Learning from Prices: Central Bank Communication and Welfare*

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Manuel Amador† and Pierre-Olivier Weill‡

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Abstract

We present a micro-founded monetary economy where agents are uncertain about both an aggregate productivity parameter and the monetary aggregate. We show that when agents learn from the distribution of prices, an increase in public information about the monetary aggregate can reduce the information content of the price system and welfare. In a simple extension to the basic model, we show that the economy can have indeterminacy, generating a new role for monetary announcements.

*We'd like to thank, for fruitful discussions and suggestions, ... All errors are ours.

†Stanford University, e-mail: amador@stanford.edu

‡Department of Economics, University of California Los Angeles, e-mail: poweill@econ.ucla.edu

1 Introduction

There is widespread agreement among economists that transparency of monetary policy is beneficial. Among the reasons, it is argued that the introduction of a well defined and transparent rule to conduct monetary policy will remove uncertainty from the economy and achieve better price stability. Also, by committing to a rule, a Central Bank can manage expectations better, and the temptation to create inflation might be alleviated. Finally, by announcing its policy in advance, the Central Bank can be held accountable for its actions. All these are valid and well understood reasons in favor of transparency and indeed, several Central Banks have adopted explicit rules as a basis for their policy making. The main objective of this paper is to investigate the limits of the pro-transparency argument.

We consider a micro-founded monetary economy composed of a continuum of islands populated by households which are uncertain about the value of an aggregate productivity parameter. There is no mobility of goods or factors across islands; the only interactions are informational. As is common in the rational expectation literature, we assume that information about the productivity parameter is initially dispersed among the households. This dispersed information gets aggregated in the price distribution of the economy, which we assume all households observe and learn from. As in Lucas (1972), the productivity parameter is not fully revealed: nominal prices are also affected by an unknown level of money supply.

When the Central Bank discloses information about the money supply, it has a direct beneficial effect: other things equal, households can extract more information about the productivity parameter from nominal prices. There is however, a countervailing equilibrium effect: after an announcement, households' decisions rely more on public information, and less on their own island specific knowledge. This change in behavior tends to reduce the endogenous informational content of prices. Indeed, we show that the distribution of nominal prices in the economy may end up aggregating less information about productivity hence reducing households' knowledge and their welfare.

In particular, we show that the observation of the price distribution is equivalent to observing two distinct signals about productivity. The first is a public signal, and can be understood as extracting information from the price distribution about productivity based on the common public prior about the money supply. The second signal is island-specific and obtained by extracting information from the prices based on the island specific-information about the money supply. Thus, households learn from both public and island specific sources. The existence of the island specific source is a necessary condition for the negative effects of public information about the money supply: public information can be welfare reducing because it hinders the generation of island specific information.

As an extension, we show that multiple equilibria can occur in a minor generalization of the basic model. We discuss a novel role for monetary policy announcements: public information releases affect the degree of indeterminacy. In particular, a release of a sufficiently precise public signal about the monetary aggregate eliminates the multiplicity. However, a mild release could instead generate it.

There are several papers that have argued that monetary transparency might be welfare reducing. Recently, Morris and Shin (2002) showed, in a beauty-contest game, that in the presence of payoff externalities, public information releases can reduce welfare (see also the subsequent analysis of Angeletos and Pavan (2007)). However, as shown by Hellwig (2005) and Roca (2006) when the payoff externalities are micro-founded in a sticky-price economy à la Woodford (2002), the adverse welfare effect of transparency disappears (see also the results of Lorenzoni (2007) in a different micro-founded model). All of these papers abstract from endogenous learning, the key element emphasized in the current paper. Note also that in our island framework, payoff externalities are ruled out: the only interactions among islands are informational.

Another closely related work is that of Morris and Shin (2005), in which the authors show that public information releases by a Central Bank can impair its ability to gather information in the future. However, in contrast with our results, in their model public information releases are always welfare improving in the absence of payoff externalities.

The rest of the paper is organized as follows. Section 2 presents the basic set up of the model. Section 3 defines and characterizes the unique linear equilibrium of the economy. The effects of public announcements about the monetary aggregate on information aggregation and welfare are analyzed in section 4. Section 5 provides an extension of the basic model where multiple equilibria can occur and public announcements interplay with indeterminacy. Section 6 discusses two issues: first, opening a bond market does not affect our equilibrium, and second, a new interpretation of the monetary announcements. Section 7 concludes.

2 Set up

Time is discrete. Although the model is essentially static, we let time be infinite so that money is valued. The economy is composed of a $[0,1]$-continuum of islands that will be affected by the same real shock. As in Townsend (1983), labor and goods do not flow between islands, but information does: the representative household of each island will observe, and learn, from the nominal prices prevailing in other islands.

Preferences and Technology

In each island, there are competitive firms operating a linear technology, transforming one unit of labor into one unit of consumption good. Firms hire labor in a competitive local labor market and sell their output in a competitive local good market. Because labor and goods are immobile, the relative wage in terms of the consumption good is unity in all islands.

At each time $t \in \{1, 2, \ldots\}$ in island $i \in [0,1]$, a representative household chooses his effort supply, L_{it}, consumption, C_{it}, and money balance M_{it}^d, in order to maximize

$$\mathrm{E}_{i1}\left[\sum_{t=1}^{\infty} \beta^{t-1}\Big(\log(C_{it}) - \Theta L_{it}\Big)\right], \tag{1}$$

where Θ represent an aggregate permanent effort cost, and subject to sequence of budget and cash-in-advance (CIA) constraints,

$$C_{it} + \frac{M_{it}^d}{P_{it}} \leq L_{it} + \frac{M_{it-1}^d}{P_{it}} \tag{2}$$

$$C_{it} \leq \frac{M_{it-1}^d}{P_{it}}. \tag{3}$$

where P_{it} denotes the nominal price level in island i at time t. The initial money balance of the representative household is $M_{i0}^d = M_i$.

Exogenous information about productivity

A key timing assumption of our model is that the cost of effort, Θ, is unknown as of time $t = 1$ but is revealed to everyone at $t = 2$. The goal of this timing assumption is to introduce a risky investment in the model: indeed, in the first period of our equilibrium, households will choose the amount L_{i1} of effort to put in their work, but the return on their investment will have a random component, $-\Theta L_{i1}$. While there are of course others and perhaps more standard models of risky investment, our timing assumption has the advantage of keeping the analysis tractable and transparent.

We assume that all households share the common and fully diffused prior that $\log(\Theta) \equiv \theta$ is normally distributed with a mean and precision of zero. (The diffuse prior assumption will be relaxed in Section 5). Households also observe an island-specific signal about the effort cost:

$$\hat{\theta}_i = \theta + \varepsilon_{\theta i} \tag{4}$$

where $\varepsilon_{\theta i}$ is normally distributed with mean zero and precision ψ_θ.

Exogenous information about money

Households do not know the aggregate money supply. Instead, they share the common prior that the logarithm of the aggregate money supply is normally distributed with mean zero and precision Ψ_m. As is standard in the literature, the precision Ψ_m represents the amount of public information about the money supply. In particular, a public information release about the money supply will translate into an increase in Ψ_m.

In practice, why would households be uncertain about the money supply? First, although the Federal Reserve Board publishes weekly data on money aggregates, M1 and M2, the estimates are subsequently revised as depository institutions either report new data or revise the data they previously reported.[1] In addition, one may argue that even error-free measures of M1 and M2 remain noisy estimates of the "true" aggregate quantity of liquidity that directly influences the nominal price level. This aggregate quantity of liquidity may include some less liquid assets omitted in M1 and M2, and could be influenced by unobserved velocity.

The initial money endowment of the representative household of island i is

$$\log M_i \equiv \hat{m}_i = m + \varepsilon_{mi} \tag{5}$$

where ε_{mi} is normally distributed across islands with mean zero and precision ψ_m. The logarithm of the aggregate money supply is thus $\log M \equiv m + (2\psi_m)^{-1}$. Note that the initial

[1]See the December 2006 Performance Evaluation of the Statistical Release about Money Stock Measures on the Federal Reserve Board website:
http://www.federalreserve.gov/releases/h6/perfeval2006.htm

money endowment provides information about the aggregate money supply.

In all the above, we assume that the random variables θ, m, $\varepsilon_{\theta i}$ and ε_{mi}, are all pairwise independent.

Information from nominal prices

The only way the islands are connected is informationally: households observe the distribution of nominal prices of the entire economy when making their labor supply and consumption decisions. Let us denote by p_t the average logarithmic price across islands:

$$p_t = \int p_{it} di \tag{6}$$

As we will show later on, the average price level will become a sufficient statistic of the entire price distribution in equilibrium.

Market Clearing Conditions

We assume that goods and labor cannot flow across the islands, so that the goods market must clear locally

$$C_{it} = L_{it} \tag{7}$$

for all $i \in [0, 1]$ and $t \in \{1, 2, \ldots\}$.

3 Equilibrium

An equilibrium is made up of a sequence of distributions of consumption, labor, and money holdings across islands, together with a distribution of prices in the economy such that, at each time,

1. given the information conveyed by the distribution of prices in the economy, and given the local price, households choose consumption, labor, and money holdings to maximize their expected utility;

2. every local good markets clear.

Before characterizing an equilibrium formally, it is convenient to first analyze the household's problem.

Solving the household problem

Consider the representative household of island $i \in [0, 1]$ and let $\beta^{t-1}\lambda_{it}$ and $\beta^{t-1}\mu_{it}$ be the non-negative Lagrange multipliers of his budget constraint (2) and CIA constraint (3). Then, the first-order conditions for consumption, labor, and money balances are

$$\frac{1}{C_{it}} = \lambda_{it} + \mu_{it} \tag{8}$$

$$\mathrm{E}_{it}\left[\Theta\right] = \lambda_{it} \tag{9}$$

$$\frac{\lambda_{it}}{P_{it}} = \beta \mathrm{E}_{it}\left[\frac{\lambda_{it+1} + \mu_{it+1}}{P_{it+1}}\right] \tag{10}$$

where the expectation operator, $\mathrm{E}_{it}\left[\,\cdot\,\right]$, is conditional on all the information available to household i as of time t.

Anticipating that the CIA constraint binds at all times, $C_{it} = M^d_{it-1}/P_{it}$, it follows that $\lambda_{it} + \mu_{it} = P_{it}/M^d_{it-1}$. In addition, plugging the good market clearing condition (7) into the binding budget constraint (2), we obtain that

$$M^d_{it} = M^d_{it-1},$$

implying that a household's money holding must stay equal to his initial money endowment at each time, i.e. $M^d_{it} = M_i$. Now, together with equations (9) and (10), these manipulations show that

$$P_{it} = \beta^{-1} M_i E_{it}\left[\Theta\right]. \tag{11}$$

Plugging this back into the binding CIA constraint, we then obtain an inverse relationship between consumption and the expected effort cost,

$$C_{it} = \beta E_{it}\left[\Theta\right]^{-1} \tag{12}$$

Plugging this into equation (8), we obtain that $\mathrm{E}_i[\Theta]/\beta = \lambda_{it} + \mu_{it}$. Together with (9), this implies that $\mu_{it} = (1-\beta)E_{it}\left[\Theta\right]/\beta$ is strictly positive, confirming our guess that the CIA constraint is binding at all times.

Note that the households are concerned only with the cost parameter Θ. Because the goods markets clear locally and the local money supply is constant and known, uncertainty about the aggregate money supply does not directly affect the household's problem. However, as seen in equation (11), nominal prices in the economy combine local expectations about the cost parameter, and local money supplies. Thus, information about money will affect households' ability to extract information about Θ from observing the economy wide prices.

Equilibrium after the second period

From period $t = 2$ onwards, households know the exact realization of Θ. Hence we have from equation (11) that, in equilibrium

$$\begin{aligned} P_{it} &= \beta^{-1} M_i \Theta \\ C_{it} &= L_{it} = \beta \Theta^{-1}, \end{aligned}$$

for all $t \geq 2$. So quantities and prices are determined from $t = 2$ onwards.

Linear equilibrium in the first period

As we proceed to study the competitive equilibrium in the first period, and given that the economy is stationary from $t = 2$ onwards, we simplify notations by removing the time subscript from all first-period variables.

Borrowing from the literature on noisy Rational Expectations in financial markets (see, among many others, Grossman (1975) and Hellwig (1980)), we will look for linear equilibria,

Definition 1 (Linear Equilibrium)**.** *A linear equilibrium is a cross sectional distribution of nominal prices P_i, consumption C_i, effort supplies L_i, and expectations about Θ, $\mathrm{E}_i[\Theta]$, such that*

i) conditional on the realization of (m, θ), the distribution of prices is log normal with constant dispersion and a mean parameter

$$p = a_0 + a_1 \theta + a_2 m \ , \tag{13}$$

for some constants a_0, a_1 and a_2.

ii) households' expectations are rational; that is, after observing their private signals $(m_i, \hat{\theta}_i)$ and the distribution of nominal prices in the economy,

$$\mathrm{E}_i[\Theta] = \mathrm{E}\left[\Theta \,|\, \hat{\theta}_i, m_i, p\right] \ ; \tag{14}$$

iii) households decisions are optimal and markets clear:

$$C_i = L_i = \beta \mathrm{E}_i [\Theta]^{-1} \tag{15}$$

$$P_i = \beta^{-1} M_i \mathrm{E}_i [\Theta] . \tag{16}$$

To understand our rational expectations condition (14), note the following: even though households observe the entire cross-sectional distribution of nominal prices, it is sufficient to condition expectations with respect to only one moment of the distribution, the average price level p. Indeed, in the equilibria we consider, the distribution of prices in the economy is log normal and hence is uniquely parameterized by its mean and its dispersion. Given the additional requirement in part (i), that the dispersion does not depend on the realization (m, θ), the mean parameter, p, thus conveys all the information embedded in the price distribution.

A unique linear equilibrium

We now proceed to construct an equilibrium and show that it is unique. We start by noting that the mean parameter p is informationally equivalent to observing a signal z such that $z = \theta + m/\alpha$, where $\alpha = a_1/a_2$; given that a_0, a_1, and a_2 are constants. In addition, the following simple transformation of households' information set allows to determine an equilibrium,

Lemma 1. *The joint observation of $\hat{\theta}_i = \theta + \varepsilon_{\theta i}$, $\hat{m}_i = m + \varepsilon_{mi}$ and $\hat{z} = \theta + m/\alpha$ is equivalent to the joint observation of,*

$$\hat{\theta}_i = \theta + \varepsilon_{\theta i} \tag{17}$$

$$\hat{z}_i \equiv \hat{z} - \hat{m}_i/\alpha = \theta - \varepsilon_{mi}/\alpha \tag{18}$$

$$\hat{z} = \theta + m/\alpha. \tag{19}$$

Proof. This follows immediately by replacing $\hat{m}_i$ by $\hat{z}_i \equiv \hat{z} - \hat{m}_i/\alpha$ in a household's information set, while keeping the two other observations, $\hat{\theta}_i$ and $\hat{z}$, the same. □

The Lemma shows that observing the price level, p, generates two independent signals about about θ. There is first a *public signal* of precision $\alpha^2 \Psi_m$, given by (19), which intuitively follows from interpreting the price level in light of the public information about the money supply. Second, the price also generates an *island specific* signal about θ of precision $\alpha^2\psi_m$, given by (18), which follows from interpreting the price in light of the island specific information, $\hat{m}_i$, about the money supply. The finding that the publicly observable price level also generates a island-specific signal is the main insight of the Lemma, and will be a key driver of our results.

Because the three signals $(\hat{\theta}_i, \hat{z}_i, \hat{z})$ about θ have independent noises, and their precisions are ψ_θ, $\alpha^2\psi_m$, and $\alpha^2\Psi_m$ respectively, it follows that

Lemma 2. *In any linear equilibrium, households posterior beliefs about Θ are log normal with mean and variance parameters:*

$$\mathrm{E}_i\left[\theta\right] = \mathrm{E}\left[\theta \,|\, \hat{\theta}_i, \hat{m}_i, p\right] = \frac{\psi_\theta\,\hat{\theta}_i + \alpha^2\psi_m\,\hat{z}_i + \alpha^2\Psi_m\,\hat{z}}{\psi_\theta + \alpha^2\psi_m + \alpha^2\Psi_m} \tag{20}$$

$$\mathrm{var}_i\left[\theta\right] = \mathrm{var}\left[\theta \,|\, \hat{\theta}_i, \hat{m}_i, p\right] = \frac{1}{\psi_\theta + \alpha^2\psi_m + \alpha^2\Psi_m}, \tag{21}$$

where $\alpha = a_1/a_2$.

Given households' diffuse prior, equation (20) and (21) are the standard Bayesian updating formula for independent signals and normal distribution. In equation (20), the posterior belief about θ is a convex sum of the three signals, where the convex weights reflect the signals relative precisions. In equation (21), the posterior precision, $1/\mathrm{var}_i\left[\theta\right]$, is obtained by adding up the precisions of the three signals.

Now, from the logarithms of (16) it follows that

$$\log(P_i) = p_i = -\log\beta + \hat{m}_i + \mathrm{E}_i[\theta] + \frac{\mathrm{var}_i[\theta]}{2} \tag{22}$$

Taking the cross-sectional average, we obtain the average log price is,

$$\begin{aligned} p &= \int p_i\,di = -\log\beta + \int \hat{m}_i\,di + \int \mathrm{E}_i\left[\theta\right]\,di + \frac{\mathrm{var}_i\left[\theta\right]}{2} \\ &= -\log\beta + \theta + \left(1 + \frac{\alpha\Psi_m}{\alpha^2\Psi_m + \alpha^2\psi_m + \psi_\theta}\right) m + \frac{\mathrm{var}_i\left[\theta\right]}{2}, \end{aligned}$$

where the second line follows from substituting the formula of equation (20) into the first line. Hence, our our linear guess that $p = a_0 + a_1\theta + a_2 m$ is verified for $a_0 = -\log\beta + \mathrm{var}_i[\theta]/2$,

$a_1 = 1$, $a_2 = 1/\alpha$, and for some α solving the fixed-point equation

$$\begin{aligned}
& 1/\alpha = 1 + \frac{\alpha\Psi_m}{\alpha^2\Psi_m + \alpha^2\psi_m + \psi_\theta} \\
\Leftrightarrow \quad & 1 = \alpha + \frac{\alpha^2\Psi_m}{\alpha^2\Psi_m + \alpha^2\psi_m + \psi_\theta} \\
\Leftrightarrow \quad & \alpha = H(\alpha) \equiv \left(1 + \frac{\alpha^2\Psi_m}{\alpha^2\psi_m + \psi_\theta}\right)^{-1}
\end{aligned} \tag{23}$$

The function $H(\alpha)$ is a positive and strictly decreasing function of α; with $H(0) = 1$ and $\lim_{\alpha\to\infty} H(\alpha) = \psi_m/(\Psi_m + \psi_m)$. Hence, it follows that:

Lemma 3. *There exists a unique solution $\alpha_\star$ to equation (23); and $\alpha_\star \in (\psi_m/(\Psi_m+\psi_m), 1)$.*

Note that $\alpha = \alpha_\star$ uniquely determines the cross-sectional distribution of log prices, which is normal with a constant dispersion, as can be seen from equation (22) after substituting in for equations (20) and (21). Also, $\alpha_\star$ determines the cross-sectional distribution of mean beliefs as implied by (20). Finally this determines a unique distribution for consumption and labor supplies, according to (15). Thus, we have shown,

Proposition 1. *There exists a unique linear equilibrium.*

Informational Efficiency and Welfare

Lemma 1 showed that the distribution of nominal prices generates two independent signals about productivity: an island-specific signal with precision $\alpha^2\psi_m$, and a public signal with precision $\alpha^2\Psi_m$. Recall also that a household also receives an island-specific exogenous signal about productivity, $\hat{\theta}_i$. Hence, the total amount of information gathered by a household from island-specific sources has a precision of

$$\psi'_\theta \equiv \psi_\theta + \alpha^2\psi_m, \tag{24}$$

the sum of all island-specific precisions. Since a household starts from a diffuse prior and receives no exogenous public signal about θ,

$$\Psi'_\theta \equiv \alpha^2\Psi_m, \tag{25}$$

measures the precision of the total amount of information it gathers from public sources. As long as ψ_m, Ψ_m and ψ_θ are finite, the equilibrium values of Ψ'_θ and ψ'_θ are also finite as

α is bounded above by 1: the price distribution does not perfectly reveal the value of the productivity parameter.

From equation (23), it follows that

$$\alpha = \left(1 + \frac{\Psi'_\theta}{\psi'_\theta}\right)^{-1} = \frac{\psi'_\theta}{\psi'_\theta + \Psi'_\theta}.$$

The observation of nominal prices increases a household's precision by

$$\alpha^2 \psi_m + \alpha^2 \Psi_m = \psi'_\theta + \Psi'_\theta - \psi_\theta.$$

This increase constitutes a natural measure of the informational efficiency of nominal prices. An important result in our island economy is that an improvement in the informational efficiency of nominal prices, $\psi'_\theta + \Psi'_\theta - \psi_\theta$, unambiguously increases *ex ante* utilitarian welfare:

Lemma 4. *The ex ante welfare of any household in the equilibrium is given by*

$$-\frac{1}{2}(\psi'_\theta + \Psi'_\theta)^{-1} + \frac{\log \beta - \beta}{1 - \beta} \tag{26}$$

Proof. The *ex ante* time-t flow welfare of a household is

$$\mathrm{E}_0[\log C_{it} - \Theta L_{it}] = \mathrm{E}_0\Big[\mathrm{E}_{it}[\log C_{it}] - \mathrm{E}_{it}[\Theta] L_{it}\Big] = -\mathrm{E}_0 \log \mathrm{E}_{it}[\Theta] + \log \beta - \beta$$

where we used that $C_{it} = L_{it} = \beta E_{it}[\Theta]^{-1}$ together with the law of iterated expectations. From period 2 onwards, $E_{it}\Theta = \Theta$. And we know that $\mathrm{E}_0[\log \Theta] = 0$ by the prior distribution assumption. So, $\mathrm{E}_0 \log \mathrm{E}_{it}[\Theta] = \mathrm{E}_0[\log \Theta] = 0$. In the first period, we have that $E_0 \log E_{i1}[\Theta] = E_0[E_{i1} \log \Theta] + \mathrm{var}_{i1}[\log \Theta]/2$. Using the law of iterated expectations and that $\mathrm{var}_{i1}[\log \Theta] = (\psi_\theta + \alpha^2 \psi_m + \alpha^2 \Psi_m)^{-1}$, and adding up through time, the result follows. $\square$

Households' *ex ante* welfare goes up with the total precision of their first-period beliefs, $\psi'_\theta + \Psi'_\theta$. This simply means that households are better off if they know more about about θ when they make their labor supply decisions. The households utility is maximized when they know Θ perfectly: either when ψ'_m or Ψ'_m are infinite. Although intuitive, this result is not a forgone conclusion: indeed, an increase in the informational efficiency of prices does not generally improves welfare.[2]

[2]See the first chapter of Brunnermeier (2001) and the references therein.

4 The Welfare Impact of Public Communication

Having characterized the unique linear equilibrium of the economy, we proceed now to analyze the question of interest: what is the impact, if any, of public information releases about m? As it is standard in the literature, we will interpret public information releases about m as increases in the precision of the prior about m, Ψ_m.

Public Communication and Island-specific Precision

Our first result concerns the value of $\alpha_\star$, the equilibrium sensitivity of prices to island-specific information.

Lemma 5. *The equilibrium value of $\alpha_\star$ is strictly decreasing in Ψ_m. It tends to 1 as Ψ_m goes to zero, and tends to zero as Ψ_m goes to infinity.*

Proof. From equation (23), we see that an increase in Ψ_m, reduces $H(\alpha)$. Given that $H(\alpha)$ is strictly decreasing, this implies that $\alpha_\star$ decreases. Given that $\alpha_\star$ decreases in Ψ_m and is bounded below by 0, $\alpha_\star$ converges to a finite limit as Ψ_m tends to infinity. Clearly, this limit cannot be positive, or else equation (23) cannot be satisfied for sufficiently high Ψ_m. Hence $\alpha_\star$ tends to 0 as Ψ_m tends to infinity. Alternative, given that $\alpha_\star$ is bounded above by 1, and decreases in Ψ_m, it follows that $\alpha_\star$ converges to a finite number as Ψ_m tends to zero, which from (23) implies that $\alpha_\star$ converges to 1. □

Hence, more public information about m will lower the sensitivity of the price to the island specific information. The intuition is straightforward: the more public information about m, the more weight households assign to their public signal, and the less weight they assign to their island specific signals when they make their labor supply decision. This lemma immediately implies our second result:

Proposition 2 (Public Crowds Out Private)**.** *The precision of the island-specific knowledge about θ, ψ'_θ, strictly decreases in Ψ_m.*

Proof. Recall that $\psi'_\theta = \psi_\theta + \alpha_\star^2 \psi_m$. Since, $\alpha_\star$ is strictly decreasing, it follows that ψ'_θ decreases. □

Public releases of information concerning m reduce the amount of island-specific information gathered about θ in the economy.

Public Communication and Public Precision

We now turn to the impact of Ψ_m on public knowledge, Ψ'_θ. From the formula that $\Psi'_\theta = \alpha_\star^2 \Psi_m$, one sees that an increase in Ψ_m has two opposite effects on Ψ'_θ. Holding $\alpha_\star$ constant, Ψ_m increases public knowledge. This is the intuitive direct beneficial effect: when households know more about money, they can extract more information from nominal prices. There is, however, a countervailing equilibrium effect: following an increase in public information about m, households put less weight on their island specific knowledge, reducing the sensitivity $\alpha_\star$. The net effect depends on the size of the release:

Proposition 3. *The precision of the public knowledge about θ, Ψ'_θ is strictly increasing in Ψ_m whenever $\psi_m < 27\psi_\theta$. When $\psi_m > 27\psi_\theta$, there exist values $\underline{\Psi}_m < \overline{\Psi}_m$, such that Ψ'_θ is strictly decreasing in Ψ_m when $\Psi_m \in [\underline{\Psi}_m, \overline{\Psi}_m]$ and strictly increasing otherwise.*

Proof. We have that

$$\Psi'_\theta = \alpha_\star^2 \Psi_m = \frac{1-\alpha_\star}{\alpha_\star}(\psi_\theta + \alpha_\star^2 \psi_m)$$

Note that

$$\frac{\partial \Psi'_\theta}{\partial \Psi_m} = \frac{\partial \alpha_\star}{\partial \Psi_m} \times \frac{\alpha_\star^2 \psi_m - 2\alpha_\star^3 \psi_m - \psi_\theta}{\alpha_\star^2} \equiv \frac{\partial \alpha_\star}{\partial \Psi_m} \times \frac{F(\alpha_\star)}{\alpha_\star^2}$$

Given that $\alpha_\star$ is strictly decreasing in Ψ_m, the derivative of Ψ'_θ with respect to Ψ_m depends on the sign of $F(\alpha)$. One see that $F(0)$ and $F(1) < 0$. In addition, one can show that $F(\alpha)$ is hump-shaped and achieves its maximum at $\alpha = 1/3$. The maximum value, $F(1/3)$, is negative if $\psi_m < 27\psi_\theta$, which immediately implies that $F(\alpha)$ is always negative. If $\psi_m > 27\psi_\theta$, then $F(\alpha)$ has two distinct positive roots, $\underline{\alpha}$ and $\bar{\alpha}$ such that $F(\alpha) > 0$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, and non-positive otherwise. Given that $\alpha_\star$ is a strictly decreasing function of Ψ_m mapping $[0, \infty]$ onto $[0, 1]$, the result follows. $\square$

Public Communication and Welfare

The final result in this section answers the question that concerns us the most: what is the effect of public information about the money supply on the total knowledge of households and welfare? As can be already inferred from Propositions 2 and 3, an increase in Ψ_m can indeed reduce the total amount known by agents in the economy, $\psi'_\theta + \Psi'_\theta$, because it can reduce both equilibrium values of ψ'_θ and Ψ'_θ. The Proposition below provides the complete characterization:

Proposition 4. *Total precision, $\psi'_\theta + \Psi'_\theta$ and welfare are U-shaped function of Ψ_m achieving their minimum at:*

$$\Psi_m^{\min} \equiv \max\left\{2\psi_m\left(\sqrt{\frac{\psi_m}{\psi_\theta}} - 1\right), 0\right\}.$$

Also, total precision $\psi'_\theta + \Psi'_\theta$ tends to infinity with Ψ_m.

Proof. ADD PROOF HERE □

Proposition 4 is the main result of this paper. It summarizes the impact of public announcements on the informational efficiency of the price distribution and welfare. In particular, it tells us that a marginal increase in public information about the money supply will reduce welfare when the private information about the aggregate money supply is very precise, or when the private information about the productivity parameter is not. The second part of the proposition implies that there always exists a sufficiently strong public release that is welfare enhancing. Clearly, whether or not such a strong announcement is possible depends on the information available to the monetary authority about the aggregate money supply.

Figure 4 shows utilitarian welfare as a function of Ψ_m, for alternative choices of exogenous parameters. As shown in the figure, $\Psi_m^{\min}$ can be equal to zero when ψ_m is small enough: then, an increase in public information is always welfare improving. Hence, an important necessary condition for public information about m to be welfare decreasing is that $\psi_m > 0$: the distribution of price must generate island-specific information. Amador and Weill (2006) have obtained a similar result relating to the importance of public and private learning for the welfare effects of public information releases, in the context of an abstract model of information diffusion.

Note also that, if ψ_θ is sufficiently small, then $\Psi_m^{\min}$ can be made arbitrarily large. This implies that, for any finite information release, there exists a small enough ψ_θ such that the release ends up welfare reducing.

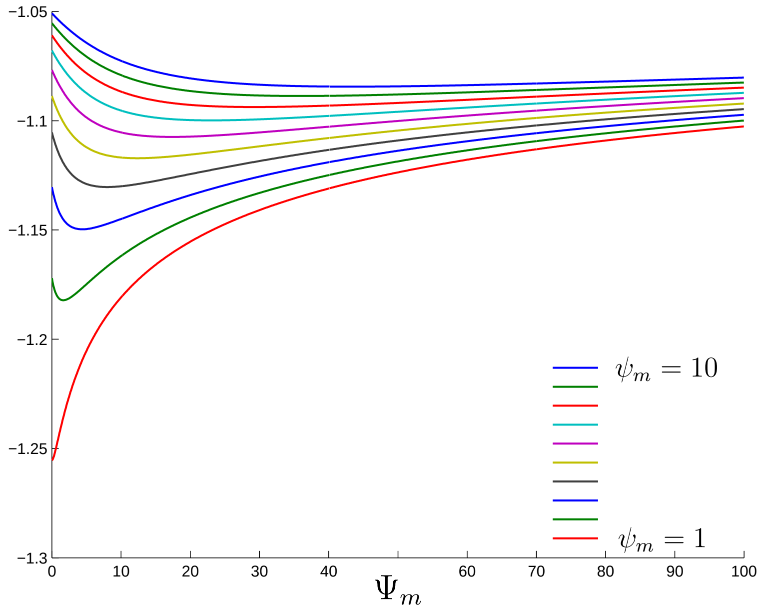

Figure 1: Utilitarian welfare as a function of Ψ_m, for alternative choices of ψ_m.

Related results from the literature

Perhaps the best known related result is that of Morris and Shin (2002) who have shown, in the context of beauty contest game, that public information can reduce welfare. Although reminiscent of their result, ours does not arise from any payoff externalities but instead from the endogenous aggregation of information through prices.

Morris and Shin (2002) emphasize that releases of public information are beneficial when the precision of the private beliefs about the state of the economy is sufficiently small. In our economy where the state has two dimensions, m and θ, this result does not always hold: while more island-specific information about θ does indeed decreases the range where public announcements are welfare increasing as it does in Morris and Shin (2002), more precise island specific information about m causes the opposite effect.

In Svensson (2006) critique of Morris and Shin (2002), it is proposed that a conservative benchmark of how likely it is that public information is welfare reducing, is when the precision of the public and the private signals are the same. This restriction on parameter values is intended to capture the fact that, in practice, the monetary authority know at least as much about its own policy than the general public. When imposing this restriction in Morris and Shin (2002)'s model, Svensson (2006) finds that public information is welfare increasing: he concludes that Morris and Shin (2002) are, in fact, pro-transparency. In our multidimensional economy, we interpret Svensson (2006)'s restriction as letting $\Psi_m = \psi_m$. A marginal public release of central-bank information will then reduce welfare if $\psi_m = \Psi_m < \Psi_m^{\min}$, which is equivalent to $\psi_m > 9/4\psi_\theta$. If this condition is satisfied, then, public information about m decreases welfare even though the precision of the public and the island specific signals about m are the same.

5 Transparency and Indeterminacy

So far we have analyzed equilibrium under the assumption of a completely diffuse prior about Θ. The reasons for imposing such assumption where twofold: first, it allows us to concentrate only on the effect of public news about the money supply; and it was a sufficient condition for uniqueness of an equilibrium. This last implication was particularly useful, given our focus on welfare analysis, as it removed the need for equilibrium selection.

In this section we relax this, and show that the model now allows for multiple equilibria. This give rise to a novel role for monetary policy announcements: they can affect the indeterminacy. In particular, a sufficiently strong release of public information about the monetary aggregate removes the indeterminacy. An important caveat is that, a mild release might instead generate it.

Accordingly, suppose then that the prior about Θ is normal with positive precision Ψ_θ. A similar argument as exposed in Section (3) now implies a new version of equation (25),

$$\Psi'_\theta = \Psi_\theta + \Psi_m \left(\frac{\psi'_\theta}{\psi'_\theta + \Psi'_\theta} \right)^2 , \tag{27}$$

whereas equation (24) remains unchanged. Letting again α be equal to $\psi'_\theta/(\psi'_\theta + \Psi'_\theta)$, we can derive a new fixed point equation,

$$\alpha = \left(1 + \frac{\alpha^2 \Psi_m + \Psi_\theta}{\alpha^2 \psi_m + \psi_\theta} \right)^{-1} , \tag{28}$$

which is equivalent to equation (23) when $\Psi_\theta = 0$. The key element to notice now is that the right hand side of the fixed equation it is not necessarily decreasing in α, and in particular, there may exists several solutions. Let us rewrite equation (28) as,

$$G(\alpha) \equiv \alpha^3(\psi_m + \Psi_m) - \alpha^2\psi_m + \alpha(\psi_\theta + \Psi_\theta) - \psi_\theta = 0 \tag{29}$$

and the following holds

Proposition 5. *For given ψ_m, ψ_θ and Ψ_θ, there exists finite values Ψ_1 and Ψ_2 with $\Psi_1 \geq \Psi_2 \geq 0$ such that a unique equilibrium is only obtained for $\Psi_m < \Psi_2$ and $\Psi_m > \Psi_1$. And for a non-empty set of values for ψ_m, ψ_θ and Ψ_θ, we have that $\Psi_1 > \Psi_2 > 0$.*

The first part of the proposition tells us that a sufficiently strong increase in Ψ_m will lead to a unique equilibrium. The second part of the proposition, tells us that for a non-empty set of parameter values, it is possible that an increase in Ψ_m will move the economy from a situation with a unique equilibrium towards indeterminacy.

Figure 2 plots a situation demonstrating how increases in Ψ_m move the economy from uniqueness to multiplicity, and eventually back to uniqueness. The figure shows the function $G(\alpha)$ for different values of Ψ_m. Equilibria are zeros of the function.

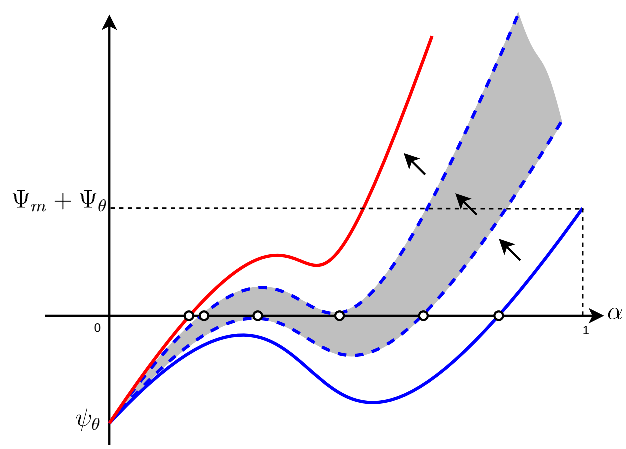

Figure 2: The figure plots the function $G(\alpha)$ for increasing values of Ψ_m, as denoted by the direction of the arrows. The shaded area is the area of indeterminacy

6 Extensions

In the first subsection below, we show that the opening a bond market in our economy will not alter the equilibrium, and hence all the previous analyses are robust to this modification of the economy. In the second subsection we reinterpret the monetary policy announcement in the form of a monetary authority who lacks complete control of the monetary aggregate.

6.1 On the Irrelevance of a Bond Market

A familiar way in which an economy aggregates dispersed private information is through asset markets. One might wonder then how robust the results regarding the social value of public announcements that we have obtained are to the introduction of a financial market where households from different island can interact. To try to answer this question, we introduce what we believe is a natural financial market in our economy: households are allow to trade claims of a zero net supply nominal bond. Our main result is that opening such a market does not provide any more information to the households, and that allocation obtained by a competitive equilibrium when the bond market is closed remains the allocation of a competitive equilibrium once it is opened.

Thus, suppose that any household at period t can buy a bond that pays a unit of the currency in the following period $t+1$. Let us denote by Q_t its nominal price.

The budget constraint of the household i in period t is now given by

$$C_{it} + \frac{M_{it}^d}{P_{it}} + \frac{B_{it}}{P_{it}} Q_t \leq L_{it} + \frac{M_{it-1}^d}{P_{it}} + \frac{B_{it-1}}{P_{it}} \tag{30}$$

where B_{it} are the amount of the bond held by household i in period t.

The bond market clears at all times, implying that

$$\int B_{it}\, di = 0 \tag{31}$$

In an equilibrium, from the Euler equation of household i we obtain that

$$Q_t = \beta \mathrm{E}_{it} \left[\frac{u'(C_{it+1}) P_{it}}{u'(C_{it}) P_{it+1}} \right].$$

We now check that the allocation without an open bond market remains an equilibrium once the market opens. First notice that equations (11) and (12) imply that $C_{it}P_{it} = M_i$, and hence,

$$Q_t = \beta \mathrm{E}_{it} \left[\frac{C_{it} P_{it}}{C_{it+1} P_{it+1}} \right] = \beta,$$

which is the same for all agents. Note as well that the price of the bond does not reveal any information: it is just equal to the discount factor. Thus any equilibrium allocation when the bond market is closed remains an equilibrium when the bond market is open with $B_{it} = 0$, and $Q_t = \beta$.

6.2 Transparency and Control: A Reinterpretation

In this section we provide an alternative interpretation of the results obtained so far.

Suppose that monetary policy results in an imperfect control of the money supply. Namely, we assume that the actual (log) money supply can be written as

$$m = m_c + m_u,$$

where m_c is the part of the money supply that the monetary authority is able to control with its various (un-modeled) policy instruments. Hence, the monetary authority has perfect knowledge of m_c. On the other hand, m_u is the part of money supply that is beyond the control of the monetary authority. We assume that m_c and m_u are independent random variables, normally distributed with mean zero, and respective precisions Ψ_c and Ψ_u.

Before the markets in each island open, we let the monetary authority be able to publicly reveal its policy, m_c, to all households in the economy. If the monetary authority chooses to be transparent about its policy, then the households believe that m is normally distributed with mean $\bar{m} = m_c$ and precision Ψ_u.

If on the other hand, the monetary authority chooses to be opaque about its policy, then households believe that m is normally distributed with a mean $\bar{m} = 0$, and a precision

$$(1/\Psi_c + 1/\Psi_u)^{-1} = \Psi_u \frac{\Psi_c}{\Psi_c + \Psi_u}.$$

One sees that, in this setup, Ψ_u measures two things: on the one hand, it measures the degree of control. Indeed, a large Ψ_u means that the variance of m_u is small, meaning that the monetary authority tightly controls the monetary aggregate. The opposite is true when Ψ_u is small. On the other hand, Ψ_u also measures the maximum amount of information that a transparent communication can reveal about the money supply.

It is also possible to show that this set up is equivalent to the release of a signal about m with an appropriately chosen precision.

An equivalence

We proceed by construction. Let $\hat{m} \equiv (1+\Psi_c/\Psi_u)m_c$. And let $\varepsilon_m \equiv \hat{m}-m = \hat{m}-m_c-m_u = \Psi_c/\Psi_u m_c - m_u$. Note that $E[\varepsilon] = 0$,

$$E[\varepsilon m] = E\left[\left(\frac{\Psi_c}{\Psi_u}m_c - m_u\right)(m_c+m_u)\right] = \frac{\Psi_c}{\Psi_u}E[m_c^2] - E[m_u^2] = 0 \ ,$$
$$E[\varepsilon\varepsilon] = E\left[\left(\frac{\Psi_c}{\Psi_u}m_c - m_u\right)^2\right] = \left(\frac{\Psi_c}{\Psi_u}\right)^2 E[m_c^2] + E[m_u^2] = \frac{\Psi_c+\Psi_u}{\Psi_u^2}.$$

So that ε_m is orthogonal to m. Hence we can reinterpret the public announcement of the monetary authority as the release of a signal $\hat{m} = m + \varepsilon$; where ε is independent from m and has precision $\Psi_u^2/(\Psi_c+\Psi_u)$.

Hence, of all the previous results we have obtained regarding the welfare effects of a public announcement and the derivation of the simple criterion to evaluate policy will hold also for this specification of the announcement.

In particular Proposition 4 implies that,

Corollary 1. *If $\Psi_u < \Psi_m^{\min}$ then transparency is welfare reducing.*

The Corollary means that transparency is more likely to reduce welfare when the monetary authority poorly controls the monetary aggregate. We also have that

Corollary 2. *For all Ψ_u, transparency is welfare reducing if ψ_θ is small enough.*

That is, for any finite degree of control, there exist situations in which transparency is welfare reducing: for instance, when the private information about the cost shock is very dispersed in the economy.

7 Conclusions

We have characterized the conditions under which announcements by the Central Bank about the state of the monetary aggregate reduce the informativeness of prices about real shocks and may actually lower welfare. Although we focus in the case where households observe nominal prices, we think is reasonable to conjecture that a similar outcome will occur in the presence of financial markets that also aggregate dispersed information in the economy.

Our model is basically static (the infinite horizon nature was just necessary for money to have value in our economy). However, similar techniques as the ones here developed may prove useful in studying the dynamic effects of information releases[3], and also in answering the timing question: *when* should the Central Bank make announcements. This is all left for future research.

[3] Amador and Weill (2006) have analyzed a dynamic version of a more abstract model.

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