Directed Search in the Housing Market*

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Abstract

In this paper, we present a directed search model of the housing market. The pricing mechanism we analyze reflects the way houses are bought and sold in the United States. Our model is consistent with the observation that houses are sometimes sold above, sometimes below and sometimes at the asking price. We consider two versions of our model. In the first version, all sellers have the same reservation value. In the second version, there are two seller types, and type is private information. For both versions, we characterize the equilibrium of the game played by buyers and sellers, and we prove efficiency. Our model offers a new way to look at the housing market from a search-theoretic perspective. In addition, we contribute to the directed search literature by considering a model in which the asking price (i) entails only limited commitment and (ii) has the potential to signal seller type.

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1 Introduction

In this paper, we present a directed search model of the housing market. We construct our model with the following stylized facts in mind. First, sellers post asking prices, and buyers observe these announcements. Second, there is not a straightforward relationship between the asking price and the final sales price. Sometimes buyers make counteroffers, and houses sell below the asking price. Sometimes houses sell at the asking price. Sometimes – typically when the market is hot – houses are sold by auction above the asking price. Third, a seller who posts a low asking price is more likely to sell his house, albeit at a lower price, than one who posts a higher asking price.[1]

Our model is one of directed search in the sense that sellers use the asking price to attract buyers. However, ours is not a standard directed search model in that we assume only limited commitment to the asking price. The specific form of commitment that we assume reflects the institutions of the U.S. housing market. Within a "selling period," buyers who view a house that is listed at a particular price can make offers on that house. A seller is free to reject any offer below her asking price, but she also has the option to accept such an offer. However, if the seller receives one or more *bona fide* offers to buy the house at her asking price (without contingencies), then she is committed to sell.[2] If the seller receives only one such offer at the asking price, then she is committed to transfer the house to the buyer at that price. If the seller receives two or more legitimate offers at her asking price, then she cannot, of course, sell the house to more than one buyer. In this case, the remaining buyers can bid against each other to buy the house. In practice, this auction often takes the form of bids with escalator clauses. For example, if a house is listed at $1 million, a buyer may submit a bid of that amount together with an offer to beat any other offer the seller might receive by $5,000 up to a maximum of $1.1 million.

Our description of the U.S. housing market is obviously stylized in the sense that there is sometimes ambiguity about what constitutes a *bona fide* offer at the asking price. For example, a seller can often reject a prospective buyer's offer at the asking price if the buyer's ability to secure a mortgage is in question. It is also important to emphasize that the institutional form of limited commitment to the asking price that we are ascribing to the U.S. housing market is not universal. For example, in the Netherlands, the asking price entails no legal commitment whatsoever. Nonetheless, since real estate agents have reputational concerns, asking prices reflect some limited commitment there as well.

Given limited commitment, what determines the asking prices that sellers post and

[1] Ortalo-Magné and Merlo (2004), using UK data, find that a lower asking price increases the number of visitors and offers that a seller can expect to receive but decreases the expected sales price.

[2] This commitment is typically written into contracts between sellers and their real estate agents.

what role do they play? We begin with a basic version of our model in which all sellers have the same reservation value. After observing all the asking prices in the market, each buyer chooses a seller to visit. Upon visiting the seller, the buyer decides how much he likes her house; that is, he observes the realization of a match-specific random variable. This realization is the buyer's private information. Based on this realization – and without knowing how many other buyers have visited this seller – the buyer chooses among accepting the seller's asking price, making a counteroffer (and, if so, at what level), or simply walking away. The seller then assembles her offers, if any. If no buyer has offered to pay the asking price, the seller decides whether or not to accept her best counteroffer. If she has received one, and only one, offer at the asking price, then she sells the house at that price. If she has received multiple offers at the asking price, she then allows the buyers who made those offers to compete for the house via an ascending bid auction. A payoff-equivalence result holds for this version of the model. All asking prices at or above the seller's reservation value give the seller the same expected payoff; asking prices below the reservation value yield a lower expected payoff. Similarly, buyers are indifferent with respect to any asking price greater than or equal to the common reservation value but strictly prefer any asking price below that level. Any distribution of asking prices greater than or equal to the common seller reservation value constitutes an equilibrium, and there are no equilibria in which any sellers post asking prices below the common reservation value. We show that these equilibria are constrained efficient; i.e., given the search frictions, equilibrium entails the optimal entry of sellers.

After that, we consider a version of our model in which sellers have different reservation values, and in which these reservation values are private information. Specifically, we examine a model in which there are two seller types – one group with a high reservation value, the other with a low reservation value. In this heterogeneous-seller version of our model, the asking price can potentially signal a seller's type. The signaling model is nonstandard in the sense that sellers have both *ex ante* and *ex post* signaling motives. *Ex ante* a seller wants to signal that her reservation value is low since buyers prefer to visit a seller who is perceived to be "weak." *Ex post*, however, i.e., once any buyers have visited, a seller prefers to have signaled a high reservation value since buyers will be less aggressive when dealing with a seller who is perceived to be "strong."[3] In separating equilibrium, the two seller types are identified by their posted asking prices. We develop the necessary and sufficient conditions for the existence of separating equilibria, and we demonstrate existence numerically. We show that separating equilibrium is constrained

[3] Note that the same mixed incentives apply in the labor market. *Ex ante* a job applicant wants to convince a prospective employer that she really wants the job; *ex post*, i.e., once she is about to be offered the job, she wants the employer to believe that she has many other good options.

efficient in two senses.[4] The equilibrium allocation of buyer visits across the two seller types is the same as the allocation that a social planner would choose, and seller entry is optimal. We also consider pooling equilibria. Given the mixed incentives that sellers face, there are potentially two types of pooling equilibria. In a pooling-on-low equilibrium, all sellers post a low asking price – the sellers with the high reservation value mimic those with the low reservation value – while in a pooling-on-high equilibrium, all sellers post a high asking price – the sellers with the low reservation value mimic those with the high reservation value. We develop the necessary and sufficient conditions for the existence of pooling equilibria and show numerically that under the appropriate equilibrium refinement (the intuitive criterion of Cho and Kreps 1987) these conditions are not satisfied. Finally, we consider the possibility of mixed equilibria. Again, there are potentially two types of equilibria. In a mixing-on-high equilibrium, some but not all of the sellers with the low reservation value mimic those with the high reservation value, and in a mixing-on-low equilibrium, some but not all of the sellers with the high reservation value mimic those with the low reservation value. We develop the necessary and sufficient conditions for such equilibria to exist and show numerically that these conditions are not satisfied.

Our paper makes two contributions. First, we add to the growing literature that uses an equilibrium search approach to understand the housing market. Search theory is a natural tool to use to analyze this market since it clearly takes time and effort for buyers to find suitable sellers and vice versa. Papers that use search theory to analyze the housing market include Wheaton (1990), Krainer (2001), Carrillo (2006), Albrecht, et al. (2007), Ngai and Tenreyro (2009), and Díaz and Jerez (2009). With the exception of Carrillo (2006) and Díaz and Jerez (2009), all of the papers in this literature assume that search is random, as opposed to directed. In some of these papers, prices are determined by Nash bargaining; in others, when a buyer and seller meet, one of the parties (typically the seller) makes a take-it-or-leave-it offer. In contrast, in our model and in Carrillo (2006) and Díaz and Jerez (2009), sellers post prices to attract buyers. In Carrillo (2006), ads are posted by sellers, stating an asking price and describing some features of the house. Buyers randomly sample one advertisement per period and then decide whether to pay a cost to visit the house or to wait to see another ad in the next period. All else equal, a lower asking price makes a prospective buyer more likely to inspect a house. Carrillo (2006) does not allow for the possibility that an ad might attract more than one buyer within a period, so the congestion effects that are central to

[4] Actually, there is a third dimension of efficiency in our model. Since more than one buyer may visit the same seller and since these buyers' reactions to the house are idiosyncratic, efficiency requires that if the house is sold, it should be sold to the buyer with the highest valuation. This efficiency criterion is trivially satisfied by the mechanism that we analyze.

models of directed search are not present in his setup. In addition, his model has more of a random search flavor since buyers can view only one ad per period. Finally, there is full commitment in Carrillo (2006) in the sense that a house never sells above its asking price.[5] Díaz and Jerez (2009) use competitive search theory (Moen 1997) to analyze the problem initially posed in Wheaton (1990), in which shocks lead to mismatch, causing a household to first search to buy a new house and then to look for a buyer for its old house. In their equilibrium, all sellers post the same asking price, the asking price and the sales price are the same, and all houses sell with the same probability The equilibrium outcome is considerably richer in our model. As in Díaz and Jerez (2009), sellers use the asking price to attract buyers. However, in our model, houses can sell below, at, or above the asking price. Finally, they consider only homogeneous sellers, whereas we also present a heterogeneous-seller version of our model.[6]

Our second contribution is to the directed search literature. In the standard directed search model, there is full commitment in the sense that all transactions must take place at the posted price. In our model, however, there is only limited commitment. The posted price "means something" and is used to attract buyers, but the final selling price need not be the same as the posted price. Camera and Selcuk (2009) also consider a model of directed search with limited commitment. As we do, they assume that sellers post prices and that buyers direct their search in response to those postings. The difference between our approach and theirs comes once buyers choose which sellers to visit. Camera and Selcuk (2009) allow for the possibility of renegotiation, i.e., that the final selling price and the posted price may differ, but they are agnostic about the specifics of the renegotiation process. Instead, they deduce some implications of assuming that the selling price is increasing in (i) the asking price and (ii) the number of buyers who visit the seller in question; e.g., they prove that all sellers post the same asking price in symmetric equilibrium.[7] Our approach differs from that of Camera and

[5] Chen and Rosenthal (1996) also treat the asking price as a price ceiling. In their model, a monopolist posts an asking price. Buyers contact the monopolist at an exogenous Poisson rate and decide whether to visit the monopolist. The tradeoff that the monopolist faces in setting her asking price is the same as in Carrillo (2006). A lower asking price makes it more likely that the buyer will visit but reduces the maximum price that the seller can get for her product.

[6] Díaz and Jerez (2009) is a dynamic model in which all sellers post the same price and all houses are sold with the same probability *within each period*, but the price and probability of sale change over time as the housing market adjusts to the effects of an exogenous aggregate shock.

[7] Our model does not fit the Camera and Selcuk (2009) framework for two reasons. First, in our model, buyers draw idiosyncratic values once they visit a seller; i.e., buyers are *ex post* heterogeneous. Second, we allow for the possibility that sellers may be *ex ante* heterogeneous in the sense of having different reservation values. Examples of models that do fit the Camera and Selcuk (2009) framework are Albrecht, Gautier and Vroman (2006) and Julien, Kennes and King (2000). These are labor market models in which the posted wage and the wage that is actually paid are the same if a worker has a single job offer, while the wage of a worker with multiple offers is bid up to the competitive level.

Selcuk (2009) in that we assume a specific price determination mechanism. We take this more specific approach because the price determination mechanism that we analyze is an important one in practice and because, as we show below, it turns out that this mechanism is efficient.

We also contribute to the directed search literature by considering the potential signaling role of the asking price. As in Delacroix and Shi (2008), we consider a model in which the asking price plays the dual role of directing buyer search and signaling seller type.[8] In their model, each seller chooses a pricing mechanism – either price posting or Nash bargaining – and whether to produce a low-quality or a high-quality good. Efficiency has two dimensions, entry and quality, and Delacroix and Shi (2008) ask under what conditions price posting or bargaining is the more efficient mechanism. The answer depends on the bargaining power parameter and on the relative quality of the two goods. They show that for almost all parameter combinations, the two mechanisms can be ranked in terms of efficiency and that in equilibrium, only the efficient mechanism is used. In the heterogeneous-seller version of our model, there are also two choice variables in the social planner problem. Seller entry should be at the efficient level, and the probability that a buyer visits a high-type seller should be at the efficient level. The pricing mechanism that we analyze is able to satisfy these two criteria simultaneously. Finally, Guerrieri, Shimer and Wright (2009) consider a directed search model where, contrary to our model, the side of the market with private information is not the side that is doing the posting. The problem they consider is one of adverse selection and they show that the equilibrium can be inefficient in this case.

In constructing our model, we have abstracted from some important features of the housing market. One obvious abstraction is that we ignore real estate agents. We do this to keep our model simple, but also because the decision about the asking price, which is the focus of our model, is ultimately the seller's to make. We also abstract from the fact that in the housing market buyers are often also sellers and their ability to buy may hinge on their ability to sell. Rather than modeling this explicitly as in Wheaton (1990) and Díaz and Jerez (2009), we capture this in the heterogeneous-seller version of our model through the reservation value. A motivated seller, one with a low reservation value, can be thought of as one who has already bought or put a contract on a new house and is thus eager to sell. Finally, houses are, of course, not identical – some are in good condition and located in desirable neighborhoods while others are not – and much of the variation in asking prices across houses reflects this intrinsic heterogeneity. In our model, we assume that buyers can identify these differences, perhaps with the

[8] See also Menzio (2007), who shows that "cheap talk" in the form of non-binding announcements about compensation can, under some circumstances, signal employer type and partially direct worker search.

help of real estate agents, perhaps by simply using the web. We are looking at the role that the asking price plays after adjusting for these differences. Sellers set different asking prices, even for houses that are intrinsically identical. These differences reflect the sellers' reservations values. Some are motivated, while others are willing to wait to get a good price.

The remainder of our paper is organized as follows. In the next section, we lay out the structure of the game that we analyze. In Section 3, we analyze the model assuming that all sellers have the same reservation value. In Section 4, we consider the heterogeneous-seller case and analyze the separating equilibrium. Pooling and mixed equilibria are relegated to the Appendix. Finally, in Section 5, we conclude.

2 Basic Model

We model the housing market as a one-shot game played by B buyers and S sellers of identical houses. We consider a large market so that both B and S go to infinity, but in such a way as to keep $\theta = B/S$, the market tightness, constant. We first analyze the market taking θ as given. Then, once the equilibrium is characterized for any given θ, we allow for free entry of sellers and examine the efficiency of market equilibrium.

The game has several stages:

1. Each seller posts an asking price a.

2. Each buyer observes all posted prices and chooses one seller to visit. There is no coordination among the buyers.

3. Upon visiting a seller, the buyer draws a match-specific value, x. The match-specific values are *iid* draws across buyer-seller pairs from a standard uniform distribution. Buyers do not observe the number of other visitors to the house.

4. The buyer can accept the asking price, a, make a counteroffer, or walk away.

5. If no buyer visits, the seller retains the value of her house.

6. If at least one buyer visits, but no buyer accepts the asking price, then the seller can accept or reject the highest counteroffer. If one or more buyers accept the asking price, then an ascending-bid auction ensues with reserve price a. In this case, the house is transferred to the highest bidder.

Buyers who fail to purchase a house receive payoffs of zero. The payoff for a buyer who draws x and then purchases the house is $x - p$, where p is the price that the buyer pays.

If no sale is made, the owner of the house retains its value, while a seller who transfers her house to a buyer at price p receives that price as her payoff.

This is a model of directed search in the sense that buyers observe all asking prices and can direct their visits to houses based on these asking prices. It differs from many directed search models in that the sellers make a limited commitment to their asking prices. That is, while in the usual directed search model, sellers (or vacancies) fully commit to their posted prices (wages) in the sense that they commit not to charge more (or pay less), here the seller makes only a limited commitment. If only one buyer shows up and accepts the asking price, then the seller agrees to sell at that price, but if more buyers show up, the price is bid up. We consider symmetric equilibria in which all buyers use the same strategy. They search optimally given the distribution of posted asking prices and given optimal directed search by other buyers. Buyers bid optimally given the bidding strategy followed by other buyers.

We first consider the case of homogeneous sellers, i.e., the case in which all sellers have the same reservation value s. In setting an asking price, each seller anticipates buyer reaction to her posted price. When sellers are homogenous, we show that the only role of the asking price is to ensure that houses do not sell below s. After considering the homogeneous case, we turn to the heterogeneous case in which sellers differ with respect to their reservation values and seller type is private information. In this case, the asking price also has the potential to signal seller type. We assume that there are two seller types: high types who have reservation value s and low types who have a reservation value that we normalize to zero.

3 Homogeneous Sellers

We begin by considering the case in which all sellers have the same reservation value, s. We first show that payoff equivalence holds in this setting. In particular, if other buyers distribute themselves randomly across sellers posting asking prices greater than or equal to s, an individual buyer finds any asking price in $[s, 1]$ equally attractive. If varying the asking price in $[s, 1]$ does not affect the expected buyer arrival rate, then any asking price $a \geq s$ generates the same expected payoff for sellers.

We next show that any distribution of asking prices on $[s, 1]$ is an equilibrium. If a single seller were to deviate and post an asking price $a' < s$, buyers would view this favorably, and the seller's expected arrival rate of visitors would increase. Nonetheless, we show that this seller's expected payoff would be lower than it would have been had she posted any $a \geq s$. That is, any configuration of asking prices greater than or equal to the common reservation value constitutes an equilibrium. So, for example, a situation in which all sellers post $a = s$ is an equilibrium, and likewise if all sellers post $a = 1$. Finally, we show that there are no equilibria in which any seller posts an asking price below s. We also examine the effects of changes in s and θ on the probability of sale and on the average selling price. The derivations of buyer and seller behavior in the homogeneous seller case carry over to the heterogeneous case and so are given in detail here. After showing the existence of equilibrium, we examine efficiency by endogenizing the number of sellers.

3.1 Buyer Side

Suppose all sellers post asking prices $a \in [s, 1]$. Suppose provisionally that buyers choose which seller to visit at random, so the number of buyers visiting any particular seller is Poisson with parameter θ. If a buyer visits a seller posting a, what bidding function should he use, and what is the expected payoff?

Let $b(x)$ denote the buyer's bid as a function of the value drawn. Let x^* be such that $x \geq x^*$ leads to a bid of $b(x) = a$.[9] The bidding function constitutes equilibrium behavior if, given that any other buyers visiting this seller set their bids using $b(x)$, it is optimal for an individual buyer to do the same. Another way to express this is: Let $b(x'; x)$ denote the bid that a buyer who draws x would make if he were to pretend to have drawn x'. Let $v(x'; x)$ be the expected payoff to buyer x who pretends to be type x'. The function $b(x) = b(x; x)$ reflects equilibrium behavior if $v(x; x) \geq v(x'; x)$ for all (x', x) pairs.

Suppose the buyer draws $x < s$. The buyer is not willing to offer a price that the seller would accept, so his bid is $b(x) = 0$; i.e., the buyer walks away, and the payoff is $v(x) = 0$.

Next suppose the buyer draws a value $s \leq x < x^*$ and that he bids $b(x'; x)$, while any other buyers visiting this seller use the bidding function $b(x)$. The number of other buyers visiting this seller is Poisson with parameter θ, so the probability that no other buyer visits this seller and draws a value above x' is $e^{-\theta(1-x')}$. Conditional on getting the house, the buyer's payoff is x minus the price paid. That is,

$$v(x'; x) = e^{-\theta(1-x')}(x - b(x'; x)).$$

Differentiating with respect to x' and evaluating at $x' = x$ gives

$$\theta e^{-\theta(1-x)}(x - b(x; x)) - e^{-\theta(1-x)} \frac{\partial b(x; x)}{\partial x'} = 0;$$

[9] Note that, for some parameter configurations, buyers may never want to bid the asking price even were they to draw $x = 1$. That is, x^* may not be relevant, i.e., $b(1) < a$. As we show below (see equations (4) and (7)), this does not affect the expected payoffs for buyers and sellers.

that is,

$$\theta(x - b(x)) = \frac{db(x)}{dx}.$$

The initial condition is $b(s) = s$ (a buyer who draws value s only buys the house if it can be purchased at price s). The solution to this equation is

$$b(x) = x - \frac{1 - e^{-\theta(x-s)}}{\theta} \text{ for } s \le x < x^*,$$

and the corresponding value is

$$v(x) = e^{-\theta(1-x)}(\frac{1 - e^{-\theta(x-s)}}{\theta}) \text{ for } s \le x < x^*.$$

Finally, for $x \ge x^*$, the buyer bids $b(x) = a$. The expected payoff is

$$v(x) = e^{-\theta(1-x)}(x - p(x)),$$

where $p(x)$ is the expected price if he draws x and wins the auction. The buyer wins the auction if and only if no other buyer visits and draws a higher value. This probability is $e^{-\theta(1-x)}$. The price that the winning buyer expects to pay depends on how many other buyers visit and draw values in $[x^*, x)$. Denote the number of other such buyers by n. This random variable is Poisson with parameter $\theta(x - x^*)$. Conditional on n, the expected price, $p(x; n)$, paid by a winning buyer who has drawn $x \ge x^*$ is

$$\begin{aligned} p(x;0) &= a \\ p(x;n) &= \frac{nx + x^*}{n+1} = x - \frac{x - x^*}{n+1} \text{ for } n = 1, 2, ... \end{aligned}$$

We then have

$$\begin{aligned} x - p(x) &= (x-a)e^{-\theta(x-x^*)} + \sum_{n=1}^{\infty} \frac{(x-x^*)e^{-\theta(x-x^*)}(\theta(x-x^*))^n}{(n+1)!} \\ &= (x^*-a)e^{-\theta(x-x^*)} + \frac{1 - e^{-\theta(x-x^*)}}{\theta} \qquad (1) \end{aligned}$$

and the expected payoff is

$$v(x) = e^{-\theta(1-x)} \left((x^* - a)e^{-\theta(x-x^*)} + \frac{1 - e^{-\theta(x-x^*)}}{\theta} \right) \text{ for } x^* \le x.$$

Before taking the final step of solving for x^*, we can summarize the problem of a buyer who visits a seller posting an asking price of $a \ge s$ as follows:

$$b(x) = \begin{cases} 0 & \text{for } 0 \le x < s \\ x - \dfrac{1 - e^{-\theta(x-s)}}{\theta} & \text{for } s \le x < x^* \\ a & \text{for } x^* \le x \end{cases} \qquad (2)$$

$$v(x) = \begin{cases} 0 & \text{for } 0 \leq x < s \\ e^{-\theta(1-x)}(\frac{1-e^{-\theta(x-s)}}{\theta}) & \text{for } s \leq x < x^* \\ e^{-\theta(1-x)}\left((x^*-a)e^{-\theta(x-x^*)} + \frac{1-e^{-\theta(x-x^*)}}{\theta}\right) & \text{for } x^* \leq x. \end{cases}$$

The continuity of $v(x)$ at x^* gives

$$x^* - \frac{1-e^{-\theta(x^*-s)}}{\theta} = a.$$

Using this to substitute for $(x^* - a)$ in (1) gives

$$x - p(x) = \frac{1-e^{-\theta(x-s)}}{\theta}.$$

That is, a buyer who draws $x \geq x^*$ can expect to pay

$$p(x) = x - \frac{1-e^{-\theta(x-s)}}{\theta} \tag{3}$$

if he is the winning bidder.

Substituting for $x^* - a$ in $v(x)$ for $x \geq x^*$ gives

$$v(x) = \begin{cases} 0 & \text{for } 0 \leq x < s \\ \frac{e^{-\theta(1-x)} - e^{-\theta(1-s)}}{\theta} & \text{for } s \leq x \end{cases}. \tag{4}$$

The expected payoff to a buyer who visits a seller posting $a \geq s$ is thus

$$V(a; s, \theta) = \int_s^1 v(x)dx = \frac{1 - e^{-\theta(1-s)} - \theta(1-s)e^{-\theta(1-s)}}{\theta^2}. \tag{5}$$

So long as all sellers posting $a \geq s$ face the same buyer arrival rate, an individual buyer has the same expected payoff if he visits any one of those sellers. In particular, the buyer's expected payoff does not depend on a. This implies that buyers can do no better than randomizing their visits over these sellers, as was assumed.

3.2 Seller Side

Assuming that buyers arrive at rate θ, we next derive the expected payoff for a seller who posts $a \geq s$. If no buyers visit this seller, she retains s. This occurs with probability $e^{-\theta}$. The seller receives at least one visitor with probability $1 - e^{-\theta}$. Given that at least one buyer visits, suppose that the highest valuation drawn is x. If $x < s$, the seller retains s. If $s \leq x < x^*$, the seller's payoff is this buyer's bid, $b(x)$. If $x \geq x^*$, the seller's expected payoff is $p(x)$.

The density of the highest x drawn at a particular seller is

$$f(x|\text{high}) = \frac{P[\text{high}|x]f(x)}{P[\text{high}]} = \frac{\theta e^{-\theta(1-x)}}{1-e^{-\theta}}. \tag{6}$$

This can be understood as follows. Consider a buyer who draws x. The probability that no other buyer visits and draws a higher value is $P[\text{high}|x] = e^{-\theta(1-x)}$. The assumption that buyer values are uniformly distributed, i.e., $f(x) = 1$, implies the *ex ante* probability (i.e., before knowing how many other buyers will visit and before drawing x) that this buyer will draw the highest value is

$$P[high] = \int_0^1 P[\text{high}|x]f(x)dx = \frac{1-e^{-\theta}}{\theta}.$$

We can summarize the above discussion as follows. If buyers arrive at rate θ, the expected payoff for a seller who posts $a \geq s$ is

$$\Pi(a;s,\theta) = e^{-\theta}s + (1-e^{-\theta})\left(\int_0^s s\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx + \int_s^{x^*} b(x)\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx + \int_{x^*}^1 p(x)\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx\right).$$

Substituting for $b(x)$ and $p(x)$ from the buyer side gives

$$\Pi(a;s,\theta) = 1 + (1-s)e^{-\theta(1-s)} - \frac{2(1-e^{-\theta(1-s)})}{\theta}. \tag{7}$$

Again, we have payoff equivalence – so long as buyers visit any seller posting $a \geq s$ with equal probability, the expected payoff associated with posting any $a \geq s$ is the same.

3.3 Equilibrium

The next step is to show that if all sellers post asking prices in $[s, 1]$, it is not in the interest of a single seller to deviate to $a' < s$. If a seller deviates, she picks the $a' < s$ that maximizes her expected payoff. How will buyers react to this deviation? First, any buyer who visits this seller who draws $x \geq a'$ will bid a'. There is no point in making a counteroffer since the seller is posting an asking price below her reservation value. Second, the buyer arrival rate adjusts to reflect the fact that the deviant seller is offering an expected payoff that may differ from $V(a)$. Let γ be the arrival rate of buyers to this deviant. This arrival rate is determined by the requirement that buyers must be indifferent between visiting a nondeviant and a deviant seller. Let $V(a';\gamma)$ be the expected payoff for a buyer who visits a seller posting a' when the expected number of other visitors to this seller is Poisson with parameter γ. Using the same argument as used to derive equation (5)

$$V(a';s,\gamma) = \int_{a'}^1 e^{-\gamma(1-x)}\left(\frac{1-e^{-\gamma(x-a')}}{\gamma}\right)dx = \frac{1-e^{-\gamma(1-a')} - \gamma(1-a')e^{-\gamma(1-a')}}{\gamma^2}. \tag{8}$$

The buyer's indifference condition is $V(a';s,\gamma) = V(a;s,\theta)$. Treating γ as a function of a' and noting that $V(a;s,\theta)$ depends neither on a' nor on γ, we have

$$\frac{\partial V}{\partial a'} + \frac{\partial V}{\partial \gamma}\frac{d\gamma}{da'} = 0; \text{ i.e., } \frac{d\gamma}{da'} = -\left(\frac{\partial V}{\partial a'}\right) / \left(\frac{\partial V}{\partial \gamma}\right).$$

Substituting for $\partial V/\partial a'$ and $\partial V/\partial \gamma$ gives

$$\frac{d\gamma}{da'} = -\frac{\gamma^3(1-a')e^{-\gamma(1-a')}}{2\left(1-e^{-\gamma(1-a')} - \gamma(1-a')e^{-\gamma(1-a')} - \frac{\gamma^2(1-a')^2}{2}e^{-\gamma(1-a')}\right)}.$$

The denominator is two times the probability that the deviant seller is visited by 3 or more buyers; thus, $d\gamma/da' < 0$, as expected.

Returning to the deviant seller's problem and using the same approach as was used to derive equation (7),

$$\Pi(a';s,\gamma) = e^{-\gamma}s + (1-e^{-\gamma})\left(\int_0^{a'} s\frac{\gamma e^{-\gamma(1-x)}}{1-e^{-\gamma}}dx + \int_{a'}^1 p(x)\frac{\gamma e^{-\gamma(1-x)}}{1-e^{-\gamma}}dx\right).$$

Here $p(x)$ is the price that the buyer who wins the auction can expect to pay. In this case, since counteroffers are not at issue, $x^* = a'$ and

$$p(x) = x - \frac{1-e^{-\gamma(x-a')}}{\gamma}.$$

Substituting and integrating gives

$$\Pi(a';s,\gamma) = 1 + (s-a')e^{-\gamma(1-a')} - \frac{2\left(1-e^{-\gamma(1-a')} - \gamma(1-a')e^{-\gamma(1-a')}\right)}{\gamma}. \tag{9}$$

Differentiating with respect to a', while taking into account that γ varies with a' via the buyer indifference condition, gives

$$\frac{d\Pi(a';s,\gamma)}{da'} = \frac{\partial \Pi(a';s,\gamma)}{\partial a'} + \frac{\partial \Pi(a';s,\gamma)}{\partial \gamma}\frac{\partial \gamma}{\partial a'}.$$

After substitution and considerable algebra,

$$\frac{d\Pi(a';s,\gamma)}{da'} = \gamma e^{-\gamma(1-a')}\left(\begin{array}{c} 1+s-2a' \\ -(1-a')\frac{1-e^{-\gamma(1-a')}-\gamma(1-a')e^{-\gamma(1-a')}-\frac{\gamma^2(1+s-2a')(1-a')}{2}e^{-\gamma(1-a')}}{1-e^{-\gamma(1-a')}-\gamma(1-a')e^{-\gamma(1-a')}-\frac{\gamma^2(1-a')^2}{2}e^{-\gamma(1-a')}} \end{array}\right).$$

This expression is positive for all $a' < s$. In short, even though buyers are more likely to visit a seller posting $a' < s$, it is not profitable for a seller to post $a' < s$.

The argument just given establishes that if all sellers post asking prices in $[s, 1]$, then no seller wants to deviate to an asking price $a' < s$. That is, any distribution of asking prices over $[s, 1]$ is consistent with equilibrium.

In fact, the same argument can be used to rule out the possibility of equilibria in which any measure of sellers posts asking prices below the common reservation value, s. Imagine a candidate equilibrium in which some sellers post asking prices below s. Buyer arrival rates would have adjusted to make each buyer indifferent as to which house he visits. That is, there is a common buyer value V associated with this putative equilibrium. The argument that we made above can be applied to show that any seller posting an asking price below s wants to increase her asking price. Consider a particular seller posting $a < s$. The expected payoff associated with posting a price below s is still given by equation (9) and the arrival rate, γ, is determined by $V(a'; s, \gamma) = V$, where the expression for $V(a'; s, \gamma)$ is still given by equation (8). The argument given above shows that this seller's expected payoff increases in her asking price.

We have thus shown:

Proposition 1 *Any configuration of asking prices over* $[s, 1]$ *constitutes an equilibrium of the homogeneous-seller model. All such equilibria are payoff equivalent. Further, there are no equilibria in which any sellers post asking prices below* s.

Proposition 1 states that there is an infinity of equilibria in the homogeneous-seller model, but we have shown that all of these equilibria are payoff equivalent. We can thus choose one of these equilibria, for example, the one in which all sellers post $a = s$, to demonstrate some of the properties of equilibrium. In particular, we now show that the probability of sale and the average selling price vary with θ and s, the exogenous parameters of the model, in the expected way.

Consider first the probability that any particular house is sold. This is

$$P[\text{Sale}] = 1 - e^{-\theta(1-s)}.$$

This probability would be the same in any equilibrium in which buyers randomize their visits across sellers. As expected, as the market gets tighter, i.e., as θ increases, the probability that a house sells increases. In addition, also as expected, as sellers become "less motivated," i.e., as s increases, the probability of a sale decreases.

The average selling price is a bit more complicated. We can write

$$E[p] = \int_s^1 \frac{p(x) f(x|\text{high})}{\int_s^1 f(u|\text{high}) du} dx.$$

When all sellers post $a = s$, the price paid for a particular house is $p(x)$ (equation 3), where x is the valuation drawn by the buyer who purchases the house. Using equation

(6), the density of x for this buyer is

$$f(x|\text{buy}) = \frac{f(x|\text{high})}{\int_s^1 f(u|\text{high})du} = \frac{\theta e^{-\theta(1-x)}}{1 - e^{-\theta(1-s)}}.$$

We thus have

$$E[p] = \frac{1}{1 - e^{-\theta(1-s)}} \left(1 + e^{-\theta(1-s)} - 2se^{-\theta(1-s)} - \frac{2}{\theta}(1 - e^{-\theta(1-s)})\right). \tag{10}$$

It is straightforward, if algebraically tedious, to verify that the average selling price is increasing in θ and in s, as one would expect.

3.4 Endogenizing θ - Efficiency

In order to address the question of efficiency, we now allow for free entry of sellers. That is, we endogenize θ.

Consider a version of the homogenous-seller model in which the measure of buyers, B, is exogenous but the measure of sellers, S, is determined by free entry. As before, let $\theta = B/S$. A seller who enters the market pays an advertising cost A and receives expected payoff $\Pi(\theta)$.[10] A prospective seller who chooses to stay out of the market retains value s. The expected payoff for a buyer is $V(\theta)$. If each seller posts an asking price $a \geq s$ and if buyers randomize their visits across sellers, the buyer and seller values are given by equations (5) and (7), respectively. Equilibrium with endogenous θ requires that the free entry condition

$$\Pi(\theta) - A = s$$

is satisfied.

The social planner's objective is to maximize total net surplus for market participants. In equilibrium, houses are sold only if the buyer's value exceeds the seller's reservation value. Further, if there are multiple visitors to a house, the house is sold to the visitor with the highest value. Thus, the only issue for the social planner is whether θ is determined efficiently given the coordination frictions in the market. Total net surplus is

$$BV(\theta) + S\left(\Pi(\theta) - A - s\right).$$

Equivalently, since B is exogenous, the social planner objective can be written on a per-buyer basis as

$$V(\theta) + \frac{\Pi(\theta) - A - s}{\theta},$$

[10] Here we consider Π and V as functions of θ. We suppress a since we earlier showed that Π and V do not depend on a. We also suppress s since it is constant.

so the efficient value of θ solves

$$V'(\theta) + \frac{\theta\Pi'(\theta) - (\Pi(\theta) - A - s)}{\theta^2} = 0.$$

Given the free entry condition, equilibrium is efficient if

$$V'(\theta) + \frac{\Pi'(\theta)}{\theta} = 0.$$

From equations (5) and (7), this holds for all s. We have thus shown:

Proposition 2 *Free entry equilibrium is efficient in the homogeneous-seller case.*

4 Heterogeneous Sellers

We next consider the case in which sellers are heterogeneous with respect to their reservation values. For simplicity, we consider two seller types. A fraction q of sellers, the high (H) type sellers, have reservation value s, as in the homogeneous case. The remaining sellers, the low (L) type or more motivated sellers, have a lower reservation value, which we normalize to 0. A seller's type is private information, but the distribution of seller types is common knowledge. Given these assumptions, there are three possible types of equilibria in which sellers follow pure strategies. In separating equilibrium, each seller posts an asking price that is type-revealing. There are two possible types of pooling equilibria - one in which all sellers post asking prices below s ("pooling-on-low") and one in which all sellers post asking prices greater than or equal to s ("pooling-on-high"). Mixed equilibria are also possible. For example, type-H sellers might randomize between posting an asking price below s and one greater than or equal to s.

Without restrictions on buyers' beliefs, many perfect Bayesian equilibria exist. We therefore apply the Intuitive Criterion (IC) of Cho and Kreps (1987) to place restrictions on buyers' out-of-equilbrium beliefs. To see how the IC works in our setting, consider a candidate pooling equilibrium. For this to be an equilibrium, neither type should have a credible deviation. What does credible mean in this context? Suppose, for example, that a type-L seller considers a deviation to some a', and suppose that if she were to do so, buyers would believe her to be type L. According to the IC, a' would be a credible deviation if

1. a' is profitable for a type-L deviant given that buyers believe the deviation signals L

2. a' is not profitable (equilibrium dominated) for a type-H deviant for any beliefs of the buyers.

As the above discussion suggests, the IC typically makes it more difficult to sustain pooling equilibria. If it is in the interest of one type t to deviate from pooling so long as that deviation is type-revealing but not in the interest of the other types t' to mimic, then the deviation should be interpreted as signaling type t. This is why only separating equilibria survive the IC in, for example, the two-type Spence (1973) model. At some point, an increase in education that would be in the interest of the high-productivity types were employers to view that deviation as a signal of high productivity becomes too costly for the low-productivity types to mimic.

Note, however, that the incentives to mimic are not straightforward in our model. *Ex ante* each seller wants buyers to believe that she is type L because this increases her expected queue length, but *ex post*, when it is time to bid, each seller wants buyers to believe that she is type H because this results in higher bids. Sellers, however, have only one signal, namely, the asking price, and must trade off the relative benefit of longer queues in the first stage versus higher bids in the second stage. This is why the two types of pooling equilibria are conceivable in our setting. If the *ex ante* benefit of longer queues dominates the *ex post* benefit of higher bids, then type-H sellers may want to mimic type-L sellers by setting a low asking price. If the *ex post* benefit of higher bids dominates, then type-L sellers may want to mimic type-H sellers by setting a high asking price. In short, even though the IC often rules out pooling equilibria in signaling models, it is not obvious that this will be the case in our setting.[11]

As it turns out, in our model, despite the conflicting incentives to mimic, no pooling equilibria exist. The *ex ante* incentive to lower the asking price to attract buyers dominates for type L; the *ex post* incentive to set a price that elicits higher bids (and ensures that the house will not be sold at a price below s) dominates for type H. Similarly, no mixed equilibria exist. That is, there are no equilibria in which any type-H seller "pretends to be a type L" nor are there equilibria in which any type-L seller "pretends to be type H." From the perspective of a social planner, there are clear benefits to the nonexistence of pooling or mixed equilibria. A social planner would prefer that the arrival rate of buyers to type-L sellers be greater than the arrival rate to type-H sellers. In a pooling equilibrium, these arrival rates would be the same since buyers cannot distinguish between seller types *ex ante*. In addition, a pooling or mixed equilibrium in which some or all type-L sellers mimic type-H sellers by setting an asking price of s or more would create a situation in which mutually beneficial trades are not realized. Consider a buyer visiting a seller posting $a \geq s$ who draws an x that is slightly greater than s. It would be rational for this buyer to make a bid below s in the hope that the seller is

[11] There are, of course, models in which pooling equilibria survive the Intuitive Criterion, e.g., Bagwell and Ramey (1988) and Angeletos, Hellwig and Pavan (2006).

type L. If it happens that this buyer has drawn the highest value of x and the seller is type H, then surplus would be left on the table.

The equilibria that do exist in our model are separating equilibria. We analyze these equilibria and their efficiency properties below. The nonexistence of pooling and mixed equilibria is discussed in more detail in the Appendix.

4.1 Separating Equilibrium

In separating equilibrium, seller types are known. Given that her type is known, a type-L seller is indifferent with respect to any asking price, $a \geq 0$. This follows from the payoff-equivalence result for homogeneous sellers. Similarly, given that her type is known, a type-H seller is indifferent with respect to any $a \geq s$. Therefore, without loss of generality, we consider a potential separating equilibrium in which type-L sellers post 0 and type-H sellers post s.

Buyers need to choose which seller type to visit. Suppose each buyer visits a seller posting s with probability r. In a finite housing market, the number of visitors to a type-H seller is then binomial with parameters rB and $\frac{1}{qS}$. As $B, S \to \infty$, with $B/S = \theta$, the number of visitors to a type-H seller is Poisson with parameter $\theta_H = r\theta/q$. Similarly, the number of visitors to a type-L seller is Poisson with parameter $\theta_L = (1-r)\theta/(1-q)$.

Let $V_L(r; q, \theta)$ be the value to a buyer of visiting a seller posting $a = 0$ given that buyers visit sellers posting $a = s$ with probability r. Similarly, let $V_H(r; q, s, \theta)$ be the value to a buyer of visiting a seller posting $a = s$ given that buyers visit sellers posting $a = s$ with probability r. If buyers visit both seller types, i.e., if $r > 0$, then buyers must be indifferent as to which seller type they visit, i.e., $V_L(r; q, \theta) = V_H(r; q, s, \theta)$ must hold. If buyers visit only type-L sellers, i.e., if $r = 0$, then $V_L(0; q, \theta) \geq V_H(0; q, s, \theta)$.

In order that a separating equilibrium exist, three conditions need to be satisfied. First, buyers should behave optimally. Second, a type-L seller should not want to deviate from $a = 0$. Third, a type-H seller should not want to deviate from $a = s$.

4.1.1 Determination of r

We start with the determination of r. Using the arguments underlying equation (5), the values of visiting a type-H seller and a type-L seller as functions of r are

$$\begin{aligned} V_H(r; q, s, \theta) &= \frac{1 - e^{-\theta_H(1-s)} - \theta_H(1-s)e^{-\theta_H(1-s)}}{\theta_H^2} \\ V_L(r; q, \theta) &= \frac{1 - e^{-\theta_L} - \theta_L e^{-\theta_L}}{\theta_L^2}. \end{aligned}$$

Using $\frac{d\theta_H}{dr} = \frac{\theta_H}{r}$ and $\frac{d\theta_L}{dr} = -\frac{\theta_L}{1-r}$, we have

$$V_H'(r;q,s,\theta) = -\frac{2}{r}\left(\frac{1 - e^{-\theta_H(1-s)} - \theta_H(1-s)e^{-\theta_H(1-s)} - \frac{\theta_H^2(1-s)^2 e^{-\theta_H(1-s)}}{2}}{\theta_H^2}\right) < 0$$

$$V_L'(r;q,\theta) = \frac{2}{(1-r)}\left(\frac{1 - e^{-\theta_L} - \theta_L e^{-\theta_L} - \frac{\theta_L e^{-\theta_L}}{2}}{\theta_L^2}\right) > 0$$

for $r \in (0,1]$. The signs of these derivatives are intuitive. All else equal, if a single buyer chooses to visit a type-H seller, he is better off doing so when fewer of his fellow buyers visit type-H sellers, i.e., when r is low.

Equivalently, $V_H(r;q,s,\theta) - V_L(r;q,\theta)$ is decreasing in r. Buyers visit both type-L and type-H sellers with positive probability iff (i) $\lim_{r\to 0} V_H(r;q,s,\theta) - V_L(r;q,\theta) > 0$ and (ii) $\lim_{r\to 1} V_H(r;q,s,\theta) - V_L(r;q,\theta) < 0$. The second condition is trivially satisfied – if all other buyers visit sellers posting $a = s$, any individual buyer is clearly better off visiting a seller posting $a = 0$. In fact, even for a smaller value of r, when $r = q$, so that $\theta_H = \theta_L$, $V_H(r;q,s,\theta) < V_L(r;q,\theta)$ so long as $s > 0$.

To find the parameter values for which condition (i) holds, note that

$$V_L(0;q,\theta) = \frac{1 - e^{-\theta/(1-q)} - (\theta/(1-q))e^{-\theta/(1-q)}}{(\theta/(1-q))^2}$$

and, using l'Hôpital's Rule, that

$$\lim_{r\to 0} V_H(r;q,s,\theta) = \frac{(1-s)^2}{2}.$$

Condition (i) is satisfied and thus buyers visit both seller types with positive probability iff

$$\frac{(1-s)^2}{2} > \frac{1 - e^{-\theta/(1-q)} - (\theta/(1-q))e^{-\theta/(1-q)}}{(\theta/(1-q))^2}. \tag{11}$$

When (11) is violated, buyers only visit the type-L sellers.

We now show that for any given values of θ and q, there are values $s > 0$ for which this inequality is satisfied; that is, there are positive values of s such that $V_H(r;q,s,\theta) = V_L(r;q,\theta)$ has a unique positive solution. Let $z = \theta/(1-q)$. Inequality (11) can be written as

$$(1-s)^2\frac{z^2}{2} > 1 - e^{-z} - ze^{-z}.$$

Suppose $s = 0$. The inequality then holds for all $z > 0$, i.e., so long as $\theta > 0$. (At $z = 0$, the two sides are equal, and the derivative of the LHS with respect to z exceeds the corresponding derivative of the RHS.) Starting from $s = 0$ and a positive value of z, if we increase s, the RHS of the inequality stays constant while the LHS falls. At $s = 1$, the inequality is clearly violated. For given values of θ and q, there is thus a critical value of s below which $V_H(r; q, s, \theta) = V_L(r; q, \theta)$ has a unique positive solution. Above that critical value, no buyers visit sellers posting $a = s$.

4.1.2 No-Mimic Conditions for Sellers

The next step is to show that if all type-L sellers post $a = 0$ and all type-H sellers post $a = s$, then no seller wants to deviate from this configuration. We first need expressions for the payoffs that the two seller types can expect to earn in equilibrium, i.e., if they do not deviate. Let $\Pi_L(0; r, q, \theta)$ be the expected payoff for a type-L seller posting 0 and $\Pi_H(s; r, q, s, \theta)$ the expected payoff for a type-H seller posting $a = s$. Using the arguments underlying equation (7), we have

$$\begin{aligned}
\Pi_L(0; r, q, \theta) &= 1 + e^{-\theta_L} - \frac{2(1 - e^{-\theta_L})}{\theta_L} \\
\Pi_H(s; r, q, s, \theta) &= 1 + (1 - s)e^{-\theta_H(1-s)} - \frac{2(1 - e^{-\theta_H(1-s)})}{\theta_H}.
\end{aligned}$$

Note that $\lim_{\theta_H \to 0} \Pi_H(s; r, q, s, \theta) = s$; i.e., if no buyers visit type-H sellers, these sellers simply retain their reservation value, s.

Deviations by Type-L Sellers

We first consider a potential deviation by a type-L seller. Such a deviation cannot increase this seller's expected payoff unless it leads buyers to believe that she is type H. If buyers continue to believe the seller is type L, then she gains nothing by deviating since her expected payoff is the same for all $a \geq 0$. There are two potential deviations to consider. First, we could imagine that a type-L seller might deviate to some $a' \in (0, s)$. This can be ruled out as follows. Observing a deviation of $a' \in (0, s)$, buyers should reason as follows. A type-L seller would gain nothing by such a deviation if buyers continued to believe that she was type L. Thus, the deviation could only be profitable if buyers believed her to be type H. However, based on the arguments in the homogeneous-seller case, such a deviation would not be profitable for a type-H seller if buyers continued to believe that she was type H.

Thus, we need only consider the second potential deviation, namely, that a type-L seller might deviate to $a' = s$. (Deviations to higher asking prices would be payoff-equivalent and thus do not need to be considered separately.) In this case, buyers would believe she is type H with probability 1 and thus visit at rate θ_H. Her expected payoff can be calculated in this case as follows. If no buyers visit, the seller retains value 0.

This occurs with probability $e^{-\theta_H}$. The seller receives at least one visitor with probability $1 - e^{-\theta_H}$. Consider a buyer who draws value x. What should this buyer do if $x < s$? The buyer believes that this seller is type L with probability 0. Consistent with our assumption in the homogeneous-seller case, we could assume that the buyer walks away, i.e., bids zero. On the other hand, there is no loss to submitting a bid; i.e., it is weakly dominant for the buyer to bid as if the seller's type were L. We therefore assume that the buyer bids $b(x)$ in this range.[12] If $s \le x \le 1$, the buyer bids s, and the seller's expected payoff is $p(x)$.

The expected payoff for a type-L seller who posts s is then

$$\begin{aligned}\Pi_L(s; r, q, \theta) &= \int_0^s \left(x - \frac{1 - e^{-\theta_H x}}{\theta_H}\right)\theta_H e^{-\theta_H(1-x)}dx + \int_s^1 \left(x - \frac{1 - e^{-\theta_H(x-s)}}{\theta_H}\right)\theta_H e^{-\theta_H(1-x)}dx \\ &= 1 + se^{-\theta_H} + (1-s)e^{-\theta_H(1-s)} - 2\left(\frac{1-e^{-\theta_H}}{\theta_H}\right),\end{aligned}$$

so the no-mimic condition for type-L sellers, i.e., $\Pi_L(0; r, q, \theta) \ge \Pi_L(s; r, q, \theta)$, is

$$1 + e^{-\theta_L} - \frac{2(1 - e^{-\theta_L})}{\theta_L} \ge 1 + se^{-\theta_H} + (1-s)e^{-\theta_H(1-s)} - 2\left(\frac{1 - e^{-\theta_H}}{\theta_H}\right) \tag{12}$$

Deviations by Type-H Sellers

A deviation to $a' < s$ is not profitable for a type-H seller if buyers continue to view the seller as a type H. Thus, we need only consider a deviation to $a' = 0$ in which case buyers would believe that the seller is type L with probability 1 and thus arrive at rate θ_L. In this case, if no buyers visit, the seller retains her value s. This occurs with probability $e^{-\theta_L}$. The seller receives at least one visitor with probability $1 - e^{-\theta_L}$. In this case, the seller's expected payoff is $p(x)$. Thus

$$\begin{aligned}\Pi_H(0; r, q, s, \theta) &= e^{-\theta_L}s + \int_0^1 \left(x - \frac{1 - e^{-\theta_L x}}{\theta_L}\right)\theta_L e^{-\theta_L(1-x)}dx \\ &= 1 + se^{-\theta_L} + e^{-\theta_L} - \frac{2(1 - e^{-\theta_L})}{\theta_L},\end{aligned}$$

and the no-mimic condition for type-H sellers, i.e., $\Pi_H(s; r, q, s, \theta) \ge \Pi_H(0; r, q, s, \theta)$, is

$$1 + (1-s)e^{-\theta_H(1-s)} - \frac{2(1 - e^{-\theta_H(1-s)})}{\theta_H} \ge 1 + se^{-\theta_L} + e^{-\theta_L} - \frac{2(1 - e^{-\theta_L})}{\theta_L}. \tag{13}$$

If $r = 0$, i.e., $\theta_H = 0$, the LHS of Inequality (13) is simply s.

[12] This assumption makes the expected payoff for the type-L deviant higher than it otherwise would be. This makes it more difficult to sustain the separating equilibrium. The assumption does not affect the expected payoff for a type-H seller posting s since a type-H seller would reject any counteroffer below s.

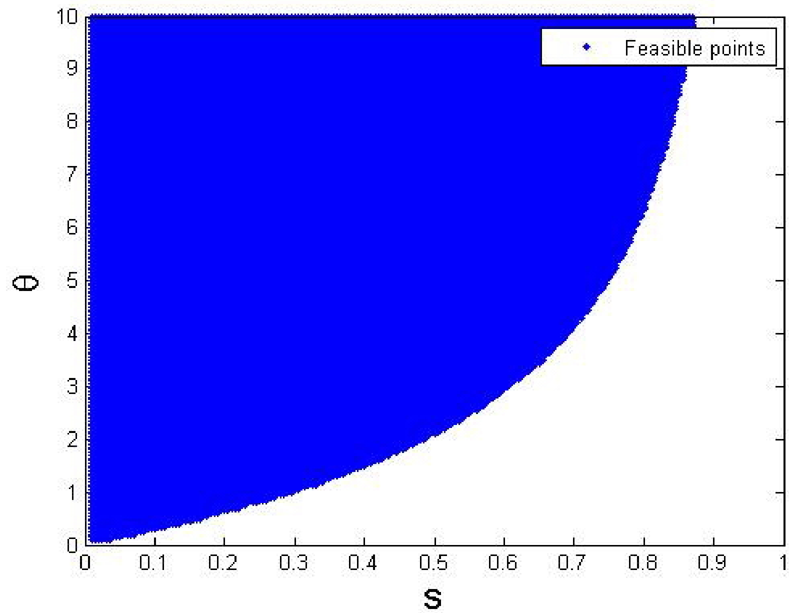

Figure 1: Existence of separating equilibrium with $r > 0$ for $q = 0.1$

4.1.3 Existence of Equilibrium

In a separating equilibrium in which $r > 0$, (11) holds as an equality and the two no-mimic conditions, inequalities (12) and (13), are both satisfied. We demonstrate the existence of equilibrium numerically.[13] Figures 1-3 show the set of (s, θ) combinations for which all three conditions hold for three different values of q. These are the shaded areas. Thus, for a wide range of q, as long as s is not too high and θ is not too low, equilibria in which buyers visit both seller types exist. This is intuitive since when s is not too high, buyers do not lose much by visiting a type-H seller, and when θ is not too low, the market is relatively tight so buyers have an incentive to visit the type-H sellers. As q increases, there are relatively fewer type-L sellers to visit so buyers have more incentive to visit the type-H sellers. In the non-shaded areas in Figures 1 to 3, where s is relatively high and/or θ is relatively low, separating equilibria exist with $r = 0$, i.e., buyers do not visit the type-H sellers.

As q increases, there are relatively fewer type-L sellers to visit so buyers have more incentive to visit the type-H sellers. In the non shaded areas in Figures 1 to 3, where s

[13] Specifically, we create a 100*100 grid for $s \in [0, 1]$ and $\theta \in [0.01, 10]$. For each (s, θ) pair, we first use the buyers' indifference condition to see if there is an interior solution for r. If there is, we solve for r and check that the no-mimic conditions for the sellers are satisfied. If no buyers choose to visit type-H sellers ($r = 0$), we check to see whether type-H sellers can gain by entering and mimicking the type-L sellers and we find that they cannot. We repeat this algorithm for several values of q.

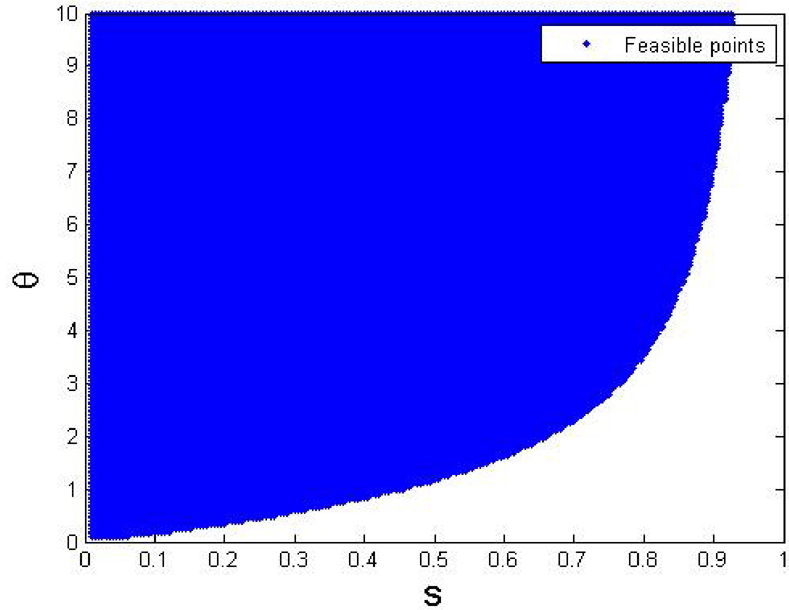

Figure 2: Existence of separating equilibrium with $r > 0$ for $q = 0.5$

is relatively high and/or θ is relatively low, separating equilibria exist with $r = 0$, i.e., buyers do not visit the type-H sellers.

We again demonstrate some of the properties of our equilibrium by looking at the probability that a house sells and the average selling price. There are now three parameters to consider varying, q, s, and θ.

The probability of sale is

$$P[\text{Sale}] = (1-q)P[\text{Sale}|\text{Type L}] + qP[\text{Sale}|\text{Type H}],$$

where

$$P[\text{Sale}|\text{Type L}] = 1 - e^{-\theta_L} \tag{14}$$

$$P[\text{Sale}|\text{Type H}] = 1 - e^{-\theta_H(1-s)}. \tag{15}$$

That is, $P[\text{Sale}]$ is a weighted average of the probabilities of sale for type-L and type-H sellers. The complication in the heterogeneous-seller case is that $\theta_H = r\theta/q$ and $\theta_L = (1-r)\theta/(1-q)$ vary with q and θ directly and with q, θ, and s indirectly because r depends on these parameters via the buyer indifference condition. The partial derivatives of r with respect to the parameters are difficult to sign analytically, but even if this were not the case, the effects are not *a priori* clear. For example, as s goes up, type-H sellers are less attractive so r falls. This means that θ_L goes up and θ_H falls as does $\theta_H(1-s)$.

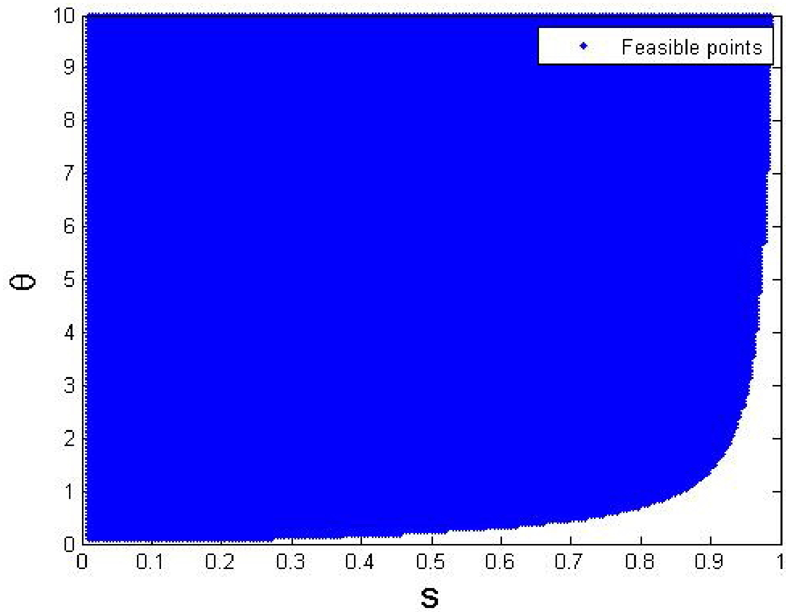

Figure 3: Existence of separating equilibrium with $r > 0$ for $q = 0.9$

Thus, as one would expect the probability of a sale for the type-H sellers is reduced while the probability for the type-L sellers increases, and the effect on the overall probability of a sale is not obvious. The effects of θ and q are more complicated since variations in these parameters affect θ_L and θ_H directly as well as via their effect on r. Thus, we calculate the comparative statics effects numerically.

Figures 4 - 6 show how the probability of sale varies with q, s, and θ. Each figure is created keeping two parameters constant from the set $\{q, s, \theta\}$ and varying the other. We experimented with many different parameter values for the fixed parameters. For each combination for which a separating equilibrium exists with $r > 0$, the signs of the slopes and the positions of the curves relative to each other do not change. First, from Figure 4 (where we set $\theta = 2$, $s = 0.5$) we see that as the fraction of type-H sellers, q, increases, the probability of sale increases both for type-H sellers and for type-L sellers. However, the overall probability of a sale falls because the weight of the type-H sellers (who have a lower sale probability than the type-L sellers) goes up.

Figure 5 (where we set $\theta = 2$, $q = 0.5$) shows that as the reservation value of the type-H sellers increases, their probability of sale falls (because r goes down) while the probability of sale for the type-L sellers rises. The first effect dominates so the overall probability of a sale goes down. Figure 6 (where we set $s = 0.5$, $q = 0.5$) shows that as the buyer/seller ratio, θ, rises, the probability of sale rises for both type-L and type-H sellers and the overall probability of a sale goes up as well.

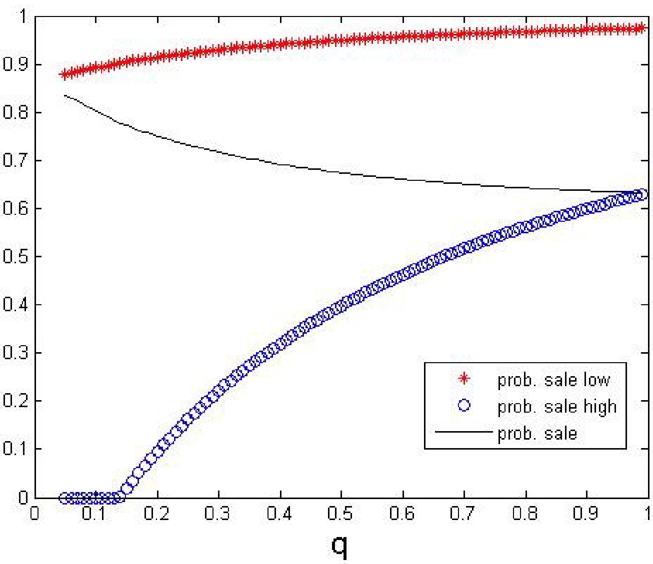

Figure 4: Probability of sale and the fraction of type-H sellers, $\theta = 2$, $s = 0.5$

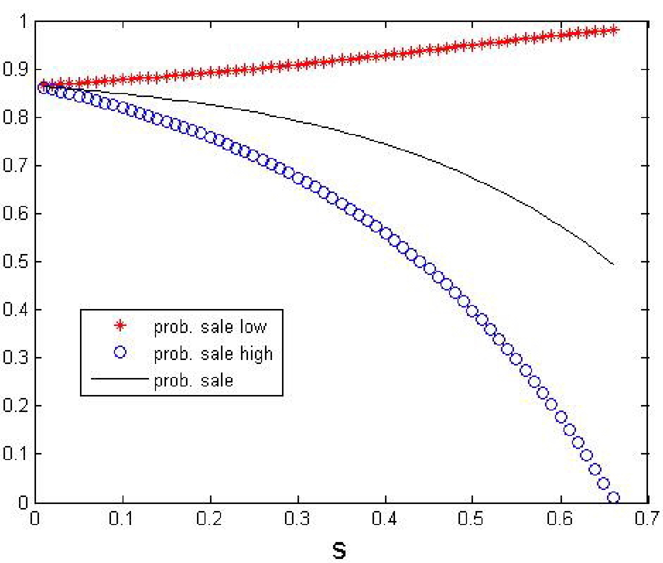

Figure 5: Probability of sale and the reservation value of type-H sellers, $\theta = 2$, $q = 0.5$

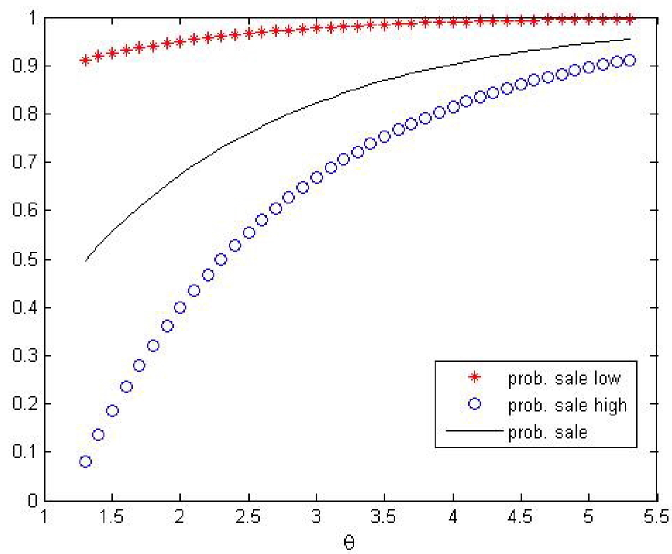

Figure 6: Probability of sale and market tightness, $s = 0.5$, $q = 0.5$

The average selling price is given by

$$E[p] = \frac{(1-q)P[\text{Sale}|\text{Type L}]E[p|\text{Type L}] + qP[\text{Sale}|\text{Type H}]E[p|\text{Type H}]}{P[\text{Sale}]},$$

which is a weighted average of the average selling prices for houses sold by type-L and type-H sellers.[14] We have also examined the comparative statics of the average selling price numerically; see Figures 7 - 9. In these Figures we set the exogenous parameters at the same values as in (Figures 4 - 6). As expected, the average selling price is increasing in the fraction of type-H sellers, the reservation value of the type-H sellers, and in the buyer/seller ratio.

4.1.4 Efficiency with Exogenous Measures of Sellers

In separating equilibrium, each buyer visits a type-H seller with probability r. Given exogenous measures of buyers and of sellers of each type and given that buyers and sellers transact efficiently once the matching pattern is determined, the only efficiency question is whether the equilibrium value of r equals the value that a social planner would choose.

Suppose the social planner chooses r to maximize total market surplus. To derive an expression for total market surplus, we argue as follows. Consider a type-L seller.

[14] $P[\text{Sale}|\text{Type L}]$ and $P[\text{Sale}|\text{Type H}]$ are given by (14) and (15) and $E[p|\text{Type L}]$ is given by (10) with $s = 0$ and θ replaced by θ_L while $E[p|\text{Type H}]$ is given by (10) with θ replaced by θ_H .

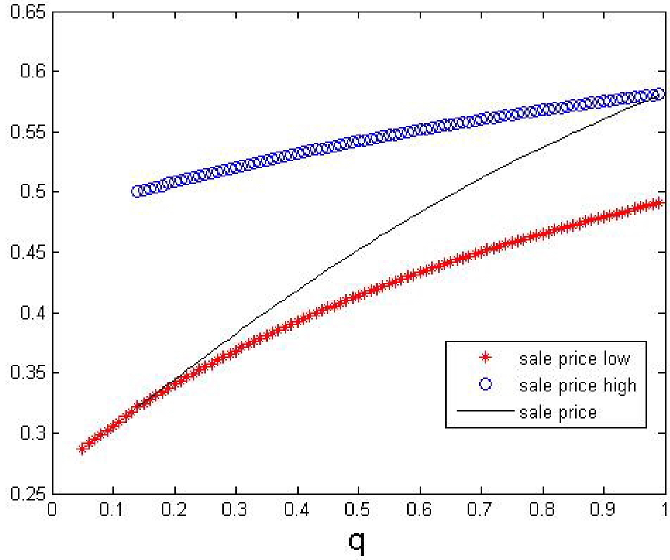

Figure 7: Average selling price and the fraction of type-H sellers, $\theta = 2$, $s = 0.5$

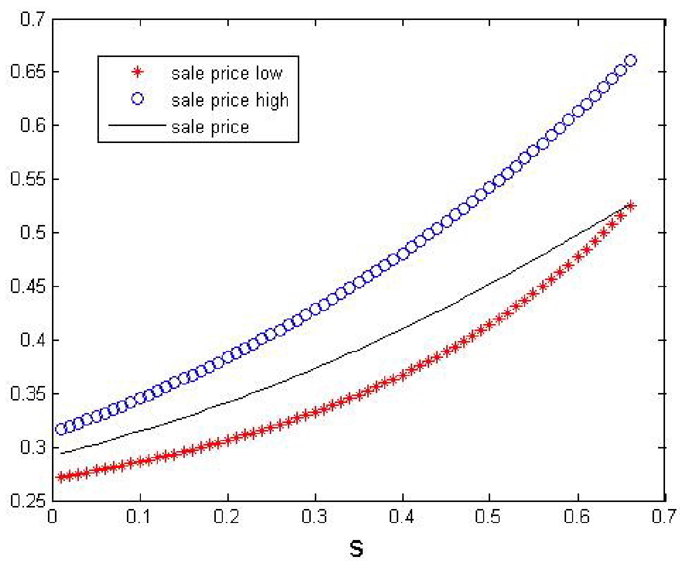

Figure 8: Average selling price and the reservation price of type-H sellers, $\theta = 2$, $q = 0.5$

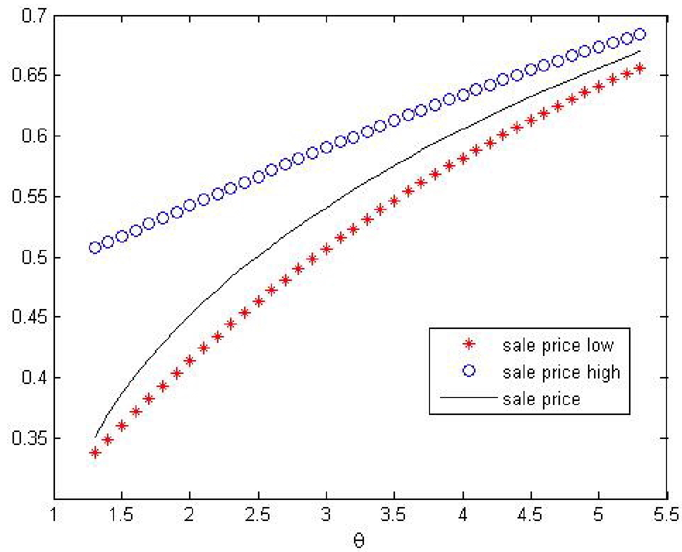

Figure 9: Average selling price and market tightness, $s = 0.5$, $q = 0.5$

If n buyers visit this seller, then the surplus associated with this seller is the highest value drawn by the n visitors. We denote this value by Y_n. Similarly, the surplus associated with a type-H seller is $\max[s, Y_n]$. The social planner's choice of r determines the distributions of the numbers of visitors for type-L and type-H sellers. Thus, r determines the expected surplus per type-L and type-H seller, and total market surplus is $((1-q)E[Y_n] + qE\max[s, Y_n])S$. Equivalently, since S is fixed, it is convenient to consider the social planner as maximizing the average market surplus per seller.

Now we turn to the problem of computing expected surplus for each type of seller as a function of r. Consider a type-L seller. Given r, the number of buyers who visit this seller is Poisson with parameter θ_L. Conditional on n, the expected surplus associated with this seller is $E[Y_n] = \dfrac{n}{n+1}$; thus, the expected surplus per type-L seller is

$$\sum_{n=0}^{\infty} \frac{n}{n+1} \frac{e^{-\theta_L}\theta_L^n}{n!} = \frac{1}{\theta_L} \sum_{u=1}^{\infty} (u-1) \frac{e^{-\theta_L}\theta_L^u}{u!} = 1 - \frac{1-e^{-\theta_L}}{\theta_L}.$$

Similarly, given r, the number of buyers visiting a type-H seller is Poisson with parameter θ_H. The expected surplus per type-H seller conditional on n is

$$\begin{aligned} E\max[s, Y_n] &= sP[Y_n < s] + E[Y_n | Y_n \geq s]P[Y_n \geq s] \\ &= s^{n+1} + \int_s^1 ny^n dy = \frac{n + s^{n+1}}{n+1}. \end{aligned}$$

Thus, the expected surplus per type-H seller is

$$\begin{aligned}\sum_{n=0}^{\infty} \frac{n + s^{n+1}}{n+1} \frac{e^{-\theta_H} \theta_H^n}{n!} &= 1 - \frac{1 - e^{-\theta_H}}{\theta_H} + \sum_{n=0}^{\infty} \frac{s^{n+1}}{n+1} \frac{e^{-\theta_H} \theta_H^n}{n!} \\ &= 1 - \frac{1 - e^{-\theta_H(1-s)}}{\theta_H}.\end{aligned}$$

The social planner's objective is thus

$$\max_{r \geq 0} \ (1-q)\left(1 - \frac{1 - e^{-\theta_L}}{\theta_L}\right) + q\left(1 - \frac{1 - e^{-\theta_H(1-s)}}{\theta_H}\right)$$

The first-order condition for the social planner's problem can be expressed as $V_L(r; q, \theta) \geq V_H(r; q, s, \theta)$ with equality if $r > 0$. This means that whatever the value of θ if all the type-H sellers are posting $a = s$ and all the type-L sellers are posting $a = 0$, the equilibrium value of r and the r that the social planner would choose coincide. That is,

Proposition 3 *With θ exogenous, separating equilibrium is efficient.*

4.1.5 Endogenizing q

We now consider a version of the heterogenous-seller model in which the measures of buyers and of type-L sellers (B and L, respectively) are exogenous but the measure of type-H sellers, H, is determined by free entry. We have in mind a situation in which the type-L sellers are "motivated" and have to be in the market, while there is a given stock of type-H homeowners of which some fraction (to be determined endogenously) wants to enter the market.[15] A type-H seller who enters the market pays an advertising cost A and in equilibrium receives an expected payoff of $\Pi_H(s; r, q, s, \theta)$. A type-H seller who chooses to stay out of the market retains value s. Let $B, L \to \infty$ with $B/L = \xi$. Let the probability that a buyer visits a type-H seller again be r. Then the number of visitors to a type-H seller is binomial with parameters rB and $\frac{1}{H}$. As $B, H \to \infty$, $rB/H = \theta_H$, so the number of visitors to a type-H seller is Poisson with parameter θ_H. Similarly, the number of visitors to a type-L seller is Poisson with parameter $\theta_L = (1-r)B/L = (1-r)\xi$.

A type-H seller only lists her house if the expected payoff exceeds the advertising cost plus the opportunity cost of selling the house, i.e., $A + s$. The extent of entry of type-H sellers is determined by the free entry condition

$$\Pi_H(s; r, q, s, \theta) = 1 + (1-s)e^{-\theta_H(1-s)} - \frac{2(1 - e^{-\theta_H(1-s)})}{\theta_H} = A + s \qquad (16)$$

[15] In any equilibrium in which type-H sellers choose to enter, all type-L sellers would also find it optimal to enter.

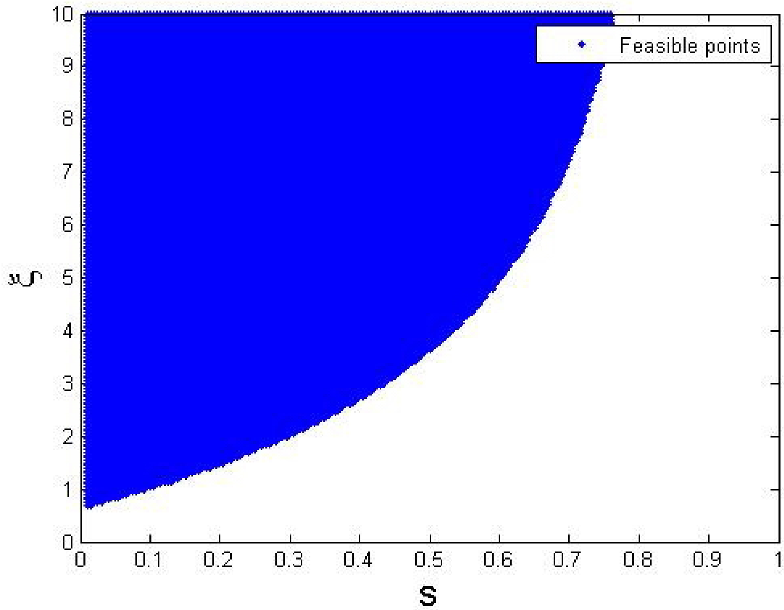

Figure 10: Existence of separating equilibrium with $r > 0$ and endogenous q

In order that a separating equilibrium with $H > 0$ exist, three additional conditions need to be satisfied. First, $V_H(r; q, s, \theta) = V_L(r; q, \theta)$ must have an interior solution; i.e., $r \in (0, 1)$. Second, a type-L seller should not want to deviate from $a = 0$, i.e., (12) is satisfied. Third, a type-H seller should not want to deviate from $a = s$, i.e., (13) must hold.

There are now two issues to consider. First, for what (ξ, s) combinations does such an equilibrium exist? Second, is entry efficient? To answer the first question, we again compute the equilibrium numerically. The shaded area of Figure 10 shows the range of (ξ, s) combinations for which a free entry separating equilibrium exists. The unshaded area contains the range of parameter values for which type-H sellers do not list their houses.

4.1.6 Efficiency with Free Entry

With free entry, we again consider the social planner's problem. It is convenient to formulate the problem on a per-buyer basis. The social planner's objective is to maximize

$$\frac{L}{B}\left(1 - \frac{1 - e^{-\theta_L}}{\theta_L}\right) + \frac{H}{B}\left(1 - \frac{1 - e^{-\theta_H(1-s)}}{\theta_H} - (A + s)\right)$$

by choosing r and q. To formulate the problem in a tractable way, we write H/B, θ_L, and θ_H in terms of (r,q). Note first that $L/B = \xi$ (exogenous). Then,

$$\begin{aligned}
\frac{H}{B} &= \frac{H}{L}\frac{L}{B} = \left(\frac{q}{1-q}\right)\frac{1}{\xi}, \\
\theta_L &= \frac{(1-r)B}{L} = (1-r)\xi, \\
\theta_H &= \frac{rB}{H} = r\xi\frac{(1-q)}{q}.
\end{aligned}$$

The social planner's problem can now be rewritten as

$$\max_{r\geq 0, q\geq 0} \xi\left(1 - \frac{1-e^{-(1-r)\xi}}{(1-r)\xi}\right) + (\frac{q}{1-q})(\frac{1}{\xi})\left(1 - \frac{1-e^{-r\xi\frac{(1-q)}{q}(1-s)}}{r\xi\frac{(1-q)}{q}} - (A+s)\right).$$

We already know that the first-order condition with respect to r gives $V_L(r;q,\theta) \geq V_H(r;q,s,\theta)$ with equality if $r>0$; i.e., the equilibrium value of r is efficient whatever the entry decision for type-H sellers. Turning to the first-order condition with respect to q, we have

$$\begin{aligned}
(\frac{1}{(1-q)\xi})^2\left\{1 + (1-s)e^{-r\xi\frac{(1-q)}{q}(1-s)} - \frac{2(1-e^{-r\xi\frac{(1-q)}{q}(1-s)})}{r\xi\frac{(1-q)}{q}} - A - s\right\} &= 0 \text{ if } q>0 \\
&\leq 0 \text{ if } q=0.
\end{aligned}$$

For an interior solution ($q>0$), the FOC is the same as (16), the free entry condition, i.e., $\Pi_H(s;r,q,s,\theta) = A+s$.

There are four conceivable situations: (i) $r>0$, $q>0$, (ii) $r=0$, $q>0$, (iii) $r>0$, $q=0$, and (iv) $r=0$, $q=0$. The most interesting is (i) and in this case, the equilibrium is clearly efficient since the FOC for r gives $V_L(r;q,\theta) = V_H(r;q,s,\theta)$ and the FOC for q gives the free entry condition. We can rule out case (ii) by noting that $\lim_{r\to 0} \Pi_H(s;0,q,s,\theta) = s$; i.e., the FOC for q cannot hold with equality. Case (iii) can be ruled out since as $q\to 0$, $V_L(r;q,\theta) > V_H(r;q,s,\theta)$ so that $r=0$. Case (iv) arises when the social planner's problem has a corner solution. Again, we have efficiency. The social planner wants to set $q=0$ when $\Pi_H(s;0,q,s,\theta) < A+s$, and when $q=0$, buyers necessarily set $r=0$. We have thus shown:

Proposition 4 *The free-entry equilibrium is efficient in the heterogeneous-seller model.*

5 Conclusions

In this paper, we construct a directed search model of the housing market. The mechanism that we analyze captures important aspects of the way houses are bought and sold in the United States. Sellers post asking prices, and buyers direct their search based on these prices. When a buyer visits a house, he can walk away, make a counteroffer, or offer to pay the asking price. If no buyers offer to pay the asking price, the seller can accept or reject the best counteroffer (if any) that she receives. If at least one buyer offers to pay the asking price, the seller is committed to sell her house at a price equal to the highest bid that follows from the competition among those buyers.

In the homogeneous-seller version of this model, that is, when we assume that all sellers have the same reservation value, s, we show that any distribution of asking prices over $[s, 1]$ is consistent with equilibrium. Furthermore, our model implies that houses sometimes sell below, sometimes at, and sometimes above the asking price. Thus, our model is consistent with equilibrium price dispersion for identical houses sold by identical sellers in terms of both asking prices and final sales prices. We also show that free-entry equilibrium is constrained efficient. That is, when sellers have to pay an advertising cost to enter the market, the free-entry and the social planner levels of market tightness coincide.

In the heterogeneous-seller version of the model, we demonstrate the existence of separating equilibria and the nonexistence of pooling and mixed equilibria numerically. In separating equilibrium, the sellers with the low (high) reservation value identify themselves by posting a low (high) asking price. That is, in addition to directing buyer search within each seller type, the asking price also plays a signaling role by allocating buyers across the two seller types. Equilibrium is again constrained efficient. The fraction of buyers who visit high-type sellers and the level of market tightness equal the values that a social planner would choose. Of course, we are not arguing that there are no inefficiencies in the housing market, but rather that the pricing mechanism and the fact that buyers do not directly observe seller types is not a source of inefficiency.

Our paper contributes both to the growing literature that uses equilibrium search theory to model the housing market and to the directed search literature. Our contribution to the housing literature is to build a directed search model that captures the main features of the house-selling process in the United States. We explain the role of the asking price and its relationship to the sales price, and we show that the mechanism we analyze is constrained efficient. Finally, our contribution to the directed search literature is to analyze a model in which there is only limited commitment and the posted price also plays a signaling role.

References

[1] Albrecht, J., A. Anderson, E. Smith and S. Vroman (2007), Opportunistic Matching in the Housing Market, *International Economic Review,* 48, 641-64.

[2] Albrecht, J., P. Gautier and S. Vroman (2006), Equilibrium Directed Search with Multiple Applications, *Review of Economic Studies,* 73, 869-91.

[3] Angeletos, G-M, C. Hellwig and A. Pavan (2006), Signaling in a Global Game: Coordination and Policy Traps, *Journal of Political Economy*, 114, 452-484,

[4] Bagwell, K. and G. Ramey (1988), Advertising and Limit Pricing, *RAND Journal of Economics,* 19, 59-71.

[5] Camera, G. and C. Selcuk (2009), Price Dispersion with Directed Search, *Journal of the European Economic Association*, 7, 1193-1224.

[6] Carrillo, P. (2006), An Empirical Two-Sided Equilibrium Search Model of the Housing Market, mimeo.

[7] Chen, Y. and R. Rosenthal (1996), Asking Prices as Commitment Devices, *International Economic Review*, 37, 129-55.

[8] Cho, I-K and D.M. Kreps (1987), Signaling Games and Stable Equilibria, *The Quarterly Journal of Economics,* 102, 179-221.

[9] Delacroix A. and S. Shi (2008), Pricing and signalling with frictions, mimeo.

[10] Díaz, A. and B. Jerez (2009), House Prices, Sales and Time on the Market: A Search-Theoretic Approach, mimeo.

[11] Guerrieri V., R. Shimer, R. Wright (2009), Adverse selection in competitive search equilibrium, mimeo.

[12] Julien, B., J. Kennes, and I. King (2000), Bidding for Labor, *Review of Economic Dynamics, 3,* 619-649.

[13] Krainer, J. (2001), A Theory of Liquidity in Residential Real Estate Markets, *Journal of Urban Economics, 49, 32-53.*

[14] Merlo, A. and F. Ortalo-Magné (2004), Bargaining over Residential Real Estate: Evidence from England, *Journal of Urban Economics*, 56, 192-216.

[15] Menzio G. (2007), A Theory of Partially Directed Search, *Journal of Political Economy*, 115, 748-769.

[16] Moen, E. (1997), Competitive Search Equilibrium, *Journal of Political Economy*, 105, 385-411.

[17] Ngai, R. and S. Tenreyro (2009), Hot and Cold Seasons in the Housing Market, mimeo, July 2009.

[18] Spence, M. A. (1973), Job Market Signaling, *Quarterly Journal of Economics,* 87, 355-374

[19] Wheaton, W. (1990), Vacancy, Search, and Prices in a Housing Market Model, *Journal of Political Economy,* 98, 1270-92.

Appendix

A Pooling and Mixed Equilibria

In this appendix, we discuss the nonexistence of pooling and mixed equilibria.

A.1 Pooling equilibria

In pooling equilibrium, all sellers post the same asking price. There are two cases to consider. First, all sellers could post a common asking price $a \in [0, s)$; second, they could all post a common asking price $a \in [s, 1]$. We refer to the two cases as "pooling on low" and "pooling on high," and we analyze them in turn.

A.1.1 Pooling on Low

Suppose all sellers post a common asking price $a \in [0, s)$. A buyer's strategy is characterized by a cutoff $\widetilde{x}$:

$0 \leq x < \widetilde{x}$ – make a bid that only L accepts

$x \geq \widetilde{x}$ – bid a

Consider first a buyer who draws $0 \leq x \leq \widetilde{x}$. Suppose the buyer bids $b(x'; x)$. All other buyers are assumed to be following $b(x)$. Only type-L sellers accept the bid and a seller is of type L with probability $(1 - q)$. Conditional on getting the house, the buyer's payoff is x (his true value) minus the price paid. That is, the buyer's expected payoff is

$$v(x'; x) = (1 - q)e^{-\theta(1-x')} (x - b(x'; x)) .$$

Using the same arguments as in the homogenous case, the bid that maximizes the payoff of a buyer who draws x is

$$b(x) = x - \frac{1 - e^{-\theta x}}{\theta} \text{ for } 0 \leq x < \widetilde{x}$$

and his expected payoff is

$$v(x) = (1 - q)e^{-\theta(1-x)} \left(\frac{1 - e^{-\theta x}}{\theta} \right) \text{ for } 0 \leq x < \widetilde{x}.$$

A buyer who draws $x \geq \widetilde{x}$ offers to pay the seller's asking price. His expected payoff in this case is

$$v(x) = e^{-\theta(1-x)}(x - p(x)),$$

where $p(x)$ is the expected price if he draws x and wins the auction. Using the same derivation as in the homogeneous case gives

$$x - p(x) = (\widetilde{x} - a)e^{-\theta(x-\widetilde{x})} + \frac{1 - e^{-\theta(x-\widetilde{x})}}{\theta}.$$

The buyer's expected payoff is then

$$v(x) = e^{-\theta(1-x)}((\widetilde{x} - a)e^{-\theta(x-\widetilde{x})} + \frac{1 - e^{-\theta(x-\widetilde{x})}}{\theta}).$$

Using the fact that $v(x)$ must be continuous at $x = \widetilde{x}$ we have

$$\widetilde{x} = a + (1-q)\left(\frac{1 - e^{-\theta\widetilde{x}}}{\theta}\right).$$

The expected payoff in this case can be rewritten as

$$v(x) = e^{-\theta(1-x)}(\frac{1 - qe^{-\theta(x-\widetilde{x})} - (1-q)e^{-\theta x}}{\theta}).$$

Thus, the value to a buyer of visiting a seller posting a is

$$\begin{aligned} V(a) &= \int_0^{\widetilde{x}} (1-q)e^{-\theta(1-x)}\left(\frac{1 - e^{-\theta x}}{\theta}\right)dx + \int_{\widetilde{x}}^1 e^{-\theta(1-x)}(\frac{1 - qe^{-\theta(x-\widetilde{x})} - (1-q)e^{-\theta x}}{\theta})dx. \\ &= \frac{1 - (1-q)(1+\theta)e^{-\theta} - q(1+\theta(1-\widetilde{x}))e^{-\theta(1-\widetilde{x})}}{\theta^2}. \end{aligned}$$

Note that $V'(a) < 0$; i.e., the buyer is worse off the higher is a.

The next step is to derive each seller's expected payoff when all sellers post $a \in [0, s)$. Now let x be the highest value drawn among the buyers visiting a particular seller. If $0 \le x < \widetilde{x}$, the type-L seller's payoff is $b(x)$ and the type-H seller's payoff is s. For $x \ge \widetilde{x}$, both sellers realize an expected payoff of $p(x)$. Let $\Pi_L(a)$ and $\Pi_H(a)$ be the expected payoffs to the two seller types. Using the same approach as in the homogenous case,

$$\begin{aligned} \Pi_L(a) &= (1 - e^{-\theta})\left(\int_0^{\widetilde{x}} b(x)\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx + \int_{\widetilde{x}}^1 p(x)\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx\right) \\ &= 1 + e^{-\theta} - \frac{2(1-e^{-\theta})}{\theta} + q(1-\widetilde{x})(e^{-\theta(1-\widetilde{x})} - e^{-\theta}) \\ \Pi_H(a) &= e^{-\theta}s + (1 - e^{-\theta})\left(\int_0^{\widetilde{x}} s\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx + \int_{\widetilde{x}}^1 p(x)\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx\right) \\ &= 1 + se^{-\theta(1-\widetilde{x})} - \frac{2(1-e^{-\theta(1-\widetilde{x})})}{\theta} + e^{-\theta(1-\widetilde{x})}[q(1-\widetilde{x}) - \widetilde{x}] + e^{-\theta}(1-q)(1-\widetilde{x}) \end{aligned}$$

Consider a deviation to $a' = 0$ by a type-L seller. Let the arrival rate of buyers to the deviant be given by γ. Buyers who visit this deviant bid $a' = 0$. Using the results from the homogeneous case, the expected payoff for the deviant is

$$\Pi_L(0;\gamma) = 1 + e^{-\gamma} - \frac{2(1-e^{-\gamma})}{\gamma}.$$

The expected payoff to a buyer who visits the deviant when the arrival rate is γ is

$$V(0;\gamma) = \frac{1-e^{-\gamma}-\gamma e^{-\gamma}}{\gamma^2}.$$

The arrival rate γ is determined by $V(0;\gamma) = V(a;\theta)$.

We establish numerically that $\Pi_L(0;\gamma) > \Pi_L(a;\theta)$ for $a < s$.[16] Note that buyers do not care about the deviant's type, so to break the candidate equilibrium at $a \in (0,s)$, it suffices to show that the type-L seller wants to deviate to $a' = 0$. Thus, of the asking prices below s, we need only consider $a = 0$.

Can $a = 0$ be an equilibrium? We need to check for potential profitable deviations. Specifically, we consider whether a type-H seller wants to deviate to $a' = 1$ assuming that buyers believe the seller to be type H.[17] We have

$$\Pi_H(0;\theta) = se^{-\theta} + 1 + e^{-\theta} - \frac{2(1-e^{-\theta})}{\theta}.$$

The profit of the deviant would be

$$\Pi_H(1;\xi) = 1 + (1-s)e^{-\xi(1-s)} - \frac{2(1-e^{-\xi(1-s)})}{\xi},$$

where ξ is determined by $V(1;\xi) = V(0;\theta)$; that is,

$$\frac{1-e^{-\xi(1-s)}-\xi(1-s)e^{-\xi(1-s)}}{\xi^2} = \frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}.$$

We again establish numerically that the deviation is profitable and thus conclude that pooling-on-low equilibria do not exist for any $a \in [0,s)$.

A.1.2 Pooling on High

The next step is to ask whether an equilibrium can exist with a common asking price $a \in [s,1]$. We derive the strategies for the buyers and the payoffs for the two types of sellers. First, we show that all candidate equilibria with $a \in [s,1]$ are payoff equivalent (for buyers, for type-L sellers and for type-H sellers). Then, we show that such equilibria do not exist.

Buyers Again let $\widetilde{x}$ be the value of x that makes buyers indifferent between making a bid that only type-L sellers accept or a bid that both types accept. As in the case of pooling on low, the bid that maximizes the payoff of a buyer who draws $0 \le x < \widetilde{x}$ is

$$b(x) = x - \frac{1-e^{-\theta x}}{\theta} \text{ for } 0 \le x < \widetilde{x}$$

[16] To establish this numerically, we take q equal to 0.1...0.9 and create a 1000*1000 grid for $s \in \langle 0,1\rangle$ and $\theta \in \langle 0.1, 30\rangle$.

[17] This assumption about buyer beliefs follows from the Intuitive Critierion.

and his expected payoff is

$$v(x) = (1-q)e^{-\theta(1-x)} \left(\frac{1-e^{-\theta x}}{\theta} \right) \text{ for } 0 \le x < \widetilde{x}. \tag{17}$$

Next, consider a buyer who draws $x \ge \widetilde{x}$. Let x^* be such that $x \ge x^*$ leads to a bid of $b(x) = a$. Note that the bid for the buyer who draws exactly $\widetilde{x}$ is s, i.e., $b(\widetilde{x}) = s$. Using the same derivation as in the homogeneous case, the bid that maximizes the payoff of a buyer who draws x in $\widetilde{x} \le x < x^*$ is

$$b(x) = x - (\widetilde{x} - s)e^{-\theta(x-\widetilde{x})} - (\frac{1-e^{-\theta(x-\widetilde{x})}}{\theta}) \text{ for } \widetilde{x} \le x < x^*$$

and his expected payoff is

$$v(x) = e^{-\theta(1-x)} \left((\widetilde{x} - s)e^{-\theta(x-\widetilde{x})} + (\frac{1-e^{-\theta(x-\widetilde{x})}}{\theta}) \right) \text{ for } \widetilde{x} \le x < x^*. \tag{18}$$

Using the continuity of the expected payoff function, we can set equations (17) and (18) equal at $x = \widetilde{x}$. This yields

$$\widetilde{x} = s + (1-q)(\frac{1-e^{-\theta\widetilde{x}}}{\theta}).$$

Substituting this into $b(x)$ and $v(x)$ gives

$$b(x) = x - (\frac{1-(1-q)e^{-\theta x} - qe^{-\theta(x-\widetilde{x})}}{\theta}) \text{ for } \widetilde{x} \le x < x^*$$

$$v(x) = e^{-\theta(1-x)}(\frac{1-(1-q)e^{-\theta x} - qe^{-\theta(x-\widetilde{x})}}{\theta}) \text{ for } \widetilde{x} \le x < x^*.$$

Finally, a buyer who draws $x \ge x^*$ offers to pay the seller's asking price. His expected payoff in this case is

$$v(x) = e^{-\theta(1-x)}(x - p(x)),$$

where $p(x)$ is the expected price if he draws x and wins the auction. Using the same derivation as in the homogeneous case, we have

$$x - p(x) = (x^* - a)e^{-\theta(x-x^*)} + \frac{1-e^{-\theta(x-x^*)}}{\theta}.$$

We can then rewrite the expected payoff for a buyer with $x \ge x^*$ as

$$\begin{aligned} v(x) &= e^{-\theta(1-x)}(x - p(x)) \\ &= e^{-\theta(1-x)}((x^* - a)e^{-\theta(x-x^*)} + \frac{1-e^{-\theta(x-x^*)}}{\theta}). \end{aligned}$$

Using the continuity of the expected payoff function, we can set the above equation and (18) equal at $x = x^*$. This yields

$$x^* = a + (\frac{1-(1-q)e^{-\theta x^*} - qe^{-\theta(x^*-\widetilde{x})}}{\theta}).$$

The expected price is then

$$\begin{aligned} p(x) &= x - (x^* - a)e^{-\theta(x-x^*)} - \frac{1-e^{-\theta(x-x^*)}}{\theta} \\ &= x - \frac{1-(1-q)e^{-\theta x} - qe^{-\theta(x-\widetilde{x})}}{\theta}. \end{aligned}$$

Substituting this into $v(x)$ gives

$$v(x) = e^{-\theta(1-x)}(\frac{1-(1-q)e^{-\theta x} - qe^{-\theta(x-\widetilde{x})}}{\theta}) \text{ for } x^* \leq x \leq 1.$$

We can summarize the problem of a buyer who visits a seller posting an asking price of $a \geq s$ as follows:

$$b(x) = \begin{cases} x - \dfrac{1-e^{-\theta x}}{\theta} & \text{for } 0 \leq x < \widetilde{x} \\ x - (\frac{1-(1-q)e^{-\theta x} - qe^{-\theta(x-\widetilde{x})}}{\theta}) & \text{for } \widetilde{x} \leq x < x^* \\ a & \text{for } x^* \leq x \leq 1 \end{cases}$$

$$v(x) = \begin{cases} (1-q)e^{-\theta(1-x)}(\frac{1-e^{-\theta x}}{\theta}) & \text{for } 0 \leq x < \widetilde{x} \\ e^{-\theta(1-x)}(\frac{1-(1-q)e^{-\theta x} - qe^{-\theta(x-\widetilde{x})}}{\theta}) & \text{for } \widetilde{x} \leq x \leq 1. \end{cases}$$

Based on the above, the expected payoff to a buyer who visits a seller posting $a \in [s, 1]$ in a pooling equilibrium is

$$V(a;\theta) = \frac{1}{\theta^2}\{1 - e^{-\theta} - \theta e^{-\theta} - q(e^{-\theta(1-\widetilde{x})}(1+\theta(1-\widetilde{x})) - e^{-\theta}(1+\theta))\}. \qquad (19)$$

Note that this does not depend on a.

Sellers We now compute the expected payoffs for the sellers. First, if no buyers visit, the seller retains her value, 0 for the type L's and s for the type H's. Given that at least one buyer visits, suppose the highest value drawn by these visitors is x. If $0 \leq x < \widetilde{x}$, the type-L seller's payoff is $b(x)$ and the type-H seller's payoff is s. For $\widetilde{x} \leq x < x^*$, both sellers realize a payoff of $b(x)$ and for $x^* \leq x \leq 1$, both sellers realize a payoff of $p(x)$. Following the derivation in the homogeneous case for the density of the highest value of x among the buyers visiting a seller, we then have

$$\Pi^L(a) = 0 + (1-e^{-\theta})\left(\int_0^{\widetilde{x}} b(x)\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx + \int_{\widetilde{x}}^{x^*} b(x)\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx + \int_{x^*}^{1} p(x)\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx\right)$$

$$\Pi^H(a) = e^{-\theta}s + (1-e^{-\theta})\left(\int_0^{\widetilde{x}} s\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx + \int_{\widetilde{x}}^{x^*} b(x)\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx + \int_{x^*}^{1} p(x)\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx\right).$$

Substitution gives

$$\Pi^L(a) = 1 - \frac{2(1-e^{-\theta})}{\theta} + (1-q(1-\widetilde{x}))e^{-\theta} + q(1-\widetilde{x})e^{-\theta(1-\widetilde{x})}$$

Note that this does not depend on a.

Finally, we can derive the payoff for the type-H sellers. From above, we have

$$\Pi^H(a) = e^{-\theta}s + (1-e^{-\theta})\left(\int_0^{\widetilde{x}} s\frac{\theta e^{-\theta(1-x)}}{(1-e^{-\theta})}dx + \int_{\widetilde{x}}^{1}(x - \frac{1-(1-q)e^{-\theta x} - qe^{-\theta(x-\widetilde{x})}}{\theta})\theta e^{-\theta(1-x)}dx\right).$$

Note that this also does not depend on a.

To show that pooling-on-high equilibria do not exist, it suffices to show that it is always in the interest of a type-L seller to deviate to $a' = 0$. Let ϕ be the arrival rate of buyers to the deviant, which is determined via a buyer indifference condition, namely, $V(0;\phi) = V(a;\theta)$, where

$$V(0;\phi) = \frac{1}{\phi^2}\{1 - e^{-\phi} - \phi e^{-\phi}\}.$$

It is in the interest of the type-L seller to deviate if $\Pi_L(0;\phi) > \Pi_L(a;\theta)$, that is, if

$$1 + e^{-\phi} - \frac{2\left(1-e^{-\phi}\right)}{\phi} > 1 - \frac{2(1-e^{-\theta})}{\theta} - q(1-\widetilde{x})e^{-\theta} + q(1-\widetilde{x})e^{-\theta(1-\widetilde{x})}.$$

We verify numerically that this inequality holds for a wide range of (θ, s, q) values and conclude that there do not exist equilibria with pooling on high.

A.2 Mixed Equilibria

We have shown numerically that pooling on either $a < s$ or $a \geq s$ is not an equilibrium. In this section, we consider the possibility of mixed equilibria. There are two types of mixed equilibria to consider – (i) an equilibrium in which all type-H sellers post $a \geq s$ but type-L sellers mix between posting $a = 0$ and posting $a \geq s$ and (ii) an equilibrium in which all type-L sellers post $a < s$ but type-H sellers mix between posting $a < s$ and posting $a = 1$.

A.2.1 Mixing by Lows

Suppose all type-H sellers post $a \geq s$ while type-L sellers mix posting with $a \geq s$ with probability m and $a = 0$ with probability $1 - m$. The question is whether this can be an equilibrium for some $m \in (0, 1)$.

Suppose buyers visit sellers posting $a \geq s$ with probability $r \in (0, 1)$. This occurs if

$$V(0; m, r) = V(a; m, r).$$

The arrival rate of buyers to sellers posting $a \geq s$ is

$$\theta_H = \frac{r\theta}{q + m(1-q)}.$$

Similarly, the arrival rate to sellers posting $a = 0$ is

$$\theta_L = \frac{(1-r)\theta}{(1-m)(1-q)}.$$

Using our results from the separating equilibrium,

$$V(0; m, r) = \frac{1 - e^{-\theta_L} - \theta_L e^{-\theta_L}}{\theta_L^2}.$$

To derive an expression for $V(a; m, r)$, let

$$\widehat{q} = \frac{q}{q + m(1-q)}$$

be the fraction of sellers posting $a \geq s$ who are type H. We can then use our results from pooling on high replacing q with $\widehat{q}$ and θ with θ_H in equation (19) to get

$$V(a; m, r) = \frac{1}{\theta_H^2}\{1 - e^{-\theta_H} - \theta e^{-\theta_H} - \widehat{q}(e^{-\theta_H(1-\widetilde{x})}(1 + \theta_H(1-\widetilde{x})) - e^{-\theta_H}(1+\theta))\},$$

where $\widetilde{x}$ is implicitly defined by

$$(1 - \widehat{q})\left(1 - e^{-\theta_H \widetilde{x}}\right) = \theta_H(\widetilde{x} - s).$$

The value of m is determined by a no-deviation condition,

$$\Pi_L(0; m, r) = \Pi_L(a; m, r);$$

that is, type-L sellers should be indifferent between posting $a = 0$ and $a \geq s$. Using our separating equilibrium results,

$$\Pi_L(0; q, \theta) = 1 + e^{-\theta_L} - \frac{2(1 - e^{-\theta_L})}{\theta_L}.$$

From our pooling-on-high results,

$$\Pi_L(a;m,r) = 1 - \frac{2}{\theta_H}(1-e^{-\theta_H}) + \widehat{q}(1-\widetilde{x})e^{-\theta_H(1-\widetilde{x})} + (1-\widehat{q}(1-\widetilde{x}))e^{-\theta_H}.$$

To check for the existence of a mixing-by-lows equilibrium, we need to see whether the two equations

$$V(0;m,r) = V(a;m,r)$$
$$\Pi_L(0;m,r) = \Pi_L(a;m,r)$$

have a solution for $(m,r) \in (0,1) \times (0,1)$.

We show numerically that this is not the case for a wide range of plausible parameter values. Thus, we conclude that this type of mixing equilibrium does not exist.

A.2.2 Mixing by Highs

Finally, we consider an equilibrium in which all type-L sellers post $a < s$ while type-H sellers post $a \geq s$ with probability n and $a < s$ with probability $1-n$. Given our results for separating equilibrium and for pooling on high, it is without loss of generality to consider only the case in which the type-H sellers mix between $a < s$ and $a = 1$. The arrival rate of buyers to sellers posting $a = 1$ is

$$\theta_H = \frac{r\theta}{nq}.$$

Similarly, the arrival rate to sellers posting $a < s$ is

$$\theta_L = \frac{(1-r)\theta}{1-q+(1-n)q} = \frac{(1-r)\theta}{1-nq}.$$

Let $\widetilde{q}$ be the fraction of sellers posting $a < s$ who are type H. We have

$$\widetilde{q} = \frac{(1-n)q}{1-nq}.$$

The buyer indifference condition is

$$V(a;n,r) = V(1;n,r).$$

The value to a buyer who visits a seller posting $a < s$ can be taken from our pooling-on-low results replacing θ with θ_L and q with $\widetilde{q}$. We have

$$V(a;n,r) = \frac{1 - \widetilde{q}e^{-\theta_L(1-\widetilde{x})}\left(1+\theta_L(1-\widetilde{x})\right) - (1-\widetilde{q})e^{-\theta_L}(1+\theta_L)}{\theta_L^2},$$

where

$$\widetilde{x} = a + (1-\widetilde{q})\left(\frac{1-e^{-\theta_L \widetilde{x}}}{\theta_L}\right).$$

The value to a buyer of visiting a seller posting $a = 1$ can be taken from our results on separating equilibrium:

$$V_H(r; q, s, \theta) = \frac{1 - e^{-\theta_H(1-s)} - \theta_H(1-s)e^{-\theta_H(1-s)}}{\theta_H^2}.$$

The seller indifference condition is

$$\Pi_H(a; n, r) = \Pi_H(1; n, r).$$

We take $\Pi_H(a; n, r)$ from our pooling-on-low results; we take $\Pi_H(1; n, r)$ from our results on separating equilibrium. Specifically,

$$\begin{aligned}
\Pi_H(a; n, r) &= 1 + se^{-\theta_L(1-\widetilde{x})} - \frac{2(1 - e^{-\theta_L(1-\widetilde{x})})}{\theta_L} + e^{-\theta_L(1-\widetilde{x})}[q(1-\widetilde{x}) - \widetilde{x}] + e^{-\theta_L}(1-q)(1-\widetilde{x}) \\
\Pi_H(1; n, r) &= 1 + (1-s)e^{-\theta_H(1-s)} - \frac{2(1 - e^{-\theta_H(1-s)})}{\theta_H}.
\end{aligned}$$

Again, for an equilibrium to exist, it must be the case that the payoffs for buyers applying to sellers posting the two different asking prices must be equal and the expected payoffs for the type-H sellers must be equal regardless of the asking price they post. That is, the two equations

$$V(a; n, r) = V(1; n, r)$$

$$\Pi_H(a; n, r) = \Pi_H(1; n, r)$$

must have a solution for $(n, r) \in (0, 1) \times (0, 1)$. We show numerically that this is not the case for a wide range of plausible parameter values. Thus, we conclude that this type of equilibrium does not exist.