z^t, and similarly for the foreign investor.

Note that in equilibrium, there will be a single market-clearing price. However, before the investors choose their portfolio positions, they will typically have different valuations of the income streams provided by different

[27]See equation 2.7 for details.

assets. We want to interpret the above formula as the value of the asset to domestic investor before he trades in the financial market. Intuitively, one can expect that the investor who values the asset more before the trade starts will end up taking a long position in the asset[28].

The first term can be though of as the present discounted value of the asset's payoffs to the consumer, and reflects his attitude towards reallocating his income over time. The second term reflects the asset's covariance, or hedging value to the consumer, and shows how well-adapted the income stream offered by the asset's payoffs is to the consumer needs.

We will argue that our first result in the previous section (Model 2) is driven by the second term in the above pricing equation (the difference in covariance, or *hedging* value of the bonds' payoff for domestic and foreign investors), while our second result (Model 3) is driven by the first term (reflecting the differences in the precautionary motives between the two investors).

Model 2. First, let's consider the effect of changing the bond payoff structure on the covariance value of bond's payoffs to both investors. We can decompose the covariance value term in the above asset-pricing equation as[29][30]:

$$Cov(\pi_h^{\sigma-1}\tilde{c}_h^{-\sigma}, r_b) \approx E(\pi_h^{\sigma-1})Cov(\tilde{c}_h^{-\sigma}, r_b) + E(\tilde{c}_h^{-\sigma})Cov(\pi_h^{\sigma-1}, r_b) \tag{5.2}$$

Note that before the domestic investor chooses his portfolio in the current period, his income next period consists of only the value of his non-traded income, so that $\tilde{c}_h = p_h w_h$ (the current portfolio that he starts the period with will only affect the constant C). Thus, the hedging value of the asset for domestic investor will depend on the covariance of the asset's payoffs with the

[28] Svensson (1988) shows that this holds exactly if only one asset is traded, he calls it a "tendency" with more than one asset.

[29] For notational simplicity, we will drop the dependence on the history of shocks, z^t, and the constant C from the expression.

[30] See Bornstedt & Goldberger (1969). The omitted term in the approximation is $E(\Delta\gamma^h \, \Delta m_h^{-\gamma} \, \Delta r_b)$, where $\Delta x \equiv x - E(x)$. It is a third-order moment, and is dominated by the first and second-order moments in our case.

domestic CPI (where positive covariance makes the asset more desirable) and the value of the non-traded income, $w_h p_h$ (where positive covariance makes the asset *less* desirable). We can expect that on one hand, the increase of the share of domestic good in the bond's payoff (in Model 2) will increase the covariance value of the bond by increasing the covariance of the bond's payoff with π_h which puts more weight on the price of the domestic good. On the other hand, the effect on the first part can also be decomposed similar to equation 5.2:

$$Cov(\tilde{c}_h, r_b) = Cov(w_h p_h, p_h) \approx E(w_h)Var(p_h) + E(p_h)Cov(w_h, p_h)$$

The last term in the above expression is likely to be negative (since non-traded income w_h is positively correlated with the total output e_h, while the increase in output drives the domestic price down), but the first term is unambiguously positive. The first term will dominate if the goods' prices are sufficiently volatile. But this is exactly what we need to account for the Backus-Smith correlation. Next, we will show that in our Model 2, this last effect (which makes the bond a bad hedge against the domestic non-traded income) dominates.

Table 6 shows how the change in the bond payoff structure from $r_b = 0.5p_h + 0.5p_f$ to $r_b = p_h$ changes the covariance values of the assets, and decomposes it into the correlation between the asset's payoffs and the corresponding non-financial incomes and CPIs in the two countries [31].

The table shows that changing payoff structure of the bond has practically no effect on the covariance values of the two stocks, but has a large effect on the covariance value of the bond. As we increase the weight on the domestic good in the bond's payoff, the bond becomes more valuable to the domestic consumer as a hedge against the CPI shocks (correlation between r_b and π_h increases from almost 0 to 0.9), and less valuable as a hedge against the

[31] The correlations are obtained by drawing several million realizations of exogenous shocks, and then using our numerical solutions to obtain the paths for the goods' and assets' prices and assets' payoffs, keeping the wealth shares constant at $\omega_h = 0.5$

Table 6: Asset payoff correlation structure

$r_b = p_h$ (Model 2)							
	r_h	r_f	r_b		r_h	r_f	r_b
$w_h p_h$	0.39	0.45	0.63	$w_f p_f$	0.44	0.38	−0.63
π_h	−0.01	0.03	0.90	π_f	0.01	−0.03	−0.90
SDF_h	−0.43	−0.48	−0.43	SDF_f	−0.47	−0.42	0.43
$r_b = 0.5p_h + 0.5p_f$ (Model 1)							
	r_h	r_f	r_b		r_h	r_f	r_b
$w_h p_h$	0.39	0.44	0.01	$w_f p_f$	0.44	0.39	0.01
π_h	−0.02	0.02	0.02	π_f	0.02	−0.02	0.02
SDF_h	−0.43	−0.47	0.01	SDF_f	−0.47	−0.43	0.01

shocks to his non-financial income (correlation between r_b and $w_h p_h$ increases from almost 0 to 0.6). The second effect dominates (correlation between r_b and SDF_h decreases from 0 to -0.4), so that the net effect is that the bond becomes less valuable to a domestic consumer.

Note that we have an opposite effect on the foreign consumer. This is due to the fact that in our simple model with just 2 countries, we have $p_f = 1 - p_h$, so that the two goods' prices are perfectly negatively correlated[32].

The above table also shows that investors' stock portfolios exhibit home bias in our model because domestic stocks offer a better hedge against the shocks to domestic non-financial income.

Even though the change in the bond payoff structure does not have a direct effect on the covariance values for the two stocks, it has an indirect effect on both agents' holdings of stocks. This can be explained as follows. Since the domestic consumer in Model 2 takes a negative position (borrows) in bond, this leads him to loose wealth over time, and effectively makes him more patient, which increases the discounted value of expected future income streams from *all* assets to him. As a result, domestic investor increases his stock holdings. The opposite happens for the foreign investor.

Model 3. A larger volatility of shocks in country 2 makes foreign in-

[32] In a model with more countries, we can expect that this last effect will be smaller.

vestor's non-traded income, and thus his stochastic discount factor, SDF_f, more volatile compared to domestic investor. With our assumed CRRA period utility function, stochastic discount factor is a convex function of non-traded income, $\tilde{c}_f$ and $\tilde{c}_h$. Thus, $E(SDF_f) > E(SDF_h)$ in equation 5.1, which effectively makes foreign investor more patient than domestic investor, driving down the NFA position in the home country. Note that foreign investor strongly prefers to have relatively safer assets in his portfolio, since more risky assets would increase the volatility of his stochastic discount factor even further.

5.3 Sensitivity analysis

In this section, we show that the features of our model that allow us to account for the Backus-Smith correlation observed in the data, are precisely the ones that are responsible for the results we obtain in our Model 2. In particular, we consider three different features of our model that can increase the volatility of goods' prices: 1) the relative volatility of preference shocks; 2) the share of spot traders; 3) the elasticity of substitution between the two consumption goods. We show that as we change any of these three features in the direction that generates greater volatility in goods' prices, it simultaneously improves our model's prediction in terms of the Backus-Smith correlation, and also generates larger negative NFA position in domestic country. In the previous section, we argued that greater volatility of goods' prices makes it more likely that the payoffs of the bond in our Model 2 will be positively correlated with the non-traded income of the domestic investor, giving him incentive to take a short position (borrow) using this bond. The following three experiments illustrate this mechanism.

Figure 8 shows the effect of changing the relative volatility of preference shocks (while retaining the share of spot traders at $\mu = 0.5$) on the Backus-Smith correlation, the proportion of equity portfolio invested in local stocks, and home country's NFA position using our Model 2.

Next, we consider the effect of changing the share of spot traders, while

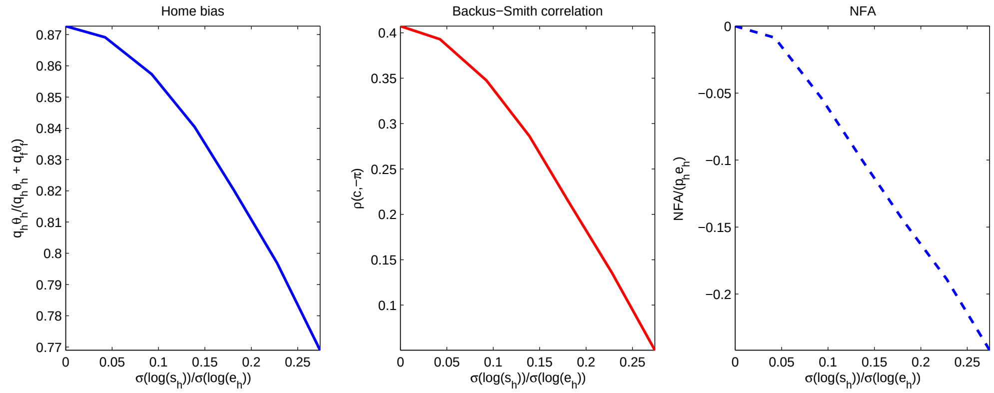


Figure 8: Changing relative volatility of preference shocks

keeping the relative volatility of preference shocks at $\frac{\sigma(log(s_h))}{\sigma(log(e_h))} = 0.3$ (our baseline parametrization). Figure 9 shows the results.

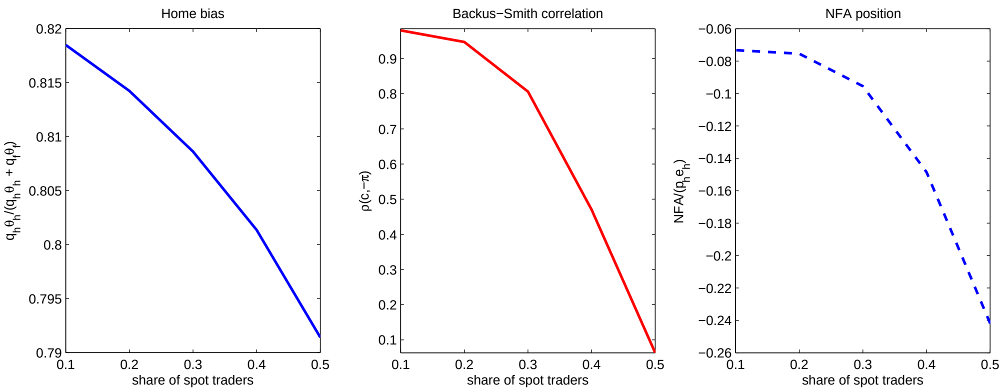


Figure 9: Changing the share of spot traders

As is clear from the figure, qualitatively, the increase in the share of spot traders has a similar effect to the increase in the relative volatility of preference shocks.

Finally, we consider the effect from changing ρ, the parameter that controls the elasticity of substitution between the two consumption goods in our model. As we decrease ρ, the elasticity of substitution decreases, and we can expect that a larger part of adjustment to external shocks will happen through price changes, making goods' prices more volatile. Figure 10 shows that this also qualitatively leads to results that are similar to our 2 previous experiments – a lower Backus-Smith correlation, and a deterioration in home country's NFA position.

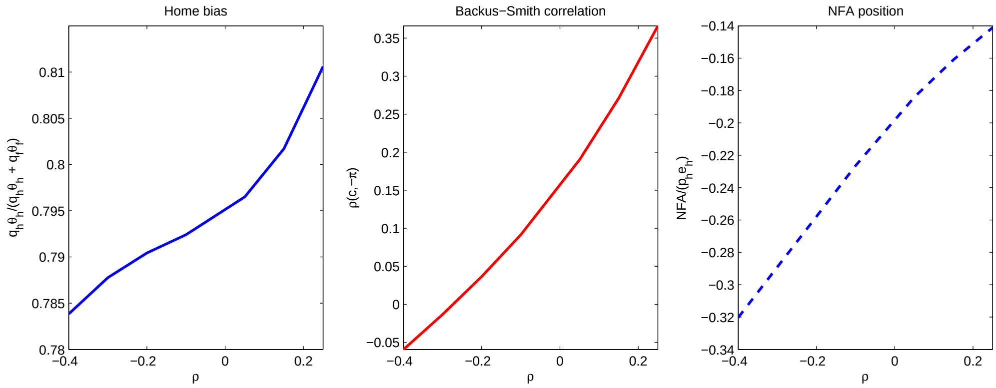


Figure 10: Changing elasticity of substitution between consumption goods

5.4 Extension: Different Debt Capacities

Reinhart, Rogoff & Savastano (2003) argue that many emerging economies have substantially lower debt capacities than the industrial countries, reaching in some cases only 15% of GDP. We test the hypothesis that lower debt capacity in some group of countries can generate global imbalances by allowing the borrowing limit to be different in our two model economies. In particular, we allow the parameter that controls the amound of debt that the country is able to accumulate to be different for the two countries. We fix it at $k = k_h = 1$ for the domestic economy, and consider the effects of lowering k_f below 1. We can expect this to have an effect similar to the one we obtain in our Model 3, where the foreign country had relatively higher volatility of exogenous shocks. With tigher borrowing constraints, we could expect foreign country to hedge against the effects of potentially binding borrowing limits through the accumulation of precautionary savings. Figure 11 shows the effect of changing k_f (using our otherwise symmetric Model 1) on the net foreign asset position of the domestic economy.

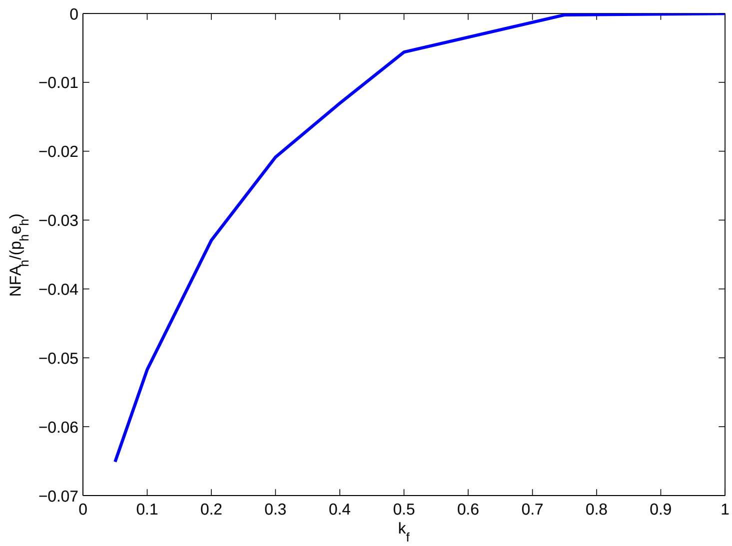


Figure 11: The effect of changing the borrowing capacity of foreign country on domestic country's NFA position

Even though qualitatively, the effect is as we would expect it to be, quantitatively we need to restrict the borrowing capacity of the foreign country substantially below $k_f = 0.5$ to achieve an economically meaningful effect.

6 Conclusions

Recent surge in international trade in financial assets has led economists to recognize that to analyze the global imbalances, one needs to incorporate portfolio choice in traditional dynamic open economy macro models. We make a step in that direction. We develop a two-country general equilibrium model with multiple internationally traded assets and incomplete financial markets. Using the method developed in Kubler & Schmedders (2003), we solve the model numerically. We choose the model's parameters so that it matches two salient features that has been stressed in the international economics literature – home bias in stock holdings, and Backus-Smith puzzle. We introduce the following two asymmetries in our model, and show that they can help explain the large and persistent negative NFA position in the U.S., and some of the features of its portfolio composition that has been noted in the literature – short position in debt, and increased share of wealth invested in risky stocks:

1. *Exorbitant privilege.* We assume that only home country can issue debt in its own currency. We show that the resulting correlation structure of the bond's payoffs and domestic non-traded income makes the bond an undesirable hedging instrument for domestic investors, leading them to take a short position (borrow) in bond.

2. *Great moderation.* Lower volatility of macroeconomic shocks in home country makes domestic investors endogenously less risk-averse. This allows us to match both the negative NFA position, and the portfolio composition structure in the home country.

We also test whether a lower borrowing capacity in the rest of the world can contribute to explaining the behavior of the NFA position in the U.S. Although we find that qualitatively, this generates the expected results, quantitatively their magnitude is much smaller than in our two previous experiments.

Some of the questions still remain unanswered. For example, we would like to know whether our results would be robust to some alternative formulations of the borrowing constraints. It is also well-known that the model with standard expected-utility preferences does not perform well in matching the equity premia observed in the data. One possible solution that has been proposed in the literature is the addition of large negative shocks, the so-called disaster states, into the model. Our numerical approach can naturally handle such a feature. We plan to address these questions in future work.

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A Appendix

A.1 Data

Table 7: Output volatility, individual countries in PWT sample

Country	$std(y^i_{hp}), \%$	$std(y^i_{\Delta}), \%$	**Weight**
ARG	3.676	4.685	0.018
AUS	1.745	1.960	0.018
AUT	1.377	1.393	0.008
BEL	0.932	1.057	0.010
BRA	3.112	3.925	0.057
CAN	2.263	1.943	0.032
CHL	1.947	2.924	0.007
CHN	4.180	4.215	0.122
COL	2.321	2.167	0.010
DNK	1.205	1.695	0.005
EGY	2.656	3.423	0.014
FIN	2.362	2.087	0.004
FRA	0.887	0.712	0.054
GER	1.291	1.353	0.083
GRC	1.572	2.310	0.008
HKG	4.245	4.610	0.008
ISL	4.556	6.221	0.000
IND	2.059	2.758	0.076
IDN	1.673	2.860	0.031
IRL	2.171	2.407	0.003
ISR	3.422	4.922	0.003
ITA	1.239	1.586	0.053
JAP	1.370	1.249	0.111

Continued on next page

Table 7 – continued from previous page

Country	$std(y^i_{hp}), \%$	$std(y^i_{\Delta}), \%$	**Weight**
KOR	1.966	2.434	0.019
LUX	3.112	3.738	0.001
MEX	1.502	1.809	0.035
NRL	1.528	1.651	0.015
NZL	1.672	2.192	0.003
NOR	2.091	2.615	0.006
PER	5.176	5.127	0.005
PHL	3.236	4.629	0.012
POR	1.505	1.390	0.006
SGP	4.785	7.294	0.003
ZAF	1.888	2.157	0.015
SPN	0.966	0.896	0.030
SWE	1.698	2.000	0.008
CHE	1.983	2.075	0.009
TWN	2.325	2.948	0.015
THA	6.490	7.616	0.012
TUR	2.632	3.249	0.014
GBR	1.553	1.429	0.057
USA	0.009	0.008	n.a.

Figure 12: Share of domestic stocks in U.S. equity portfolio

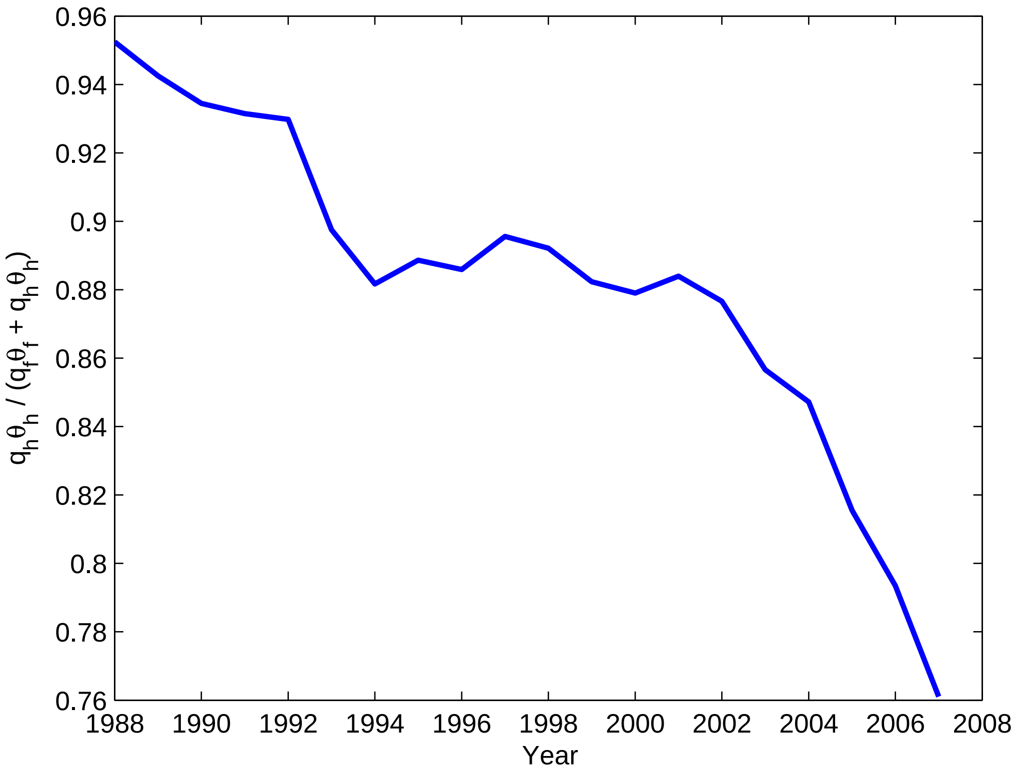

Table 8: Summary statistics for the output process, OECD.Stat sample

Statistic	**Value**
HP-filtered series	
$std(y_{hp}^{us}), \%$	1.00
$std(y_{hp}^{row}), \%$	2.09
$cor(y_{hp}^{us}, y_{hp}^{row})$	0.09
Growth rate series	
$std(y_{\Delta}^{us}), \%$	0.89
$std(y_{\Delta}^{row}), \%$	2.19
$cor(y_{\Delta}^{us}, y_{\Delta}^{row})$	0.10

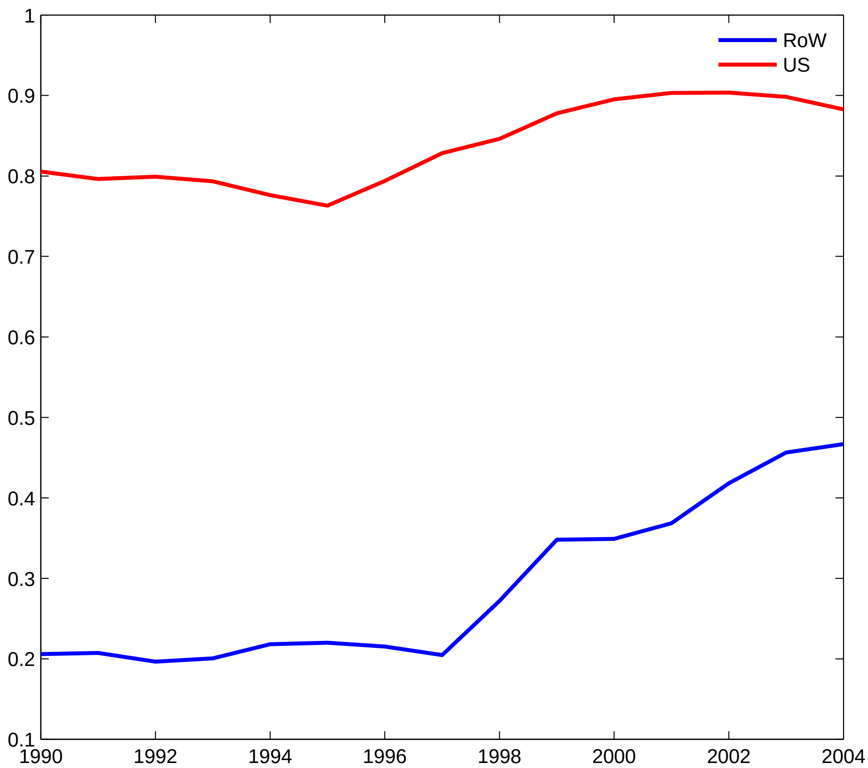

Figure 13: Share of debt liabilities issued in own currency, US and Rest of the World

A.2 Numerical algorithm

We approximate the policy and price functions with splines. We experimented with both cubic and piecewise linear splines, which produced practically identical solutions, and we ultimately chose piecewise linear functions for speed considerations. Since our model has only 2 countries, our endogenous state variable, *wealth share*, can be represented with a single number, $\omega \in [0, 1]$. We form a grid M over $[0, 1]$, and start with an initial guess for the policy and price functions, f^0. Good choice of f^0 is important for both convergence and speed of the algorithm. We choose either a solution to the static ("last-period") problem, or a solution for a "nearby" economy as our guess.

Given the function f^{n-1} that determines the behavior of the economy next period, we obtain a new function f^n by solving the following system of static and short-term dynamic optimality conditions ("expectations correspondence") + the "law of motion for wealth share" equation for each gridpoint $(\omega, z) \in M \times Z$:

$$p_f/p_h = (s_f/s_h)(c_h/c_f)^{1-\rho} = (s^*_f/s^*_h)(c^*_h/c^*_f)^{1-\rho}, \tag{A.1}$$

$$p_f/p_h = (s_f/s_h)(c^{st}_h/c^{st}_f)^{1-\rho} = (s^*_f/s^*_h)(c^{*st}_h/c^{*st}_f)^{1-\rho}, \tag{A.2}$$

$$q_h = \beta E_{z^+}\left(\frac{\lambda_h^{n-1}(\omega^+, z^+)}{\lambda_h} r_h^{n-1}(\omega^+, z^+)\right) + \phi_h, \tag{A.3}$$

$$q_h = \beta E_{z^+}\left(\frac{\lambda_f^{n-1}(\omega^+, z^+)}{\lambda_f} r_h^{n-1}(\omega^+, z^+)\right) + \phi^*_h, \tag{A.4}$$

$$q_f = \beta E_{z^+}\left(\frac{\lambda_h^{n-1}(\omega^+, z^+)}{\lambda_h} r_f^{n-1}(\omega^+, z^+)\right) + \phi_f, \tag{A.5}$$

$$q_f = \beta E_{z^+}\left(\frac{\lambda_f^{n-1}(\omega^+, z^+)}{\lambda_f} r_f^{n-1}(\omega^+, z^+)\right) + \phi^*_f, \tag{A.6}$$

$$q_b = \beta E_{z^+}\left(\frac{\lambda_h^{n-1}(\omega^+, z^+)}{\lambda_h} r_b^{n-1}(\omega^+, z^+)\right) + \nu\frac{\partial B_h(b)}{\partial b}, \tag{A.7}$$

$$q_b = \beta E_{z^+}\left(\frac{\lambda_f^{n-1}(\omega^+, z^+)}{\lambda_f} r_b^{n-1}(\omega^+, z^+)\right) + \nu^* \frac{\partial B_f(b^*)}{\partial b^*}, \tag{A.8}$$

$$\phi_h \geq 0,\ \theta_h \geq 0,\ \phi_h\theta_h = 0, \tag{A.9}$$

$$\phi_f \geq 0,\ \theta_f \geq 0,\ \phi_f\theta_f = 0, \tag{A.10}$$

$$\phi_h^* \geq 0,\ \theta_h^* \geq 0,\ \phi_h^*\theta_h^* = 0, \tag{A.11}$$

$$\phi_f^* \geq 0,\ \theta_f^* \geq 0,\ \phi_f^*\theta_f^* = 0, \tag{A.12}$$

$$\nu \geq 0,\ B_h(b) \geq 0,\ \nu B_h(b) = 0, \tag{A.13}$$

$$\nu^* \geq 0,\ B_f(b^*) \geq 0,\ \nu^* B_f(b^*) = 0, \tag{A.14}$$

$$\tilde{I} = k(1-\mu)(w_h p_h + w_f p_f) + r_h + r_f, \tag{A.15}$$

$$p_h c_h + p_f c_f + q_h\theta_h + q_f\theta_f + q_b b = \omega\tilde{I} + (1-k)(1-\mu)w_h p_h, \tag{A.16}$$

$$p_h c_h^* + p_f c_f^* + q_h\theta_h^* + q_f\theta_f^* + q_b b^* = (1-\omega)\tilde{I} + (1-k)(1-\mu)w_f p_f, \tag{A.17}$$

$$p_h c_h^{st} + p_f c_f^{st} = \mu w_h p_f, \tag{A.18}$$

$$p_h c_h^{*st} + p_f c_f^{*st} = \mu w_f p_f, \tag{A.19}$$

$$\lambda_h = g_h^{1-\rho-\gamma}\left(s_h c_h^{\rho-1} + s_f c_f^{\rho-1}\right) \tag{A.20}$$

$$\lambda_f = g_f^{1-\rho-\gamma}\left(s_h^*(c_h^*)^{\rho-1} + s_f^*(c_f^*)^{\rho-1}\right) \tag{A.21}$$

$$p_h + p_f = 1 \tag{A.22}$$

$$c_h + c_h^* + c_h^{st} + c_h^{*st} = e_h \tag{A.23}$$

$$\theta_h + \theta_h^* = 1, \quad \theta_f + \theta_f^* = 1, \quad b + b^* = 0 \tag{A.24}$$

$$\omega^+ = \frac{r_h^{n-1}(\omega^+, z^+)\theta_h + r_f^{n-1}(\omega^+, z^+)\theta_h + r_b^{n-1}(\omega^+, z^+)b + k(1-\mu)w_h(z^+)p_h^{n-1}(\omega^+, z^+)}{r_h^{n-1}(\omega^+, z^+) + r_f^{n-1}(\omega^+, z^+) + k(1-\mu)\left(w_h(z^+)p_h^{n-1}(\omega^+, z^+) + w_f(z^+)p_f^{n-1}(\omega^+, z^+)\right)} \tag{A.25}$$

Equations A.1 and A.2 are static optimality conditions. Equations A.3 – A.8 are short-term dynamic optimality (Euler) equations. Equations A.9 – A.14 are Kuhn-Tucker complementarity conditions. Equation A.15 defines total financial wealth. Equations A.16 – A.19 are the budget constraints. Equations A.20 and A.21 define marginal utility of income for domestic and foreign investors. Equation A.22 is a price normalization. Equations A.23 and A.24 are market-clearing conditions. Finally, equation A.25 implicitly defines next-period wealth share ω^+ as a function of today's portfolio choices.

Note that λ_h^{n-1}, λ_f^{n-1}, $r_h^{n-1} = q_h^{n-1} + p_h^{n-1} d_h$, $r_f^{n-1} = q_f^{n-1} + p_f^{n-1} d_f$, $r_b^{n-1} = \alpha p_h^{n-1} + (1-\alpha) p_f^{n-1}$, p_h^{n-1} and p_f^{n-1} are components of f^{n-1}. Also, note that $B_h(b)$ and $B_f(b^*)$ are the borrowing limits defined in 2.3.

We keep updating the policy and pricing functions until $||f^n - f^{n-1}|| < \epsilon$ for some small $\epsilon > 0$.

Garcia-Zangwill trick. Kuhn-Tucker complementarity conditions that involve inequalities can be transformed into equalities using the so-called Garcia-Zangwill trick. It is basically a change of variables. We will demonstrate how it works using the borrowing limit and the optimality condition for the bond holdings. Fix some positive integer $k \geq 2$ and define

$$\alpha^+ = (\max[0, \alpha])^k, \alpha^- = (\max[0, -\alpha])^k$$

Note that $\alpha^+ \geq 0$, $\alpha^- \geq 0$ and $\alpha^+ \alpha^- = 0$. We can now reformulate the Euler equation and Kuhn-Tucker conditions of domestic consumer for bonds as follows:

$$q_b = \beta E_{z^+} \left(\frac{\lambda_h^{n-1}(\omega^+, z^+)}{\lambda_h} r_b^{n-1}(\omega^+, z^+) \right) + \alpha^+ \frac{\partial B_h(b)}{\partial b}$$

$$\alpha^- - B_h(b) = 0$$

Similarly, we can transform all other Kuhn-Tucker conditions into equalities.