Household Leverage and the Recession*

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Abstract

A salient feature of the recent U.S. recession is that regions (states, counties) that have experienced the largest declines in household leverage have also experienced the largest declines in output and employment, a pattern difficult to explain with standard models of financing frictions. We propose a theory that can account for these cross-sectional facts. We study a cash-in-advance economy in which home equity borrowing, alongside public money, is used to conduct transactions. A decline in home equity borrowing tightens the cash-in-advance constraint, thus triggering a recession. We show that the evidence on house prices, leverage and employment across US regions identifies the key parameters of the model. Models estimated with cross-sectional evidence display high sensitivity of real activity to nominal credit shocks. Since home equity borrowing and public money are, in the model, perfect substitutes, our counter-factual experiments suggest that monetary policy actions have significantly reduced the severity of the recent recession.

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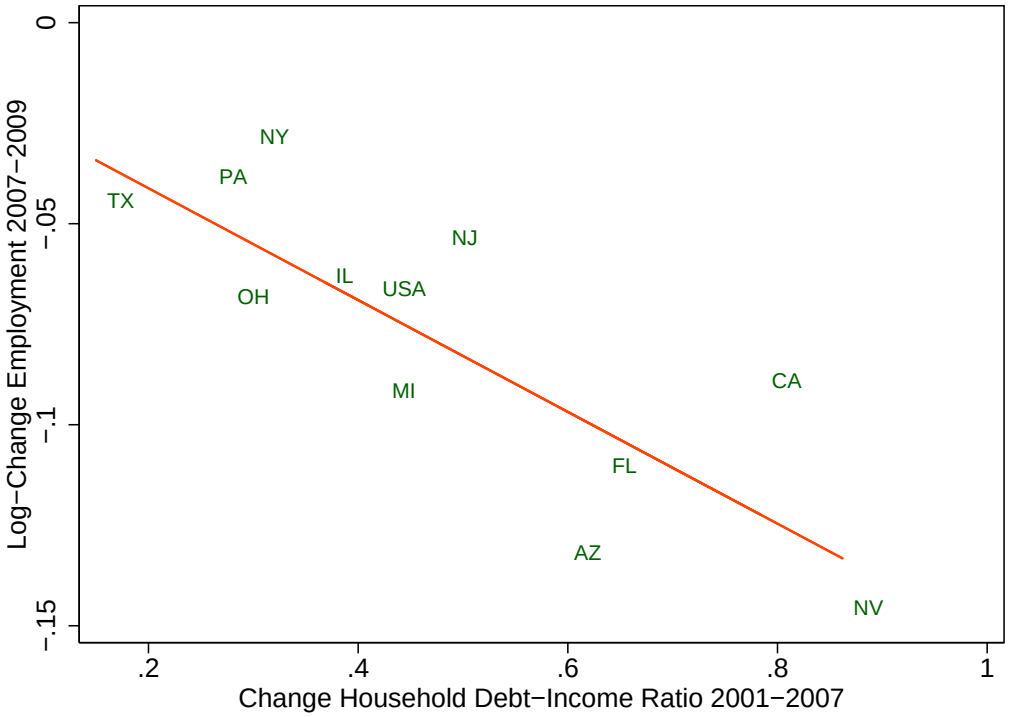
A striking feature of the recent recession is that regions of the U.S. (states or counties) that have experienced the largest swings in household borrowing have also experienced the largest declines in employment and output. Figure 1 illustrates this feature of the data, by showing the relationship between household borrowing during the credit boom and the change in employment during the subsequent credit bust.[1]

Figure 1: Borrowing predicts Employment across U.S. States



This pattern in the data is at odds with the predictions of standard models of financing frictions. Such models predict that a tightening of borrowing constraints at the household level leads to a decline in consumption but, due to wealth effects, to an increase in the supply of labor.[2] Non-standard preferences that attenuate wealth effects can mute the counterfactual responses of output and employment, but cannot, on their own, reproduce the striking correlation between household debt and employment in the data.

Our goal in this paper is to propose a theory that can account for the evidence in Figure 1, as well as other important cross-sectional features of the recent U.S. recession. We parametrize the theory to allow

[1] See also Mian and Sufi (2010a) andMian and Sufi (2010b) who carefully document this pattern in a cross-section of MSAs in the U.S.

[2] See Chari, Kehoe, and McGrattan (2005). The small open economy "Sudden Stop" literature (see, e.g. Mendoza (2010)) addresses this issue by postulating that the tightening of credit reduces the firms' ability to finance working capital. We discuss these issues below.

it to account for the salient features of the dynamics of leverage and employment in a cross-section of U.S. states and study the aggregate response of our model economy to credit shocks.

The model we study is a cash-in-advance economy with a continuum of islands that trade with each other. Each island produces tradeable and non-tradeable goods subject to a constant-returns technology. Tradeable goods produced on different islands are imperfectly substitutable.

Our key departure from standard cash-in-advance models is that, in addition to public (government-issued) money, households can use home equity borrowing in order to conduct transactions. The amount of home equity borrowing is limited by a collateral constraint: households can only borrow up to a fraction, θ, of the value of their home. These assumptions have two important consequences. First, homes provide liquidity services in addition to housing services. The liquidity services depend on the price of homes relative to consumption goods, the shadow value of liquidity, and the value of the collateral constraint, θ. Home prices therefore depend on current and expected values of θ. Second, home prices affect the amount of nominal balances that can be used to finance consumption expenditures. From a monetary perspective, an increase in real estate wealth effectively increases the velocity of money. This is the channel through which our model generates business cycles from nominal credit shocks. A decline in borrowing tightens the cash-in-advance constraint and amplifies the transactions frictions, thus leading to a recession. Absent the cash-in-advance constraint such a decline would involve no real transfers of resources from one island to another and would have no effect on real activity.[3]

In addition to the cash-in-advance and collateral constraints, we introduce two frictions that allow our model to account for the pattern of the data presented in Figure 1. First, nominal wage rigidities translate the decline of nominal consumption expenditures into real consumption spending. Second, we introduce frictions that prevent the immediate re-allocation of labor from the non-tradeable to the tradeable goods sector. Without this friction a negative credit shock leads to an expansion of the tradeable sector which can quickly undo the effect of the credit tightening by increasing the inflow of public money from other islands.

In our model, three parameters determine the aggregate and cross-sectional responses to the large swings in housing wealth observed in the data. The first two parameters are the degree of wage stickiness and the degree of labor mobility. As discussed above, both of these frictions amplify the response of employment to a decline in home equity borrowing. We therefore pin down the size of these parameters by requiring that the model reproduces the relationship between measures of real activity across U.S. states (construction and

[3]We note that the cash-in-advance constraint is not critical to our results. An alternative model with a tightening of the ability to borrow intertemporally in order to smooth consumption would produce similar results. We choose the cash-in-advance formulation because it allows us to discuss monetary policy responses.

non-construction employment) and measures of household leverage.

The third key parameter, θ, determines the fraction of consumption expenditures that were financed out of home equity borrowing during the upturn preceding the recession. To pin down the size of this parameter, we turn to the evidence from Mian and Sufi (2010a). These researchers argue that borrowing against the value of one's home accounts for a significant fraction of the rise in U.S. household leverage from 2002 to 2006. They use household-level data for a sample of 74,000 homeowners in different geographic regions of the U.S. and instrument house price growth using proxies for housing supply elasticities at the MSA level. In doing so they find that a 1$ increase in house prices *causes* a $0.25 increase in home equity debt. We use their findings to pin down the third key parameter in our model. To give a sense of the magnitude of this parameters, our calibration implies a marginal propensity to consume out of housing wealth of 6.6 cents on the dollar.

We use the model to study its predictions about the effect of the credit boom of 2001-2007 (a 50% increase in the debt-to-income ratio) and subsequent bust on measure of aggregate economic activity. We study two experiments. In the first experiment, we assume that monetary policy (the supply of public money) is constant throughout. In the second experiment we assume that public money expands by 7% of GDP, in line with the expansion of the Fed's balance sheet in the U.S. Absent the Fed intervention, the model predicts an 8.5% drop in non-construction employment (5.3% in the data) and a 7.2% drop in non-durable consumption (2.7% in the data). As in the data, the response of durable consumption (20.5% the model vs 14% in the data, respectively), is a lot more severe due to the much stronger intertemporal substitution for durables. The model thus over-predicts the decline in real activity observed in the data.

In contrast, when we allow for a 7% expansion of public money, the model's predictions for the decline in real activity are in line with the data: a decline of non-construction employment of 5.5% (5.3% in the data), non-durable consumption of 3.8% (2.7%) in the data and durable consumption of 13% (14% in the data). Our model thus suggests that absent the Fed intervention the employment and consumption declines would have been more than 50% larger.

Relation to the literature

Our paper is related to four lines of research: (i) macroeconomic models with credit frictions, (ii) monetary economics, (iii) real estate wealth; (iv) determinants of consumer spending. We discuss the connections of our paper to each topic. Following Bernanke and Gertler (1989), most macroeconomic papers introduce credit constraints at the entrepreneur level (Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999)). In all these models, the availability of credit limits *corporate* investment. As a result, credit constraints affect

the economy by affecting the size of the capital stock. Gertler and Kiyotaki (2010) study a model where shocks that hit the financial intermediation sector lead to tighter borrowing constraints for entrepreneurs. We model shocks in a similar way. The difference is that our borrowers are households, not entrepreneurs, and, we argue, this makes a difference for the model's cross-sectional implications. Models that emphasize firm-level frictions cannot reproduce the strong correlation between household-leverage and employment at the micro-level, unless the banking sector is island-specific, as in the small open economy "Sudden Stop" literature (Mendoza (2010)). This "local lending channel" does not appear to be operative across U.S. states, however, presumably because business lending is not very localized.[4]

On the monetary side, we follow the cash-in-advance literature of Lucas (1980) and Lucas and Stokey (1987). We introduce home equity borrowing and show that velocity becomes a function of home prices. In the model, stricter lending standards lead to a drop in real estate value, which decreases the spending power of consumers. In terms of classical monetary economics our model interpretes the recession as a large drop in velocity. We also study monetary responses to the crisis, and in particular non-standard interventions as in Gertler and Karadi (2009) and Curdia and Woodford (2009).

Our paper is related to the literature on housing wealth and consumption. Like Iacoviello (2005) we study a model where housing wealth can be used as collateral for loans. In his model, these are loans to entrepreneurs, while in our model, these are loans to households. Moreover, as emphasized above, the role of credit in our model is to facilitate transactions, not to smooth consumption intertemporally. Our paper is also related to two recent papers that focus on home prices dynamics. Favilukis, Ludvigson, and Van Nieuwerburgh (2009) emphasize the role of time-varying risk premia in observing the recent increase and declines in housing prices. Burnside, Eichenbaum, and Rebelo (2011) emphasize heterogeneous expectations about long-run fundamentals and "social dynamics." Our paper is less concerned with the exact source of house price movements, but rather with their their effects on real activity in the aggregate and in the cross-section.

Finally, our paper is related to the literature on the determinants of consumer spending, in particular the responses to fiscal stimulus. Our model is consistent with the findings of Johnson, Parker, and Souleles (2006) and Parker, Souleles, Johnson, and McClelland (2011).[5] A convenient assumption in the model is

[4]For instance, Mian and Sufi (2010b) find that the predictive power of household borrowing remains the same in counties dominated by national banks. It is also well known that businesses entered the recession with historically strong balanced sheets and were able to draw on existing credit lines (Ivashina and Scharfstein, 2008).

[5]Johnson, Parker, and Souleles (2006) find that, in 2001, "households spent 20 to 40 percent of their rebates on nondurable goods during the three-month period in which their rebates arrived, and roughly two-thirds of their rebates cumulatively during this period and the subsequent three-month period." Parker, Souleles, Johnson, and McClelland (2011) find that, in 2008, "households spent about 12-30% of their stimulus payments on non- durable expenditures during the three-month period in

that each island is populated by one representative household. A potential issue with this assumption is that "all" consumers are liquidity constrained. While we make this assumption for tractability, it does not need to imply implausible consumption patterns. Parker, Souleles, Johnson, and McClelland (2011) find that even self-reported savers spend a significant fraction of the payments received from the government, and recent work by Kaplan and Violante (2011) show that agents can be at the same time wealthy and liquidity constrained. Our paper is also related to the recent work of Guerrieri and Lorenzoni (2010) who provide a rationale for the sharp drop in interest rates due to the household-level credit crunch. Unlike these researchers, we emphasize the cross-sectional facts and focus on understanding the dynamics of employment.

Methodologically, we share our emphasis on cross-sectional information with Nakamura and Steinsson (2011). They study the effect of military procurement spending across U.S. regions, and they also emphasize the role of nominal rigidities and the power of cross-sectional evidence for identifying key model parameters. In both models differences in island-level employment dynamics are unaffected by aggregate-level shocks which are difficult to isolate: for example productivity shocks, changes in monetary policy, or foreign capital flows.[6] As a result, both our and their paper argue, cross-sectional statistics impose sharp restrictions on the set of parameter values that allow the model to match the data.

In Section 1 we present the model and we define the equilibrium. In Section 2 we study the qualitative and theoretical properties of the model in simplified setup. In Section 3 we propose a quantitative calibration and we study the response of the economy to various shocks.

1 Model

We study a closed economy with a continuum of islands that trade with each other. Each island produces tradeable and non-tradeable goods and is populated by a representative household. Means of payment are provided by the government and by private lenders (banks and shadow banks).

Our model can be interpreted as a large country with a collection of regions (e.g., USA), or a monetary union with a collection of states (e.g., EU). The key assumption are that these regions share a common currency, and that agents live and work in only one region.

which the payments were received," and that "there was also a substantial and significant increase in spending on durable goods, in particular vehicles, bringing the average total spending response to about 50-90% of the payments."

[6] It is worth emphasizing that these shocks would create first order issues in interpreting aggregate data. For instance, Favilukis, Ludvigson, and Van Nieuwerburgh (2009) show that foreign inflows can have a significant impact on aggregate house price dynamics. Similarly, calibrating the model's parameters using only with aggregate data would require to take a stand on controversial issues of monetary policy (Taylor, 2011).

1.1 Households

The household's preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u\left(\bar{c}_{i,t}, \bar{d}_{i,t}, h_{i,t}, \bar{l}_{i,t}\right)$$

where $\bar{c}_{i,t}$ denotes non durable consumption, $\bar{d}_{i,t}$ and $h_{i,t}$ are the stocks of durable goods and housing owned by the household, and $\bar{l}_{i,t}$ is an index of labor supplied. We motivate the demand for money with a constraint à la Clower (1967). An important feature of our model is that households have two sources of liquidity: cash and private credit. We assume that credit is collateralized by housing wealth while cash is not.

As in all cash-in-advance models, we must specify the timing of trades within a period. We follow the timing proposed by Lucas (1980).[7] Each period is divided into three stages. Money and banking markets open first. Households bring in pre-existing cash balances $X_{i,t-1}$ and obtain a credit line from private lenders, while the government engages in open market operations. We call M_{it} the government-issued cash in the hands of consumers after the open markets operations at time t, and B_{it} the amount of private credit available. In the second stage, each household splits into a worker and a shopper. The shopper can spend no more than $M_{i,t} + B_{i,t}$, while the worker supplies her labor. In the last stage of the period, the household receives its labor income and the profits distributed by the firms, repays the private lenders and carries over $X_{i,t}$ units of currency to the next period. Notice that that $B_{i,t}$ is within-period credit. The timing of the model is summarized in Table 1.

Table 1: Timing of Households Cash and Credit Flows

	Financial Trading	Shopping & Production	Payment Collection
Cash	$M_{i,t} = X_{i,t-1} + T_{i,t}$	$M_{i,t}$	$X_{i,t}$
Credit	0	$B_{i,t}$	$-(1+r)\,B_{i,t}$
Spending	0	$\bar{P}_{i,t}\bar{c}_{i,t} + Q_{i,t}y^h_{i,t} + \bar{V}_{i,t}\bar{e}_{i,t}$	0
Income	$T_{i,t}$	0	$\Pi_{i,t} + W_{i,t}\cdot l_{i,t}$

Let $Q_{i,t}$ be the price of houses on island i at time t, and let $y^h_{i,t} = h_{i,t} - (1-\delta_h)\,h_{i,t-1}$ denote the purchase of housing. Similarly, let $\bar{V}_{i,t}$ denote the price index for durable goods and $\bar{P}_{i,t}$ denote the price index for non-durable consumption. Let $\bar{e}_{i,t} = \bar{d}_{i,t} - (1-\delta_d)\,\bar{d}_{i,t-1}$ denote purchases of durable goods. The consumer

[7] Sargent and Smith (2009) discuss the importance of the timing of tax collection. This issue does not matter when we perform our cross-sectional analysis since we set taxes to zero. It can matter, however, when we consider various monetary policy responses in the last section of the paper. See also Lucas and Stokey (1987).

spends his balances on non-durables, durables and housing, subject to the cash & credit in advance constraint:

$$\bar{P}_{i,t}\bar{c}_{i,t} + Q_{i,t}y^h_{i,t} + \bar{V}_{i,t}\bar{e}_{i,t} \leq M_{i,t} + B_{i,t}, \tag{1}$$

Equation (1) says that firms accept to sell goods in exchange for bills printed by the government as well as units of credit backed by banks.[8] We assume that private credit for consumption must be collateralized by housing wealth. The amount of private credit is subject to the collateral constraint:

$$B_{i,t} \leq \theta_{i,t}Q_{i,t}h_{i,t}. \tag{2}$$

The parameter $\theta_{i,t}$ is exogenous, potentially island-specific, and the only source of shocks in this economy.

The household supplies three types of labor: to the non-tradeable, tradeable, and housing sectors. Each is industry-specific and aggregates into a final composite labor supply as:

$$\bar{l}_{i,t} = \left[\alpha_\tau \left(l^\tau_{i,t}\right)^\phi + \alpha_n \left(l^n_{i,t}\right)^\phi + \alpha_h \left(l^h_{i,t}\right)^\phi\right]^{\frac{1}{\phi}} \tag{3}$$

where $\phi \geq 1$ is a parameter that governs how substitutable different types of labor are and determines the degree to which labor can be reallocated across sectors. If $\phi = 1$, we have the model with perfect substitutability (mobility) across sectors, while as ϕ tends to ∞, the total amount of labor supplied is the maximum of what is supplied in each sector.

Since labor is sector-specific, wages differ across sectors. Let $W_{i,t}$ denote the vector of nominal wages in each sector and let $\Pi_{i,t}$ be the profits paid by private firms. At the end of the period, the liquidity position of the household is therefore: $X_{i,t} = \Pi_{i,t} + W_{i,t} \cdot l_{i,t} + M_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t} - Q_{i,t}y^h_{i,t} - \bar{V}_{i,t}\bar{e}_{i,t} - rB_{it}$. Finally, government implements monetary policy by printing new bills at the beginning of time t, and distributing them across islands: $M_{i,t+1} = X_{i,t} + T_{i,t+1}$. The flow budget constraint of the consumer is therefore

$$M_{i,t+1} = \Pi_{i,t} + W_{i,t} \cdot l_{i,t} + M_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t} - Q_{i,t}y^h_{i,t} - \bar{V}_{i,t}\bar{e}_{i,t} - rB_{it} + T_{i,t+1}. \tag{4}$$

The total amount printed by the government is simply $T_{t+1} = \int T_{i,t+1}$. In the remaining of the paper, we

[8] An equivalent interpretation of (1) is that houses are purchased with credit, and goods with both cash M_{it} and left-over credit $B_{it} - Q_{i,t}y^h_{i,t}$.

use the following specification for the utility function:

$$u\left(\bar{c}_{i,t},\bar{d}_{i,t},h_{i,t},\bar{l}_{i,t}\right)=\log\bar{c}_{i,t}+\xi\log\bar{d}_{i,t}+\eta\log h_{i,t}-\frac{\bar{l}_{i,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}.$$

1.2 Credit

We let $B_{i,t}$ is the credit provided by banks. Consumers use this credit, together with their holdings of public money, to purchase goods from firms. As in the search theory of money (see Lagos (2010) for a discussion and references), the idea is that consumers are anonymous to firms, but not to banks. Firms therefore cannot trust consumers to repay but they can go after the banks. Banks can keep track of consumers and seize a fraction $\theta_{i,t}$ of the collateral in case of default.

At the end of the period, the consumer repays $(1+r)B_t$ to the bank, and the bank pays B_t to the firm, thus making a profit equal to $\Pi_t^B = rB_t$. We assume free entry in the banking sector, thus in equilibrium we have $r=0$.[9] Finally, we assume that β and $\theta_{i,t}$ are low enough for the constraints (1) and (2) to bind in all islands at all times.

1.3 Wages

So far we have described the program of households as if there were no frictions in the labor market. In the quantitative experiments below we assume that wages are sticky.[10] The wage in sector k in island i at time t is given by

$$W_{i,t}^k=\left(W_{i,t-1}^k\right)^{\lambda}\left(W_{i,t}^{k*}\right)^{1-\lambda} \tag{5}$$

where $W_{i,t}^{k*}$ is the the frictionless nominal wage, implicitly defined by the labor-leisure choice:

$$\beta E_t\left[\frac{u_{\bar{c},it+1}}{\bar{P}_{i,t+1}}\right]W_{i,t}^{k*}=-u_{l^k,it}.$$

The parameter λ measures the degree of nominal rigidity. When $\lambda=1$ wages are fixed, and when $\lambda=0$ wages are fully flexible. Given the assumptions we have made on preferences, we can write the frictionless

[9] Also recall that B is within-period credit, i.e. credit flowing from workers to shoppers subject to the cash-advance-constraint. In that sense, B is really private money. The distinction between multi-period credit and within-period credit is not important as long as there are no dead weight losses from default. Analyzing costly defaults is important but clearly beyond the scope of this paper. In our calibration, we assume that home equity loans have a maturity of 5 years and we use the correct accounting to translate stocks into flows.

[10] What matters for our cross-sectional result is the stickiness of relative wages across islands. Our interpretation of rigidities as being nominal – denominated in currency common to all islands and controlled by a central bank – only matters in the last part of the paper when we analyze counter-factual monetary experiments.

wage as:

$$W_{i,t}^{k*} = \alpha_k \left(\bar{l}_{i,t}\right)^{\frac{1}{\nu}} \left(\frac{l_{i,t}^k}{\bar{l}_{i,t}}\right)^{\phi-1} \left(\beta E_t \left[\frac{1}{\bar{P}_{i,t+1}\bar{c}_{i,t+1}}\right]\right)^{-1}. \tag{6}$$

Our specification of wage rigidities is thus that of a partial-adjustment model in which a fraction $1-\lambda$ of the gap between the actual and desired wage is closed every period. Note that an alternative would be to explicitly model households as being represented by unions who face a constant hazard of resetting their wages, as in the Calvo model. Since we study the effect of permanent shocks, our conjecture is that this alternative specification, though more notationally burdensome, would produce very similar results. Notice finally that a higher ϕ makes it costlier for sectoral labor to adjust, by increasing the disutility for work and therefore the sectoral wage.

1.4 Housing

We next discuss the housing market. Let $\mu_{i,t}$ be the multiplier on the cash-in-advance constraint. The housing Euler equation is:

$$\frac{\eta}{h_{i,t}} + \mu_{i,t}\theta_{i,t}Q_{i,t} = \frac{Q_{i,t}}{\bar{P}_{i,t}\bar{c}_{i,t}} - \beta\left(1-\delta_h\right)E_t\left[\frac{Q_{i,t+1}}{\bar{P}_{i,t+1}\bar{c}_{i,t+1}}\right] \tag{7}$$

This equation is intuitive. Without the second term on the LHS, it would be a standard durable demand equation. $\frac{\eta}{h_{i,t}}$ is the marginal benefit of one extra unit of housing, and the RHS is the user cost. In our model, however, houses also provide liquidity services. The value of these services is $\mu_{i,t}$ and each unit of housing provides $\theta_{i,t}Q_{i,t}$ units of liquidity. Note that, using the consumption Euler equation we have that the shadow value of liquidity is $\mu_{i,t} = \frac{1}{\bar{P}_{i,t}\bar{c}_{i,t}} - \beta E_t \frac{1}{\bar{P}_{i,t+1}\bar{c}_{i,t+1}}$.

There is a housing construction sector on each island. Firms on each island can produce new houses using a decreasing return technology

$$y_{i,t}^h = \left(l_{i,t}^h\right)^{\chi}, \tag{8}$$

where χ determines the degree of decreasing returns. We allow decreasing returns in order to capture the role of land as a fixed factor in housing production. The aggregate stock of houses evolves according to:

$$h_{i,t} = \left(1-\delta\right)h_{i,t-1} + y_{i,t}^h \tag{9}$$

Since the price of new housing goods is $Q_{i,t}$, profit maximization by construction firms implies

$$W_{i,t}^h = \chi Q_{i,t} \left(l_{i,t}^h\right)^{\chi-1} \tag{10}$$

Profits of construction firms are simply $\Pi_{i,t}^h = (1-\chi) Q_{i,t} y_{i,t}^h$, and we assume for simplicity that construction firms are locally owned, so that $\Pi_{i,t}^h$ is paid to the household of island i.

1.5 Non-Durable Consumption

Household's consumption is an aggregate over the consumption of different varieties of tradable and non-tradable goods. We assume that the aggregation function has a constant elasticity of substitution σ between tradables and non tradables:

$$\bar{c}_{i,t} = [\omega_c^{\frac{1}{\sigma}} \left(\bar{c}_{i,t}^{\tau}\right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_c)^{\frac{1}{\sigma}} \left(c_{i,t}^n\right)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}},$$

where $\bar{c}_{i,t}^{\tau}$ is the consumption of the tradable good, $c_{i,t}^n$ is the consumption of the non-tradable good, and $\omega_c \in (0,1)$ is the weight on tradeables in the aggregator. The tradable good is itself an aggregate of the goods produced on different islands, with elasticity of substitution γ between goods produced on different islands:

$$\bar{c}_{i,t}^{\tau} = \left(\int_j c_{i,t}^{\tau}(j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

where j denotes the island where the good is produced. Let $\bar{P}_t^{\tau}$ denote the price index for tradable goods. It is common to all islands since we assume no trade costs, and it given by $\bar{P}_t^{\tau} \equiv \left(\int_i \left(P_{i,t}^{\tau}\right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$, where $P_{i,t}^{\tau}$ denotes the price at which the tradables produced on island i are sold. Let $P_{i,t}^n$ denote the price of non-tradable goods in island i. The total consumption price index on island i is: $\bar{P}_{i,t} \equiv \left[\omega_c \left(\bar{P}_t^{\tau}\right)^{1-\sigma} + (1-\omega_c) \left(P_{i,t}^n\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$. Demand for non-tradables is:

$$c_{i,t}^n = (1-\omega_c) \left(\frac{P_{i,t}^n}{\bar{P}_{i,t}}\right)^{-\sigma} \bar{c}_{i,t} \tag{11}$$

The demand on island i for tradables produced by island j is:

$$c_{i,t}^{\tau}(j) = \omega_c \left(\frac{P_{j,t}^{\tau}}{\bar{P}_t^{\tau}}\right)^{-\gamma} \left(\frac{\bar{P}_t^{\tau}}{\bar{P}_{i,t}}\right)^{-\sigma} \bar{c}_{i,t} \tag{12}$$

1.6 Durables consumption

Investment in durables is also an aggregator over purchases of different varieties of tradable and non-tradable goods. We assume that the aggregation function has the same constant elasticity of substitution σ between tradables and non tradables as for consumption goods:

$$\bar{e}_{i,t} = [\omega_d^{\frac{1}{\sigma}} \left(\bar{e}_{i,t}^{\tau}\right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_d)^{\frac{1}{\sigma}} \left(e_{i,t}^{n}\right)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}},$$

where $\bar{e}_{i,t}^{\tau}$ are purchases of the tradable good to be used for investment, $e_{i,t}^{n}$ are purchases of the non-tradable good to be used for investment, and $\omega_d \in (0,1)$ is the weight on tradeables in the investment aggregator. The tradable investment good is itself an aggregate of the goods produced on different islands, with elasticity of substitution γ between goods produced on different islands:

$$\bar{e}_{i,t}^{\tau} = \left(\int_j e_{i,t}^{\tau}(j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

where j denotes the island where the good is produced. Let $\bar{V}_t^{\tau}$ denote the price index for tradable goods. This price index is common to all islands since we assume no trade costs, and it given by $\bar{V}_t^{\tau} \equiv \left(\int_i \left(P_{i,t}^{\tau}\right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} = \bar{P}_t^{\tau}$, where, recall, $P_{i,t}^{\tau}$ denotes the price at which the tradables produced on island i are sold. Also recall that $P_{i,t}^{n}$ is the price of non-tradable goods in island i. The total investment price index on island i is: $\bar{V}_{i,t} \equiv \left[\omega_d \left(\bar{P}_t^{\tau}\right)^{1-\sigma} + (1-\omega_d)\left(P_{i,t}^{n}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$. Notice the only reason the durable and non-durable indices may differ is because of the differences in the weight of tradeables in the two aggregators. Demand for non-tradable goods used for investment is:

$$e_{i,t}^{n} = (1-\omega_d)\left(\frac{P_{i,t}^{n}}{\bar{V}_{i,t}}\right)^{-\sigma} \bar{e}_{i,t} \tag{13}$$

The demand on island i for tradables produced by island j is:

$$e_{i,t}^{\tau}(j) = \omega_d \left(\frac{P_{j,t}^{\tau}}{\bar{P}_t^{\tau}}\right)^{-\gamma} \left(\frac{\bar{P}_t^{\tau}}{\bar{V}_{i,t}}\right)^{-\sigma} \bar{e}_{i,t} \tag{14}$$

tradeable and non-tradeable. Finally, investment in durables satisfies the Euler equation

$$\frac{\xi}{\bar{d}_{i,t}} = \frac{\bar{V}_{i,t}}{\bar{P}_{i,t}\bar{c}_{i,t}} - \beta(1-\delta_d)E_t\left[\frac{\bar{V}_{i,t+1}}{\bar{P}_{i,t+1}\bar{c}_{i,t+1}}\right], \tag{15}$$

and durable goods accumulate according to

$$\bar{d}_{i,t} = \bar{e}_{i,t} + (1 - \delta_d)\, \bar{d}_{i,t-1} \tag{16}$$

1.7 Production and Market Clearing

We assume perfect competition in both tradables and non-tradables, as well as the housing construction sector. Each island is inhabited by a continuum of firms that produce a tradable good, and a continuum of firms that produce a non-tradable good. We also assume that labor is the only factor and that production is constant returns to scale:

$$y^n_{i,t} = l^n_{i,t} \quad \text{and} \quad y^\tau_{i,t} = l^\tau_{i,t} \tag{17}$$

Because of perfect competition the price of both tradable and non-tradable goods is equal to the nominal marginal cost in each sector on the island: $P^\tau_{i,t} = W^\tau_{i,t}$, and similarly $P^n_{i,t} = W^n_{i,t}$. Tradeable and non-tradeable goods are used for consumption and investment. Market clearing therefore requires

$$y^n_{i,t} = c^n_{i,t} + e^n_{i,t}, \tag{18}$$

in the non tradable sector, and

$$y^\tau_{i,t} = \int_{j \in [0,1]} \left(c^\tau_{j,t}(i) + e^\tau_{j,t}(i) \right), \tag{19}$$

in the tradable sector.

1.8 Equilibrium

We assume exogenous shocks to the tightness of borrowing constraints $\theta_{i,t}$. We will later discuss the interpretation of these shocks. To complete the description of the economy, we need to specify the monetary and fiscal policy. In equilibrium an island's cash holdings evolve according to (4). The transfers $\{T_{i,t}\}_{i,t}$ and money supplies $\{M_{i,t}\}_{i,t}$ must be consistent with the budget constraints of the government, and island-level money holdings follow the process

$$M_{i,t} = M_{i,t-1} + P^\tau_{i,t-1} y^\tau_{i,t-1} - \bar{P}^\tau_{t-1} \left(\bar{c}^\tau_{i,t-1} + \bar{e}^\tau_{i,t-1} \right) + T_{i,t}. \tag{20}$$

For most of our analysis we simply assume that the aggregate stock of currency remains constant, and we

normalize it to $M_t = 1$ and $T_{i,t} = 0$ for all i and t.[11]

An equilibrium is a collection of prices and allocations. Since the list is long, it is more convenient to use some equilibrium conditions to limit the number of equilibrium objects. From the pricing conditions $P^\tau_{i,t} = W^\tau_{i,t}$ and $P^n_{i,t} = W^n_{i,t}$, we can define the tradable price index $\bar{P}^\tau_t$ and the island specific price indices $\bar{P}_{i,t}, \bar{V}_{i,t}$, as a function of wages. Therefore we only need to include $Q_{i,t}, W^\tau_{i,t}, W^n_{i,t}, W^h_{i,t}$, in the list of equilibrium prices. Given these prices, real non durable expenditures $\bar{c}_{i,t}$ determine local demand $c^n_{i,t}$ and bilateral demands $c^\tau_{i,t}(j)$ by (11) and (12). Similarly, real durable expenditures $\bar{e}_{i,t}$ determine $e^n_{i,t}$ and $e^\tau_{i,t}(j)$ by (13) and (14). Labor inputs determine production in (8) and (17) and the labor index $\bar{l}_{i,t}$ in (3). Finally, the two stock variables $h_{i,t}, \bar{d}_{i,t}$ are simply pinned down by (9) and (16).

The equilibrium is thus defined by the four prices listed above and seven quantities: two for the credit market $B_{i,t}, M_{i,t}$, three for the labor market $l^n_{i,t}, l^\tau_{i,t}, l^h_{i,t}$, and two for the goods market $\bar{c}_{i,t}, \bar{e}_{i,t}$. The intuition for how we pin down the equilibrium is as follows. The three labor supply equations in (5), together with (6), pin down $W^\tau_{i,t}, W^n_{i,t}, W^h_{i,t}$. House prices $Q_{i,t}$ are pinned down by (7). (1), (2) pin down consumption $\bar{c}_{i,t}$ and borrowing $B_{i,t}$. (10), (19), (18) pin down $l^n_{i,t}, l^\tau_{i,t}, l^h_{i,t}$. (15) pins down $\bar{e}_{i,t}$, and (20) pins down $M_{i,t}$.

2 Qualitative Properties of a Simplified Model

We now study a special case to build some intuition about the effect of credit shocks in our model economy. In particular, we explain the difference between aggregate and island-level responses to credit shocks. To do so we consider a model without construction ($h = 1$ given exogenously and $\delta_h = 0$), with perfect labor mobility across sectors ($\phi = 1$), and without durable consumption ($\xi = 0$). Also, let $\omega = \omega_c$ denote the weight on tradeables in the consumption basket.

2.1 Nominal Credit and Velocity

Combining the CIA constraint (1) with the collateral constraint equation (2) we obtain a collateralized-credit-in-advance (CCIA) constraint: $\bar{P}_{i,t}\bar{c}_{i,t} = M_{i,t} + \theta_{i,t}Q_{i,t}h_{i,t}$. We define $x_{i,t}$ as nominal consumption spending in island i at time t, $x_{i,t} \equiv \bar{P}_{i,t}\bar{c}_{i,t}$, and $q_{i,t}$ as the housing wealth to spending ratio, $q_{i,t} \equiv \frac{Q_{i,t}h_i}{\bar{P}_{i,t}\bar{c}_{i,t}}$. The CCIA constraint then becomes

$$x_{i,t} = \frac{M_{i,t}}{1 - \theta_{i,t}q_{i,t}} \tag{21}$$

[11] Since in the aggregate we have $\int \left(P^\tau_{i,t-1}y^\tau_{i,t-1} - \bar{P}^\tau_{i,t-1}\left(\bar{c}^\tau_{i,t-1} + \bar{e}^\tau_{i,t-1}\right)\right) di = 0$ by the resource constraint, we have $\int T_{i,t} = M_t - M_{t-1}$. Nothing pins down transfers to individual islands, however and we use the no transfer case as a benchmark.

With these new variables, we can rewrite the house price equation (7) as

$$\eta + \beta E_t \left[q_{i,t+1} \right] = \left(1 - \theta_{i,t} \left(1 - \beta E_t \left[\frac{x_{i,t}}{x_{i,t+1}} \right] \right) \right) q_{i,t} \tag{22}$$

Equations (21) and (22) provide a lot of intuition for the model. Given processes for $M_{i,t}$ and $\theta_{i,t}$ we could solve for $x_{i,t}$ and $q_{i,t}$ using (21) and (22). This is what we do in a one-island economy with aggregate money supply M_t controlled by a central bank. Note that $\theta_{i,t} q_{i,t}$ acts as a shock to *velocity* in equation (21).

Across islands, however, $M_{i,t}$ evolves endogenously, for two reasons. First the central bank does not control the allocation of money across industries or locations within a country, and even less across countries in a monetary union. Second, islands accumulate or decumulate government money depending on the private credit shocks that they experience. In particular, it would never be optimal for a government to reset $M_{i,t} = 1$ at the beginning of each period. In our benchmark model, we set $T_{it} = 0$. Each island's money holdings are then an island-specific state variable.

2.2 Labor Markets and Consumption

Nominal wage setting is given by (5), and labor market clearing in each island implies $l_{i,t} = l^n_{i,t} + l^\tau_{i,t}$. Using $x_{i,t}$, we can rewrite the labor supply (6) as

$$\left(l^n_{i,t} + l^\tau_{i,t} \right)^{\frac{1}{\nu}} = W^*_{i,t} \beta E_t \left[x^{-1}_{i,t+1} \right]. \tag{23}$$

Trade and technology pin down labor demands. For local goods, we have $l^n_{i,t} = c^n_{i,t}$, which we can rewrite as

$$l^n_{i,t} = (1 - \omega) \frac{x_{i,t} W^{-\sigma}_{i,t}}{\bar{P}^{1-\sigma}_{i,t}}. \tag{24}$$

For traded goods, we have $l^\tau_{i,t} = \int_j c^\tau_{j,t}(i) dj$ which we can rewrite as

$$l^\tau_{i,t} = \omega W^{-\gamma}_{i,t} \left(\bar{P}^\tau_t \right)^{\gamma - \sigma} \int_j \frac{x_{j,t}}{\bar{P}^{1-\sigma}_{j,t}} \tag{25}$$

The price indexes are such that

$$\left(\bar{P}_{i,t} \right)^{1-\sigma} = \omega \left(\bar{P}^\tau_t \right)^{1-\sigma} + (1 - \omega) \left(W_{i,t} \right)^{1-\sigma} \tag{26}$$

and

$$\left(\bar{P}_t^{\tau}\right)^{1-\gamma} = \int_j \left(W_{j,t}\right)^{1-\gamma} \tag{27}$$

In this simplified system, we now have nine equations (5, 20, 21, 22, 23, 24, 25, 26, 27) and nine unknowns $\left\{q_{i,t}, x_{i,t}, M_{i,t}, l_{i,t}^n, l_{i,t}^{\tau}, W_{i,t}, W_{i,t}^*, \bar{P}_{i,t}, \bar{P}_t^{\tau}\right\}$.

2.3 One island economy

We first consider an economy without heterogeneity. In steady state, the resource constraint is: $\bar{c} = l$ and the labor-leisure condition implies $\bar{c}\bar{l}^{1/\nu} = \beta$. Therefore $l = \bar{c} = (\beta)^{\frac{\nu}{1+\nu}}$ and the only steady state distortion is the intertemporal wedge introduced by the cash-in-advance constraint. Equation (22) implies $\bar{q} = \frac{\eta}{(1-\beta)(1-\theta)}$, and (21) implies $x = \frac{M}{1-\theta\bar{q}}$, and the price level must be such that

$$\frac{M}{\bar{P}\bar{c}} = 1 - \bar{\theta}\bar{q} \tag{28}$$

The parameters must be such that $\bar{\theta}\bar{q} < 1$, or $(1-\beta)\left(1-\bar{\theta}\right) > \eta\bar{\theta}$. In particular, β, η and θ must all be small enough.

Consider the dynamics of credit first. Given processes $\{M_t\}_t$ and $\{\theta_t\}_t$ for aggregate money supply and credit tightness, the system can be solved for $\{x_t, q_t\}_t$ using (21) and (22) without reference to the rest of the model, i.e., independently of technology, nominal rigidity, and labor supply preferences. When $\theta = 0$, the solution is always $x_t = M_t$ as in the standard cash-in-advance model. When $\theta > 0$, house price or collateral shocks to are transmitted by the collateral constraint. In the one island economy, we have $W_t = \bar{P}_t$ and the equations for the price levels are trivial. We also have $\bar{c}_t = l_t$. Once we have solved for x_t and q_t we can therefore solve for W_t and l_t by using $W_t l_t = x_t$, $W_t = W_{t-1}^{\lambda}\left(W_t^*\right)^{1-\lambda}$, and $(l_t)^{\frac{1}{\nu}} = \beta W_t^* E_t\left[x_{t+1}^{-1}\right]$. Note that the labor shares are constant in the one island economy.[12]

Consider next the impact of a permanent, unanticipated shock to θ. When M and θ are constant, we have $q(\theta) = \frac{\eta}{(1-\beta)(1-\theta)}$ and $x(\theta) = \frac{M}{1-\theta q}$. After a permanent shock to the borrowing constraint, if monetary policy is unchanged, the economy evolves along a path with constant nominal spending. If the shock is positive, nominal spending jumps up and remains constant. We see that q is increasing in θ: if credit is easier to obtain, housing value must increase relative to consumption spending because the collateral dimension of housing services makes houses more valuable. Spending must go up because of both θ and q.

[12] Since $l_t^n = (1-\omega)\frac{x_t}{P_t}$ and $l_t^{\tau} = \omega\frac{x_t}{P_t}$, we always have $\frac{l_t^n}{l_t^n + l_t^{\tau}} = 1 - \omega$.

Going back to q, this means that housing prices must also increase so that even though spending goes up, house prices increase more than spending. Following a permanent shock, x is constant and since $W_t l_t = x$ and employment is

$$\ln(l_t) = \frac{\lambda\nu}{1-\lambda+\nu}\left(\ln(x) - \ln(W_{t-1})\right) + \frac{(1-\lambda)\nu}{1-\lambda+\nu}\ln(\beta),$$

while W satisfies

$$(1-\lambda+\nu)\ln W_t = \lambda\nu \ln W_{t-1} + (1-\lambda)\left((1+\nu)\log x - \nu\log\beta\right).$$

Without nominal rigidities (i.e., $\lambda = 0$) wages adjust immediately to nominal credit shocks and employment remains constant.[13] More generally, the persistence of the real effects following a permanent credit shock is given by $\frac{\lambda\nu}{1-\lambda+\nu}$. Persistence thus depends on the degree of nominal rigidity and on the elasticity of labor supply. If wages are fixed (i.e., $\lambda = 1$) the real impact of *aggregate* nominal credit shocks is permanent. We will show that this result does not hold in the cross-section.

2.4 Cross-sectional responses

Consider an economy in which islands differ in the tightness of the borrowing constraint, θ_i. Two issues arise at the island level. First, $M_{i,t}$ is endogenous since islands can accumulate more or less public money. Second, $W_{i,t}l_{i,t} \neq \bar{P}_{i,t}\bar{c}_{it}$ since some goods are traded. Both of these issues are reflected in the money accumulation equation: $M_{i,t+1} - M_{i,t} = W_{i,t}l_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t}$. Credit dynamics satisfy (22) and (20). The eight equilibrium conditions have been described earlier. It is easy to check that the steady state allocations satisfy $l_i = \bar{c}_i = (\beta)^{\frac{\nu}{1+\nu}} = \bar{c}$. Since $l_i^n = (1-\omega)\bar{c}$ and $l_i^\tau = \omega\bar{c}$, we always have $\frac{l_i^n}{l_i^n + l_i^\tau} = 1-\omega$. All wages are the same and $W_i = \bar{P}$. Therefore all x_i are equal in all islands The following Lemma summarizes the steady state prices and quantities

Lemma 1. *In the steady state, all islands have the same real allocations, the same wages, prices and the same nominal spending. Only house prices differ across islands. The aggregate price level solves*

$$\frac{M}{\bar{P}\bar{c}} = \int_i \left(1 - \frac{\eta\theta_i}{(1-\beta)(1-\theta_i)}\right) di. \tag{29}$$

[13] When $\lambda = 0$, we have $(l_t)^{\frac{1+\nu}{\nu}} = \beta E_t\left[\frac{x_t}{x_{t+1}}\right]$ so transitory shocks would still matter. This reflects the intertemporal distortion coming from the CIA constraint. The model without nominal friction is neutral with respect to permanent nominal credit shocks. It is not super-neutral because θ is not constant, then x moves around, and this creates intertemporal disturbances in labor supply but these distorsions are small.

The CIA constraints determine the money balances $M_i = (1 - \theta_i q_i)\,\bar{P}\bar{c}$ that implement these allocations. With constant x, we have $q_i = \frac{\eta}{(1-\beta)(1-\theta_i)}$. In the aggregate, we must have, $\int M_i = M$ so the price level must solve Equation (29) which is the generalization of (28) to an economy with heterogeneous nominal credit supplies. The Lemma states that differences in θ_i across islands do not translate into differences in prices or allocations. The reason is that islands with tighter constraints private credit accumulate public money. Since money and private credit are perfect substitutes, both prices and allocations (with the exception of house prices) are unaffected by the cross-sectional dispersion in θ_i.

Consider next the effect of an unanticipated, one-time shock to θ_i in any particular island. To study the responses to such a shock, we find it useful to study log-linear approximations to the equilibrium conditions. For any variable z_{it} we write $z_{i,t} = \bar{z}\,(1 + \hat{z}_t + \hat{z}_{it})$, where $\hat{z}_t$ is the solution to the one-island log-linear model, and the total log-change is $d\ln z_{i,t} = \hat{z}_t + \hat{z}_{it}$. We note that, up to a first-order approximation, the evolution of the aggregates in our model with heterogeneous islands is equivalent to the evolution of the one-island economy. Hence, we first characterize the one-island (aggregate) responses and then compute log-deviations of each island from the aggregate responses. From now on, we use the term "one-island" and "aggregate" interchangeably.

Consider first the island-level response of trade and labor demand. In the aggregate, we have that $P_t = W_t$. Around these aggregate dynamics, we have $\hat{l}^n_{i,t} = \hat{x}_{i,t} - \sigma\hat{W}_{i,t} - (1-\sigma)\,\hat{\hat{P}}_{i,t}$, $\hat{\hat{P}}_{i,t} = (1-\omega)\,\hat{W}_{i,t}$, and $\hat{l}^\tau_{i,t} = -\gamma\hat{W}_{i,t}$. We therefore have that $\hat{l}^n_{i,t} = \hat{x}_{i,t} - (1 - \omega\,(1-\sigma))\,\hat{W}_{i,t}$. Since $\hat{l}_{i,t} = (1-\omega)\,\hat{l}^n_{i,t} + \omega\hat{l}^\tau_{i,t}$, we obtain

$$\hat{l}_{i,t} = (1-\omega)\,\hat{x}_{i,t} - (\omega\gamma + (1-\omega)\,(1 - \omega\,(1-\sigma)))\,\hat{W}_{i,t}. \tag{30}$$

This equation links island-level employment to island-level nominal spending on non tradeable goods and island-specific wages. Compared to the aggregate economy, employment is less sensitive to (local) spending. The wage elasticity of labor demand depends on both elasticities γ and σ, and on the importance of traded goods ω.

Consider next the island-level responses of labor supply and the dynamics of wages: $\hat{W}_{i,t} = \lambda\hat{W}_{i,t-1} + (1-\lambda)\,\hat{W}^*_{i,t}$, and $\hat{l}_{i,t} = \nu\left(\hat{W}^*_{i,t} - E_t\left[\hat{x}_{i,t+1}\right]\right)$. Solving for the desired wage, we obtain an equation that describes wages dynamics as a function of total spending:

$$(1-\lambda)\left(\hat{l}_{i,t} + \nu E_t\left[\hat{x}_{i,t+1}\right]\right) = \nu\left(\hat{W}_{i,t} - \lambda\hat{W}_{i,t-1}\right). \tag{31}$$

Equation (31) is relevant only when $\lambda < 1$. When $\lambda = 1$, wages are fixed at their steady-state values.

The third and last part of the system describes credit dynamics. In the aggregate, we have $x_t = W_t l_t$. At the island level, we have:

$$\left(1-\bar{\theta}\bar{q}\right)\hat{x}_{i,t} - \bar{\theta}\bar{q}\hat{q}_{i,t} = \hat{W}_{i,t-1} + \hat{l}_{i,t-1} - \bar{\theta}\bar{q}\left(\hat{q}_{i,t-1} + \hat{x}_{i,t-1}\right) + \bar{\theta}\bar{q}\left(\hat{\theta}_{i,t} - \hat{\theta}_{i,t-1}\right), \tag{32}$$

and

$$\beta E_t\left[\hat{q}_{i,t+1} + \bar{\theta}\hat{x}_{i,t+1}\right] = \left(1-\left(1-\beta\right)\bar{\theta}\right)\hat{q}_{i,t} + \bar{\theta}\beta\hat{x}_{i,t} - \left(1-\beta\right)\bar{\theta}\hat{\theta}_{i,t}. \tag{33}$$

We therefore have a system of four equations (30, 31, 32, 33) in four endogenous unknowns $(\hat{W}_{i,t}, \hat{l}_{i,t}, \hat{x}_{i,t}, \hat{q}_{i,t})$ and one exogenous processes for $\theta_{i,t}$. We calibrate and solve the system numerically in Section 3, but much intuition can be gained by considering the special case of fixed wages.

We consider permanent shocks to $\theta_{i,t}$ so after the initial shock $\theta_{i,0}$ at $t = 0$, we have $\hat{\theta}_{i,t} = \hat{\theta}_{i,t-1}$ for $t = 1, ..., \infty$ and the credit system (32,33) is simplified. We also assume that relative wages do not change: $\hat{W}_{i,t} = 0$.[14] With constant relative wages we have $\hat{l}_{i,t} = \left(1-\omega\right)\hat{x}_{i,t}$, and the money accumulation equation (32) becomes:

$$\left(1-\bar{\theta}\bar{q}\right)\hat{x}_{i,t} - \bar{\theta}\bar{q}\hat{q}_{i,t} = \left(1-\omega-\bar{\theta}\bar{q}\right)\hat{x}_{i,t-1} - \bar{\theta}\bar{q}\hat{q}_{i,t-1}.$$

We 'guess and verify' a solution of the type:

$$\hat{q}_{i,t} = \tilde{q}_i - a\hat{x}_{i,t}. \tag{34}$$

The intuition for this guess comes from the model's implications for aggregate dynamics and the steady state cross section. In the aggregate, we know that permanent shocks to θ lead to constant values for x and q. This is not going to be the case here, so x will move, and q will be affected. In the cross sectional steady state, we have $q_i = \frac{\eta}{(1-\beta)(1-\theta_i)}$ so it is easy to guess that there must be a time invariant component to q. The money accumulation equation implies

$$\hat{x}_{i,t} = \left(1 - \frac{\omega}{1-\bar{\theta}\bar{q}\left(1-a\right)}\right)\hat{x}_{i,t-1}. \tag{35}$$

In the special case $\omega = 0$, we go back to the one island economy with constant x. The house pricing equation

[14] This could be either because wages are rigid in nominal terms, $\lambda = 1$, or because relative wages are fixed across islands. In the first case, we can drop equation (31). In the second case, we are simply saying $W_{it} = W_t$ in all islands. Empirically, this appears to be a reasonable approximation to the data. Theoretically, we know that $W_{it} = W_t$ in the long run. See below for a discussion of what happens if relative wages move.

becomes

$$\beta\left(\bar{\theta}-a\right)E_t\left[\hat{x}_{i,t+1}\right]+\beta\tilde{q}_i=\left(1-\left(1-\beta\right)\bar{\theta}\right)\left(\tilde{q}_i-a\hat{x}_{i,t}\right)+\bar{\theta}\beta\hat{x}_{i,t}-\left(1-\beta\right)\bar{\theta}\hat{\theta}_i$$

We can now identify the constant terms and the dynamic terms. For the constant term we get $\tilde{q}_i=\frac{\bar{\theta}}{1-\bar{\theta}}\hat{\theta}_i$. This is what we expected since the long run value for $\hat{q}_i$ implies $d\log q_i=-d\log\left(1-\theta_i\right)=\frac{\bar{\theta}}{1-\bar{\theta}}\hat{\theta}_i$. For the dynamic terms we get

$$E_t\left[\hat{x}_{it+1}\right]=\left(1-\frac{a\left(1-\beta\right)\left(1-\bar{\theta}\right)}{\beta\left(\bar{\theta}-a\right)}\right)\hat{x}_{it}$$

Under perfect foresight and using the law of motion (35), we obtain an equation for a:

$$\omega\left(\bar{\theta}-a\right)\beta=a\left(1-\beta\right)\left(1-\bar{\theta}\right)\left(1-\bar{\theta}\bar{q}\left(1-a\right)\right) \tag{36}$$

We can find a solution for a, which validates our initial guess in equation (34). If $\omega=0$, we have $a=0$ as in the one-island economy. When $\omega>0$, the LHS of (36) decreases and reaches zero when $a=\bar{\theta}$, while the RHS is zero when $a=0$ and increases afterward. There is therefore a unique solution $0<a<\bar{\theta}$. Equation (35) shows that the system is stable and $\lim_{t\to\infty}\hat{x}_{it}=0$.

In the cross section, permanent shocks have temporary consequences because money can flow across islands. The persistence of shocks at the island level does not depend much on the degree of nominal rigidity. This is in sharp contrast with the response of the aggregate economy. The reason is that islands that are hard hit by the nominal credit shock accumulate money balances

$$M_{i,t+1}-M_{i,t}=\bar{x}\left(\hat{l}_{i,t}-\hat{x}_{it}\right)=-\omega\bar{x}\hat{x}_{i,t}.$$

This shows again the role of trade in smoothing the cross-sectional shocks. The impact response, assuming we start from steady state with $\hat{\theta}_{i,t-1}=0$, is $\left(1-\bar{\theta}\bar{q}\right)\hat{x}_{i,0}-\bar{\theta}\bar{q}\hat{q}_{i,0}=\bar{\theta}\bar{q}\hat{\theta}_i$ and since $\hat{q}_{i,0}=\tilde{q}_i-a\hat{x}_{i,0}$ we have

$$\left(1-\left(1-a\right)\bar{\theta}\bar{q}\right)\hat{x}_{i,0}=\frac{\bar{q}}{1-\bar{\theta}}\bar{\theta}\hat{\theta}_i.$$

A positive shock to credit increases spending in the island.[15]

[15]Finally we can come back to our assumption of constant wages. If relative wages can move, they will help smooth the transition by making hard hit islands temporarily more competitive. Without this we force all the adjustment through consumption and nominal spending. But the main intuition should not change much. We can see which way wages want to adjust by looking at equation (31). Since $\hat{l}_{i,t}=\left(1-\omega\right)\hat{x}_{i,t}$ and since $x_{i,t}$ follows an AR(1) process, wages would like to follow an AR(2) process. We thus expect the response of wages to be hump-shaped. Following a negative shock, relative wages fall first, then rise back to one, the long run value. As long as labor supply is somewhat elastic, the response of wages is small, and the dynamics derived under the assumption of fixed wages give a good approximation.

2.5 Comparison of Time Series and Cross-Section

We finally compare the time-series and cross sectional responses of the economy to permanent shocks to credit supply. In the aggregate we have $q(\theta) = \frac{\eta}{(1-\beta)(1-\theta)}$ and $x(\theta) = \frac{M}{1-\theta q}$. Therefore, on impact, we have $d\ln q = \frac{\bar{\theta}}{1-\theta} d\ln\theta$ and thus $\frac{\partial \ln(x)}{\partial \ln(\theta)} = \frac{\bar{\theta}\bar{q}}{(1-\bar{\theta})(1-\bar{\theta}\bar{q})}$. Across islands, relative housing wealth evolves as $d\ln q_{i,t} = \tilde{q}_i - a\hat{x}_{i,t}$. The permanent component, $\tilde{q}_i = \frac{\bar{\theta}}{1-\theta}\hat{\theta}_i$, is the same as in the aggregate case. Because of the temporary component, however, the adjustment of relative housing wealth is gradual. Spending reacts according to: $\frac{\partial \ln(x_i)}{\partial \ln(\theta_i)} = \frac{\bar{\theta}\bar{q}}{(1-\bar{\theta})(1-(1-a)\bar{\theta}\bar{q})}$. The response of local spending to local credit is muted by a. For employment, we have $\frac{\partial \ln(l_{i,0})}{\partial \ln(x_{i,0})} = 1-\omega$. We summarize the employment responses in Table 2.

Table 3: Elasticities with Fixed Wages and Permanent Credit Shocks

$\lambda = 1$, $\rho = 1$	Aggregate	Across Islands
Spending to Credit $\frac{\partial \ln(x)}{\partial \ln(\theta)}$	$\frac{\bar{\theta}\bar{q}}{(1-\bar{\theta})(1-\bar{\theta}\bar{q})}$	$\frac{\bar{\theta}\bar{q}}{(1-\bar{\theta})(1-(1-a)\bar{\theta}\bar{q})}$
Labor to Spending $\frac{\partial \log(l)}{\partial \log(x)}$	1	$1-\omega$
Persistence	Permanent	Temporary

With fixed wages, spending is equal to real consumption. So Table 2 also shows that in the cross section, employment reacts by a fraction ω less than consumption, while in the aggregate it responds by as much as consumption does.

We summarize our results in the following Proposition.

Proposition 1. ***Positive Properties****. Cross sectional responses to credit shocks are muted in three ways relative to aggregate responses: (i) local spending reacts less to local credit because velocity effects are smaller; (ii) employment is less sensitive to local spending because of trade; and (iii) the effects dissipate over time because of endogenous adjustment in money balances.*

The following figures illustrate the proposition. We report some impulse responses to further illustrate the workings of the model. Figure 2 shows impulse responses to a 1% aggregate (common to all islands) drop in θ_t in this economy[16]. W^* drops immediately while actual wages adjust more gradually due to nominal rigidities. As a result consumption and employment drop. House prices drop because nominal spending drops and because the drop in θ_t makes houses less useful in undoing the borrowing constraints. The drop in B is therefore larger than the drop in θ and we have an amplification mechanism.

[16]We report the parameter values used in this calculation in Table 3 below.

Figure 2: Aggregate Response to Credit Tightening

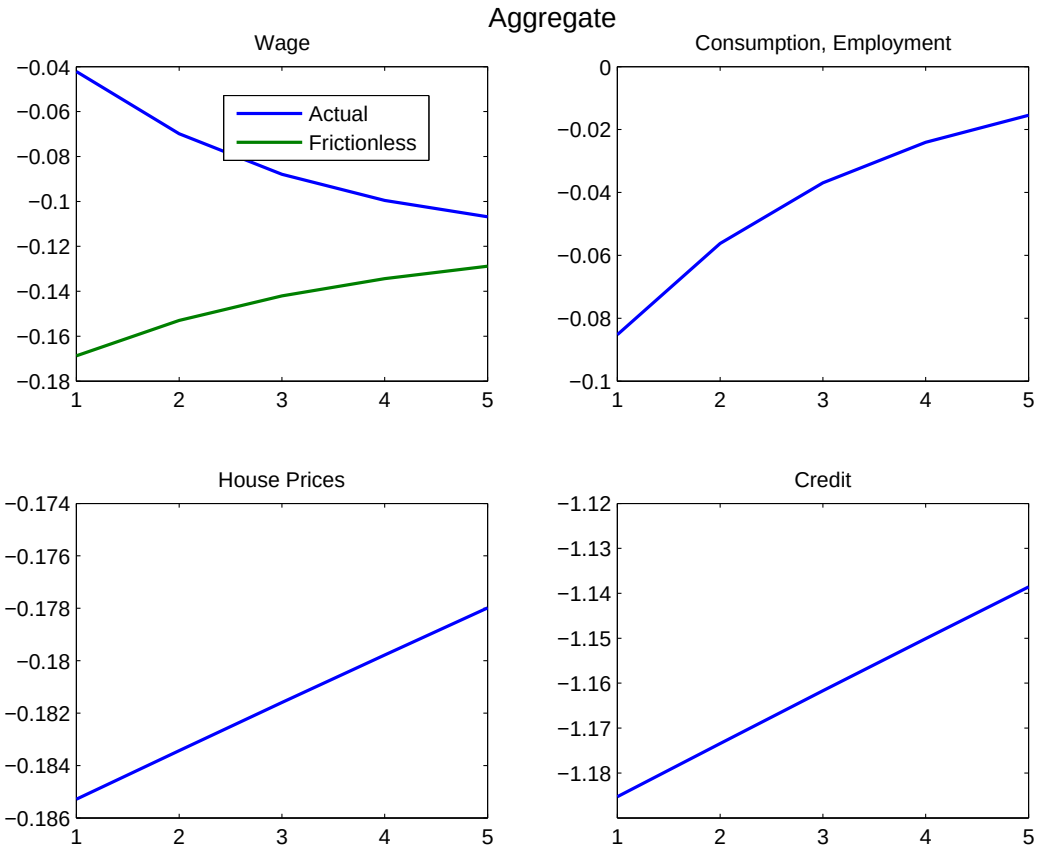
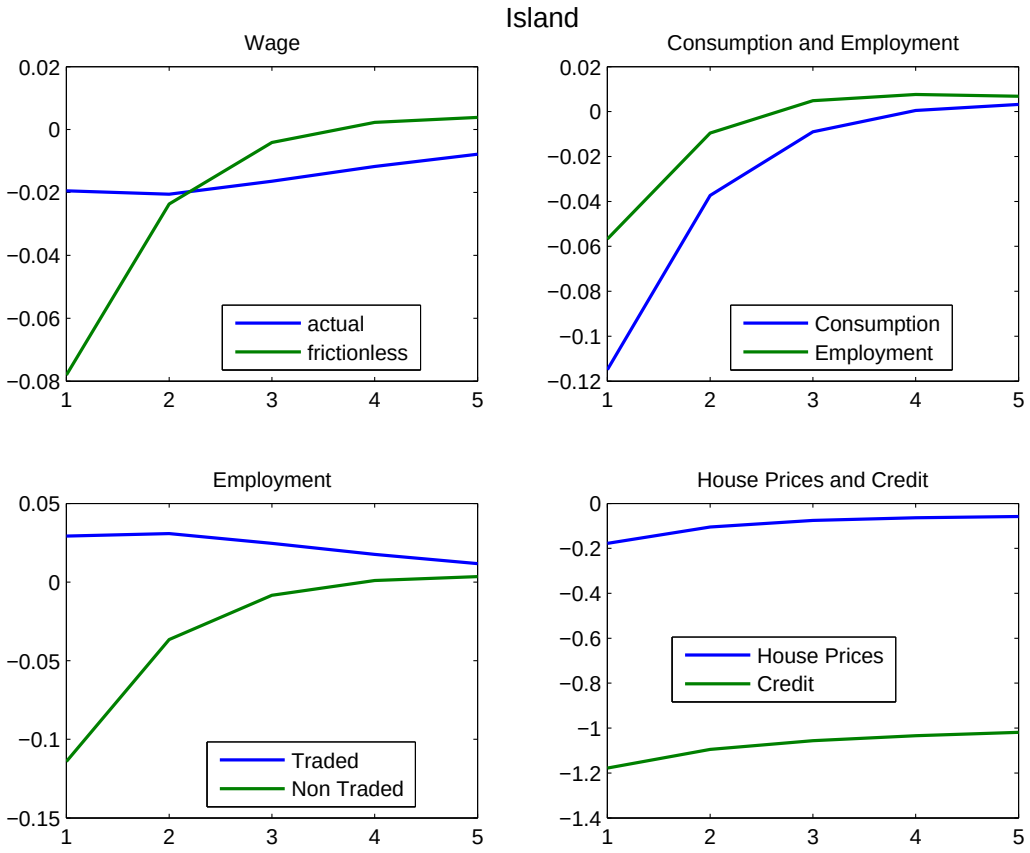


Figure 3 reports similar responses to an island-specific shock, $\theta_{i,t}$, assuming all other islands are at their steady-state values. Consumption responds by more (-0.9% on impact) than employment does (-0.45% on impact) because wages decrease in the island and hence demand for its tradeables increases. From the results of the previous section, we know that when shocks are permanent and wages rigid, the ratio of the response of l to that of c is equal to $1-\omega$, which is 0.58 for our benchmark value of $\omega = 0.42$. In the actual simulation, the ratio is 0.51, which is close to 0.58 but, as expected, slightly smaller since wages do adjust.

Figure 4 illustrates why all series are less persistent in the cross-section than in the aggregate by showing the evolution of nominal variables. The fact that consumption drops more than employment implies that the island accumulates public money, M, immediately after the shock. This increase in M compensates the decline in private credit, so that nominal spending reverts to the steady-state faster than in the aggregate.

Figure 3: Island Response to Credit Tightening



2.6 Some Normative Implications

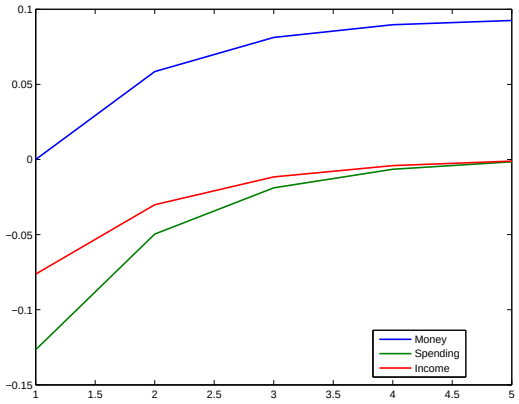
The focus of our paper is on the positive and quantitative properties of the model. However, in the interest of building intuition, it is useful to state two simple normative proposition.

The first normative proposition is that, absent any frictions on monetary and fiscal policies, the government can always maintain the steady state allocations by targeting nominal spending.

Proposition 2. ***Perfect Stabilization****. Let $\bar{x}$ be the steady state value of nominal spending (common to all islands from Lemma 1). If the government adjusts its island-level transfers and its aggregate money supply so that $M_{i,t} = \bar{x}\left(1 - \theta_{i,t} q_{i,t}\right)$, then real allocations remain at their steady state levels after any history of credit shocks.*

Proof. Only the path of nominal spending matters for real allocations in equations (5 23, 24, 25, 26, 27). If $x_{i,t} = \bar{x}$ then $\left\{l^n_{i,t}, l^\tau_{i,t}, W_{i,t}, W^*_{i,t}, \bar{P}_{i,t}, \bar{P}^\tau_t\right\}$ all remain constant at their steady state values. Local house

Figure 4: Island Monetary Response



prices are pinned down by 22, and $M_{i,t} = \bar{x}\,(1 - \theta_{i,t} q_{i,t})$ ensures that $x_{i,t} = \bar{x}$. Finally, the implicit transfer payments are given by (20). □

In the one island case, the steady state implementation only requires open market operations to stabilize aggregate nominal spending. With heterogeneity across islands, the implementation requires transfers across islands, presumably involving fiscal authorities.

The second normative proposition concerns corrective taxes on labor income and home construction. Before describing Pigouvian taxes, we note that the Friedman rule would be optimal in our economy without island level shocks. By deflating at rate β the government could reduce the multiplier μ_t to zero and eliminate all distortions in the economy. Our model is silent on the reasons that might make the Friedman rule undesirable, or that might prevent its implementation. Instead, we simply assume that prices are constant in steady state. This creates a wedge in the steady state labor supply. This can be corrected by a labor income subsidy. Now imagine an economy similar to the one we have described, but with endogenous housing supply, and let δ_h be the depreciation rate of houses. In order to understand the nature of optimal taxes, we allow the government to use two separate instruments: a subsidy on labor income, and a specific tax on home construction. We obtain the following results:

Proposition 3. ***Efficient Taxes and Home Construction.*** *The steady state allocation with constant money supply is efficient when labor income is subsidized at the rate* $\beta^{-1} - 1$ *and home construction is taxed at the rate* $\frac{\theta(1-\beta)}{1-\beta(1-\delta_h)}$.

Proof. See appendix. □

The subsidy $\beta^{-1}-1$ means that the steady state allocation of a model with exogenous housing would be efficient. The key to understanding the proposition is to see that houses are used as a form of commodity money. For the standard reasons identified in the monetary literature (Sargent and Wallace, 1983), when we introduce a housing construction sector there is excessive production of commodity money, i.e., excessive construction of new houses. The tax rate equals the liquidity services from housing θ times the steady state value of money (the Lagrange multiplier on the CIA constraint, or the opportunity cost of holding money, which is $1-\beta$). The denominator of the tax rate is simply an adjustment for the durability of housing (since $\delta_h < 1$).

3 Calibration

3.1 Complete model

If we combine the cash-in-advance constraint and the collateral constraint we now obtain $\bar{P}_{i,t}\bar{c}_{i,t} + Q_{i,t}y^h_{i,t} + V_{i,t}e_{i,t} = M_{i,t} + \theta_{i,t}Q_{i,t}h_{i,t}$. Defining as in Section 2, $x_{i,t} \equiv \bar{P}_{i,t}\bar{c}_{i,t}$ and $q_{i,t} \equiv \frac{Q_{i,t}h_{i,t}}{x_{i,t}}$, and the corresponding ratio for durable goods $v_{i,t} = \frac{V_{i,t}d_{i,t}}{x_{i,t}}$, we see that equation (21) becomes

$$x_{i,t}\left(1-\left(\theta_{i,t}-\frac{y^h_{i,t}}{h_{i,t}}\right)q_{i,t}+\frac{\bar{e}_{i,t}}{d_{i,t}}v_{i,t}\right)=M_{i,t}. \tag{37}$$

The velocity interpretation still applies, but now we need to take into account housing construction and spending of durable goods. We can write the house price equation (7) as

$$\eta+\beta\left(1-\delta_h\right)E_t\left[q_{i,t+1}\frac{h_{i,t}}{h_{i,t+1}}\right]=\left(1-\theta_{i,t}\left(1-\beta E_t\left[\frac{x_{i,t}}{x_{i,t+1}}\right]\right)\right)q_{i,t}.$$

Similarly, the Euler equation for durables is $\xi+\beta(1-\delta_d)E_t\left[v_{i,t+1}\frac{d_{i,t}}{d_{i,t+1}}\right]=v_{i,t}$. Trade and technology pin down labor demands. Market clearing for non-tradable goods (18) becomes

$$l^n_{i,t}=\left(\left(1-\omega\right)\bar{P}^{\sigma-1}_{i,t}+\left(1-\omega_d\right)v_{i,t}\frac{\bar{e}_{i,t}}{d_{i,t}}\bar{V}^{\sigma-1}_{i,t}\right)\left(W^n_{i,t}\right)^{-\sigma}x_{i,t},$$

and for tradable goods (19) becomes

$$l^\tau_{i,t}=\left(W^\tau_{i,t}\right)^{-\gamma}\left(\bar{P}^\tau_t\right)^{\gamma-\sigma}\int_j\left(\omega\bar{P}^{\sigma-1}_{j,t}+\omega_d v_{j,t}\frac{\bar{e}_{j,t}}{d_{j,t}}\bar{V}^{\sigma-1}_{j,t}\right)x_{j,t}.$$

For convenience, the complete set of equilibrium conditions is provided in the Appendix.

3.2 Steady State and Static Parameters

We consider a steady state in which $\theta_i = \theta$ is the same in all islands. We have two sets of parameters in our model economy. The first set, referred to as *static parameters*, mostly determines the steady state of our model economy. We choose these parameters to ensure that our model matches salient features of the U.S. data. The second set of parameters, referred to as *dynamic parameters*, consists of (λ, ϕ, θ), the parameters that govern the degree of nominal and real labor market rigidities, as well as the size of the collateral constraint. These parameters mostly affect the dynamic responses of the model in response to shocks. We pin down these parameter values by requiring that the model accounts for the cross-sectional dynamics of debt, house prices, and employment in the data.

Here we briefly describe how we have chosen the static parameters of our model . We describe our choice of the dynamic parameters in the next section, after we describe the cross-sectional experiments that we conduct. Table 3 reports the parameter values we use and the moments of the data that pin down each parameter.

We assume that a period is one year. For the borrowing constraints to bind in equilibrium, households must be sufficiently impatient. We therefore set $\beta = 0.95$, at the lower end of the range of values $(0.95 - 0.98)$ used in the literature.

The ratio of residential investment spending to the housing stock is equal to 3.6% in the data. In the steady state of our model we have that

$$\frac{y^h}{h} = \delta_h = 0.036,$$

so this pins down the rate at which the housing stock depreciates, δ_h. The value of housing stock relative to consumption expenditure, q, is equal to 2.11 in the data. In the steady state of our model we have that

$$q = \frac{\eta}{1 - \beta\left(1 - \delta_h\right) - \theta\left(1 - \beta\right)},$$

so we choose η accordingly.

In a similar fashion, δ_d pins down the ratio of spending on durables to their stock (equal to 0.27 in the data) since

$$\frac{e}{d} = \delta_d = 0.27$$

in our model. Moreover, the value of the durables relative to consumption expenditures, is equal to 0.5 in the data and equal to

$$v = \frac{\xi}{1 - \beta(1 - \delta_d)}$$

in the steady-state of our model. We thus choose ξ to ensure our model matches the value of v in the data.

We normalize the stock of money, M, equal to 1. Since all nominal variables (including the price of houses, Q) are proportional to M in the model, this is simply a convenient normalization that only determines the price level in this economy. The CCIA constraint (37) therefore gives:

$$x = \frac{M}{1 - (\theta - \delta_h)\, q + \delta_d v} = \frac{1}{1 - (\theta - \delta_h)\, q + \delta_d v}$$

The parameters α_τ and α_n are not separately identified from ω_c and ω_d since we have assumed constant returns to labor and both sets of weights simply pin down the share of each sector's expenditure/labor. We therefore normalize α_τ and α_n to ensure that wages are equal to unity in the two sectors in the steady-state: $W^n = W^\tau = 1$. Given this normalization, goods prices are also equal to 1. We then choose α_h to ensure that $l^h = s_h \left(l^\tau + l^n\right)$, i.e. so that the steady-state share of labor in construction is s_h that in the goods-producing sectors. In the data, the ratio of labor in construction to 6.6% so we set the value of α_h to hit this target.

Table 5: Parameters

Parameter Name		*Value*	*Source/Target*
Annual Discount Factor	β	0.95	
Home Value over Non Durable Spending	q	2.11	Value in 2001. BEA, Flow of Funds
Home Depreciation Rate	δ_h	0.036	Residential investment spending over housing stock
Labor Share Construction	χ	0.6	Construction Wages over Residential Investment
Durable Stock Value over Non Durable Spending	$\bar{v}$	0.5	Value in 2001. BEA, Flow of Funds
Durable Depreciation Rate	δ_d	0.27	Spending on durables relative to durable stock
Employment Share of Construction	s_h	0.066	Value in 2001. BEA.
Trade weight in non durable consumption	ω_c	0.25	Trade literature. Distribution adjusted.
Trade weight in durable consumption	ω_d	0.6	Trade literature. Distribution adjusted.
Labor Supply Elasticity	ν	2	Hall (2010)
Elasticity of substitution among traded goods	γ	1.5	Trade literature
Elasticity of substitution traded/non traded	σ	0.1	Own estimate

We calibrate the shares of tradeable goods as in the international trade literature. We assume that the distribution margin accounts for 40% of the retail price of the good. We assume that all durable goods are tradeable. Adjusting for local distribution, this gives $\omega_d = 0.6$. For non durable goods, we use the BEA

data on Personal Consumption Expenditure. We identify tradeables with "goods" and non-tradeables with "services excl. housing". The share of tradeables shows a trend decline over time and is around 0.4 in 2002. Adjusting for distribution costs gives $\omega_c = 0.25$. Finally, we choose an elasticity of substitution between tradeables and non-tradeables, σ, in order to match the comovement of the relative price of tradeables to non-tradeables and the share of tradeables in the data. In the data, there was a substantial decline in the relative price of tradeables and only a modest increase in real tradeables consumption. A value of σ equal to 0.1 fits this evidence best. It is more difficult to pin down the elasticity of substitution between tradeables produced on different islands, γ. In the international trade and macro literature, estimates of trade elasticities range from 0.5 to 4. We consider below a value equal to $\gamma = 1.5$, the typical value used in the international macro literature. It turns out that the exact value of γ is not critical in our model as long as wages are sticky.

Finally, we follow Hall (2010) and set the labor supply elasticity, ν, equal to 2.

4 Quantitative Cross Sectional Experiments

We next describe the cross-sectional experiments we conduct, as well as our choice of the dynamic parameters, λ, ϕ, θ, that allow our model to match the cross-sectional dynamics in the data.

4.1 The Experiment

We study an experiment in which all islands start in the (identical) steady-state with the same credit parameter, $\theta = \bar{\theta}$, in 2001. From 2001 to 2007 each island experiences a gradual, equally-sized, island-specific increase in θ. Finally, in 2008 and 2009 the collateral constraint in each island returns to $\bar{\theta}$ in two equally-sized steps. Hence, as in the data, islands that experience the largest booms prior to 2007 also experience the largest busts after 2007.

It turns out that changes in the current value of θ cannot replicate some important features of the cross sectional dispersion that we observe. Specifically, the cross-sectional dispersion of home prices is too large to be explained simply by the current value of θ. The basic issue is the following: θ drives both x and Q, but with reasonable parameters, if the only shock is the current value of θ, the change in house prices cannot be more than 1.5 times the change in nominal spending. This is not a severe constraint with aggregate data, but it is not enough for the cross section. In the Appendix we describe one way to explain the cross-section: news shocks to future values of θ. News shock can change q without changing the current value of θ. As a

result Q can move by more that 1.5 nominal income. All our results can be interpreted using this approach.

This "news" interpretation is formally consistent with our model, but is really not crucial for our results.[17] For the sake of simplicity, and since our goal is to study an experiment that accounts simultaneously for the dynamics of credit and housing prices, we simply introduce a wedge in the housing Euler equation that allows us to reproduce the behavior of house prices in the data. In particular, we now have

$$\eta + \beta\left(1-\delta_h\right) E_t\left[q_{i,t+1}\frac{h_{i,t}}{h_{i,t+1}}\right] = \left(1-\theta_{i,t}\left(1-\beta E_t\left[\frac{x_{i,t}}{x_{i,t+1}}\right]\right)\right) q_{i,t} + \omega_{i,t}$$

We choose the wedge $\omega_{i,t}$ so that our model reproduces the response of house prices in the data from 2001 to 2007. As with the collateral parameter, each island experiences a gradual, equally-sized, island-specific increase in Q_i, the price of houses, from 2001 to 2007. In 2008 and 2009 house prices revert to the initial steady-state in two equally-sized steps. We continue to assume a one-shock model so that θ_i and Q_i are perfectly correlated. In the news interpretation of the model this implies that current changes in θ are perfectly correlated with expectations of further future increases in θ.

To map the model to the data, we will compute elasticities of island-level employment to island-level changes in debt-to-income ratios. Changes in debt-to-income arise in the model from two sources: changes in the collateral constraint, θ, and changes in house prices. It turns out that in our model the size of these elasticities only depends on the relative size of the change in θ to that of changes in Q, not on the absolute size of these changes, since the model is approximately linear. The relative size matters since changes in house prices affect the returns to construction, and therefore the dynamics of employment, differently than changes in the collateral requirement. The fact that the absolute size of such changes is irrelevant implies that the elasticities we compute are unaffected by the standard deviation and higher order moments of the distribution of changes in debt-to-income in the data.

To pin down the size of changes in θ and Q, we require that the model matches two key moments that describe the credit boom reported in Table 7. The first moment is the average increase in the debt to income ratio of 0.46. (from 0.86 to 1.32 in the cross-section of 12 states in Figure 1 for which data is available). The second moment is the cross-sectional elasticity of house prices to leverage. To compute this moment, we run a regression of the log-change in house prices, $\Delta \log Q_i$, from 2001 to 2007 on the change in the debt-to-income ratio, $\Delta B_i/Y_i$ in this same period and find an elasticity equal to 0.86. Intuitively, the first

[17] To be precise, what is crucial is to set up the experiment with the correct initial conditions, that is, the correct distribution of B/Y and Q/Y. How we obtain these initial conditions does not matter. We could use preference shocks to capture the interactions of demographics (retirement of given age cohorts) and state-level characteristics (weather, etc.). We could use different prices dynamics (Burnside, Eichenbaum, and Rebelo (2011)).

moment pins down the size of the credit boom (the increase in θ), while the second pins down the size of house price increases necessary to allow the model to reproduce the response of house prices to changes in debt-to-income at the state level.

Table 7 reports that the model requires a 21% average increase in the collateral parameter, θ, and a 40% increase in the average house price, in order to match these two moments of the data. Note that the increase in house prices is slightly smaller than in the data (40% vs. 45%). Since our calibration of the dynamic parameters below relies on the model's predictions for cross-sectional elasticities, we prefer a parametrization that accounts for the cross-sectional elasticity of house price changes to changes in leverage in the data, rather than the average change in house prices, though the discrepancy between the model and the data is clearly negligible.

Table 7: Island Credit Boom

	Data	Model
Targets		
$\Delta(B/Y)$	0.461	0.461
$\Delta \log(Q) / \Delta(B/Y)$	0.862	0.861
Parameters		
average $\Delta \log Q$	0.453	0.397
average $\Delta \log \theta$	-	0.211

4.2 Calibration of dynamic parameters

The parameters θ, λ, ϕ are the key parameters in our model since they determine the economy's response to credit shocks. Intuitively, a higher θ implies that a higher fraction of nominal consumption spending is financed out of private credit and is therefore sensitive to credit shocks. A higher degree of wage stickiness, λ, implies a greater extent with which nominal shocks affect real activity. Finally, a greater degree of labor market rigidities, as captured by ϕ, implies that it is costlier to reallocate labor from the non-tradeable to the tradeable goods sector. Labor market rigidities amplify the island-specific shock by preventing islands from accumulating public money.

4.2.1 Calibration of the steady-state collateral constraint, θ

To pin down the steady-state value of the collateral constraint, θ, we use micro evidence from Mian and Sufi (2010a). Mian and Sufi (2010a) argue that borrowing against the value of home equity accounts for a significant fraction of the rise in US household leverage from 2002 to 2006. They follow from 1997 to 2008

a random sample of 74,000 U.S. homeowners (who owned their homes as of 1997) in 2,300 zip codes located in 68 MSAs. As of 1997, median total debt is $100,000 of which $88,000 is home debt (home equity plus mortgages), and the debt to income ratio is 2.5. Total debt grows by 8.6% between 1998 and 2002, and by 34.4% between 2002 and 2006. These changes are accounted for by home debt growth. The debt to income ratio does not change from 1998 to 2002 and then increases by 0.75.

Mian and Sufi argue that there is a causal link from house price growth to borrowing. The critical issue is that house price growth is endogenous. An omitted factor, such as expected income growth, could be driving both house prices and current borrowing (and consumption). To identify a causal link they use instruments for house price growth based on proxies for housing supply elasticities at the MSA level.

In their estimates, a $1 increase in house prices *causes* a $0.25 increase in home equity debt. Two issues arise when we map this number into our model. The first issue is maturity. Our model assumes that debt is repaid at the end of each year, while home lines of credit have an average maturity of 5 years. We show in the appendix, using a simple model in which debt has a maturity of N years, households borrow every N years (in a staggered fashion), and repay a fraction $\frac{1}{N}$ of the debt each year, that the conversion factor from the stock measure to the flow measure is $N/2$. This is intuitive, since if the initial amount of debt is equal to B, then the average debt position of all households is equal to $\frac{B}{N/2}$. With an average maturity of 5 years, $0.25 translates into $0.1 in our model with one-period debt.

The second issue has to do with the fact that θ changes over time in our model. To see this, note that Mian and Sufi report that a $1 increase in house prices from 2001 to 2007 leads to a $0.25 increase in debt, or:

$$B_{2007} - B_{2001} = 0.25 \left(Q_{2007} - Q_{2001} \right)$$

If θ were constant, then, since in our model $B = \theta Q$ we would have (ignoring the maturity adjustment) $\theta = 0.25$. We assume however, that θ changes over time, and that the increase in θ is perfectly correlated with changes in house prices. Hence, the Mian-Sufi elasticity does not recover the steady-state collateral constraint, $\theta_{2001} = \theta$. To recover this parameter, note that, according to our model $B_{2007} - B_{2001} = \theta_{2007} Q_{2007} - \theta_{2001} Q_{2001}$. Up to a first-order, we can thus write:

$$B_{2007} - B_{2001} \approx \frac{(\theta_{2007} - \theta_{2001})}{\theta_{2001}} \theta_{2001} Q_{2001} + \frac{(Q_{2007} - Q_{2001})}{Q_{2001}} \theta_{2001} Q_{2001}$$

Since, as shown in Table 7, we assume that θ increases by $1/2$ as much as Q does (0.21 vs. 0.40), we have

that

$$B_{2007} - B_{2001} \approx \frac{3}{2}\theta_{2001}\left(Q_{2007} - Q_{2001}\right)$$

This implies that the Mian-Sufi elasticity is related to the steady-state collateral constraint θ by a factor of about 3/2. Accounting for the two sources of bias, we have that $\bar{\theta}_{equity} = 0.25 * 2/3 * 2/5 = 0.067$. Recall also that in our model we allow for housing construction. Assuming a loan-to-value (LTV) ratio of 80% for new mortgages, and noting that in the steady state the annual flow of spending on new homes is equal to δ_h, we have that the collateral on mortgage debt is equal to $\bar{\theta}_{mort} = 0.8\delta_h = .0288$. The total amount of debt (mortgage and home equity lines of credit) is thus bounded above by $\bar{\theta} = \bar{\theta}_{mort} + \bar{\theta}_{equity}$ and is equal to $\bar{\theta} = 0.0955$. This number is quite reasonable a priori. Since $1 = \frac{M}{x} + \theta q$, and since $\theta q \approx 0.2$, the calibration implies that about 20% of consumer spending is sensitive to real estate wealth. Alternatively, 20% of the household consumption spending in our model is financed using debt (private money), and the rest using public money.

4.2.2 Calibration of the degree of labor market rigidities, λ and ϕ

To pin down λ and ϕ, the parameters that govern the degree of real and nominal labor market frictions, we require that the model accounts for the cross-sectional elasticities of changes in employment in the construction and non-construction sectors during the bust (2007 to 2009) to the change in the debt-to-income ratio, $\Delta B/Y$ during the boom (2001-2007), as in Figure 1. We compute these elasticities in the data using the 12 states in Figure 1. Table 8 reports the moments in the model and in the data. We note that the elasticity of changes in non-construction employment during the bust to changes in debt-to-income during the boom is equal to -0.099 in the model and -0.098 in the data. Thus, the large decline in employment in states that have experienced the largest booms is not accounted for by a decline in construction employment alone. Though housing employment was a lot more sensitive to changes in debt (the elasticity is -0.59 in the model and -0.52 in the data), the share of construction employment is fairly small so that declines in non-construction employment account for the bulk, 70% (.098/.139) of the overall drop in employment in the data. The last few rows of the table report several additional predictions of the model which we discuss below.

We note that the model, not surprisingly since we use two parameters to fit these facts, does a good job at reproducing the cross-sectional elasticities in the data. The implied parameter values are equal to $\lambda = 0.74$ and $\phi = 4$, suggesting a very large degree of wage stickiness (only a 26% fraction of the gap between

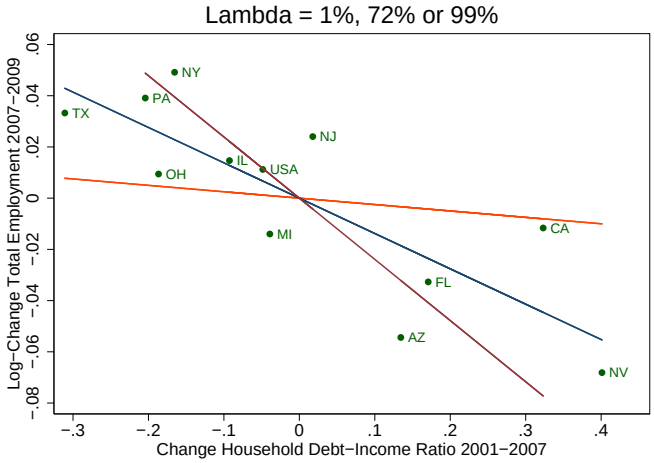
the current and frictionless wage is covered each period) and very large costs of reallocating labor across industries.

Table 8: Island Credit Crunch

	Model	Data
Targets (elasticities to debt-income)		
total employment	-0.137	-0.139
non-construction employment	-0.105	-0.098
construction employment	-0.594	-0.524
Additional Testable Predictions		
Leverage	-0.292	-0.248
Home Prices	-0.858	-1.058
Consumption Spending	-0.214	-0.243
Durable Consumption Spend.	-0.679	-0.692
Non Durable Consum. Spend.	-0.185	-0.174
Non Cons. Wages	-0.053	0.007
Construction Wages	-0.596	-0.063

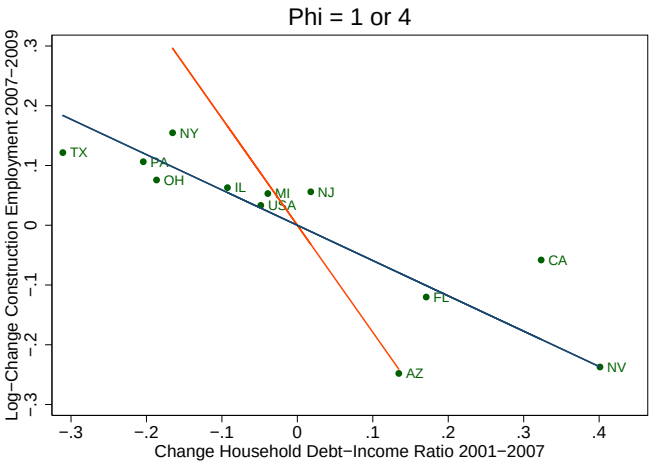
Figures (5) and (6) give a sense of how the data identifies the parameters λ and ϕ. The lines are the prediction of the model with the credit boom and bust simulated as in Table 7. Without wage rigidities ($\lambda = 0$), Figure (5) shows that total employment barely moves in the cross section.

Figure 5: Identifying Wage Rigidities from the Cross-Section



Without sectoral reallocation costs ($\phi = 0$), Figure (6) shows that labor moves too much across sectors. A similar picture emerges if we use durable versus non durable employment.

Figure 6: Identifying Reallocation Costs from the Cross-Section



Consider finally several additional predictions our model makes for the cross section. Figures 7 and 8 show the prediction of the model for the credit and housing markets, also reported in Table 8. The model does an excellent job at reproducing these features of the data. In particular, the elasticity of house price changes in the bust to the change in debt-to-income during the boom is equal to -1.06, thus only slightly higher than the -0.86 in the data. This suggests that our assumption that house prices revert to their steady-state values after the bust is in line with the data. Similarly, our model reproduces well the elasticity of the log-change in debt-to-income, $\Delta \log B/Y$ in the bust to the change in debt-to-income $\Delta B/Y$ in the boom (-0.29 in the model vs. -0.25 in the data), suggesting that our assumption that the collateral constraint returns to its steady-state level is reasonable as well.

Our model also has implications regarding the cross-sectional response of consumption. Testing these implications is difficult, however, since state-level consumption data is unavailable. For lack of a better measure, we construct measures of consumption expenditures across states using the Consumer Expenditure Survey (CEX). The results are reported in Table 8 and in Figure 9. One should be careful in interpreting these results since CEX was never designed to properly measure consumption across states and the state identifier is in some cases coded with noise. For this reasons the cross sectional correlation of consumption with changes in leverage is smaller than for the other variables in the Table. Nonetheless, the model predicts elasticities that are consistent with the data. The elasticity of consumption spending to changes in debt-to-income is equal to -0.21 in the model and -0.24, implying that consumption is 1.56 times more responsive to credit shocks than employment (-0.214/-0.137) in the model, and 1.75 times in the data. Durable goods

spending is approximately three times more volatile than total consumption spending, both in the model and in the data.

Notice finally that there is one dimension along which our model does not fit the data well, namely the cross-sectional responses of construction (and to a lesser extent) non-construction wages. In particular, the model predicts that wages decline much more than they do in the data (the elasticity is -5.3% in the non-construction sectors compared to 0.7% in the data and equal to -60% in the construction sector vs. -6.3% drop in the data). Accounting for these moments would require unreasonable amounts of wage stickiness, and, more importantly, would overstate the employment responses in the data. We also note that the cross sectional fit of the wage regression is poor. There appears to be a lot of noise in cross sectional wages that the model cannot replicate that is essentially uncorrelated with the extent of the credit boom.

Figure 7: Cross-Sectional Prediction - Home Prices

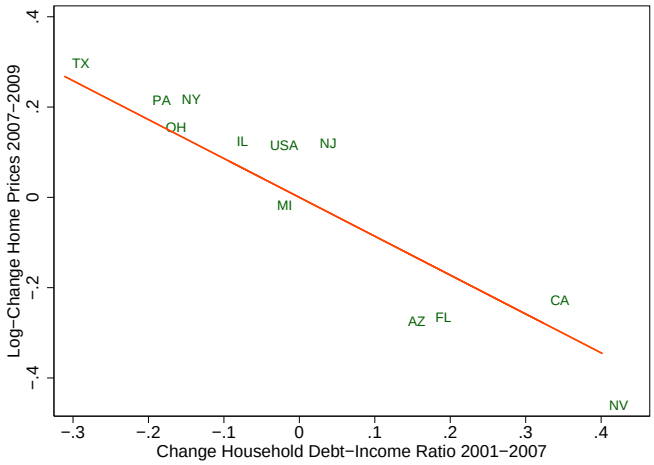


Figure 8: Cross-Sectional Prediction - Deleveraging

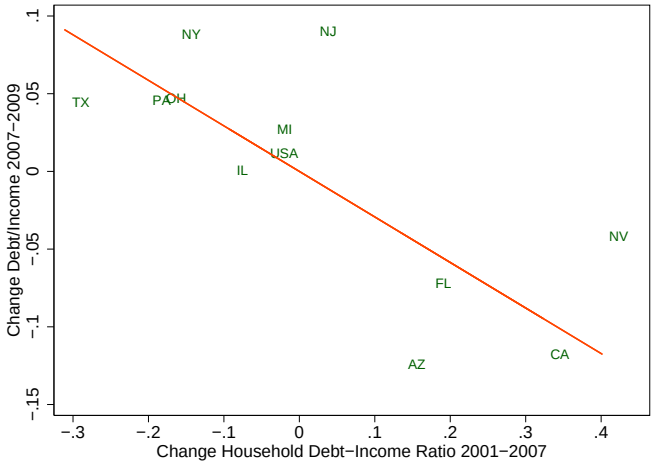
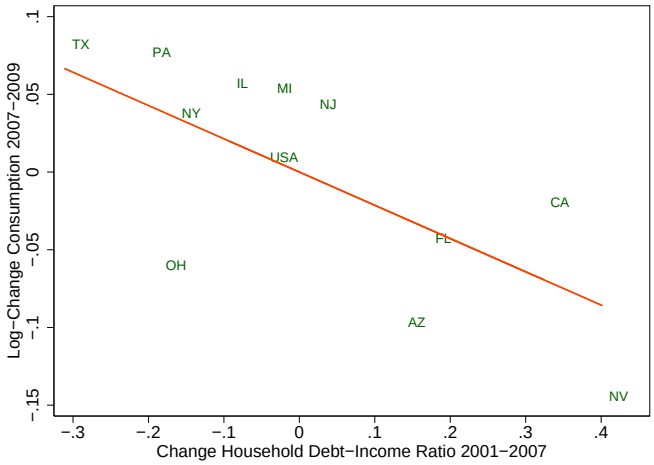


Figure 9: Cross-Sectional Prediction - Consumption



5 Quantitative Aggregate Experiments

We next study the model's aggregate implications. We first describe the experiment that replicates the aggregate credit boom. The data sources we use to quantify the size of the aggregate credit boom are slightly different from those used in the state-level analysis, hence there are some minor discrepancies from what we have reported earlier. As in the island experiment, we generate a credit boom by matching key moments of the data. We then let the model return to the initial value of $\bar{\theta}$ in a two-year period.

Table 9 reports the aggregate moments that we use to pin down the dynamics of q_t and θ_t during the

boom and bust cycle. We ask the model to replicate the 49% increase in the debt-to-income ratio from 2001 to 2007, as well as the dynamics (an initial 20% increase and a subsequent 30% bust) of the ratio of home values to consumption spending, q. We match these statistics by feeding the model a path for θ_t and wedges ω_t that match these statistics exactly. As above, we model the bust as a gradual, two-period long, equally-sized decline in θ from its value at the peak (2007) to the steady-state value. As for ω_t, we ask the model to reproduce the dynamics of q_t in each of the years of the bust.

Table 9: Aggregate Experiment

	Data	Model
$d(B/Y)$ from 2001 to 2007	0.49	0.49
q in 2001	2.11	2.11
q in 2007	2.53	2.53
q in 2008	2.05	2.05
q in 2009	1.85	1.85

When computing the responses of aggregate variables in these experiments, we entertain two sets of assumptions about the path of monetary policy. In the first experiment, we set $M = 1$. In the other experiment, we allow the central bank to expand its balance sheet by 7% of GDP. This is an upper bound on the potential effects of the first round of non standard monetary policies (see Gertler and Karadi (2009), Gertler and Kiyotaki (2010), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010)).

Table 10 reports the results of the experiments with no changes in monetary policy (referred to as No QE). The first column reports the change, from 2007 to 2009, of key aggregate variables in the data. The second column reports the evolution of aggregate variables in our model economy, referred to as Benchmark. We note that our model predicts a recession that is significantly worse than the one in the data. For example, non-construction employment declines by 8.6% in the model versus 5.3% in the data. Non-durable consumption declines by 7.2% in the model versus 2.7% in the data. Durable consumption declines by about 20.0% in the model versus 13.8% in the data.

The last columns of Table 10 illustrate the role of the key parameter values we have used in accounting for these results. When we set $\lambda = 0$, we find that the model produces a much milder recession. For example, non-construction employment declines by about 0.6% compared to 8.6% in the Benchmark economy. The recession is mostly accounted for by the contraction in the construction sector, combined with the frictions on labor mobility that we have assumed.

When we set $\bar{\theta} = 0$, credit shocks no longer affect nominal consumption spending. The model once again

produces a mild recession, again driven by the contraction in the housing sector, associated with the decline in house prices.

Finally, when we set $\phi = 1$, we note that the model produces a contraction in the housing sector much more severe than in the data (a 72% decline in employment compared to a 21% decline in the data), but the effect on total employment is muted by the fact that non-construction employment declines by less (5.4% compared to 8.6%) than in the benchmark model. We also note that labor immobility affects the response of consumption: absent such frictions (i.e., when $\phi = 1$), non-durable consumption declines by only 4.2% (7.2% in the benchmark model), while non-durable consumption declines by about 15.5% (20% in the benchmark model).

We thus conclude that the three dynamic parameters play a crucial role in determining the response of our model economy to credit shocks. This reinforces the need to identify such parameters using a richer set of cross-sectional moments, rather than relying solely on aggregate statistics. The latter reflect the stance of monetary and fiscal policy, international capital flows, as well as other real shocks, and use of the aggregate data alone precludes a sharp identification of the key frictions in the model.

Table 10: Aggregate Outcomes and Counter-Factuals

	Data, 2007-09	Model, No QE			
		Benchmark	$\lambda = 0$	$\bar{\theta} = 0$	$\phi = 1$
Non Cons Employment	-0.053	-0.086	-0.006	0.007	-0.054
Construc. Emp.	-0.207	-0.272	-0.090	-0.146	-0.722
NonDur Consumption	-0.027	-0.072	-0.006	0.006	-0.042
Durable Consumption	-0.138	-0.200	-0.011	0.013	-0.155
Debt/Income	-0.043	-0.075	-0.080	-0.180	-0.066
Home Value/Income	-0.260	-0.279	-0.298	-0.219	-0.202
Non Cons. Wages	-0.011	-0.036	-0.126	0.016	-0.032
Construc. Wages	-0.007	-0.316	-0.416	-0.219	-0.032

We next study the effect of monetary interventions in our model economy. We assume here a 7% expansion (relative to the size of GDP) of the stock of public money, equally distributed across islands, and gradually implemented in 2008 and 2009. Table 11 reports the results of this expansion under two assumptions. The first column assumes that the Fed simply lends directly to households. The second column assumes that the Fed issues public money, M.

Table 11: Aggregate Outcomes and Counter-Factuals

	QE 7% GDP		Data	No QE
	with B	with M		Benchmark
Non Cons Employment	-0.043	-0.043	-0.053	-0.086
Construc. Emp.	-0.218	-0.220	-0.207	-0.272
NonDur Consumption	-0.038	-0.038	-0.027	-0.072
Durable Consumption	-0.132	-0.131	-0.138	-0.200
Debt/Income	-0.005	-0.136	-0.043	-0.075
Home Value/Income	-0.288	-0.289	-0.260	-0.279
Non Cons. Wages	0.002	0.002	-0.011	-0.036
Construc. Wages	-0.265	-0.265	-0.007	-0.316

Note that the model predicts responses that are much more in line with the actual dynamics in the data. Not surprisingly, since public and private money are substitutes in the model, the response of real variables is independent of the exact source of the expansion of the Fed's balance sheet. Non-construction employment declines by almost as much as in the data (4.3% versus 5.3%), as does construction employment (22% vs. 20.7%). The model slightly overstates the decline in non-durable consumption (a drop of 3.8% vs. 2.7%) in the data, and slightly understates the drop in durable consumption (13.2% vs. 13.8%), but overall accounts for the response of consumption and employment fairly well. In particular, our model is consistent with the observation that durable-goods spending declined by much more than non-durable spending.

Comparing the first two columns of Table 11 with the last column of Table 11, which reproduces the statistics for the economy with no monetary intervention, we can gauge the effect a monetary expansion of 7% in the two years of the credit crunch. Absent the monetary intervention, the drop in non-construction employment would have been about twice larger, while the drop in non-durable consumption would have been 90% larger and that in durable-goods spending would have been about 50% larger. Of course, a monetary expansion of the right magnitude can completely offset the effect of the decline in household borrowing, as shown earlier. An interesting extension of our analysis would consider a more realistic description of monetary policy and the constraints that have prevented the Fed from expanding the supply of public money. We relegate such an extension to future work.

6 Conclusions

We have studied a cash-in-advance economy in which home equity borrowing, together with public money, is used to conduct transactions. We calibrated the model to account for the evidence on the dynamics of credit and employment in a cross-section of U.S. states and have argued that a model capable of matching

the cross-sectional facts implies strong sensitivity of real activity to credit shocks. We interpret these results as suggesting that a sharp reduction in credit at the household level accounts to a non-negligible extent for the collapse of output and employment in the recent recession. Expansionary monetary policy can, in this framework, significantly reduce the severity of a recession.

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Appendix

Consumers' First Order Conditions

The first-order conditions for money holdings, consumption, labor, and housing and non-durables are:

$$\begin{aligned}
\frac{u_{\bar{c},it}}{\bar{P}_{i,t}} &= \beta\left(1+r\right)E_t\frac{u_{\bar{c},it+1}}{\bar{P}_{i,t+1}}+\mu_{i,t},\\
-\frac{u_{l^k,it}}{W^k_{i,t}} &= \beta E_t\frac{u_{\bar{c},it+1}}{\bar{P}_{i,t+1}} \text{ for } k=n,\tau,h,\\
u_{h,it}+\mu_{i,t}\theta_{i,t}Q_{i,t} &= \frac{Q_{i,t}}{\bar{P}_{i,t}}u_{\bar{c},it}-\beta(1-\delta_h)E_t\frac{Q_{i,t+1}}{\bar{P}_{i,t+1}}u_{\bar{c},it+1},\\
u_{\bar{d},it} &= \frac{\bar{V}_{i,t}}{\bar{P}_{i,t}}u_{\bar{c},it}-\beta(1-\delta_d)E_t\frac{\bar{V}_{i,t+1}}{\bar{P}_{i,t+1}}u_{\bar{c},it+1},
\end{aligned}$$

where $\mu_{i,t}$ is the multiplier on the borrowing constraint.

Credit and Home Price Dynamics with News

In this section we briefly explain how anticipated changes in θ affect current home prices and credit. This is important to account for the relative volatility of credit and home prices.

Unexpected Shocks (benchmark model)

This is what we have been doing to far. Imagine that we start from a steady state with $\theta=\theta_0$, $q_0=\frac{\eta}{(1-\beta)(1-\theta_0)}$ and $x_0=\frac{1}{1-\theta_0 q_0}$. The time line of events is:

- $t=0$. Steady state with θ_0
- $t\geq 1$. Permanent shock realized, $\theta_t=\theta_1$ remains constant.

For small values of θ, in steady state, we have $x_t\approx 1+\theta_t q_t$ and $q_t\approx\frac{\eta}{1-\beta}\left(1+\theta_t\right)$. Start from $q_0=2$ and $\theta_0=0$ so $\frac{\eta}{1-\beta}=2$ while $x_0=1$. Consider a small change in θ, then $q_t\approx 2\left(1+\theta_t\right)$ and $\hat{q}_t=\frac{q_t-q_0}{q_0}\approx\theta_t$ so that $x_t\approx 1+2\theta_t$ and $\hat{x}_t=\frac{x_t-x_0}{x_0}\approx 2\theta_t$. House prices are

$$Q_t=q_t x_t$$

Hence

$$\hat{Q}_1 = \hat{q}_1 + \hat{x}_1 = \frac{3}{2}\hat{x}$$

Say we want x to move up by 10% in our calibration this implies that Q moves up by 15%. In this model, house prices cannot move up more than 1.5 times nominal spending. So spending should move by at least 2/3 of house price appreciation. In the cross section, however, spending moves by 0.14 to 0.18 times the log change in house prices. This suggests we need "anticipated" shocks as well.

Expected Shock (news model)

Now we had a "news shock":

- $t = 0$. Steady state with θ_0
- $t = 1$. News that $\{\theta_t\}$ will permanently jump to $\theta_2 > \theta_0$ at time 2. We still have $\theta_1 = \theta_0$, but $\{q_t\}$ jumps to $q_1 > q_0$, and therefore x will also jump.
- $t \geq 2$. Permanent shock realized, $\theta_t = \theta_2$ remains constant.

It is easy to see that from $t = 2$ onwards, we are back to steady state with $q_2 = \frac{\eta}{(1-\beta)(1-\theta_2)}$ and $x_2 = \frac{1}{1-\theta_2 q_2}$. What is more interesting is what happens at time 1. We have $x_1 = \frac{1}{1-\theta_0 q_1}$ and $\eta + \beta q_2 = \left(1 - \theta_0\left(1 - \beta\frac{x_1}{x_2}\right)\right) q_1$. So we can solve for q_1 exactly using

$$\eta + \beta q_2 = \left(1 - \theta_0\left(1 - \beta\frac{1-\theta_2 q_2}{1-\theta_0 q_1}\right)\right) q_1$$

For small θ, we have $x_1 \approx 1 + \theta_0 q_1$ and $q_1 \approx \frac{\eta+\beta q_2}{1-\theta_0(1-\beta)}$. We cannot literally start from $\theta_0 = 0$ because we would need infinite q_1 to move x_1. Since $x_0 \approx 1 + \theta_0 q_0$, we have $\hat{x}_1 \approx \frac{\theta_0(q_1-q_0)}{1+\theta_0 q_0} = \frac{q_0\theta_0}{1+\theta_0 q_0}\hat{q}_1$. With our usual calibration of $q_0 = 2$ and $\theta_0 = 5\%$, we get $\hat{x}_1 = \frac{1}{11}\hat{q}_1$. Now we get $\hat{Q}_1 = \hat{q}_1 + \hat{x}_1 = 12\hat{x}_1$. If x moves up by 10%, house prices move up by 120%.

Calibrating θ

What is the right value for θ given the Mian-Sufi estimates? We need to map a "5 year" regression estimate into an annual model, taking into account the maturity of HELOCs and the sources of the shock. The Mian-Sufi result says:

$$B_T - B_1 = 0.25\,(Q_T - Q_1)$$

First issue: the source of shock

In our model, normalizing $h = 1$, we have $B = \theta Q$ so

$$B_T - B_1 = \theta_T Q_T - \theta_1 Q_1 \approx (\theta_T - \theta_1) Q_1 + (Q_T - Q_1) \theta_1$$

In the news model, have $\theta_T = \theta_1$ so we can indeed use the Mian-Sufi estimate of 0.25 to calibrate θ_1. If the shock is a move in current θ, it is not so clear. With current shocks only, we have shown that $\frac{Q_T - Q_1}{Q_1} \approx 3(\theta_T - \theta_1)$ so $B_T - B_1 \approx \frac{Q_T - Q_1}{3Q_1} Q_1 + (Q_T - Q_1)\theta_1$ and $\Delta B = \left(\theta_1 + \frac{1}{3}\right) \Delta Q$. Thus it is not possible to have a coefficient of 0.25 in this case. It must be at least 0.33. But suppose we think that the current shock has moved by some amount $\theta_T - \theta_1$. The rest is anticipated shocks. The anticipated shocks move house prices. If we assume that anticipated shocks are proportional to realized current ones, then house price movements will be proportional to shocks:

$$\frac{\theta_T - \theta_1}{\theta_1} = m \frac{Q_T - Q_1}{Q_1}$$

which implies for debt

$$B_T - B_1 \approx (\theta_T - \theta_1) Q_1 + (Q_T - Q_1) \theta_1 = (Q_T - Q_1) \theta_1 (1 + m)$$

So the bias is m. Note that the pure news model has effectively $m = 0$ since it imposes $\theta_T - \theta_1 = 0$. Now we can get a sense of how large m is by looking at macro data. If we write $\theta_T = (1 + g)\theta_1$. The macro data suggests $g \approx 0.2$ since debt went up by 20% more than house value (in aggregate 50% versus 40%). Then we have $g = m \frac{Q_T - Q_1}{Q_1}$. Since house value went up 40%, we get $m = 0.5$, which implies that $\theta_1 = 2/3 * 0.25$

Second issue: maturity

Imagine the following economy. There are N households. Household 1 borrow B at the *beginning* of time 1, and spends it immediately. Then it repays B/N at the *end* of time $1, 2..N$. Then at time $N + 1$, it starts the same cycle. Household 2 does it at 2 and $N + 2$ and so on. This economy is stationary and the maturity of debt is 5 years (note that we abstract from interest rates for simplicity, as in our model). Moreover, in any period, the beginning of period spending is B (by the one household who just took out the loan). The total repayment at the end of the period is $N \times B/N = B$. So this matches exactly our model in terms of

flows. But the outstanding balances are:

- Beginning of period outstanding debt: $B+B\left(1-\frac{1}{N}\right)+B\left(1-\frac{2}{N}\right)\ldots+\frac{B}{N}=NB-\frac{N(N-1)}{2}B=\frac{N+1}{2}B$
- End of period is beginning of period minus B: $\frac{N-1}{2}B$

So the average balance during the period is exactly $\frac{NB}{2}$. Our θ relates to spending within the period, which is B. If we measure θ as Mian-Sufi, we get an upward bias of $N/2$. Since average maturity is 5 years for HELOC, the bias is 2.5. If we take the "news" model, we want to calibrate θ by scaling it down from 0.25 to 0.1.

If we take the mixed model, we obtain our "structural" estimate of θ_1 as

$$\theta_1 = 0.25/2.5 * 2/3 = 0.067.$$

Proofs of Proposition 3

The simplest way to understand the optimal plan is to solve for the plan without CiA constraints, and then to show it can be implemented with the right taxes. Consider for simplicity, a linear technology for home construction. Without CiA, the Lagrangian of the Planner's program is

$$\mathbb{L}=\mathbb{E}\sum\beta^t\left\{u\left(c_t,h_t,l_t\right)+\lambda_t\left(l_t-c_t-h_t+\left(1-\delta_h\right)h_{t-1}\right)\right\}.$$

The optimal labor supply requires $u_c\left(t\right)+u_l\left(t\right)=0$ and the optimal housing investment requires

$$u_h\left(t\right)=u_c\left(t\right)-\beta\left(1-\delta_h\right)\mathbb{E}_t\left[u_c\left(t+1\right)\right].$$

We can compare with the decentralized equilibrium with taxes and constant prices. Let τ^l be the tax on labor income and τ^h be the tax on home construction. Optimal labor supply requires $\left(1-\tau^l\right)\beta=1$. As expected, the planner would choose a negative labor income tax to correct the intertemporal distortion. The steady state housing equation of the Planner's program is $\frac{h}{c}=\frac{\eta}{1-\beta(1-\delta_h)}$ while in the decentralized we have $\frac{h}{c}=\left(1-\tau^h\right)q$ and $q=\frac{\eta}{1-\beta(1-\delta_h)-\theta(1-\beta)}$. So optimality requires

$$\tau^h=\frac{\theta\left(1-\beta\right)}{1-\beta\left(1-\delta_h\right)}.$$