

# Establishments Dynamics, Vacancies and Unemployment: A Neoclassical Synthesis\*

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**Abstract:** This paper develops a Walrasian equilibrium theory of establishment dynamics and matching frictions and uses it to analyze business cycle fluctuations. Two scenarios are considered: one in which the matching process is subject to congestion externalities and another in which it is not. The paper finds that the scenario with congestion externalities replicates U.S. business cycle dynamics much better than the scenario with efficient matching.

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# 1 Introduction

The purpose of this paper is to evaluate if standard neoclassical theory can be used to explain the observed behavior of establishment dynamics, vacancies and unemployment both at growth and business cycle frequencies. To this end, the paper constructs a real business cycle model that blends three important strands in the literature: 1) the Hopenhayn and Rogerson (15) model of establishment dynamics, 2) the Mortensen and Pissarides (18) matching model, and 3) the Lucas and Prescott (16) islands model. A key feature of the model is that it fully relies on classical price theory: All prices, including that of labor, are determined in Walrasian markets.

The economy is populated by a representative household that values consumption and leisure. Output, which can be consumed or invested, is produced by a large number of spatially separated establishments that are subject to aggregate and idiosyncratic productivity shocks. The amount of hiring that an establishment can undertake is constrained by the number of recruitment opportunities that it has available. Unemployed workers can become employed only if they gain employment opportunities. Recruitment opportunities for establishments and employment opportunities for workers are jointly produced by a neoclassical recruitment technology that uses unemployed workers and the consumption good as inputs of production. Following the matching literature, the recruitment technology is allowed to be subject to production externalities: The total number of unemployed workers in the economy and the aggregate amount of recruitment expenditures affect its productivity. Any of the workers with employment opportunities can be hired by any of the establishments with recruitment opportunities.

The paper defines and fully characterizes a recursive competitive equilibrium for this economy. It shows that an equilibrium can be constructed by solving a social planning problem with side conditions. The social planner solves a standard utility maximization problem subject to feasibility constraints, except that it takes as given the total number of unemployed workers and the aggregate amount of recruitment expenditures that enter the recruitment technology as external effects. At equilibrium, these variables must be generated by the social planner's optimal decision rules. The recruitment opportunities and employment decision rules of the establishments are also characterized. In particular, they are shown to be of the (S,s) variety. This, together with the assumptions that the idiosyncratic productivity shocks take a finite number of values and that the aggregate productivity shocks are sufficiently small, implies that the distribution over establishment types has a finite support. As a consequence, the social planner's problem can be formulated in terms of a finite number of state and decision variables. This is an important result: Despite the model's complexity, simple linear-quadratic methods can be used for computing a recursive competitive equilibrium.

The paper then evaluates how well the model is able to explain the data. Two versions are considered: A version without external effects in the recruitment technology and a version with external effects. Both versions are calibrated to identical U.S. long-run observations. Some parameter values are closely related to the neoclassical growth model and are calibrated to reproduce similar observations (e.g. the capital/output ratio, the investment/output ratio, etc.). The rest of the parameters are chosen to reproduce observations on establishment dynamics (e.g. the size distribution of establishments, job creation and destruction rates, etc.), worker flows (e.g. the separation rates, the hazard rate from unemployment, etc.), and vacancies (e.g. the vacancy rate, recruitment costs, etc.). When an aggregate productivity shock of empirically relevant magnitude is introduced, the paper finds that the version without external effects in the recruitment technology fails to reproduce the data: Fluctuations in unemployment and recruitment expenditures are too small and the model is unable to

generate a clear Beveridge curve. On the contrary, the version with external effects is quite successful in these dimensions. Thus the paper indicates that, when looked through the eyes of neoclassical theory, the data supports the hypothesis of congestion externalities in the matching process.

While the paper has a strong empirical focus it also makes a theoretical contribution to the literature on equilibrium unemployment. This literature has been dominated by two main strands: the Mortensen-Pissarides (18) matching model and the Lucas-Prescott (16) islands model. The Mortensen-Pissarides model is extremely useful for analyzing vacancies and unemployment and has been extended to incorporate business cycle fluctuations (e.g. Andolfatto (4), Merz (17), Shimer (20), Hall (13), Hagedorn and Manovskii (9), etc.) and, more recently, establishment dynamics (e.g. Acemoglu and Hawkins (1), Cooper et. al (6), etc.). However, the model has a significant drawback: It introduces free parameters in the wage determination process. Even in the simplest version of the model it is unclear what value to use for the Nash bargaining parameter. In versions with aggregate fluctuations and establishment dynamics, the degrees of freedom multiply since it is possible for the Nash bargaining parameter to vary systematically with the state of the economy or of an individual establishment. The Lucas-Prescott model does not suffer from these difficulties since wages are determined in Walrasian markets.<sup>1</sup> However, there is no notion of vacancies in that model: Firms behave as if they could hire any number of workers at the island specific competitive wage rate. That is, firms do not need to undertake any type of active recruitment effort in order to fill their job openings. This paper avoids these limitations: By blending together the Mortensen-Pissarides model and the Lucas-Prescott model, it delivers a framework for analyzing vacancies and unemployment in which all prices are fully determined by preferences and technology. Incorporating the Hopenhayn-Rogerson model is also important since establishments dynamics are the counterpart to worker flows and vacancies. The result is a comprehensive theory of labor market dynamics.

The paper is organized as follows. Section 2 describes the economy. Section 3 describes a recursive competitive equilibrium. Section 4 characterizes a recursive competitive equilibrium and describes how to compute it. Section 5 calibrates the two versions of the model. Finally, Section 6 presents the results. An appendix provides proofs to the most important claims made in the paper.

## 2 The economy

The economy is endowed with a measure one of workers. A worker is a capital good that does not depreciate and can not be produced. During any period of time a worker can be in either of two states: *employed* or *unemployed*. Employed workers produce the consumption good while unemployed workers produce home goods. Employed workers can be freely transformed into unemployed workers. However, unemployed workers can only be transformed into employed workers using a costly technology. All workers are subject to an idiosyncratic productivity shock called a *quit* shock, that makes them temporarily unproductive as employed workers. A worker that quits needs to spend a full period of time unemployed before regaining his productive capacity. The probability that a worker quits at the beginning of the following period depends on

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<sup>1</sup>The Lucas-Prescott model has been used, among other things, to study the effects of labor market policies (e.g. Alvarez and Veracierto (3)), business cycle dynamics (e.g. Veracierto (23)), occupational mobility (e.g. Kambourov and Manovskii (11)), and rest unemployment (e.g. Alvarez and Shimer (2)).

his current employment status: It is equal to  $\pi_n$  if the worker is currently employed and it is equal to  $\pi_u$  if the worker is currently unemployed.

The economy is populated by a representative household with preferences given by

$$\mathcal{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \varphi U_t \right] \right\}, \quad (1)$$

where  $C_t$  is consumption,  $U_t$  is the total number of unemployed workers,  $\varphi > 0$ ,  $\sigma > 0$  and  $0 < \beta < 1$ .

The consumption good is produced by a large number of establishments. Each establishment has a production function given by

$$y_t = e^{z_t} s_t F(n_t, k_t),$$

where  $z_t$  is an aggregate productivity shock,  $s_t$  is an idiosyncratic productivity shock,  $n_t$  is the number of employed workers,  $k_t$  is physical capital, and  $F$  is a continuously differentiable, strictly increasing, strictly concave and decreasing returns to scale production function that satisfies the Inada conditions. The idiosyncratic productivity shock  $s_t$  takes values in a finite set  $S$  and follows a Markov process with monotone transition matrix  $Q$ . Realizations of  $s_t$  are independent across establishments and  $s_t = 0$  is an absorbing state. Since there are no fixed costs of operation, exit takes place only when the idiosyncratic productivity level becomes zero. In every period of time a measure  $\varrho$  of new establishments is exogenously born. Their distribution over initial productivity shocks is given by  $\psi$ . The aggregate productivity shock follows an AR(1) process given by

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \quad (2)$$

where  $0 \leq \rho < 1$ , and  $\varepsilon_{t+1}$  is i.i.d., normally distributed, with variance  $\sigma_\varepsilon^2$  and zero mean.

The number of employed workers  $n_t$  at an establishment is given by

$$n_t = n_{t-1} + h_t - f_t,$$

where  $h_t$  are the gross employment increases (i.e. *hirings*) and  $f_t$  are the gross employment reductions (i.e. *firings*). All the workers that are fired become unemployed. Because of the exogenous quit of employed workers,  $f_t$  is effectively constrained as follows

$$\pi_n n_{t-1} \leq f_t \leq n_{t-1}.$$

The number of new hires  $h_t$  is limited by the number of *recruiting opportunities*  $j_t$  that the establishment has at the beginning of the period, i.e.

$$h_t \leq j_t. \quad (3)$$

Unemployed workers can become employed only if they are transformed into workers with *employment opportunities*. Workers with employment opportunities  $e_{t+1}$  and recruiting opportunities  $j_{t+1}$  are jointly produced using the following *recruitment technology*:

$$e_{t+1} = G(a_t, u_t, A_t, U_t), \quad (4)$$

$$j_{t+1} = H(a_t, u_t, A_t, U_t), \quad (5)$$

where  $a_t$  are recruitment expenditures (in the consumption good),  $u_t$  are unemployed workers,  $A_t$  is the aggregate amount of recruitment expenditures in the economy, and  $U_t$  is the total number of unemployed workers in the economy.<sup>2</sup>

The recruitment technology satisfies the following assumptions: 1)  $G$  and  $H$  are continuously differentiable, 2)  $G$  and  $H$  are increasing in  $(a_t, u_t)$ , 3)  $G$  and  $H$  are homogenous of degree one with respect to  $(a_t, u_t)$  and homogeneous of degree zero with respect to  $(A_t, U_t)$ , 4)  $G$  and  $H$  are concave in  $(a_t, u_t)$ , 5) and  $G$  satisfies that

$$G(a_t, u_t, A_t, U_t) \leq u_t, \text{ for every } (a_t, u_t, A_t, U_t). \quad (6)$$

Observe from equation (6) that not all unemployed workers that enter the recruitment technology are transformed into workers with employment opportunities:

$$x_{t+1} = u_t - G(a_t, u_t, A_t, U_t),$$

is the number of *unsuccessful candidates* that the recruitment technology generates.

### 3 Recursive competitive equilibrium

The state of the economy is given by the quintuple  $(z, K, E, X, \mu)$ , where  $z$  is the aggregate productivity level,  $K$  is the aggregate stock of capital,  $E$  is the aggregate number of workers with employment opportunities,  $X$  is the aggregate number of unsuccessful candidates,  $\mu(s, l \times j)$  is a measure of establishments over individual states  $(s, l, j)$ , and  $(E, X, \mu)$  satisfies that<sup>3</sup>

$$\int l \mu(s, dl \times dj) + E + X = 1. \quad (7)$$

There are three competitive sectors in the economy: a households sector, an establishments sector, and a recruitment industry.

Households earn income from renting capital to the establishments and from the aggregate profits made by the establishments sector.<sup>4</sup> They spend their income on consumption, on investment and on renting unemployed workers. The individual state of a household is the amount of capital that it owns  $\kappa$ . The household's problem is described by the following Bellman equation:

$$B(\kappa, z, K, E, X, \mu) = \max_{\{c, i, m\}} \left\{ \frac{c^{1-\sigma} - 1}{1-\sigma} + \varphi m + \beta \mathcal{E} [B(\kappa', z', K', E', X', \mu') \mid z] \right\} \quad (8)$$

subject to:

$$c + i + r^u(z, K, E, X, \mu) m \leq r^k(z, K, E, X, \mu) \kappa + \Pi(z, K, E, X, \mu), \quad (9)$$

$$\kappa' = (1 - \delta) \kappa + i \quad (10)$$

$$(K', E', X', \mu') = L(z, K, E, X, \mu). \quad (11)$$

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<sup>2</sup>Observe that when  $G_A$ ,  $G_U$ ,  $H_A$  or  $H_U$  are strictly positive, the recruitment technology is subject to production externalities.

<sup>3</sup>Equation (7) implies that either  $E$  or  $X$  could be removed from the aggregate state vector. However, this would complicate the definition of a recursive competitive equilibrium.

<sup>4</sup>Each household is assumed to own one share of each establishment in the economy.

where  $r^u$  is the rental rate of an unemployed worker,  $r^k$  is the rental rate of capital,  $\Pi$  are the aggregate profits made by the establishments sector,  $i$  is investment,  $m$  is the number of unemployed workers that the household rents, and  $L$  is the law of motion for the endogenous state of the economy. Equation (9) is the budget constraint of the household, and equation (10) is the law of motion for its stock of capital. The household's optimal decisions are  $c = c(\kappa, z, K, E, X, \mu)$ ,  $i = i(\kappa, z, K, E, X, \mu)$ , and  $m = m(\kappa, z, K, E, X, \mu)$  for consumption, investment and unemployed workers, respectively.

The establishments rent capital, purchase workers with employment opportunities (up to the number of recruitment opportunities that they have at the beginning of the period), sell unemployed workers (up to their previous-period employment level), and purchase next-period recruitment opportunities. The individual state of an establishment is given by a triple  $(s, l, j)$ , where  $s$  is its current idiosyncratic productivity level,  $l$  is its previous-period employment level and  $j$  is its recruitment opportunities at the beginning of the period. The establishment's problem is described by the following Bellman equation:

$$W(s, l, j, z, K, E, X, \mu) = \max_{\{f, h, k, n, v\}} \left\{ e^z s F(n, k) + p^u(z, K, E, X, \mu) f - p^e(z, K, E, X, \mu) h - r^k(z, K, E, X, \mu) k - p^v(z, K, E, X, \mu) v + \mathcal{E} \left[ \sum_{s'} q(z, K, E, X, \mu, z') W(s', l', j', z', K', E', X', \mu') Q(s, s') \mid z \right] \right\} \quad (12)$$

subject to

$$n = l + h - f \quad (13)$$

$$\pi_n l \leq f \quad (14)$$

$$f \leq l \quad (15)$$

$$h \leq j \quad (16)$$

$$l' = n \quad (17)$$

$$j' = v \quad (18)$$

$$(K', E', X', \mu') = L(z, K, E, X, \mu). \quad (19)$$

where  $p^u$  is the price of an unemployed worker,  $p^e$  is the price of a worker with employment opportunities,  $p^v$  is the price of a next-period recruitment opportunity,  $q(\cdot, z')$  is the price of an Arrow security that delivers one unit of the consumption good if the next-period aggregate productivity level is equal to  $z'$ ,  $n$  is the number of employed workers,  $k$  is the capital level,  $f$  are the firings,  $h$  are the hirings, and  $v$  are the purchases of next-period recruitment opportunities. The constraints (13)-(16) have been described in the previous section. The establishment's optimal decisions are  $n = n(s, l, j, z, K, E, X, \mu)$ ,  $k = k(s, l, j, z, K, E, X, \mu)$ ,  $f = f(s, l, j, z, K, E, X, \mu)$ ,  $h = h(s, l, j, z, K, E, X, \mu)$ , and  $v = v(s, l, j, z, K, E, X, \mu)$ , for employed workers, capital, firings, hirings and next-period recruitment opportunities, respectively.

The recruitment companies sell workers with employment opportunities and next-period recruitment opportunities. They also buy and sell unemployed workers and rent them to the households sector. The individual state of a recruitment company is a pair  $(e, x)$ , where  $e$  is its number of workers with employment opportunities at the beginning of the period,

and  $x$  is its number of unsuccessful candidates. The problem of a recruitment company is given as follows:

$$\begin{aligned}
R(e, x, z, K, E, X, \mu) = \max_{\{a, b, d, u\}} & \{p^e(z, K, E, X, \mu) d + p^v(z, K, E, X, \mu) b \\
& + p^u(z, K, E, X, \mu) [x + e - d - u] + r^u(z, K, E, X, \mu) u - a \\
& + \mathcal{E} [q(z, K, E, X, \mu, z') R(e', x', z', K', E', X', \mu') \mid z]\}
\end{aligned} \tag{20}$$

subject to

$$\begin{aligned}
d & \leq (1 - \pi_u) e \\
b & = H(a, u, A, U) \\
e' & = G(a, u, A, U) \\
x' & = u - G(a, u, A, U) \\
A & = A(z, K, E, X, \mu) \\
U & = U(z, K, E, X, \mu) \\
(K', E', X', \mu') & = L(z, K, E, X, \mu).
\end{aligned}$$

where  $d$  is the number of workers with employment opportunities that the recruitment company sells,  $b$  is the number of next-period recruitment opportunities that the recruitment company sells,  $u$  is the number of unemployed workers that the recruitment company owns,  $a$  are the expenditures that the recruitment company makes,  $A$  are the aggregate recruitment expenditures in the economy, and  $U$  is the aggregate number of unemployed workers. Observe that, since unemployed workers quit at the rate  $\pi_u$ ,  $d$  cannot exceed  $(1 - \pi_u) e$ . Also observe that the recruitment company can sell as unemployed workers all of its unsuccessful candidates  $x$  and any of its unsold workers with employment opportunities  $e - d$ . The recruitment company's optimal decisions are  $a = a(e, x, z, K, E, X, \mu)$ ,  $b = b(e, x, z, K, E, X, \mu)$ ,  $d = d(e, x, z, K, E, X, \mu)$ , and  $u = u(e, x, z, K, E, X, \mu)$ , for recruitment expenditures, next-period recruitment opportunities, sales of workers with employment opportunities, and unemployed workers, respectively.<sup>5</sup>

A recursive competitive equilibrium can now be defined.

**Definition 1** *A recursive competitive equilibrium (RCE) is a set of value functions  $B(\kappa, z, K, E, X, \mu)$ ,  $W(s, l, j, z, K, E, X, \mu)$ ,  $R(e, x, z, K, E, X, \mu)$ , a set of individual decision rules  $c(\kappa, z, K, E, X, \mu)$ ,  $i(\kappa, z, K, E, X, \mu)$ ,  $m(\kappa, z, K, E, X, \mu)$ ,  $n(s, l, j, z, K, E, X, \mu)$ ,  $k(s, l, j, z, K, E, X, \mu)$ ,  $f(s, l, j, z, K, E, X, \mu)$ ,  $h(s, l, j, z, K, E, X, \mu)$ ,  $v(s, l, j, z, K, E, X, \mu)$ ,  $a(e, x, z, K, E, X, \mu)$ ,  $b(e, x, z, K, E, X, \mu)$ ,  $d(e, x, z, K, E, X, \mu)$ ,  $u(e, x, z, K, E, X, \mu)$ , a pair of aggregate decision rules  $A(z, K, E, X, \mu)$ ,  $U(z, K, E, X, \mu)$ , an aggregate law of motion  $L(z, K, E, X, \mu)$ , an aggregate profits function  $\Pi(z, K, E, X, \mu)$ , and a set of price functions  $r^k(z, K, E, X, \mu)$ ,  $r^u(z, K, E, X, \mu)$ ,  $p^u(z, K, E, X, \mu)$ ,  $p^e(z, K, E, X, \mu)$ ,  $p^v(z, K, E, X, \mu)$ ,  $q(z, K, E, X, \mu, z')$ , such that:*

(i) *the value function  $B(\kappa, z, K, E, X, \mu)$  solves the households' Bellman equation and  $c(\kappa, z, K, E, X, \mu)$ ,  $i(\kappa, z, K, E, X, \mu)$ , and  $m(\kappa, z, K, E, X, \mu)$  are the associated decision rules,*

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<sup>5</sup>Sections 1.1-1.3 in the Technical Appendix provide first-order and envelope conditions for the household, establishment and recruitment company's decision problems, respectively.

(ii) the value function  $W(s, l, j, z, K, E, X, \mu)$  solves the establishments' Bellman equation and  $n(s, l, j, z, K, E, X, \mu)$ ,  $k(s, l, j, z, K, E, X, \mu)$ ,  $f(s, l, j, z, K, E, X, \mu)$ ,  $h(s, l, j, z, K, E, X, \mu)$ , and  $v(s, l, j, z, K, E, X, \mu)$  are the associated decision rules,

(iii) the value function  $R(e, x, z, K, E, X, \mu)$  solves the Bellman equation of the recruitment companies and  $a(e, x, z, K, E, X, \mu)$ ,  $b(e, x, z, K, E, X, \mu)$ ,  $d(e, x, z, K, E, X, \mu)$ , and  $u(e, x, z, K, E, X, \mu)$  are the associated decision rules,

(iv) the prices of the Arrow securities satisfy that

$$q(z, K, E, X, \mu, z') = \beta \frac{c(K, z, K, E, X, \mu)^\sigma}{c(K', z', K', E', X', \mu')^\sigma},$$

where  $(K', E', X', \mu') = L(z, K, E, X, \mu)$ ,

(v) the capital rental market clears, i.e.

$$K = \sum_s \int k(s, l, j, z, K, E, X, \mu) \mu(s, dl \times dj)$$

(vi) the rental market for unemployed workers clears, i.e.

$$u(E, X, z, K, E, X, \mu) = m(K, z, K, E, X, \mu)$$

(vii) the ownership market for unemployed workers clears, i.e.

$$\begin{aligned} u(E, X, z, K, E, X, \mu) &= X + E - d(E, X, z, K, E, X, \mu) \\ &\quad + \sum_s \int f(s, l, j, z, K, E, X, \mu) \mu(s, dl \times dj) \end{aligned}$$

(viii) the market for workers with employment opportunities clears, i.e.

$$d(E, X, z, K, E, X, \mu) = \sum_s \int h(s, l, j, z, K, E, X, \mu) \mu(s, dl \times dj)$$

(ix) the market for next-period recruitment opportunities clears, i.e.

$$b(E, X, z, K, E, X, \mu) = \sum_s \int v(s, l, j, z, K, E, X, \mu) \mu(s, dl \times dj)$$

(x) the market for the consumption good clears, i.e.

$$\begin{aligned} &c(K, z, K, E, X, \mu) + i(K, z, K, E, X, \mu) + a(E, X, z, K, E, X, \mu) \\ &= \sum_s \int e^z s F [n(s, l, j, z, K, E, X, \mu), k(s, l, j, z, K, E, X, \mu)] \mu(s, dl \times dj) \end{aligned}$$

(xi) the aggregate decision rules are generated by the optimal individual decisions, i.e.

$$A(z, K, E, X, \mu) = a(E, X, z, K, E, X, \mu)$$

$$U(z, K, E, X, \mu) = u(E, X, z, K, E, X, \mu)$$

(xii) the aggregate law of motion is generated by the optimal individual decisions, i.e.

$$(K', E', X', \mu') = L(z, K, E, X, \mu)$$



is given as follows:

$$\begin{aligned}
K' &= (1 - \delta) K + i(K, z, K, E, X, \mu) \\
E' &= G[a(E, X, z, K, E, X, \mu), u(E, X, z, K, E, X, \mu), A(z, K, E, X, \mu), U(z, K, E, X, \mu)] \\
X' &= u(E, X, z, K, E, X, \mu) - E' \\
\mu'(s', \mathcal{L} \times \mathcal{J}) &= \sum_s \int_{\mathcal{B}(s, \mathcal{L} \times \mathcal{J})} Q(s, s') \mu(s, dl \times dj) + \varrho \psi(s') \mathcal{I}(\mathcal{L} \times \mathcal{J})
\end{aligned}$$

where

$$\mathcal{B}(s, \mathcal{L} \times \mathcal{J}) = \{(l, j) : n(s, l, j, z, K, E, X, \mu) \in \mathcal{L} \text{ and } v(s, l, j, z, K, E, X, \mu) \in \mathcal{J}\}$$

and where  $\mathcal{I}(\mathcal{L} \times \mathcal{J})$  is an indicator function which takes a value of one if  $(0, 0) \in \mathcal{L} \times \mathcal{J}$ , and a value of zero otherwise.<sup>6</sup>

## 4 Characterization and computation of a RCE

Due to the external effects in the recruitment technology a RCE is generally inefficient and must be solved for directly. The high dimensionality of the state space makes this a daunting task. However, it can be simplified considerably. This section provides a solution method that can be easily implemented in actual computations. The method relies on two key properties of a RCE. First, that it can be characterized as the solution to a dynamic programming problem with side conditions.<sup>7</sup> Second, that in a neighborhood of the deterministic steady state, the dynamic programming problem can be represented as having a finite number of state and decision variables. The following subsections explain these properties in detail.

### 4.1 The myopic social planner's problem

Consider the problem of a social planner that seeks to maximize utility subject to the economy's feasibility constraints. However, the social planner is *myopic* in the sense that he does not fully internalize the effects of his decisions on the output produced by the recruitment technology. In particular, the myopic social planner takes the recruitment technology as being the following:

$$\begin{aligned}
E' &= G(A, U, \hat{A}, \hat{U}) \\
J' &= H(A, U, \hat{A}, \hat{U})
\end{aligned}$$

where  $E'$  are next-period workers with employment opportunities,  $J'$  are next-period recruitment opportunities,  $A$  are recruitment expenditures,  $U$  are unemployed workers, and  $\hat{A}$  and  $\hat{U}$  are exogenous productivity shocks. The shocks  $\hat{A}$  and  $\hat{U}$  evolve according to the following stochastic process:

$$\hat{A} = \hat{A}(z, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$$

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<sup>6</sup>It is straightforward to verify that if  $(E, X, \mu)$  satisfy equation (7), then  $(E', X', \mu')$  also satisfy equation (7).

<sup>7</sup>The dynamic optimization problem depends on exogenous parameters, which in turn depend on the solution to the dynamic optimization problem. Finding a RCE is then reduced to solving a fixed point problem on those parameters. This basic strategy for solving for a competitive equilibrium in an economy with externalities is already familiar to the literature, though in much simpler contexts (e.g. Kehoe, Levine and Romer (12), Jones and Manuelli (10), etc.).

$$\begin{aligned}\hat{U} &= \hat{U}(z, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) \\ (\hat{K}', \hat{E}', \hat{X}', \hat{\mu}') &= \hat{L}(z, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})\end{aligned}$$

where  $z$  is the aggregate productivity level, and  $(\hat{K}, \hat{E}, \hat{X}, \hat{\mu})$  are variables that lie in the same space as  $(K, E, X, \mu)$ .

The state of the myopic social planner is then given by the state of the economy  $(z, K, E, X, \mu)$  and by the variables  $(\hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ , which are sufficient statistics for predicting the future behavior of  $\hat{A}$  and  $\hat{U}$ . The problem of the myopic social planner facing a stochastic process  $(\hat{A}, \hat{U}, \hat{L})$  is described by the following Bellman equation:

$$V(z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) = \max \left\{ \frac{C^{1-\sigma} - 1}{1-\sigma} + \varphi U + \beta \mathcal{E} \left[ V(z', K', E', X', \mu', \hat{K}', \hat{E}', \hat{X}', \hat{\mu}') \mid z \right] \right\} \quad (21)$$

subject to

$$C + I + A \leq \sum_s \int e^z s F[n(s, l, j), k(s, l, j)] \mu(s, dl \times dj) \quad (22)$$

$$\sum_s \int k(s, l, j) \mu(s, dl \times dj) \leq K \quad (23)$$

$$\sum_s \int v(s, l, j) \mu(s, dl \times dj) \leq H(A, U, \hat{A}, \hat{U}) \quad (24)$$

$$U \leq X + E - \sum_s \int h(s, l, j) \mu(s, dl \times dj) + \sum_s \int f(s, l, j) \mu(s, dl \times dj) \quad (25)$$

$$\sum_s \int h(s, l, j) \mu(s, dl \times dj) \leq (1 - \pi_u) E \quad (26)$$

$$n(s, l, j) = l + h(s, l, j) - f(s, l, j) \quad (27)$$

$$h(s, l, j) \leq j \quad (28)$$

$$\pi_n l \leq f(s, l, j) \quad (29)$$

$$f(s, l, j) \leq l \quad (30)$$

$$K' = (1 - \delta) K + I \quad (31)$$

$$E' = G(A, U, \hat{A}, \hat{U}) \quad (32)$$

$$X' = U - G(A, U, \hat{A}, \hat{U}) \quad (33)$$

$$\mu'(s', \mathcal{L} \times \mathcal{J}) = \sum_s \int_{\{(l, j): n(s, l, j) \in \mathcal{L} \text{ and } v(s, l, j) \in \mathcal{J}\}} Q(s, s') \mu(s, dl \times dj) + \varrho \psi(s') \mathcal{I}(\mathcal{L} \times \mathcal{J}) \quad (34)$$

$$\hat{A} = \hat{A}(z, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) \quad (35)$$

$$\hat{U} = \hat{U}(z, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) \quad (36)$$

$$(\hat{K}', \hat{E}', \hat{X}', \hat{\mu}') = \hat{L}(z, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}). \quad (37)$$

where equations (22)-(34) are feasibility constraints and equations (35)-(37) describe the stochastic process that  $\hat{A}$  and  $\hat{U}$  follow over time.<sup>8</sup> The myopic social planner's decision rules are  $C = C^m(z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ ,  $I = I^m(z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ ,  $n = n^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ ,  $k = k^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ ,  $f = f^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ ,  $h = h^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ ,  $v = v^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ ,  $U = U^m(z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ ,  $A = A^m(z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ , for consumption, investment, establishment employment, establishment capital, establishment firings, establishment hirings, establishment recruitment opportunities, unemployment and recruitment expenditures, respectively.

The following proposition provides a characterization of the decision rules to the myopic social planner's problem.

**Proposition 2** *Let  $\{C^m, I^m, n^m, k^m, f^m, h^m, v^m, U^m, A^m\}$  be the solution to the MSP's with exogenous stochastic process  $(\hat{A}, \hat{U}, \hat{L})$ . Then, there exist thresholds  $\underline{n}^m(s, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ ,  $\bar{n}^m(s, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$  and  $\bar{v}^m(s, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$  and a shadow capital price function  $r^k(z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$  such that, for every  $s > 0$  and  $l + j > 0$ :*

$$n^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) = \max \left\{ \begin{array}{l} \min \left\{ (1 - \pi_n)l + j, \underline{n}^m(s, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) \right\}, \\ \min \left\{ (1 - \pi_n)l, \bar{n}^m(s, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) \right\} \end{array} \right\},$$

$$v^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) = \max \left\{ \bar{v}^m(s, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) - (1 - \pi_n)n^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}), 0 \right\},$$

$$h^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) = \max \left\{ n^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) - l, 0 \right\}$$

$$f^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) = \max \left\{ l - n^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}), 0 \right\}$$

$$e^z s F_k \left[ n^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}), k^m(s, l, j, z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}) \right] = r^k(z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu}),$$

**Proof.** *In the economy in which  $(\hat{A}, \hat{U}, \hat{L})$  truly represent exogenous productivity shocks to the recruitment technology, the Welfare Theorems apply. In this case the problem described by equation (21) is the social planner's problem and its solution can be decentralized as a recursive competitive equilibrium in which prices are functions of the state  $(z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$ . The claim then follows from characterizing the optimal decision rules to the associated establishments' problem given by equation (12).<sup>9</sup> ■*

This proposition is important because it can be used to reduce the dimensionality of the decision variables in the myopic social planner's problem: Instead of choosing functions  $n^m$ ,  $v^m$ ,  $h^m$ ,  $f^m$  and  $k^m$  defined over the infinite number of triples  $(s, l, j)$ , the myopic social planner can be restricted to choose thresholds  $\underline{n}^m$ ,  $\bar{n}^m$  and  $\bar{v}^m$  defined over the finite number of singletons  $s$ .

The next proposition states that if the solution to a myopic planner's problem satisfies certain side conditions, then it is a RCE.

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<sup>8</sup>Observe that if equations (35)-(37) were substituted by  $\hat{A} = A$  and  $\hat{U} = U$ , the solution to this planning problem would be the Pareto optimal allocation.

<sup>9</sup>For details see Sections 2 in the Technical Appendix.

**Proposition 3** Let  $\{C^m, I^m, n^m, k^m, f^m, h^m, v^m, U^m, A^m\}$  be the solution to the myopic social planner's problem with exogenous stochastic process  $(\hat{A}, \hat{U}, \hat{L})$ .

Suppose that,

$$\hat{A}(z, K, E, X, \mu) = A^m(z, K, E, X, \mu, K, E, X, \mu),$$

$$\hat{U}(z, K, E, X, \mu) = U^m(z, K, E, X, \mu, K, E, X, \mu).$$

In addition, suppose that

$$(\hat{K}', \hat{E}', \hat{X}', \hat{\mu}') = \hat{L}(z, K, E, X, \mu).$$

satisfies that

$$\hat{K}' = (1 - \delta)K + I^m(z, K, E, X, \mu, K, E, X, \mu),$$

$$\begin{aligned} \hat{E}' = G & [A^m(z, K, E, X, \mu, K, E, X, \mu), U^m(z, K, E, X, \mu, K, E, X, \mu), \\ & A^m(z, K, E, X, \mu, K, E, X, \mu), U^m(z, K, E, X, \mu, K, E, X, \mu)], \end{aligned}$$

$$\hat{X}' = U^m(z, K, E, X, \mu, K, E, X, \mu) - \hat{E}',$$

$$\hat{\mu}'(s', \mathcal{L} \times \mathcal{J}) = \sum_s \int_{\mathcal{B}(s, \mathcal{L} \times \mathcal{J})} Q(s, s') \mu(s, dl \times dj) + \varrho \psi(s') \mathcal{I}(\mathcal{L} \times \mathcal{J}),$$

where

$$\mathcal{B}(s, \mathcal{L} \times \mathcal{J}) = \{(l, j) : n^m(s, l, j, z, K, E, X, \mu, K, E, X, \mu) \in \mathcal{L} \text{ and } v^m(s, l, j, z, K, E, X, \mu, K, E, X, \mu) \in \mathcal{J}\}.$$

Then, there exists a RCE  $\{B, W, R, c, i, m, n, k, f, h, v, a, b, d, u, A, U, L, \Pi, r^k, r^u, p^u, p^e, p^v, q\}$  such that

$$\begin{aligned} c(K, z, K, E, X, \mu) &= C^m(z, K, E, X, \mu, K, E, X, \mu) \\ i(K, z, K, E, X, \mu) &= I^m(z, K, E, X, \mu, K, E, X, \mu) \\ n(s, l, j, z, K, E, X, \mu) &= n^m(s, l, j, z, K, E, X, \mu, K, E, X, \mu) \\ f(s, l, j, z, K, E, X, \mu) &= f^m(s, l, j, z, K, E, X, \mu, K, E, X, \mu) \\ h(s, l, j, z, K, E, X, \mu) &= h^m(s, l, j, z, K, E, X, \mu, K, E, X, \mu) \\ v(s, l, j, z, K, E, X, \mu) &= v^m(s, l, j, z, K, E, X, \mu, K, E, X, \mu) \\ A(z, K, E, X, \mu) &= A^m(z, K, E, X, \mu, K, E, X, \mu) \\ U(z, K, E, X, \mu) &= U^m(z, K, E, X, \mu, K, E, X, \mu). \end{aligned}$$

**Proof.** It follows from comparing the necessary and sufficient conditions for a RCE with the necessary and sufficient conditions to the myopic social planner's problem.<sup>10</sup> ■

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<sup>10</sup>For the details, see Section 3.4 in the Technical Appendix.

## 4.2 State space characterization

When there are no aggregate productivity shocks to the economy (i.e. when  $z$  is identical to zero), a deterministic steady state can be defined. In particular, a *myopic steady state* is given by an aggregate state  $(K^*, E^*, X^*, \mu^*, \hat{K}^*, \hat{E}^*, \hat{X}^*, \hat{\mu}^*)$  that replicates itself under the myopic planner's optimal decision rules.<sup>11</sup> Characterizing the invariant distribution  $\mu^*$  of a myopic steady state will turn to be crucial for characterizing the state-space when the economy is subject to small aggregate productivity shocks.

From equation (34) and Proposition 2, observe that the invariant distribution  $\mu^*$  must satisfy the following equation:

$$\mu^*(s', \mathcal{L} \times \mathcal{J}) = \sum_s \int_{\{(l,j): n^*(s,l,j) \in \mathcal{L} \text{ and } v^*(s,l,j) \in \mathcal{J}\}} Q(s, s') \mu^*(s, dl \times dj) + \varrho \psi(s') \mathcal{I}(\mathcal{L} \times \mathcal{J}),$$

where

$$n^*(s, l, j) = \max \left\{ \begin{array}{l} \min \{(1 - \pi_n) l + j, \underline{n}^*(s)\}, \\ \min \{(1 - \pi_n) l, \bar{n}^*(s)\} \end{array} \right\}, \quad (38)$$

$$v^*(s, l, j) = \max \{\bar{v}^*(s) - (1 - \pi_n) n^*(s, l, j), 0\}. \quad (39)$$

The following proposition characterizes a support to the invariant distribution  $\mu^*$  in terms of the finite number of steady state thresholds  $\underline{n}^*$ ,  $\bar{n}^*$  and  $\bar{v}^*$ .<sup>12</sup>

**Proposition 4** *Let  $M$  be a natural number satisfying that*

$$(1 - \pi_n)^M \max \{\bar{n}^*(s_{\max}), \bar{v}^*(s_{\max})\} < \min \{\underline{n}^*(s_{\min}), \bar{v}^*(s_{\min})\}. \quad (40)$$

*Define the set  $\mathcal{N}^*$  as follows:*

$$\mathcal{N}^* = \left\{ \bigcup_{s \in S} \bigcup_{k=0}^{M-1} \left\{ (1 - \pi_n)^k \underline{n}^*(s), (1 - \pi_n)^k \bar{n}^*(s), (1 - \pi_n)^k \bar{v}^*(s) \right\} \right\} \cup \{0\}.$$

*Then, the set*

$$\mathcal{P}^* = \left\{ (s, l, j) : s \in S, l \in \mathcal{N}^*, \text{ and } j \in \bigcup_{s' \in S} \left\{ \max [\bar{v}^*(s') - (1 - \pi_n) l, 0] \right\} \right\} \cup \left\{ \bigcup_{s \in S} \{(s, 0, 0)\} \right\}$$

*is a support of the invariant distribution  $\mu^*$ .*

**Proof.** *See Appendix A. ■*

Observe that Proposition 4 not only constructs a support  $\mathcal{P}^*$  for the invariant distribution  $\mu^*$ , but determines that it is a finite set.

In order to analyze off-steady state dynamics it will be useful to define  $\underline{n}_t$ ,  $\bar{n}_t$ , and  $\bar{v}_t$ , as the threshold functions chosen at date  $t$ . In addition, it will be useful to define the following minimum distance:

$$\varepsilon = \min |a - b| \quad (41)$$

<sup>11</sup>From Proposition 3 we know that if  $(K^*, E^*, X^*, \mu^*) = (\hat{K}^*, \hat{E}^*, \hat{X}^*, \hat{\mu}^*)$ , this myopic steady-state constitutes a steady-state equilibrium. See Sections 4.1 and 4.3 in the Technical Appendix for explicit steady state equilibrium conditions and a computational algorithm.

<sup>12</sup>In the statement of the proposition  $s_{\max}$  and  $s_{\min}$  denote the largest and smallest positive values for  $s$ , respectively.

subject to

$$a, b \in \mathcal{D}^* \text{ and } a \neq b,$$

where

$$\mathcal{D}^* = \mathcal{N}^* \cup \left\{ \bigcup_{s \in S} \left\{ (1 - \pi_n)^M \underline{n}^*(s), (1 - \pi_n)^M \bar{n}^*(s), (1 - \pi_n)^M \bar{v}^*(s) \right\} \right\}.$$

The following proposition characterizes the distribution  $\mu_{t+1}$  under the assumptions that  $\mu_t$  and the finite history of thresholds  $\{\underline{n}_{t-k}, \bar{n}_{t-k}, \bar{v}_{t-k}\}_{k=0}^{M+1}$  are sufficiently close to their steady-state counterparts.

**Proposition 5** *Let  $M$  be defined by equation (40) and  $\varepsilon$  by equation (41).*

*Suppose that*

$$|\underline{n}_{t-k}(s) - \underline{n}^*(s)| < \varepsilon/2, \quad (42)$$

$$|\bar{n}_{t-k}(s) - \bar{n}^*(s)| < \varepsilon/2, \quad (43)$$

$$|\bar{v}_{t-k}(s) - \bar{v}^*(s)| < \varepsilon/2, \quad (44)$$

for every  $s$  and every  $0 \leq k \leq M+1$ .

*Suppose that the distribution  $\mu_t$  has a finite support  $\mathcal{P}_t$  given by*

$$\mathcal{P}_t = \left\{ (s, l, j) : s \in S, l \in \mathcal{N}_t, \text{ and } j \in \bigcup_{s' \in S} \{\max[\bar{v}_{t-1}(s') - (1 - \pi_n)l, 0]\} \right\} \cup \left\{ \bigcup_{s \in S} \{(s, 0, 0)\} \right\} \quad (45)$$

where

$$\mathcal{N}_t = \left\{ \bigcup_{s \in S} \bigcup_{k=0}^{M-1} \left\{ (1 - \pi_n)^k \underline{n}_{t-k-1}(s), (1 - \pi_n)^k \bar{n}_{t-k-1}(s), (1 - \pi_n)^k \bar{v}_{t-k-2}(s) \right\} \right\} \cup \{0\}. \quad (46)$$

*In addition, suppose that for every  $(s, l, j) \in \mathcal{P}_t$ :*

$$\mu_t(s, l, j) = \mu^*(s, l^*, j^*), \quad (47)$$

where  $(s, l^*, j^*)$  is the unique element of  $\mathcal{P}^*$  satisfying that  $|l - l^*| < \varepsilon/2$  and  $|j - j^*| < \varepsilon/2 + (1 - \pi)\varepsilon/2$ .

*Then, the distribution  $\mu_{t+1}$  has a finite support  $\mathcal{P}_{t+1}$  given by*

$$\mathcal{P}_{t+1} = \left\{ (s, l, j) : s \in S, l \in \mathcal{N}_{t+1}, \text{ and } j \in \bigcup_{s' \in S} \{\max[\bar{v}_t(s') - (1 - \pi_n)l, 0]\} \right\} \cup \left\{ \bigcup_{s \in S} \{(s, 0, 0)\} \right\}$$

where

$$\mathcal{N}_{t+1} = \left\{ \bigcup_{s \in S} \bigcup_{k=0}^{M-1} \left\{ (1 - \pi_n)^k \underline{n}_{t-k}(s), (1 - \pi_n)^k \bar{n}_{t-k}(s), (1 - \pi_n)^k \bar{v}_{t-k-1}(s) \right\} \right\} \cup \{0\}.$$

*Moreover, for every  $(s, l, j) \in \mathcal{P}_{t+1}$ :*

$$\mu_{t+1}(s, l, j) = \mu^*(s, l^*, j^*)$$

where  $(s, l^*, j^*)$  is the unique element of  $\mathcal{P}^*$  satisfying that  $|l - l^*| < \varepsilon/2$  and  $|j - j^*| < \varepsilon/2 + (1 - \pi)\varepsilon/2$ .

**Proof.** See Appendix A. ■

Proposition 5 plays a crucial role in the solution method to be described below. It implies that if the economy starts at the deterministic steady-state at  $t = 0$  and the thresholds  $\underline{n}_t$ ,  $\bar{n}_t$  and  $\bar{v}_t$  thereafter fluctuate within a sufficiently small neighborhood of their steady state values, then the distribution  $\mu_t$  will always have a finite support  $\mathcal{P}_t$  determined by the

finite history of thresholds  $\{\underline{n}_{t-k}, \bar{n}_{t-k}, \bar{v}_{t-k}\}_{k=1}^{M+1}$  (equations 45 and 46) and its mass at each point in  $\mathcal{P}_t$  will be given by the mass of the invariant distribution  $\mu^*$  at the corresponding point in  $\mathcal{P}^*$  (equation 47). As a result the state to the myopic planner problem can be defined in terms of the finite history of thresholds  $\{\underline{n}_{t-k}, \bar{n}_{t-k}, \bar{v}_{t-k}\}_{k=1}^{M+1}$  instead of the distribution  $\mu_t$ .

### 4.3 Solution method

This section redefines the myopic social planner's problem so that standard solution methods can be applied. For this purpose, it will be convenient to return to a recursive formulation and define  $\underline{n}_k$ ,  $\bar{n}_k$  and  $\bar{v}_k$  as the thresholds that were chosen  $k$  periods ago (relative to the current period).

Recall from Section 4.2 that the finite history  $\{\underline{n}_k, \bar{n}_k, \bar{v}_k\}_{k=1}^{M+1}$  can be used to construct the current distribution  $\mu$  (as long as fluctuations are sufficiently small). Moreover, Proposition 2 states that the current thresholds  $(\underline{n}_0, \bar{n}_0, \bar{v}_0)$  fully describe the employment rule  $n$ , the vacancies rule  $v$ , the hiring rule  $h$  and the firing rule  $f$ . In turn, the employment decision rule  $n$  and the aggregate stock of capital  $K$  are sufficient for determining the capital allocation rule  $k$ .<sup>13</sup> This suggests that the state vector  $(z, K, E, X, \mu, \hat{K}, \hat{E}, \hat{X}, \hat{\mu})$  in the myopic planner's problem can be replaced by the vector  $(z, K, E, \{\underline{n}_k, \bar{n}_k, \bar{v}_k\}_{k=1}^{M+1}, \hat{K}, \hat{E}, \{\hat{\underline{n}}_k, \hat{\bar{n}}_k, \hat{\bar{v}}_k\}_{k=1}^{M+1})$  and that the decision variables  $(k, n, v, h, f)$  can be replaced by  $(\underline{n}_0, \bar{n}_0, \bar{v}_0)$ .<sup>14</sup>

Also, observe from equation (32) that  $A$  can be written as  $A = g_1(E', U, \hat{A}, \hat{U})$  for some differentiable function  $g_1$  and, since at equilibrium  $A = \hat{A}$  and  $U = \hat{U}$ , that  $\hat{A}$  can be written as  $\hat{A} = g_2(\hat{E}', \hat{U})$  for some differentiable function  $g_2$ . Moreover, at equilibrium we have that  $U = 1 - \int n d\mu$  and that  $\hat{U} = 1 - \int \hat{n} d\hat{\mu}$ . Substituting these expressions and equations (22)-(33) into the return function in equation (21), the myopic planner's problem can then be written as follows:<sup>15</sup>

$$\begin{aligned} V & \left( z, K, E, \{\underline{n}_k, \bar{n}_k, \bar{v}_k\}_{k=1}^{M+1}, \hat{K}, \hat{E}, \{\hat{\underline{n}}_k, \hat{\bar{n}}_k, \hat{\bar{v}}_k\}_{k=1}^{M+1} \right) \\ & = \max \left\{ R \left( z, K, E, \{\underline{n}_k, \bar{n}_k, \bar{v}_k\}_{k=1}^{M+1}, \hat{K}, \hat{E}, \{\hat{\underline{n}}_k, \hat{\bar{n}}_k, \hat{\bar{v}}_k\}_{k=1}^{M+1}, K', E', \underline{n}_0, \bar{n}_0, \bar{v}_0, \hat{E}', \hat{\underline{n}}_0, \hat{\bar{n}}_0, \hat{\bar{v}}_0 \right) \right. \\ & \quad \left. + \beta \mathcal{E} \left[ V \left( z', K', E', \{\underline{n}'_k, \bar{n}'_k, \bar{v}'_k\}_{k=1}^{M+1}, \hat{K}', \hat{E}', \{\hat{\underline{n}}'_k, \hat{\bar{n}}'_k, \hat{\bar{v}}'_k\}_{k=1}^{M+1} \right) \mid z \right] \right\} \end{aligned} \quad (48)$$

subject to

$$\underline{n}'_k = \underline{n}_{k-1}, \text{ for } k = 1, \dots, M+1 \quad (49)$$

$$\bar{n}'_k = \bar{n}_{k-1}, \text{ for } k = 1, \dots, M+1 \quad (50)$$

$$\bar{v}'_k = \bar{v}_{k-1}, \text{ for } k = 1, \dots, M+1 \quad (51)$$

<sup>13</sup>Since capital is freely movable, the myopic social planner allocates the aggregate stock of capital  $K$  to equate the marginal productivity of capital across all types of islands  $(s, l, j)$ , subject to the feasibility constraint (23).

<sup>14</sup>The variables  $X$  and  $\hat{X}$  can be removed from the state vector because they are actually redundant (see equation 7).

<sup>15</sup>Observe that equation (24) must be used to remove some vacancy threshold  $\bar{v}$  (e.g.  $\bar{v}(s_{\min})$ ) from the formulation of the problem, since it always hold with equality. Similarly, when the deterministic steady state is such that equation (26) holds with equality, it must be used to remove some lower employment threshold  $\underline{n}$  (e.g.  $\underline{n}(s_{\min})$ ) from the formulation of the problem. Otherwise, equation (26) must be ignored.

$$\left( \hat{K}', \hat{E}', \left\{ \hat{\underline{n}}'_k, \hat{\bar{n}}'_k, \hat{\bar{v}}'_k \right\}_{k=1}^{M+1} \right) = \hat{L} \left( z, \hat{K}, \hat{E}, \left\{ \hat{\underline{n}}_k, \hat{\bar{n}}_k, \hat{\bar{v}}_k \right\}_{k=1}^{M+1} \right), \quad (52)$$

where the vector of decision variables is  $\left( K', E', \left\{ \underline{n}'_k, \bar{n}'_k, \bar{v}'_k \right\}_{k=1}^{M+1} \right)$ .

Let the optimal decision rule to the above problem be given by

$$\left( K', E', \left\{ \underline{n}'_k, \bar{n}'_k, \bar{v}'_k \right\}_{k=1}^{M+1} \right) = D \left( z, K, E, \left\{ \underline{n}_k, \bar{n}_k, \bar{v}_k \right\}_{k=1}^{M+1}, \hat{K}, \hat{E}, \left\{ \hat{\underline{n}}_k, \hat{\bar{n}}_k, \hat{\bar{v}}_k \right\}_{k=1}^{M+1} \right).$$

The condition for a RCE in Proposition 3 then becomes:

$$\hat{L} \left( z, K, E, \left\{ \underline{n}_k, \bar{n}_k, \bar{v}_k \right\}_{k=1}^{M+1} \right) = D \left( z, K, E, \left\{ \underline{n}_k, \bar{n}_k, \bar{v}_k \right\}_{k=1}^{M+1}, K, E, \left\{ \underline{n}_k, \bar{n}_k, \bar{v}_k \right\}_{k=1}^{M+1} \right). \quad (53)$$

Observe that there are a finite number of arguments to the return function in equation (48) and that all their values are strictly positive at the deterministic steady state (except for the aggregate productivity level  $z$ ). Since  $R$  is differentiable, a Taylor expansion at the deterministic steady state can then be performed to obtain a quadratic objective function. Since the constraints in equations (49)-(51) are linear, this delivers a standard linear-quadratic RCE structure that can be solved using standard methods (e.g. Hansen and Prescott (14)).<sup>16</sup> The linear decision rule  $D$  thus obtained is a good local approximation and, as long as fluctuations in the aggregate productivity shock  $z$  are small, it can be used to simulate and analyze equilibrium business cycle fluctuations.

## 5 Calibration

Throughout the rest of the paper the recruitment technology will be given a matching function interpretation in which employment and recruitment opportunities are produced in pairs at the aggregate level. In particular, the recruitment technology will be restricted to satisfy that

$$G(A, U, A, U) = H(A, U, A, U), \quad (54)$$

for every  $(A, U)$ .

Two version of the model economy are considered: One where the matching technology is subject to congestion externalities and another where it isn't. The matching technology with congestion externalities is given by

$$G(a, u, A, U) = u \frac{A}{[U^\phi + A^\phi]^{\frac{1}{\phi}}}, \quad (55)$$

$$H(a, u, A, U) = a \frac{U}{[U^\phi + A^\phi]^{\frac{1}{\phi}}}, \quad (56)$$

and the matching technology with no externalities is given by

$$G(a, u, A, U) = H(a, u, A, U) = \frac{u \cdot a}{[u^\phi + a^\phi]^{\frac{1}{\phi}}}. \quad (57)$$

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<sup>16</sup>Strictly speaking, the linear-quadratic structure is obtained only when the aggregate law of motion  $\hat{L}$  in equation (52) is linear. However, this will be true in equilibrium. In fact, Hansen and Prescott (14) update the linear law of motion  $\hat{L}$  at each value function iteration by imposing the RCE condition (53) on the linear decision rule  $D$  obtained from the iteration.



Both matching technologies satisfy equation (54) and all the assumptions made in Section 2. Moreover, they both aggregate into a standard den Haan-Ramey-Watson (8) matching function

$$G(A, U, A, U) = \frac{U.A}{[U^\phi + A^\phi]^{\frac{1}{\phi}}}. \quad (58)$$

Also, observe that the matching technology with congestion externalities given by equations (55) and (56) captures a standard assumption in the matching literature: That the aggregate market-tightness ratio  $A/U$  determines the rate at which individual unemployed workers find employment opportunities and the rate at which individual help-wanted ads find recruitment opportunities.

The rest of this section calibrates the steady states of both versions of the model economy to identical long-run U.S. observations. Before proceeding it will be necessary to select a model time-period that is both convenient and consistent with observations.

The Job Openings and Labor Turnover Survey (JOLTS) conducted by the Bureau of Labor Statistics is an important source of information for two key features of the model: the creation of recruitment opportunities (i.e. job openings) and the worker turnover process. JOLTS, which is a monthly survey of continuing nonagricultural establishments, defines job openings as positions for which there is work available, for which a job could start within 30 days, and for which there is an active recruitment effort taking place (such as advertisement in newspapers, radio and television, posting “help wanted signs”, interviewing candidates, etc.). Job openings are measured on the last business day of the month. On the contrary, hirings, which are defined as all additions to the establishments’ payrolls, are measured over the entire month. The *vacancy yield rate* defined as the average monthly ratio of hirings to job openings over the entire period 2000-2005 is equal to 1.3 (see Davis et. al, (7)).

Since hirings cannot exceed recruitment opportunities in the model economy (see equation 3), a vacancy yield rate greater than one can only be obtained through time aggregation. This suggests calibrating to a short time period. However, computational convenience requires making the time period as large as possible. The largest time period consistent with the above observation is 3 weeks. The reason is simple: if total hirings turned out to be approximately equal to total recruitment opportunities, a monthly vacancy yield rate close to 1.3 would be obtained from the simple fact that a month contains 4/3 three-weeks periods. Observe that, since equation (54) implies that recruitment opportunities are equal to  $E$ , equation (26) indicates that a small  $\pi_u$  is a necessary condition for total hirings to be close to total recruitment opportunities. In what follows the time period will thus be tentatively selected to be 3 weeks and  $\pi_u$  will be set to zero.<sup>17</sup> Moreover, it will be assumed that total hirings are approximately equal to total recruitment opportunities. Later on it will be verified that this assumption is correct and that the monthly vacancy yield rate obtained is indeed consistent with the JOLTS measurement.

The next issue that must be addressed is what actual measure of capital should the model capital correspond to. Since the focus is on establishment level dynamics, it seems natural to abstract from capital components such as land, residential structures, and consumer durables. The empirical counterpart for capital is then identified with plant, equipment, and

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<sup>17</sup>Observe that, since establishments invest in recruitment opportunities one period in advance and some of them end-up transiting to lower idiosyncratic productivity levels (or even exiting), not all existing recruitment opportunities end-up being exercised. Thus, when  $\pi_u$  is equal to zero equation (26) holds with strict inequality. This in turn implies that  $p^e$  must be equal to  $p^u$ , since owners of employable workers must be made indifferent between selling them or keeping them as unemployed workers.

inventories. As a result, investment is associated in the National Income and Product Accounts (NIPA) with nonresidential investment plus changes in business inventories. The empirical counterpart for consumption is identified with personal consumption expenditures in nondurable goods and services. Output is then defined as the sum of these investment and consumption measures. The quarterly capital-output ratio and the investment-output ratio corresponding to these measures are 6.8 and 0.15, respectively. Since at steady state  $I/Y = \delta(K/Y)$ , these ratios require that  $\delta = 0.005515$ .

The production function is assumed to have the following functional form:

$$y = sn^\gamma k^\theta,$$

where  $0 < \gamma + \theta < 1$ . Calibrating to an annual interest rate of 4 percent, which is a standard value in the macro literature, requires a time discount factor  $\beta$  equal to 0.99755. Given this value for  $\beta$ , the above value for  $\delta$ , and given that the capital share satisfies that

$$\theta = \frac{(1/\beta + \delta)K}{Y},$$

matching the U.S. capital-output ratio requires choosing a value of  $\theta$  equal to 0.2168. Similarly,  $\gamma = 0.64$  is selected to reproduce the share of labor in National Income.<sup>18</sup> Observe that, since workers are capital goods, the “wage rate” used in calculating the labor share is given by the user cost  $(1 - \beta)p^e$ . In what follows, the value of  $p^e$  will be normalized to an arbitrary value and the utility of leisure parameter  $\varphi$  will be selected to generate that value.

The values for the idiosyncratic productivity levels  $s$ , the distribution over initial productivity levels  $\psi$  and the transition matrix  $Q$  are key determinants of the job-flows generated by the model. As a consequence I choose them to reproduce observations from the Business Employment Dynamics (BED) data set, which is a virtual census of establishments level dynamics. Since BED data across establishment sizes can be found for the nine employment ranges shown in the first column of Table 1, I restrict the idiosyncratic productivity levels  $s$  to take nine positive values and choose them so that all establishments with a same idiosyncratic productivity level choose employment levels in the same range.

The average size of new entrants can be obtained by dividing the total gross job gains at opening establishments by the total number of opening establishments. Using data between 1992:Q3 and 2005:Q4, I find that the average size of new entrants is equal to 5.3 employees. Since this is a small number, I restrict the distribution over initial productivity levels  $\psi$  to put positive mass on only the two lowest values of  $s$  and choose  $\psi(s_1)$  to reproduce that average size.

Similarly, the average size at exit can be obtained by dividing the total gross job losses at closing establishments by the total number of closing establishments. Using data for the same time period as above, I find that the average size at exit is equal to 5.2 employees. Since this is also a small number, I restrict the probabilities of transiting to a zero productivity level  $Q(s, 0)$  to take positive values only at the two lowest values of  $s$  and choose  $Q(s_1, 0)/Q(s_2, 0)$  to reproduce that average size. The level for  $Q(s_1, 0)$  is then chosen to reproduce the average quarterly rate of gross job losses due to closing establishments (JLD) over the same time period, which is equal to 1.6%.<sup>19</sup>

The rest of the transition matrix  $Q$  is parameterized with enough flexibility to reproduce important establishment level

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<sup>18</sup>In the model,  $\gamma$  is not strictly the same as the share of labor in National Income. However, under  $\gamma = 0.64$  the labor share turns out to be 0.6364.

<sup>19</sup>Since the model time period is three weeks, quarterly statistics are constructed following establishments over four consecutive time periods.

observations. The only restriction that I impose is that  $Q(s_i, s_j) > 0$  only if  $j = i - 1$ ,  $j = i$  or  $j = i + 1$ . Since the rows of  $Q$  add to one this introduces 16 parameters (2 parameters each, for  $i = 2, \dots, 8$ , and 1 parameter each, for  $i = 1, 9$ ). Eight of these parameters are chosen to reproduce the shares in total employment across size classes (which provide eight independent observations). The other eight parameters are chosen to reproduce the shares in total gross job gains across size classes (which also provide eight independent observations). I must point out that the BED does not tabulate statistics across size classes in its regular reports. However, these statistics can be found in Okolie (19) (Tables 1 and 3) for the first two quarters of 2000. These statistics together with the corresponding model statistics are shown in the first panel of Table 1. The second panel reports the average sizes at entry and exit both in the model and the data. We see that the model does a good job at reproducing these observations. As a test of the model, Table 1 also includes the shares in total gross job losses across size classes for the first two quarters of 2000 in Okolie (19), and the average quarterly rates of gross job gains due to expanding establishments (JGE), gross job gains due to opening establishments (JGB), gross job losses due to contracting establishments (JLC), and exit reported by the BED for the period 1992:Q3-2005:Q4. Although the fit is not perfect, we see that the model also does a good job at reproducing these statistics.<sup>20</sup>

The exogenous separation rate  $\pi_n$  and the number of establishments created every period  $\rho$  are important determinants of the worker flows in and out of unemployment, so I calibrate them to reproduce this type of observations. In particular, I target an average monthly separation rate from employment equal to 3.5% and an average monthly hazard rate from unemployment equal to 46%, which were estimated by Shimer (21) using CPS data between 1948 and 2004. Since the separation rate of 3.5% is significantly larger than the rate of job losses experienced by establishments, I select a positive value of  $\pi_n$  to reproduce the excess worker reallocation.<sup>21</sup> Also, observe that the separation and hazard rates estimated by Shimer (21) imply a steady state unemployment rate equal to 7.1%. The average size of establishments implied by the distribution reported in Table 1 thus determine the entry rate of establishments  $\rho$  that is needed to generate an aggregate employment level  $N$  equal to 0.929.

Based on evidence in Barron et. al (5) and Silva and Toledo (22), Hagedorn and Manovskii (9) determined that the costs of hiring a worker are equivalent to 4.5% of quarterly wages.<sup>22</sup> Since total hirings are assumed to be approximately equal to total recruitment opportunities, the cost of hiring a worker is approximately equal to the price of a next-period recruitment opportunity  $p^v$ . This suggest calibrating parameter values to reproduce the following relation:

$$p^v = 0.045 \times 4 \times (1 - \beta) p^e, \tag{59}$$

where  $(1 - \beta) p^e$  represents 3-weeks wages and a factor of 4 is needed to convert them into quarterly wages. Recall that the price of a worker  $p^e$  was normalized to an arbitrary value. Thus, equation (59) solely imposes a restriction on  $p^v$ .

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<sup>20</sup>The main discrepancy is with the shares in total gross job losses for the size ranges (5, 10) and (10, 20), which are too large in the first case and too small in the second. This could be remedied by allowing for a positive  $Q(s_3, 0)$  and by lowering  $Q(s_2, 0)$ , since the range (5, 10) accounts for a large fraction of the establishments closings. However, I do not expect that such modification would affect the main results of the paper.

<sup>21</sup>Not surprisingly, my calibrated value of  $\pi_n$  is smaller than the quit rate of workers measured by JOLTS (equal to 2% a month), since many of those separations entail job-to-job transitions that the model abstracts from.

<sup>22</sup>Since in their model capital is iddle while a job is open, Hagedorn and Manovskii (9) add an imputed opportunity cost of capital to the total costs of hiring a worker. I do not make such adjustment because there is no iddle capital in my model.

The steady-state price of a recruitment opportunity  $p^v$  depends on  $A$ ,  $U$  and the matching function curvature parameter  $\phi$ .<sup>23</sup> Also, since (by assumption) total hirings are approximately equal to total recruitment opportunities, we have from equation (58) that

$$\frac{\text{Hirings}}{U} \approx \frac{A}{[U\phi + A\phi]^{\frac{1}{\phi}}}, \quad (60)$$

which must be equal to the hazard rate of unemployment that we are calibrating to (i.e. the monthly hazard rate of 46%, estimated by Shimer (21)). Since we are calibrating to a known value of  $U$  (equal to 0.071), equations (59) and (60) can be used to solve for  $A$  and  $\phi$ . The values thus obtained are quite reasonable. In particular, the implied elasticity of the hazard rate from unemployment  $G(A, U, A, U)/U$  to the unemployment-help-wanted-ads ratio  $U/A$  turn out to be 0.52 in the case of congestion externalities and 0.64 in the case of efficient matching. This elasticities are within the range estimated by previous studies (e.g. Shimer (20), Hall (13), etc.).

As a test of the model Table 2 reports a set of basic monthly statistics both for JOLTS and the model economy that were not used as calibration targets (except for the vacancy yield rate).<sup>24</sup> We see that the model does a reasonable job at reproducing not only the vacancy yield rate, but the hiring and separation rates for continuing establishments, the fraction of vacancies with zero hirings and the fraction of hires with zero vacancies. The low rate of exogenous separations  $\pi_n$  explains the model's success in reproducing the fraction of vacancies with zero hirings. The reason is that a significant number of establishments reach the lower thresholds  $\underline{n}$  and start hiring just enough workers to replenish the exogenous separation of workers. Since the monthly rate of exogenous separation is less than 1%, following Davis et al. (7), I classify these establishments (and their corresponding vacancies) as having zero hirings.

Observe that the model's ability at reproducing the JOLTS vacancy yield rate confirms that the strategy of calibrating to a three weeks time-period and setting  $\pi_u$  to zero was justified. In fact, the assumption that total hirings are approximately equal to total recruitment opportunities is verified: Total hirings turn out to be 94% as large as total recruitment opportunities.

Finally, the parameters  $\rho_z$  and  $\sigma_\varepsilon^2$  governing the aggregate productivity shock process are selected to reproduce the empirical behavior of measured Solow residuals in the U.S. economy.<sup>25</sup> Defining output and capital as above and using civilian employment as the labor input, I find that measured Solow residuals are highly persistent and that their quarterly proportionate changes have a standard deviations equal to 0.0064 over the period 1951:1-2004:4.<sup>26</sup> It turns out that values of  $\rho_z = 0.95$  and  $\sigma_\varepsilon = 0.0041$  are needed to reproduce these observations using the artificial data generated by both versions of the model economy.<sup>27</sup>

All calibrated parameter values are summarized in Table 3.

<sup>23</sup>For the details, see Section 4.1 in the Technical Appendix.

<sup>24</sup>JOLTS statistics are from Davis et al. (7).

<sup>25</sup>Let  $\gamma^e$  denote the empirical labor share implicit in the National Income and Product Accounts. Proportionate changes in measured Solow residuals are then defined as the proportionate change in aggregate output minus the sum of the proportionate change in labor times  $\gamma^e$ , minus the sum of the proportionate change in capital times  $(1 - \gamma^e)$ .

<sup>26</sup>Solow residuals are constructed using an empirical labor share  $\gamma^e$  equal to 0.64.

<sup>27</sup>In both model economies quarterly Solow residuals are measured with the same empirical labor share  $\gamma^e$  used to measure Solow residuals in

## 6 Business cycles

This section uses both versions of the model economy calibrated in the previous section to address three important questions: 1) Can the neoclassical theory developed so far account for U.S. business cycle observations?, 2) Which scenario for the matching process is empirically more plausible: The efficient matching or the congestion externalities scenarios?, and 3) Is the model consistent with microeconomic adjustments at the establishment level?

Before turning to these questions it will be useful to describe salient features of U.S. business cycle fluctuations. Table 4 reports business cycle statistics for a number of U.S. time series corresponding to the period 1951:1 to 2004:4. All time series were logged and detrended using the Hodrick-Prescott filter with smoothing parameter 1,600 before computing any statistics. The upper panel of Table 4 reports standard deviations and correlations with output for GDP (Y), consumption (C), investment (I), capital (K), civilian employment (N), and labor productivity (Y/N). These statistics are standard in the RBC literature. They show that consumption, employment and labor productivity fluctuate roughly 61% as much as output, that capital fluctuates only 43% as much as output, that investment fluctuates 3.3 times more than output, and that all variables are strongly procyclical except for capital, which is acyclical. The lower panel of Table 4 reports standard deviations and the cross-correlation matrix for GDP (Y), employment (N), unemployment (U), help-wanted ads (A), market tightness (A/U), job creation (JC) and job destruction (JD).<sup>28</sup> These statistics have been emphasized in the labor literature (e.g. Hagedorn and Manovskii (9), Shimer (20), Davis and Haltiwanger, etc.). They show that unemployment and help wanted ads fluctuate about 7.3 times more than output, that job creation fluctuates 3.7 times more than output and that job destruction is about 70% more variable than job creation. Unemployment is strongly countercyclical, help-wanted ads are strongly procyclical, job creation is acyclical and job destruction is weakly countercyclical. Also observe that job creation and job destruction are weakly negatively correlated and that a “Beveridge curve” is obtained: help-wanted ads and unemployment are strongly negatively correlated.

### 6.1 Efficient matching vs. congestion externalities

Table 5 reports business cycle statistics for the model economy with efficient matching. Time series of length equal to 864 time periods were computed for 100 simulations and then aggregated into a quarterly frequency to obtain 216 quarters of data (the same length as the U.S. series). The reported statistics are averages across these simulations. With regard to standard RBC statistics, we see from the upper-panels of Tables 4 and 5 that the model with efficient matching reproduces the comovements with output quite well: Except for capital, which is acyclical, all other variables are procyclical. The model’s performance is not as good in terms of standard deviations, though. We see that investment fluctuations are as large as in the data but the rest of the variables are much smoother. The largest difference is with consumption, which fluctuates only 30% as much as in the data. However, this is a standard problem with RBC models. The most disappointing

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the U.S. economy.

<sup>28</sup>The job creation and job destruction rates are from Davis, Faberman and Haltiwanger (7). Job creation corresponds to the sum of gross job gains due to expanding establishments (JGE) and gross job gains due to opening establishments (JGB). Job destruction corresponds to the sum of gross job losses due to contracting establishments (JLC) and job losses due to closing establishments (JLD).

performance is with employment, which fluctuates only 46% as much as in the data. This smoothness is in turn inherited by output, which fluctuates only 72% as much as in the U.S. The failure of the model with efficient matching to account for labor market dynamics is evident in the lower panel. We see that unemployment, help-wanted ads, job creation and job destruction fluctuate too little compared with the data. Moreover, the model fails to generate a strong “Beveridge curve”: the correlation of unemployment with help-wanted ads is only -0.40.

We now turn to the model with congestion externalities in the matching process. Table 6 shows the results. We see that in terms of standard RBC statistics that this version of the model replicates U.S. business cycle observations at least as well as the economy with efficient matching. Comovements with output are still very similar with the data: consumption, investment, employment and labor productivity are all procyclical while capital is acyclical. The dimension in which the economy with congestion externalities outperforms the economy with efficient matching is in standard deviations: Employment, capital and output go from being 46%, 66% and 72% as volatile as the data to being 101%, 87%, 94% as volatile, respectively.<sup>29</sup> The relative success of the model with congestion externalities is even more striking when measured in terms of the labor market statistics reported in the lower panel of Table 6. Unemployment and help-wanted ads go from being 49% and 26% as volatile as the data to being 111% and 106% as volatile, respectively. Although job creation and job destruction still fluctuate too little compared with the US. economy, they are twice as volatile as in the economy with efficient matching. Also, observe that help-wanted ads have become much more procyclical, bringing them closer to the data, and that a Beveridge curve is now obtained: the correlation of help-wanted ads with unemployment is -0.71 (which is not as strong as the data but is a substantial improvement over the economy with efficient matching). Moreover, note that job creation is acyclical, that job destruction is weakly countercyclical and that job creation and job destruction are negatively related; properties all observed in the U.S. economy. Overall we can say that the economy with congestion externalities, while not a perfect fit, does a surprisingly good job at accounting for salient features of U.S. business cycle fluctuations. This is particularly true when compared with the economy with efficient matching.

There are two differences between the economy with efficient matching and the economy with congestion externalities. First, as Table 3 indicates, the economies have different parameter values (in particular, the curvature of the matching function  $\phi$  and the utility of leisure  $\varphi$  are different). Second, although their aggregate matching functions have identical functional forms (see equation 58), their individual matching technologies are different (compare equations 55- 56 with equation 57). In order to determine which of these differences drives the result that the economy with congestion externalities outperforms the economy with efficient matching, Table 7 reports the business cycle statistics for the Pareto optimal allocation of the economy with congestion externalities. Since the social planner fully internalizes the effects of the congestion externalities, any differences between these statistics and those of the economy with efficient matching reported in Table 5 can be solely attributed to differences in parameter values. Since Table 7 is very similar to Table 5, we conclude that the bulk of the differences in business cycle fluctuations between the economy with efficient matching and the economy with congestion externalities is not due to different parameter values but to the different individual matching technologies.<sup>30</sup>

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<sup>29</sup>The only drawback is with labor productivity, which goes from being 71% as volatile as the data to being only 54% as volatile.

<sup>30</sup>From Tables 6 and 7 we also conclude that introducing policies that achieve the first-best allocation would significantly reduce aggregate fluctuations in the economy with congestion externalities. See the working paper version (Veracierto xx) for an analysis of such policies.

In order to determine what feature of the matching technology with congestion externalities is essential for generating relatively large business cycle fluctuations, it will be useful to consider the following linear matching technology:

$$G(a, u, A^*, U^*) = u \frac{A^*}{\left[ (U^*)^\phi + (A^*)^\phi \right]^{\frac{1}{\phi}}}, \quad (61)$$

$$H(a, u, A^*, U^*) = a \frac{U^*}{\left[ (U^*)^\phi + (A^*)^\phi \right]^{\frac{1}{\phi}}}, \quad (62)$$

where  $A^*$  and  $U^*$  are positive parameters. Observe that if an economy had identical parameter values as the economy with congestion externalities but its matching technology was described by equations (61) and (62) with  $A^*$  and  $U^*$  given by the steady-state values of help-wanted ads and unemployment in the economy with congestion externalities, respectively, its steady state would be identical to the steady state of the economy with congestion externalities. However, its business cycles would be different. The reason is that the linear matching technology in equations (61) and (62) has a constant productivity while the linear matching technology faced by the myopic social planner in the economy with congestion externalities is subject to productivity shocks given by the realizations of the market tightness ratio  $A/U$ . Table 8 reports the business cycle statistics for this economy. We see that its business cycles are in fact much larger than in the economy with congestion externalities: Except for labor productivity and help-wanted ads, all variables are significantly more volatile than in Table 6. This indicates that the crucial feature generating the relatively large business cycles in the economy with congestion externalities is the linearity of the individual matching technology given by equations (55) and (56): The external effects from endogenous variations in the market tightness ratio  $A/U$  only serve to dampen the aggregate fluctuations generated by the economy. This is not surprising. Since equation (26) holds with strict inequality, the technology for creating workers with employment opportunities in equation (55) is irrelevant. On the contrary, the technology for creating recruitment opportunities in equation (56) binds the amount of hiring that the economy can undertake. Since aggregate market tightness  $A/U$  is strongly procyclical in Table 6, the productivity of this technology turns out to be countercyclical. This reduces the response of aggregate employment to aggregate productivity shocks, leading to lower employment fluctuations in Table 6 than in Table 8. This also explains why help-wanted ads are more volatile in Table 6 than in Table 8: Help-wanted ads need to respond more to aggregate shocks to partially compensate for the countercyclical productivity.

## 6.2 Reallocation shocks

Section 6.1 showed that the economy with congestion externalities is quite successful at reproducing salient features of U.S. business cycle fluctuations. However, it has a noticeable drawback: The model fails to reproduce the cyclical properties of job creation and job destruction. While Table 6 shows that the correlations of job creation and job destruction with all other variables are in line with the data, their standard deviations are too small: Job creation and job destruction fluctuate only 66% and 42% as much as the data, respectively. Moreover, Table 6 shows that job destruction is as volatile as job creation while in the data it is 69% more volatile.

The purpose of this section is to explore to what extent the model's performance could be improved by introducing reallocation shocks that affect the idiosyncratic productivity shocks process. A wide variety of reallocation shocks may be analyzed using the computational approach developed in this paper. For instance, the reallocation shock considered could

affect the dispersion of the idiosyncratic productivity levels  $s$  while leaving the transition matrix  $Q$  unchanged. Another possible reallocation shock could leave the idiosyncratic productivity levels  $s$  unchanged while affecting the persistence  $Q(s, s)$  of the different idiosyncratic productivity levels  $s$ . It turns out that these types of reallocations shocks do not improve the model's performance. The reason is that they synchronize the changes in job creation and job destruction, breaking their negative correlation and failing to generate their asymmetric volatilities.

In order to break that synchronization the following reallocation shock will be considered. Let  $S^*$  be the set of idiosyncratic productivity levels and  $Q^*$  the transition matrix that were calibrated in Section 5. The reallocation shock  $r_t$  leaves the set of values for the idiosyncratic productivity levels unchanged at  $S^*$  but affects the transitions matrix  $Q_t$  as follows. For every  $s$  and  $s'$  in  $S^*$ ,

$$Q_t(s, s') = \begin{cases} Q^*(s, s'), & \text{if } s' > s, \\ e^{r_t} Q^*(s, s'), & \text{if } s' < s, \\ 1 - \sum_{s'' < s} e^{r_t} Q^*(s, s'') - \sum_{s'' > s} Q^*(s, s''), & \text{if } s' = s. \end{cases} \quad (63)$$

Observe that the reallocation shock  $r_t$  affects the probabilities of transiting to lower productivity levels but not the probabilities of transiting to higher productivity levels. Any variations in the probabilities of transiting to lower productivity levels are absorbed by the probabilities of no-change.<sup>31</sup>

The reallocation shock  $r_t$  and the aggregate productivity shock  $z_t$  follow a joint autoregressive process given by

$$\begin{bmatrix} z_{t+1} \\ r_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_{zz} & \rho_{zr} \\ \rho_{rz} & \rho_{rr} \end{bmatrix} \begin{bmatrix} z_t \\ r_t \end{bmatrix} + \begin{bmatrix} \sigma_{zz} & \sigma_{zr} \\ \sigma_{rz} & \sigma_{rr} \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1}^z \\ \varepsilon_{t+1}^r \end{bmatrix}, \quad (64)$$

where  $\varepsilon_{t+1}^z$  and  $\varepsilon_{t+1}^r$  are normally distributed with zero mean and unit standard deviation.

Allowing the reallocation shock  $r_t$  to be negatively correlated with the aggregate productivity shock  $z_t$  is crucial for generating asymmetries in the job creation and job destruction process. To see this, suppose that the economy is hit by a negative aggregate productivity shock that is accompanied by higher transition probabilities to lower idiosyncratic productivity levels. Since these higher transition probabilities generate job destruction, the response of job destruction to the negative aggregate productivity shock will thus be amplified. On the contrary, if the larger transition probabilities are short-lived (i.e. if  $\rho_{rr}$  is close to zero) the response of job creation to the negative aggregate productivity shock will be dampened. The reason, is that after its initial fall, the distribution of idiosyncratic productivity levels will be reverting towards the invariant distribution generated by the transition matrix  $Q^*$ , creating job creation over time. Both effects work in the same direction: making job destruction relatively more volatile.<sup>32</sup>

Given the above discussion, the reallocation shocks will be restricted to be short-lived and perfectly negatively correlated with innovations in aggregate productivity. In turn, the aggregate productivity shock will be allowed to have the same

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<sup>31</sup>Restrictions to ensure that the probabilities of no-change  $Q(s, s)$  remain positive for every possible realization  $Q$  of the reallocation shock  $r_t$  are ignored in equation (63) since these restrictions turn out to be non-binding.

<sup>32</sup>This result may not be obtained if  $\rho_{rr}$  is close to one. If after a negative aggregate productivity shock hits the economy establishments expect good idiosyncratic productivity levels to be much more transient than before, they will have fewer incentives to invest in recruitment opportunities after a high idiosyncratic productivity level is realized. As a consequence, the drop in job creation after the negative aggregate productivity shock hits the economy may actually be amplified.



persistence level as in the benchmark case. Under these assumptions the general process in equation (64) reduces to the following specification:

$$\begin{bmatrix} z_{t+1} \\ r_{t+1} \end{bmatrix} = \begin{bmatrix} 0.95 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_t \\ r_t \end{bmatrix} + \begin{bmatrix} \sigma_{zz} & 0 \\ \sigma_{rz} & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1}^z \\ \varepsilon_{t+1}^r \end{bmatrix}. \quad (65)$$

The parameters  $\sigma_{zz}$  and  $\sigma_{rz}$  in equation (65) are selected to reproduce two important observations. First, that the standard deviation of measured Solow residuals is equal to 0.0064 (the same observation that was used in the benchmark case). Second, that the standard deviation of job destruction is 69% larger than the standard deviation of job creation. The parameter values consistent with these observations turn out to be  $\sigma_{zz} = 0.003683$  and  $\sigma_{rz} = -0.095853$ . Table 9 reports the business cycle statistics for this economy. We see that in terms of standard RBC statistics, that the economy with reallocation shocks is virtually identical to the benchmark economy with congestion externalities (see Table 6). Despite of the fact that the aggregate productivity shock is 10% less variable and that the reallocation shock is i.i.d., the aggregate dynamics of the economy with reallocation shocks are essentially the same to those of the benchmark case. In terms of labor market statistics, we also see some similarities with the benchmark economy: job creation and job destruction are weakly negatively correlated, job destruction is weakly countercyclical, and the model generates a Beveridge curve (unemployment and help-wanted are negatively correlated). The significant improvement over the benchmark economy is that (by construction) job destruction is now much more volatile than job creation. However, this comes at a cost: job creation is now weakly countercyclical (instead of acyclical) and the standard deviation of help-wanted ads has decreased. The reason for this is that after an aggregate negative productivity shock hits the economy and the distribution of idiosyncratic shocks falls on impact, job creation and help-wanted ads are kept relatively high as the distribution of idiosyncratic shocks reverts towards the invariant distribution. We also see that the economy with reallocation shock continues to share a serious drawback with the benchmark economy: the standard deviations of job creation and job destruction continue to be too small.

We conclude that the reallocation shocks, while useful in generating asymmetries in job creation and job destruction, fail short of completely accounting for the cyclical behavior of establishment dynamics and vacancies. Extending the model to incorporate job-to-job transitions that uncouple job creation and job destruction from aggregate employment fluctuations may improve the model's performance. However, this is left for future research.

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# Appendix A

**Proof of Proposition 4:** From equations (38) and (39) we know that an establishment of type  $(s, l, j)$  transits to a next-period type  $(s', l', j')$ , with  $s'$  randomly determined,

$$l' = n^*(s, l, j), \quad (66)$$

and

$$j' = \max\{\bar{v}^*(s) - (1 - \pi_n)l', 0\}. \quad (67)$$

Define

$$\mathcal{P}^{(0)} = \bigcup_{s \in S} \{(s, 0, 0)\}.$$

Since establishments are created with  $(l, j) = (0, 0)$ ,  $\mathcal{P}^{(0)}$  describes the set of all possible types  $(s, l, j)$  of establishments of zero age.

Define

$$\mathcal{N}^{(0)} = \{0\}.$$

Since  $n^*(s, l, j) = 0$  whenever  $(l, j) = (0, 0)$ ,  $\mathcal{N}^{(0)}$  describes the set of all possible employment levels of establishments of zero age.

Starting from  $\mathcal{N}^{(0)}$ , define recursively a sequence of sets  $\mathcal{P}^{(m)}$  and  $\mathcal{N}^{(m)}$  as follows:

$$\begin{aligned} \mathcal{P}^{(m)} &= \left\{ (s, l, j) : s \in S, l \in \mathcal{N}^{(m-1)}, \text{ and } j \in \bigcup_{s_{-1} \in S} \{\max[\bar{v}^*(s_{-1}) - (1 - \pi_n)l, 0]\} \right\} \\ \mathcal{N}^{(m)} &= \left\{ \bigcup_{s \in S} \{\underline{n}^*(s), \bar{n}^*(s), \bar{v}^*(s)\} \right\} \cup \left\{ n : n = (1 - \pi_n)n_{m-1} \text{ for some } n_{m-1} \in \mathcal{N}^{(m-1)} \right\}, \end{aligned}$$

for  $m = 1, 2, \dots, \infty$ .

From equations (38), (39), (66) and (67) we know that  $\mathcal{P}^{(m)}$  contains the set of all possible types  $(s, l, j)$  of establishments of age  $m$ , and that  $\mathcal{N}^{(m)}$  contains the set of all possible employment levels of establishments of age  $m$ .<sup>33</sup>

By induction, it can be shown that:

$$\mathcal{N}^{(m)} = \left\{ \bigcup_{s \in S} \bigcup_{k=0}^{m-1} \left\{ (1 - \pi_n)^k \underline{n}^*(s), (1 - \pi_n)^k \bar{n}^*(s), (1 - \pi_n)^k \bar{v}^*(s) \right\} \right\} \cup \{0\}, \quad (68)$$

for  $m = 1, 2, \dots, \infty$ .

A direct consequence of equation (68) is that  $\mathcal{N}^{(m-1)} \subset \mathcal{N}^{(m)}$ , for every  $m \geq 1$ . Thus, the set  $\mathcal{N}^{(m)}$  in fact contains all the possible employment levels of establishments of age  $m$  or younger and the set  $\mathcal{P}^{(m)} \cup \mathcal{P}^{(0)}$  contains all the possible types of establishments of age  $m$  or younger. Moreover,

$$\mathcal{N}^{(m)} / \mathcal{N}^{(m-1)} = \bigcup_{s \in S} \left\{ (1 - \pi_n)^{m-1} \underline{n}^*(s), (1 - \pi_n)^{m-1} \bar{n}^*(s), (1 - \pi_n)^{m-1} \bar{v}^*(s) \right\}, \quad (69)$$

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<sup>33</sup>Observe that the “max” and “min” operators in equation (38) have been disregarded in the construction of the sets  $\mathcal{P}^{(m)}$  and  $\mathcal{N}^{(m)}$ . Thus, the set of actual types of establishments of age  $m$  and the set of actual employment levels of establishments of age  $m$  are smaller than  $\mathcal{P}^{(m)}$  and  $\mathcal{N}^{(m)}$ , respectively.

for  $m = 1, 2, \dots, \infty$ , where “/” denotes set difference.

In what follows it will be shown that there exists a  $M < \infty$  such that  $\mathcal{N}^{(M)}$  contains the set of all possible employment levels of establishments of all ages  $m = 0, 1, \dots, \infty$ . To prove this it suffices to show that there exists a  $M < \infty$  such that no establishment of age  $M + 1$  will choose an employment level in the set  $\mathcal{N}^{(M+1)}/\mathcal{N}^{(M)}$ , i.e. all establishments of age  $M + 1$  will choose an employment level in the set  $\mathcal{N}^{(M)}$ .<sup>34</sup>

Let  $M$  satisfy equation (40). Since  $0 < \pi_n < 1$ , such a  $M$  exists.

Let  $(s, l, j) \in \mathcal{P}^{(M+1)}$ .

Suppose that  $n^*(s, l, j) \in \mathcal{N}^{(M+1)}/\mathcal{N}^{(M)}$ . Since  $\mathcal{N}^{(M+1)}/\mathcal{N}^{(M)}$  satisfies equation (69), and  $M$  satisfies equation (40), it follows that

$$n^*(s, l, j) \leq (1 - \pi_n)^M \max \{ \bar{n}^*(s_{\max}), \bar{v}^*(s_{\max}) \} < \min \{ \underline{n}^*(s_{\min}), \bar{v}^*(s_{\min}) \}. \quad (70)$$

Also, since  $n^*(s, l, j)$  satisfies equation (38) and  $(s, l, j) \in \mathcal{P}^{(M+1)}$ , we have that

$$n^*(s, l, j) \in \{ \underline{n}^*(s), \bar{n}^*(s), (1 - \pi_n)l \} \cup \left\{ \bigcup_{s_{-1} \in S} \bar{v}^*(s_{-1}) \right\}. \quad (71)$$

From equation (71) and the last inequality in equation (70), we then have that

$$n^*(s, l, j) = (1 - \pi_n)l.$$

Suppose, first, that  $j = 0$ .

Suppose that some establishment of age  $M$  transits to  $(s, l, j)$ . From equation (67), this can be the case only if

$$0 = \max \{ \bar{v}^*(s_{-1}) - (1 - \pi_n)l, 0 \},$$

for some  $s_{-1} \in S$ .

But, from equation (70)

$$n^*(s, l, j) = (1 - \pi_n)l < \bar{v}^*(s_{-1}),$$

for all  $s_{-1} \in S$ . A contradiction.

Hence,  $(s, l, j) \in \mathcal{P}^{(M+1)}$  does not correspond to an establishment of age  $M + 1$ .

Suppose now that  $j > 0$ .

Let  $s_{-1}$  be such that  $(1 - \pi_n)l + j = \bar{v}^*(s_{-1})$  (since  $(s, l, j) \in \mathcal{P}^{(M+1)}$ , such an  $s_{-1}$  exists).

Then, from equation (38) we have that

$$n^*(s, l, j) = \max \left\{ \begin{array}{l} \min \{ \bar{v}^*(s_{-1}), \underline{n}^*(s) \}, \\ \min \{ (1 - \pi_n)l, \bar{n}^*(s) \} \end{array} \right\},$$

and, therefore, that

$$n^*(s, l, j) = (1 - \pi_n)l \leq \bar{n}^*(s) \text{ and } n^*(s, l, j) = (1 - \pi_n)l \geq \min \{ \bar{v}^*(s_{-1}), \underline{n}^*(s) \}. \quad (72)$$

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<sup>34</sup>This condition is sufficient because whenever an establishment reaches age  $M + 1$ , its age can be reset to  $M$  without consequence. This procedure can be repeated an infinite number of times.

The second inequality in equation (72) contradicts equation (70).

We conclude that no establishment of age  $M + 1$  chooses an employment level in the set  $\mathcal{N}^{(M+1)}/\mathcal{N}^{(M)}$ . It follows that the set  $\mathcal{P}^* = \mathcal{P}^{(M+1)} \cup \mathcal{P}^{(0)}$  is a support of the invariant distribution  $\mu^*$ . ■

**Proof of Proposition 5:** Observe that the optimal decision rules at period  $t - k$  are given by

$$n_{t-k}(s, l, j) = \max \left\{ \begin{array}{l} \min \{(1 - \pi_n)l + j, \underline{n}_{t-k}(s)\}, \\ \min \{(1 - \pi_n)l, \bar{n}_{t-k}(s)\} \end{array} \right\}, \quad (73)$$

and

$$v_{t-k}(s, l, j) = \max \{\bar{v}_{t-k}(s) - (1 - \pi_n)n_{t-k}(s, l, j), 0\}, \quad (74)$$

for  $k = 0, 1, \dots, M + 1$ .

a) We will first show that  $\mathcal{P}_{t+1}$  is a support of the distribution  $\mu_{t+1}$ .

Define the sets  $\mathcal{A}_t$  and  $\mathcal{B}_{t+1}$  as follows:

$$\begin{aligned} \mathcal{A}_t &= \bigcup_{s \in S} \left\{ (1 - \pi_n)^{M-1} \underline{n}_{t-M}(s), (1 - \pi_n)^{M-1} \bar{n}_{t-M}(s), (1 - \pi_n)^{M-1} \bar{v}_{t-M-1}(s) \right\}, \\ \mathcal{B}_{t+1} &= \{l' : l' = (1 - \pi_n)l, \text{ for some } l \in \mathcal{N}_t/\mathcal{A}_t\}. \end{aligned} \quad (75)$$

Observe that

$$\mathcal{N}_{t+1} = \mathcal{B}_{t+1} \cup \left\{ \bigcup_{s \in S} \{\underline{n}_t(s), \bar{n}_t(s), \bar{v}_{t-1}(s)\} \right\}. \quad (76)$$

To show that  $\mathcal{P}_{t+1}$  is a support of the distribution  $\mu_{t+1}$  it suffices to show that

$$(s, l, j) \in \mathcal{P}_t \implies n_t(s, l, j) \in \mathcal{N}_{t+1} \text{ and } v_t(s, l, j) \in \bigcup_{s' \in S} \{\max[\bar{v}_t(s') - (1 - \pi_n)n_t(s, l, j), 0]\}. \quad (77)$$

Let  $(s, l, j) \in \mathcal{P}_t$ .

Suppose, first, that  $(l, j) = (0, 0)$ .

From equation (73) we then have that  $n_t(s, l, j) = 0$  and, from equation (74), that  $v_t(s, l, j) = \bar{v}_t(s)$ . Therefore, equation (77) is satisfied.

Suppose, now, that  $(l, j) \neq (0, 0)$ .

Then,

$$s \in S, l \in \mathcal{N}_t \text{ and } j \in \bigcup_{s' \in S} \{\max[\bar{v}_{t-1}(s') - (1 - \pi_n)l, 0]\}. \quad (78)$$

From equations (73) and (78), we then have that

$$n_t(s, l, j) = \max \left\{ \begin{array}{l} \min \{\max[\bar{v}_{t-1}(s'), (1 - \pi_n)l], \underline{n}_t(s)\}, \\ \min \{(1 - \pi_n)l, \bar{n}_t(s)\} \end{array} \right\}, \quad (79)$$

for some  $s' \in S$ .

As a consequence,

$$n_t(s, l, j) \in \{(1 - \pi_n)l, \bar{n}_t(s), \underline{n}_t(s)\} \cup \left\{ \bigcup_{s' \in S} \{\bar{v}_{t-1}(s')\} \right\}. \quad (80)$$

From equations (75), (76) and (80) we have that

$$l \in \mathcal{N}_t / \mathcal{A}_t \Rightarrow n_t(s, l, j) \in \mathcal{N}_{t+1}.$$

Suppose that  $l \in A_t$ . Without loss of generality assume that

$$l = (1 - \pi_n)^{M-1} \underline{n}_{t-M}(\hat{s})$$

for some  $\hat{s} \in S$  (the cases  $l = (1 - \pi_n)^{M-1} \bar{n}_{t-M}(\hat{s})$  and  $l = (1 - \pi_n)^{M-1} \bar{v}_{t-M-1}(\hat{s})$  can be handled in exactly the same way).

Then, equation (79) becomes

$$n_t(s, l, j) = \max \left\{ \begin{array}{l} \min \left\{ \max \left[ \bar{v}_{t-1}(s'), (1 - \pi_n)^M \underline{n}_{t-M}(\hat{s}) \right], \underline{n}_t(s) \right\}, \\ \min \left\{ (1 - \pi_n)^M \underline{n}_{t-M}(\hat{s}), \bar{n}_t(s) \right\} \end{array} \right\}. \quad (81)$$

for some  $s' \in S$ .

But from equation (40) and equations (42)-(44), we have that

$$(1 - \pi_n)^M \underline{n}_{t-M}(\hat{s}) < (1 - \pi_n)^M \bar{n}_{t-M}(\hat{s}) \leq (1 - \pi_n)^M \bar{n}_{t-M}(s_{\max}) < \bar{v}_{t-1}(s_{\min}) \leq \bar{v}_{t-1}(s'),$$

and that

$$(1 - \pi_n)^M \underline{n}_{t-M}(\hat{s}) < (1 - \pi_n)^M \bar{n}_{t-M}(\hat{s}) \leq (1 - \pi_n)^M \bar{n}_{t-M}(s_{\max}) < \underline{n}_t(s_{\min}) \leq \underline{n}_t(s) < \bar{n}_t(s).$$

Therefore equation (81) becomes

$$\begin{aligned} n_t(s, l, j) &= \max \left\{ \begin{array}{l} \min \{ \bar{v}_{t-1}(s'), \underline{n}_t(s) \}, \\ (1 - \pi_n)^M \underline{n}_{t-M}(\hat{s}) \end{array} \right\} \\ &= \min \{ \bar{v}_{t-1}(s'), \underline{n}_t(s) \}. \end{aligned}$$

Thus, from equations (76),  $n_t(s, l, j) \in \mathcal{N}_{t+1}$ .

From equation (74), observe that

$$v_t(s, l, j) = \max \{ \bar{v}_t(s) - (1 - \pi_n) n_t(s, l, j), 0 \}$$

Thus,

$$v_t(s, l, j) \in \bigcup_{s' \in S} \{ \max [ \bar{v}_t(s') - (1 - \pi_n) n_t(s, l, j), 0 ] \}.$$

Therefore,  $\mathcal{P}_{t+1}$  is a support of the distribution  $\mu_{t+1}$ .

b) To prove the second part of the Proposition it will be convenient to define the following (one-to-one and onto) functions.

For every  $(s, l, j) \in \mathcal{P}_t$ :

$$\begin{aligned} l_t^*(l, j) &= l^* \\ j_t^*(l, j) &= j^* \end{aligned}$$

where  $(s, l^*, j^*)$  is the unique element of  $\mathcal{P}^*$  satisfying that  $|l - l^*| < \varepsilon/2$  and  $|(1 - \pi_n)l + j - [(1 - \pi_n)l^* + j^*]| < \varepsilon/2$ .

Similarly, for every  $(s', l', j') \in \mathcal{P}_{t+1}$ :

$$\begin{aligned} l_{t+1}^*(l', j') &= l^* \\ j_{t+1}^*(l', j') &= j^* \end{aligned}$$

where  $(s', l^*, j^*)$  is the unique element of  $\mathcal{P}^*$  satisfying that  $|l' - l^*| < \varepsilon/2$  and  $|(1 - \pi_n)l' + j' - [(1 - \pi_n)l^* + j^*]| < \varepsilon/2$ .

Observe that, by assumption, we have that for every  $(s, l, j) \in \mathcal{P}_t$ :

$$\mu_t(s, l, j) = \mu^*(s, l_t^*(l, j), j_t^*(l, j)). \quad (82)$$

We need to show that for every  $(s', l', j') \in \mathcal{P}_{t+1}$ :

$$\mu_{t+1}(s', l', j') = \mu^*(s', l_{t+1}^*(l', j'), j_{t+1}^*(l', j')). \quad (83)$$

Let  $(s', l', j') \in \mathcal{P}_{t+1}$ .

Using equation (82), we have that

$$\mu_{t+1}(s', l', j') = \sum_{(s, l, j) \in \mathcal{G}_t(l', j')} Q(s, s') \mu^*(s, l_t^*(l, j), j_t^*(l, j)) + \varrho\psi(s') \mathcal{I}(l', j'),$$

where

$$\mathcal{G}_t(l', j') = \{(s, l, j) \in \mathcal{P}_t: n_t(s, l, j) = l' \text{ and } v_t(s, l, j) = j'\}.$$

Also observe that

$$\mu^*(s', l_{t+1}^*(l', j'), j_{t+1}^*(l', j')) = \sum_{(s, l^*, j^*) \in \mathcal{G}^*(l_{t+1}^*(l', j'), j_{t+1}^*(l', j'))} Q(s, s') \mu^*(s, l^*, j^*) + \varrho\psi(s') \mathcal{I}(l_{t+1}^*(l', j'), j_{t+1}^*(l', j')),$$

where

$$\mathcal{G}^*(l_{t+1}^*(l', j'), j_{t+1}^*(l', j')) = \{(s, l^*, j^*) \in \mathcal{P}^*: n^*(s, l^*, j^*) = l_{t+1}^*(l', j') \text{ and } v^*(s, l^*, j^*) = j_{t+1}^*(l', j')\}.$$

To show that equation (83) holds, it then suffices to show that

$$(l', j') = (0, 0) \Leftrightarrow (l_{t+1}^*(l', j'), j_{t+1}^*(l', j')) = (0, 0), \quad (84)$$

$$(s, l, j) \in \mathcal{G}_t(l', j') \implies (s, l_t^*(l, j), j_t^*(l, j)) \in \mathcal{G}^*(l_{t+1}^*(l', j'), j_{t+1}^*(l', j')), \quad (85)$$

$$(s, l^*, j^*) \in \mathcal{G}^*(l_{t+1}^*(l', j'), j_{t+1}^*(l', j')) \implies (s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*)) \in \mathcal{G}_t(l', j'). \quad (86)$$

where  $([l_t^*]^{-1}, [j_t^*]^{-1})$  is the inverse function of  $(l_t^*, j_t^*)$ .

b.1) Proof of equation (84).

It is a direct consequence of how  $l_{t+1}^*$  and  $j_{t+1}^*$  were defined and equations (42)-(44).

b.2) Proof of equation (85).

Let  $(s, l, j) \in \mathcal{G}_t(l', j')$ . Then,  $(s, l, j) \in \mathcal{P}_t$ ,

$$l' = \max \left\{ \begin{array}{l} \min \{(1 - \pi_n)l + j, \underline{n}_t(s)\} \\ \min \{(1 - \pi_n)l, \bar{n}_t(s)\} \end{array} \right\},$$



and

$$(1 - \pi_n) l' + j' = \max \{ \bar{v}_t(s), (1 - \pi_n) l' \}.$$

Observe that  $(s, l_t^*(l, j), j_t^*(l, j)) \in \mathcal{P}^*$ ,

$$n^*(s, l_t^*(l, j), j_t^*(l, j)) = \max \left\{ \begin{array}{l} \min \{ (1 - \pi_n) l_t^*(l, j) + j_t^*(l, j), \underline{n}^*(s) \} \\ \min \{ (1 - \pi_n) l_t^*(l, j), \bar{n}^*(s) \} \end{array} \right\},$$

and

$$\begin{aligned} & (1 - \pi_n) n^*(s, l_t^*(l, j), j_t^*(l, j)) + v^*(s, l_t^*(l, j), j_t^*(l, j)) \\ &= \max \{ \bar{v}^*(s), (1 - \pi_n) n^*(s, l_t^*(l, j), j_t^*(l, j)) \}. \end{aligned}$$

Since

$$\begin{aligned} & |(1 - \pi_n) l - (1 - \pi_n) l_t^*(l, j)| < \varepsilon/2, \\ & |[ (1 - \pi_n) l + j ] - [ (1 - \pi_n) l_t^*(l, j) + j_t^*(l, j) ]| < \varepsilon/2, \\ & | \underline{n}_t(s) - \underline{n}^*(s) | < \varepsilon/2, \end{aligned}$$

and

$$| \bar{n}_t(s) - \bar{n}^*(s) | < \varepsilon/2,$$

it follows that

$$| n^*(s, l_t^*(l, j), j_t^*(l, j)) - l' | < \varepsilon/2, \tag{87}$$

and, therefore, that

$$| [ (1 - \pi_n) n^*(s, l_t^*(l, j), j_t^*(l, j)) + v^*(s, l_t^*(l, j), j_t^*(l, j)) ] - [ (1 - \pi_n) l' + j' ] | < \varepsilon/2. \tag{88}$$

Since  $(s', l', j') \in \mathcal{P}_{t+1}$  and  $[s', n^*(s, l_t^*(l, j), j_t^*(l, j)), v^*(s, l_t^*(l, j), j_t^*(l, j))] \in \mathcal{P}^*$ , equations (87) and (88) imply that

$$\begin{aligned} l_{t+1}^*(l', j') &= n^*(s, l_t^*(l, j), j_t^*(l, j)), \\ j_{t+1}^*(l', j') &= v^*(s, l_t^*(l, j), j_t^*(l, j)). \end{aligned}$$

Since  $(s, l_t^*(l, j), j_t^*(l, j)) \in \mathcal{P}^*$  it follows that

$$(s, l_t^*(l, j), j_t^*(l, j)) \in \mathcal{G}^*(l_{t+1}^*(l', j'), j_{t+1}^*(l', j')).$$

b.3) Proof of equation (86).

Let  $(s, l^*, j^*) \in \mathcal{G}^*(l_{t+1}^*(l', j'), j_{t+1}^*(l', j'))$ . Then,  $(s, l^*, j^*) \in \mathcal{P}^*$ ,

$$l_{t+1}^*(l', j') = n^*(s, l^*, j^*) \tag{89}$$

$$= \max \left\{ \begin{array}{l} \min \{ (1 - \pi_n) l^* + j^*, \underline{n}^*(s) \} \\ \min \{ (1 - \pi_n) l^*, \bar{n}^*(s) \} \end{array} \right\}, \tag{90}$$

and

$$(1 - \pi_n) l_{t+1}^*(l', j') + j_{t+1}^*(l', j') = (1 - \pi_n) n^*(s, l^*, j^*) + v^*(s, l^*, j^*) \quad (91)$$

$$= \max \{ \bar{v}^*(s), (1 - \pi_n) l_{t+1}^*(l', j') \}. \quad (92)$$

Observe that  $(s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*)) \in \mathcal{P}_t$ ,

$$n_t(s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*)) = \max \left\{ \begin{array}{l} \min \left\{ (1 - \pi_n) [l_t^*]^{-1}(l^*, j^*) + [j_t^*]^{-1}(l^*, j^*), \underline{n}_t(s) \right\} \\ \min \left\{ (1 - \pi_n) [l_t^*]^{-1}(l^*, j^*), \bar{n}_t(s) \right\} \end{array} \right\},$$

and

$$\begin{aligned} & (1 - \pi_n) n_t(s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*)) + v_t(s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*)) \\ &= \max \left\{ \bar{v}_t(s), (1 - \pi_n) n_t(s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*)) \right\}. \end{aligned}$$

Also, from equation (77), we have that

$$\left[ s', n_t \left[ s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*) \right], v_t \left[ s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*) \right] \right] \in \mathcal{P}_{t+1}$$

for every  $s'$ .

Moreover,

$$\begin{aligned} & l_{t+1}^*(n_t \left[ s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*) \right], v_t \left[ s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*) \right]) \\ &= n^*(s, l^*, j^*) \end{aligned}$$

and

$$\begin{aligned} & j_{t+1}^*(n_t \left[ s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*) \right], v_t \left[ s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*) \right]) \\ &= v^*(s, l^*, j^*) \end{aligned}$$

Hence, from equations (89) and (91), we have that

$$\begin{aligned} & l_{t+1}^*(n_t \left[ s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*) \right], v_t \left[ s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*) \right]) \\ &= l_{t+1}^*(l', j') \end{aligned}$$

and

$$\begin{aligned} & j_{t+1}^*(n_t \left[ s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*) \right], v_t \left[ s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*) \right]) \\ &= j_{t+1}^*(l', j') \end{aligned}$$

It follows that

$$\begin{aligned} l' &= n_t(s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*)), \\ j' &= v_t(s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*)). \end{aligned}$$

Since  $(s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*)) \in \mathcal{P}_t$  it follows that

$$(s, [l_t^*]^{-1}(l^*, j^*), [j_t^*]^{-1}(l^*, j^*)) \in \mathcal{G}_t(l', j') \blacksquare$$

# Table 1

Quarterly observations

Panel A: BED data, March 2000 to June 2000						
Size	Data			Model		
Classes* (employees)	Shares in Employment	Shares in Job Gains	Shares in Job Losses	Shares in Employment	Shares in Job Gains	Shares in Job Losses
[1, 5)	6.4%	16.9%	9.7%	9.4%	14.6%	9.6%
[5, 10)	8.1%	13.1%	11.6%	6.8%	16.1%	18.4%
[10, 20)	10.7%	14.9%	13.7%	11.0%	15.6%	3.8%
[20, 50)	16.6%	18.3%	18.2%	16.5%	17.4%	15.6%
[50, 100)	13.1%	11.6%	12.6%	12.1%	11.6%	15.5%
[100, 250)	16.5%	11.9%	14.6%	16.5%	12.5%	13.2%
[250, 500)	9.8%	5.9%	8.5%	9.4%	4.6%	11.8%
[500, 1000)	7.3%	3.5%	5.4%	7.1%	7.7%	5.0%
[1000, $\infty$ )	11.6%	4.2%	5.9%	11.3%	0.0%	7.1%

  

Panel B: BED data, 1992:3-2005:4		
	Data	Model
size at entry	5.3	5.3
size at exit	5.2	5.3
JGB	1.7%	1.7%
JGE	6.2%	6.2%
JLD	1.6%	1.7%
JLC	6.0%	6.2%
Exit Rate	5.2%	6.9%

## Table 2

Monthly observations

Panel A: CPS data, 1948-2004		
	Data	Model
Separation rate	3.5%	3.5%
Hazard rate	46%	45%
Panel B: JOLTS data, 2000-2005		
	Data	Model
Vacancy rate	2.2%	2.3%
Hiring rate	3.2%	2.9%
Separation rate	3.1%	2.9%
Vacancies yield rate	1.3	1.2
% Vacancies with zero hiring	18.7%	19.6%
% Hiring with zero vacancies	42.3%	37.4%
% Establishments with zero hiring	81.6%	86.5%
% Establishments with zero vacancies	87.6%	93.1%

### Table 3

Calibrated parameter values

Parameter	Description	Value
$\varrho$	entry of establishments	0.000856916
$\delta$	capital depreciation rate	0.0055147
$\beta$	discount factor	0.9975517
$\theta$	capital share	0.216757
$\gamma$	labor share	0.64
$\pi_u$	quit rate, unemployed workers	0
$\pi_n$	quit rate, employed workers	0.00675
$\phi$	curvature, matching function	1.01614466 (efficient matching) 0.734344 (congestion externalities)
$\varphi$	utility of leisure	0.788949363 (efficient matching) 0.8098743 (congestion externalities)
$\rho_z$	persistence aggregate shocks	0.95
$\sigma_\varepsilon$	standard deviation aggregate shocks	0.0041

## Table 3 (Continued)

Calibrated idiosyncratic process

Idiosyncratic Productivity levels:

$$\begin{aligned}
 s_0 &= 0.00 & s_1 &= 6.2 & s_2 &= 6.9 & s_3 &= 7.5 & s_4 &= 8.6 \\
 s_5 &= 9.4 & s_6 &= 10.9 & s_7 &= 12.1 & s_8 &= 13.1 & s_9 &= 14.3
 \end{aligned}$$

Initial distribution:

$$\begin{aligned}
 \psi_0 &= 0.00 & \psi_1 &= 0.017 & \psi_2 &= 0.983 & \psi_3 &= 0.00 & \psi_4 &= 0.00 \\
 \psi_5 &= 0.00 & \psi_6 &= 0.00 & \psi_7 &= 0.00 & \psi_8 &= 0.00 & \psi_9 &= 0.00
 \end{aligned}$$

Transition matrix:

$$Q = \begin{pmatrix}
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0.0028 & 0.8157 & 0.1815 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0.0544 & 0.1905 & 0.7500 & 0.0051 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0.0105 & 0.9703 & 0.0192 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0.0330 & 0.9309 & 0.0361 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0.0777 & 0.9119 & 0.0104 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0.0245 & 0.9571 & 0.0184 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0.0612 & 0.9262 & 0.0126 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0325 & 0.9335 & 0.0340 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0356 & 0.9644
 \end{pmatrix}$$

# Table 4

Business Cycle Statistics: U.S. economy

A. Macroeconomic variables: 1951:1-2004:4							
Standard deviations							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.58	0.90	6.76	0.68	1.00	0.99		
Correlations with output							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.00	0.80	0.91	0.05	0.80	0.79		
B. Labor market variables: 1951:1-2004:4							
Standard deviations							
<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>	
1.58	1.00	12.32	13.89	25.66	7.47	12.6	
Correlations matrix							
	<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>
<i>Y</i>	1.00	0.80	-0.84	0.90	0.89	0.16	-0.62
<i>N</i>		1.00	-0.87	0.88	0.89	-0.10	-0.36
<i>U</i>			1.00	-0.92	-0.98	-0.03	0.47
<i>A</i>				1.00	0.98	0.11	-0.59
<i>A/U</i>					1.00	0.08	-0.55
<i>JC</i>						1.00	-0.59

# Table 5

Business Cycle Statistics: Efficient Matching

A. Macroeconomic variables							
Standard deviations							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.14	0.27	6.58	0.45	0.46	0.71		
Correlations with output							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.00	0.75	0.99	0.18	0.96	0.99		
B. Labor market variables							
Standard deviations							
<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>	
1.14	0.46	5.97	3.56	8.08	2.37	2.49	
Correlations matrix							
	<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>
<i>Y</i>	1.00	0.97	-0.96	0.56	0.96	-0.17	-0.60
<i>N</i>		1.00	-1.00	0.39	0.91	-0.34	-0.47
<i>U</i>			1.00	-0.40	-0.91	0.34	0.47
<i>A</i>				1.00	0.74	0.58	-0.88
<i>A/U</i>					1.00	0.08	-0.74
<i>JC</i>						1.00	-0.40



# Table 6

Business Cycle Statistics: Matching externalities

A. Macroeconomic variables							
Standard deviations							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.49	0.32	8.63	0.59	1.01	0.54		
Correlations with output							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.00	0.68	0.99	0.16	0.98	0.92		
B. Labor market variables							
Standard deviations							
<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>	
1.49	1.01	13.67	14.70	26.22	4.96	5.29	
Correlations matrix							
	<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>
<i>Y</i>	1.00	0.98	-0.97	0.73	0.91	-0.19	-0.51
<i>N</i>		1.00	-0.99	0.67	0.89	-0.29	-0.43
<i>U</i>			1.00	-0.71	-0.92	0.29	0.42
<i>A</i>				1.00	0.93	0.34	-0.79
<i>A/U</i>					1.00	0.04	-0.66
<i>JC</i>						1.00	-0.45

# Table 7

Business Cycle Statistics: Efficient allocation for economy with matching externalities

A. Macroeconomic variables							
Standard deviations							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.22	0.29	7.09	0.49	0.59	0.67		
Correlations with output							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.00	0.73	0.99	0.18	0.96	0.97		
B. Labor market variables							
Standard deviations							
<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>	
1.22	0.59	5.80	4.50	8.46	2.79	2.98	
Correlations matrix							
	<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>
<i>Y</i>	1.00	0.96	-0.96	0.52	0.93	-0.28	-0.50
<i>N</i>		1.00	-1.00	0.33	0.86	-0.21	-0.34
<i>U</i>			1.00	-0.34	-0.86	0.21	0.34
<i>A</i>				1.00	0.76	0.71	-0.87
<i>A/U</i>					1.00	0.23	-0.74
<i>JC</i>						1.00	-0.70

# Table 8

Business Cycle Statistics: Efficient allocation for economy with linear matching technology

A. Macroeconomic variables							
Standard deviations							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.80	0.38	10.82	0.71	1.47	0.44		
Correlations with output							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.00	0.70	0.98	0.16	0.99	0.82		
B. Labor market variables							
Standard deviations							
<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>	
1.80	1.47	20.99	9.40	24.47	8.36	7.89	
Correlations matrix							
	<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>
<i>Y</i>	1.00	0.99	-0.96	0.22	0.91	0.18	-0.07
<i>N</i>		1.00	-0.98	0.17	0.90	0.16	-0.04
<i>U</i>			1.00	-0.18	-0.92	-0.16	0.04
<i>A</i>				1.00	0.54	0.73	-0.67
<i>A/U</i>					1.00	0.42	-0.30
<i>JC</i>						1.00	-0.74

# Table 9

Business Cycle Statistics: Reallocation shocks

A. Macroeconomic variables							
Standard deviations							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.50	0.35	8.63	0.58	1.03	0.53		
Correlations with output							
<i>Y</i>	<i>C</i>	<i>I</i>	<i>K</i>	<i>N</i>	<i>Y/N</i>		
1.00	0.73	0.99	0.17	0.98	0.93		
B. Labor market variables							
Standard deviations							
<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>	
1.50	1.03	11.90	7.97	18.31	4.00	6.53	
Correlations matrix							
	<i>Y</i>	<i>N</i>	<i>U</i>	<i>A</i>	<i>A/U</i>	<i>JC</i>	<i>JD</i>
<i>Y</i>	1.00	0.98	-0.98	0.67	0.93	-0.39	-0.42
<i>N</i>		1.00	-1.00	0.66	0.93	-0.44	-0.39
<i>U</i>			1.00	-0.68	-0.95	0.43	0.38
<i>A</i>				1.00	0.88	0.27	-0.83
<i>A/U</i>					1.00	-0.16	-0.61
<i>JC</i>						1.00	-0.44