Customer Capital*

Preliminary

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Abstract

Firms spend significant resources on creating and maintaining long-term customer relationships. We explore the role of this customer capital - a form of intangible capital - for firm valuation and investment. We build a search theoretic general equilibrium model of long-term customer relationships, where frictional product markets require firms to spend resources on customer acquisition, and cause existing customers to be partially locked-in. We show that the customer expansion friction causes physical investment to respond to productivity shocks with a lag. This allows the model to reproduce the low correlation of investment with Tobin's Q found in data, in addition to generating in a natural way the positive correlation of investment with cash flows observed. We provide evidence from Compustat supporting our theory: sorting industries according to the degree of friction – proxied by the degree of selling expenses – we find that in industries with greater frictions: i) investment is more correlated with lagged Tobin's Q, and ii) investment is less correlated with Tobin's Q and more with cash flows.

Firms spend substantial resources on marketing and sales costs. Viewing these costs as evidence of frictions in product markets, which require firms to spend resources on customer acquisition, also implies that existing customers are valuable to firms. This paper studies the implications of such *customer capital* for firm investment behavior and valuation. Our starting point is the neoclassical adjustment cost model of investment, where firms accumulate physical capital, such as plants and equipment, to maximize the present discounted value of profits.[1] The value of the firm is driven by its quantity of capital, and there is a one-to-one relationship between investment and Tobin's Q, the ratio of firm value to its capital stock.

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[1] Important early contributions include Abel (1982), Hayashi (1982) and Summers (1981).

This simple prediction of the model has been widely rejected by the data – a puzzling finding because it appears to hold *quantitatively* even in extensions of the model designed to break the theoretical relationship.[2] We show that if firm expansion is constrained by the costs of expanding the customer base: i) the correlation between investment and Tobin's Q falls because investment responds to productivity shocks with a lag, and ii) the volatility of investment relative to Tobin's Q falls because of the additional adjustment cost associated with customer base expansion. Both factors contribute toward reconciling the evidence found in investment regressions with theory. Moreover, cashflow effects are a natural prediction of our model. We present evidence from Compustat supporting our theory. Sorting industries according to measures of sales costs, we find that in high friction industries: i) investment is more correlated with lagged values of Tobin's Q, and ii) investment regressions yield a lower coefficient on Q, a higher coefficient on cashflow, and a lower R^2.

We propose a search theoretic general equilibrium model of customer capital. In the model frictions arise from product differentiation, reminiscent of the Dixit-Stiglitz framework. We depart from that standard framework by assuming that: i) customers cannot observe product characteristics without inspecting products, and ii) the inspection process is costly for both buyers and sellers. Firms seeking to attract new customers must maintain costly sales personnel/facilities, which become congested if they attempt to expand quickly. Pricing decisions reflect the desire to attract and maintain customers: we assume firms can use discounts to attract new customers, but cannot commit to low prices once the customer has adopted the product.[3] By causing customers to be partially locked in, search frictions lead to long-term relationships between buyers and sellers, allowing the firm to derive positive profits from existing customers.[4]

We show that product market frictions lead to a Tobin's Q above one, as the firm derives value from its customer base. As frictions diminish the model approaches the neoclassical adjustment cost model with Q equal to one. Our main finding is that product market frictions generate a time-varying wedge between marginal Q and Tobin's Q which: i) reduces the correlation of investment with Tobin's Q, and ii) gives firm cash flow predictive power over Tobin's Q in an investment regression. When frictions are small, Q-theory regressions work perfectly in the model with an R^2 of 100% and a slope coefficient reflecting the physical adjustment costs. As frictions grow, firms become constrained by their customer base in the short-run. The first order of business following a positive productivity shock is to expand

[2]See Gomes (2001). While there are many potential mechanisms to break the Q-theory, such as fixed costs, financing constraints, or decreasing returns to scale in production, to replicate the failure of investment regressions in data studies generally need to appeal to measurement error in Q (e.g. Erickson and Whited 2000, Eberly, Rebelo, and Vincent 2008). There is a large literature on Q-theory and the cash flow sensitivity. See e.g. Caballero (1999) for a survey.

[3]Our model allows two-part pricing, differentiating between new and existing customers.

[4]Examples of products motivating the model are newspapers subscriptions and cell phones services. Newspapers offer discounts to entice new customers. They are then able to charge a price above the marginal cost of production for an extended period of time. Cell phone providers also compete for customers by offering contracts with an initial discount (the phone is offered free of charge). In such industries it appears to be common practice to evaluate the value by measuring the number of customers, the retention rate, and the margin per customer. However, we believe the key insights of our analysis are more general, and apply also to markets where contracts are implicit.

the customer base rather than physical production capacity, with investment rising only as the accumulating customer base allows. While marginal Q always traces the hump-shaped response of investment to the shock, Tobin's Q rises on impact, reflecting the increased value of the existing customer base of the firm. Firm profits can be a better predictor of investment because they share the hump-shaped response of marginal Q, both because profits grow together with the customer base, and because the costs of this expansion are deducted from profits per standard accounting rules.

Finally, we turn to Compustat data for evidence of our proposed mechanism. The basic prediction of the model is that cross-sectional variation in the degree of product market frictions should matter both for how well investment regressions work, as well as how quickly investment responds to Tobin's Q. We sort industries according to the long-run degree of selling expenses,[5] identifying high selling expense industries as high friction industries. We find that in high friction industries Tobin's Q has less predictive power for investment, and cash flows more. Moreover, we find that in high friction industries investment appears more correlated with lagged values of Tobin's Q than in lower friction industries.

The paper is organized as follows. Section 1 presents our model and Section 2 studies its implications. Section 3 discusses the empirical evidence. Section 4 discusses related literature.

1 The Model

This section presents the model: first the firms and then the representative household.

1.1 Firms

Production is carried out by a continuum of firms. Each firm decides each period: i) how much to spend on sales effort, ii) what kind of offer to make to attract new customers, and iii) how much to invest in production capacity. Together with the customer base inherited from the previous period, the first two choices determine how many customers the firm has this period. As each customer buys one unit, this determines how much output needs to be produced.

Production technologies are Cobb-Douglas: $y = zk^{\alpha_k}l^{1-\alpha_k}$, with $\alpha_k \in (0,1)$. Producers rent labor l at a competitive market with wage w. Capital k is accumulated via investment i according to $k_{t+1} = (1-\delta_k)k_t + i_t$, where δ_k is the depreciation rate of capital. Investment goods are purchased at a frictionless goods market at price normalized to one, but installing this new capital entails an adjustment cost $\phi(i,k)$. Firm-specific productivity z follows a Markovian stochastic process with a bounded support and a continuous and monotone transition function Q.

The products these firms produce are differentiated in such a way that each potential new customer must inspect the product in person to determine whether it fits their needs. To

[5] Measured using Compustat's "selling, general and administrative expenses."

allow this inspection to take place, firms must spend resources on marketing and sales efforts, measured by s, the sales personnel of the firm. We assume s is associated with a convex cost $\kappa(s)$, where convexity captures the idea that effective sales personnel/locations are in limited supply, making it costly to expand.[6]

Product inspection is affected by congestion. If a firm's pricing attracts x potential customers this period, the measure of new customer relationships generated is given by a constant returns to scale matching function $M(x,s) = \xi x^{\alpha} s^{1-\alpha}$, with $\xi > 0, \alpha \in (0,1)$. This measure increases in potential customers, but at a diminishing rate if sales personnel is fixed. The measure also increases in sales personnel, but at a diminishing rate if the number of potential customers is fixed.[7] Using $\theta = \frac{x}{s}$ to denote potential customers per sales person, the rate at which the producer gets new customers (per sales person) is $q(\theta) = \xi\theta^{\alpha}$, and the rate at which potential customers find suppliers is $\mu(\theta) = \xi\theta^{\alpha-1}$.

We assume that a firm seeking to attract new customers can communicate its prices to everyone costlessly. The firm can influence the measure of potential customers arriving to inspect products through pricing, and may have an incentive to offer low prices to attract more customers. Once a customer has adopted the product, this incentive is gone, and the firm maximizes profits by raising the price for that customer to a level that makes the customer indifferent between staying with the firm and finding a new supplier.[8] As this value is the same for all customers with all firms, the price p charged will also be the same. The model thus has two-part pricing, where all customers pay p for one unit of the good, but new customers get an initial discount Δ.

Firms anticipate that offering a larger discount will attract more potential customers per sales person i.e. θ is an increasing function of Δ. To capture this relationship, we write $\theta = \Theta(\Delta)$, where the function $\Theta : \mathbb{R} \to \mathbb{R}_+$ is an equilibrium object shown in Section 1.2 to be strictly increasing and convex.[9]

A firm with an existing customer base n that chooses discount Δ and sales personnel s, attracts $sq(\Theta(\Delta))$ new customers this period. Such a firm generally produces for $y = n + sq(\Theta(\Delta))$ customers this period, hiring exactly the amount of labor needed to produce this

[6] We think of customers as coming from different sales locations, some better than others.

[7] Stevens (2007) describes a plausible process which delivers such a matching function.

[8] The firm cannot commit to lower prices after the customer has adopted the product. However, ex ante the customer understands this, and because he is indifferent with respect to the timing of prices, it does not affect the allocation. It does affect firm value however, raising Tobin's Q. It is assumed that producers have a large number of customers, leaving individual customers with no bargaining power to negotiate a lower price with the firm.

[9] Customers prefer larger discounts, but because producers offering them attract many potential customers per sales person, it becomes time-consuming for the consumer to inspect the product and establish a customer relationship with a producer offering a large discount. Customers take into account this time cost in choosing among discounts and in the limit customer flows adjust such that customers are indifferent between higher and lower discounts. The higher the discount, the more potential customers per sales person, i.e. higher θ (see Section 1.2 for more).

output, given the existing stock of capital. The firm's Bellman equation reads

$$V(k,n,z;w,\Theta) = \max_{s\geq 0,\Delta,p,i,l,y} py - lw - \kappa(s) - sq(\Theta(\Delta))\Delta - \phi(i,k) + \beta \int V(k',n',z';w,\Theta)Q(z,dz') \tag{1}$$

$$y \leq n + sq(\Theta(\Delta)), \tag{2}$$

$$y = zk^{\alpha_k} l^{1-\alpha_k}, \tag{3}$$

$$n' = (1-\delta_n)y, \tag{4}$$

$$k' = (1-\delta_k)k + i, \tag{5}$$

$$p \leq 1. \tag{6}$$

The firm sells the output y to its customers at price p. It pays the wages of production labor, the costs of sales personnel, and gives the discounts promised to new customers. It also pays the costs of investing into production capacity, in place next period. As the firm's sales are constrained by the size of its customer base, equation (2) states that the firm's production output must be less or equal to that customer base. Equation (3) determines the labor needed to produce the desired output y. Equation (4) is the law of motion for the customer base, where δ_n is the depreciation rate of customers. Finally, equation (5) is the law of motion for capital.

When deciding how to set the price p, the firm understands that there is a cutoff price level above which the customer will be driven to search for another supplier. As long as the price remains weakly below this cutoff, however, raising the price has no effect on demand. To maximize profit, the firm sets the price equal to this cutoff, which is shown in Section 1.2 to equal one.

While in general firms choose positive levels of sales personnel, those facing a sufficiently bad productivity shock may choose not to seek new customers, allowing their customer base to shrink with attrition. If the shock is particularly bad, they may even choose to produce less than their existing stock of customers would allow selling.[10] How prevalent this situation is depends on the customer retention rate $1-\delta_n$ and how large and persistent firm level shocks are. It is less likely to happen if the retention rate is low, or if negative shocks are small and/or temporary.

Problem (1) determines the decision rules $s(k,n,z;w,\Theta)$, $\Delta(k,n,z;w,\Theta)$, $l(k,n,z;w,\Theta)$, $i(k,n,z;w,\Theta)$, $y(k,n,z;w,\Theta)$ for each firm, together with the value function $V(k,n,z;w,\Theta)$.[11]

Choice of Sales Personnel The optimal level of sales personnel is characterized by the first order condition

$$\frac{\kappa'(s)}{q(\theta)} + \Delta = 1 - wl_2(k,y,z) + \beta(1-\delta_n)\int V_2(k',n',z';w,\Theta)Q(z,dz'). \tag{7}$$

[10] Customers are indifferent between staying with the current producer and finding a new one, so they will simply find another supplier. We assume that customers retain no memory of past suppliers and their products.

[11] We assume the value function is differentiable. There may be an issue with differentiability of V w.r.t. n when $s=0$ is optimal. We conjecture that the argument in Campbell and Fisher (2000) can be adapted, i.e. the conditional expectation of the value is differentiable in n, which is enough for our purposes.

The left-hand side represents the marginal cost of acquiring an additional customer: the first term is the marginal increase in sales costs and the second the discount. The right-hand side represents the marginal increase in firm value from an additional customer: today's revenue increases by the price, 1, today's production costs increase according to the marginal cost $wl_2(k, y, z)$, and tomorrow's customer base is larger by $1 - \delta_n$ customers.

The optimal discount is characterized by the first-order condition:

$$1 = \frac{q'(\theta)\Theta'(\Delta)}{q(\theta)}[1 - wl_2(k, y, z) - \Delta + \beta(1 - \delta_n) \int V_2(k', n', z'; w, \Theta)Q(z, dz')]. \tag{8}$$

The left hand side represents the marginal cost of increasing the discount, while the right hand side represents the corresponding marginal increase in firm value: the rate at which the firm establishes new customer relationships increases according to $\frac{q'(\theta)\Theta'(\Delta)}{q(\theta)}$,[12] with each new customer increasing firm value by the term in the brackets. While a larger discount makes new customers less profitable, by attracting more customers per sales person it also increases the number of new customers per costly sales person. How large the discount should be depends crucially on how severely congestion affects the formation of new customer relationships, captured by the elasticity of the matching function. If sales personnel cannot accommodate more customers per period, there is no point in increasing discounts to attract more customers.

Combining equations (7) and (8) yields

$$\frac{\kappa'(s)}{q(\theta)} = \frac{q(\theta)}{q'(\theta)\Theta'(\Delta)}. \tag{9}$$

Once we specify the function for $\theta = \Theta(\Delta)$ in Section 1.2, we will see that firms choosing a higher level of sales personnel also choose bigger discounts.

Optimal Investment The first-order condition for investment i is familiar,

$$\phi_1(i, k) = \beta \int V_1(k', n', z'; w, \Theta)Q(z, dz'), \tag{10}$$

relating investment today to expected marginal Q next period, $\int V_1(k', n', z; w, \Theta)Q(z, dz')$. If the adjustment cost is quadratic, as generally specified, optimal $\frac{i}{k}$ becomes a linear function of marginal Q. Moreover, in the absence of product market frictions, marginal Q equals average Q, $V(k', n', z'; w)/k'$, readily measurable in the data. We will show in Section 2 that frictional product markets introduce a systematic wedge between marginal and average Q, which can help explain the failure of this simple testable prediction of the neoclassical adjustment cost model of investment in the data.

[12]The firm gets new customers at rate $q(\theta)$ per sales person.

1.2 Representative Household

The representative household's preferences over consumption C_t and leisure L_t are

$$\sum_{t=0}^{\infty} \beta^t u(C_t, L_t). \tag{11}$$

The household's budget constraint reads, at each date $t \geq 0$,

$$C_t + B_t \leq w_t(1 - L_t) + D_t + (1 + r_t)B_{t-1}, \tag{12}$$

The household is endowed with one unit of time each period and purchases consumption at a frictionless market with price normalized to one and leisure at the market wage w_t. The non-leisure time is divided between market work L^m and search for suppliers of consumption goods L^s, with both activities yielding the same return w_t in equilibrium. Moreover, in a stationary equilibrium this wage is constant over time. The household owns all firms in the economy, receiving the aggregated dividends D_t each period. The household has access to a risk-free bond B_t, with return r_t. In a stationary equilibrium, $1 + r_t = \frac{1}{\beta}$ and $B_t = 0$ for all t.

Product search Product search is necessary for establishing and maintaining long-term customer relationships with producers, in order to procure consumption goods for the household.[13] For each unit of time spent searching, the household must decide which firm's product to try. The household anticipates that firms offering larger discounts attract more potential customers per sales person, increasing the time cost of finding a suitable product. Given discount Δ and a ratio of potential customers to sales person θ, the return to spending a marginal unit of time in product search reads

$$\mu(\theta)[1 - p + \Delta - K_s + \beta(1 - \hat{\delta}_n)\frac{1 - p}{1 - \beta(1 - \hat{\delta}_n)}].$$

New customer relationships are born at rate $\mu(\theta)$, and once this happens, the firm delivers one unit of the good to the buyer each period for as long as the relationship lasts. In return, the household pays the producer p units of consumption each period. In the first period, the buyer also receives the discount[14] and incurs a switching cost K_s in adapting to the new supplier. Purchasing relationships end at rate $\hat{\delta}_n$, either for idiosyncratic reasons or because the firm restricts output due to a severe negative shock.

After the customer relationship has been established and the promised initial discount paid, the firm maximizes profits by raising the price to make the buyer indifferent between staying and finding a new supplier. This implies that $p = 1$, making the return to search $\mu(\theta)[\Delta - K_s]$.

Because the household chooses how to divide searching time among firms, this return is equalized across all firms with a positive measure of sales people. Moreover, the return equals the opportunity cost, w :

$$w = \mu(\theta)[\Delta - K_s] \tag{13}$$

[13]Part of these goods will also be sold as investment goods to firms.
[14] It may be greater than p.

This equation implicitly defines an increasing relationship between the discount a firm offers and the resulting measure of potential customers per sales person. Solving the equation for θ yields the function: $\Theta(\Delta) = \mu^{-1}(\frac{w}{\Delta - K_s})$.[15]

Given a firm's sales personnel s, equations (9) and (13) determine that firm's discount and measure of potential customers per sales person as function of s. Assuming $\kappa(s)$ is strictly convex, they imply that firms choosing larger a sales force also offer a larger discount.[16]

1.3 Aggregation

We denote the cross-sectional distribution of firms by $\nu(k, n, z)$. Over time this distribution evolves according to a law of motion determined by the productivity process and firm decision-making: $\nu' = T(\nu; w, \Theta)$. Our focus is on a stationary distribution ν.

Total output in any period is $Y(\nu; w, \Theta) = \int y(k, n, z; w, \Theta) d\nu(k, n, z)$. The total labor demand of firms is $L^p(\nu; w, \Theta) = \int l(k, n, z; w, \Theta) d\nu(k, n, z)$, while the labor devoted to product search by the household is[17]

$$L^s(\nu; w, \Theta) = \int s(k, n, z; w, \Theta)\Theta(\Delta(k, n, z; w, \Theta)) d\nu(k, n, z).$$

Total investment is $I(\nu; w, \Theta) = \int i(k, n, z; w, \Theta) d\nu(k, n, z)$ with costs of investing $\Xi_I(\nu; w, \Theta) = \int \phi(i(k, n, z; w, \Theta), k) d\nu(k, n, z)$. Total sales effort is $S(\nu; w, \Theta) = \int s(k, n, z; w, \Theta) d\nu(k, n, z)$ with costs $\Xi_{SA}(\nu; w, \Theta) = \int \kappa(s(k, n, z; w, \Theta)) d\nu(k, n, z)$.

The dividend income of the household can be written as

$$\begin{aligned} D(\nu; w, \Theta) =& Y(\nu; w, \Theta) - wL^p(\nu; w, \Theta) - \Xi_{SA}(\nu; w, \Theta) \\ & - \int s(k, n, z; w, \Theta) q(\Theta(\Delta(k, n, z; w, \Theta)))\Delta(k, n, z; w, \Theta) d\nu(k, n, z) - \Xi_I(\nu; w, \Theta). \end{aligned}$$

Focusing on a stationary distribution, the aggregate customer base is constant and the number of customers incurring switching costs equals $\hat{\delta}_n Y(\nu; w, \Theta) = \int s(k, n, z; w, \Theta) q(\theta(\Delta(k, n, z; w, \Theta))) d\nu(k, n, z)$. Total switching costs are $\Xi_{SW}(\nu; w, \Theta) = \hat{\delta}_n Y(\nu; w, \Theta) K_s$.

1.4 Equilibrium

We embed the competitive search equilibrium of Moen (1997) into the stationary equilibrium defined e.g. in Gomes (2001).

[15] With the Cobb-Douglas matching technology, where $q(\theta) = \xi\theta^\alpha$ and $\mu(\theta) = \xi\theta^{\alpha-1}$, we have $\Theta(\Delta) = (\frac{\xi(\Delta - K_s)}{w})^{\frac{1}{1-\alpha}}$.

[16] They imply a positive relationship between $\frac{\kappa'(s)}{w}$, and θ. In particular, for the Cobb-Douglas case, $\Delta = K_s + [\kappa'(s)\frac{\alpha}{1-\alpha}]^{1-\alpha} w^\alpha \frac{1}{\xi}$, and $\theta = \frac{\kappa'(s)}{w}\frac{\alpha}{(1-\alpha)}$.

[17] Notice that because θ is the ratio of potential customer to sales person, $s\theta$ is the measure of potential customers.

DEFINITION 1. A stationary competitive search equilibrium specifies: i) for the household: decision rules $L^m(w, D(w, \Theta))$, $L^s(w, D(w, \Theta))$, $C(w, D(w, \Theta))$, ii) for the firms: decision rules $s(k, n, z; w, \Theta)$, $\Delta(k, n, z; w, \Theta)$, $i(k, n, z; w, \Theta)$, $l(k, n, z; w, \Theta)$, $y(k, n, z; w, \Theta)$ and value function $V(k, n, z; w, \Theta)$, iii) aggregate quantities $S(w, \Theta)$, $I(w, \Theta)$, $L^p(w, \Theta)$, $Y(w, \Theta)$, $D(w, \Theta)$, iv) wage w, v) the function $\Theta(\Delta)$, and vi) distribution of firms ν, such that

1. **Consumers optimize:**
   (a) The decision rules $L^m(w, D(w, \Theta))$, $L^s(w, D(w, \Theta))$, $C(w, D(w, \Theta))$, solve the household problem to maximize (11) subject to (12).
   (b) For all Δ,
      i. the tradeoff $\Theta(\Delta)$ satisfies $-w + \mu(\Theta(\Delta))[\Delta - K_s] = 0$ whenever $\Theta(\Delta) > 0$,
      ii. and $-w + \mu(\Theta(\Delta))[\Delta - K_s] \leq 0$ whenever $\Theta(\Delta) = 0$.
2. **Firms optimize:** The decision rules and value function solve the production firms' Bellman equation.
3. **Labor market clears:** $L^m(w, \Theta) = L^p(w, \Theta)$.
4. **Consistency:** The distribution of firms ν follows the law of motion $\nu' = T(\nu; w, \Theta)$, with $\nu' = \nu$, and the aggregate variables are consistent with the definitions in Section 1.3. (Add:$\hat{\delta}_n$)

By Walras' law, also the goods market clears: $Y = C + \Xi_{SA} + \Xi_I + \Xi_{SW}$.[18]Production output is used up by consumption, sales costs, investment costs, and switching costs.

2 The Consequences of Frictions

This section parameterizes the model and studies its implications for investment dynamics. Studying the steady-state of the model, we first show that customer capital raises firm value above the value of physical capital alone. Studying the dynamics of the model, we then show that product market frictions lead to lagged responses of investment to shocks, and can help explain the patterns observed in investment-regressions.

[18]Note that $\int sq(\theta)\Delta d\nu = \int s\theta\mu(\theta)\Delta d\nu = \int s\theta[w + \mu(\theta)K_s]d\nu = w\int s\theta d\nu + \Xi_{SW}$, by equation (13). Hence, substituting dividends into the household budget gives

$$C = w(1-L) + D = w(L^m + L^s) + pY - wL^p - \Xi_{SA} - w\int s\theta d\nu - \Xi_{SW} - \Xi_I = w(L^m + L^s) + Y - wL^s - wL^p - \Xi_{SA} - \Xi$$

$$= Y - \Xi_{SA} - \Xi_{SW} - \Xi_I.$$

2.1 Calibration

We parameterize the model as described in Table 1 (the model is monthly, but we report annual values). While many of the parameters are standard, and taken from the literature, there are some involving the frictional product market which are not. We set the depreciation of the customer base to $\delta_n = 20\%$ per year following Hall (2008). The convex cost of sales effort is assumed quadratic: $\kappa(s) = \kappa\frac{s^2}{2}$. We set the coefficient κ of the adjustment cost function, and the coefficient ξ and elasticity α of the matching function ($M(x, s) = \xi x^{\alpha} s^{1-\alpha}$) to accomodate the following targets: First, the share of marketing expenditures in GDP is on average 4% (Hall 2008). Second, time use surveys suggest that time spent in product research is roughly one hour per week. Finally, we set the average markup to 10%.[19]

Table 1: Calibration

β	discount rate	.95
ρ_z	persistence of productivity	.8
σ_z	st.dev. of productivity	.1
α_K	share of capital	.3
δ_K	depreciation of capital	.1
η_I	elasticity of adj. cost	1
δ_N	depreciation of customers	.2
κ	sales cost parameter	107.4
α	matching function parameter	0.14
ξ	matching function parameter	0.87

2.2 Changes in the Steady-state

The extent of frictions in the product market can be thought of as measured by ξ, the coefficient on the matching function. As ξ grows, frictions diminish and the model approaches the neoclassical model. Along the way both the resources spent on sales effort and the time spent searching diminish, and the customer base loses its value. In the frictionless limit the value of the firm equals the value of physical capital, so Tobin's Q equals one. Figure 1 illustrates these effects, with the frictionless limit on the left and the friction increasing toward the right as resources spent on sales effort per output grow. The benchmark calibration is marked by the vertical line, with implied Tobin's Q approximately 1.7. This substantial value associated with the customer base derives from the front-loaded costs of attracting customers: both the costs of sales effort as well as the discounts used. The present discounted value of future profits per customer (the excess of price over marginal cost) exactly compensates for these costs.

[19]We set the switching cost K_s to zero for now.

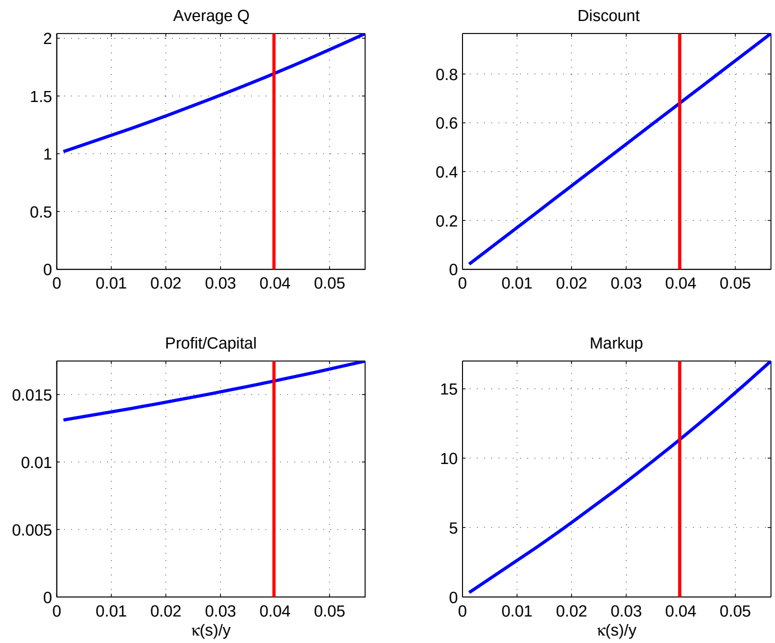

Figure 1: Impact of friction on steady-state

Notes: The figure plots the steady-state of the model varying the degree of search friction via the matching function parameter ξ. As ξ falls, matching rates fall and the share of selling expenses in GDP $\kappa(s)/y$ rise. The vertical line indicates the benchmark calibration described in the text.

2.3 Changes in Responses to Shocks

Regressions of investment on Tobin's Q generally yield a small coefficient on Q and a low R^2. Moreover, contrary to what the neoclassical adjustment cost model predicts, firm cash flow appears to matter for investment. Frictional product markets can help reconcile these findings with the model, and to understand the mechanism, it is useful to consider impulse responses to a productivity shock. When product market frictions are negligible, a positive productivity shock causes a firm to expand, as shown in Figure 2, constrained only by the physical adjustment cost. In response to the shock, the shadow value of additional capital jumps up on impact. Consistent with the first order condition governing investment (10), the investment rate responds exactly as this shadow value, jumping up on impact. Average Q has a similar response, implying that the investment rate and Tobin's Q are highly correlated.

Figure 3 shows what happens instead when frictions are non-negligible. In this case firm growth is constrained also by the product market frictions, making expanding the customer base costly. The first order of business following the productivity shock is no longer to expand physical production capacity, but to expand the customer base. Indeed, the increase in productivity allows the firm to produce more output even with the existing capacity, calling for an increase in the number of customers. The customer base expansion is both costly and time consuming, leading to slower growth in sales. In response to the shock, marginal Q and investment rise, but reach their peaks only as the customer base expansion allows. A similar pattern holds for profit rates, which rise as the customer base grows and as

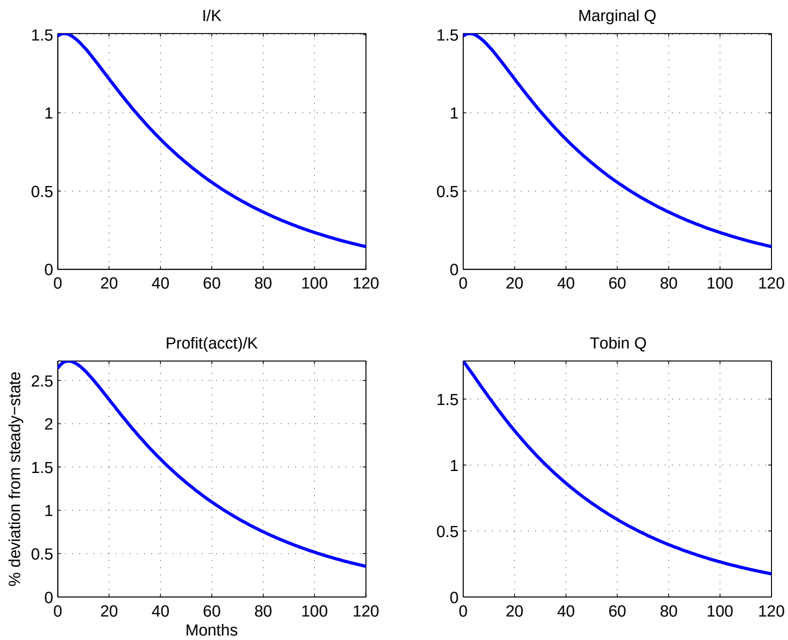

Figure 2: Responses to tfp-shock with low frictions

Notes: Impulse responses when frictions are small. The variables are all in % deviation from the non-stochastic steady-state and the period is a month.

the initially extensive costs of customer expansion begin to subside. Tobin's Q, on the other hand, rises without lag as the drop in production costs makes the existing customer base more valuable to the firm. Leading to the investment regression results discussed next, this figure suggests that investment rates are likely to be more strongly correlated with profits than Tobin's Q.[20]

2.4 Implications for Investment Regressions

To study the implications of the model for investment regressions more concretely, we simulate a balanced panel of firms from the model and run the following Q-regressions, as well as Q-regressions augmented with cash flows, in this simulated data:

$$\frac{I_{it}}{K_{it}} = b_0 + b_1 Q_{it} + \varepsilon_{it}, \tag{14}$$

$$\frac{I_{it}}{K_{it}} = b_0 + b_1 Q_{it} + b_2 \frac{CF_{it}}{K_{it}} + \varepsilon_{it}, \tag{15}$$

[20]Product market frictions reduce the overall sales volatility as well – a potential limitation in light of the high volatility observed in data. Note however that due to constant returns in production, the volatility of sales in the model tends to be high.

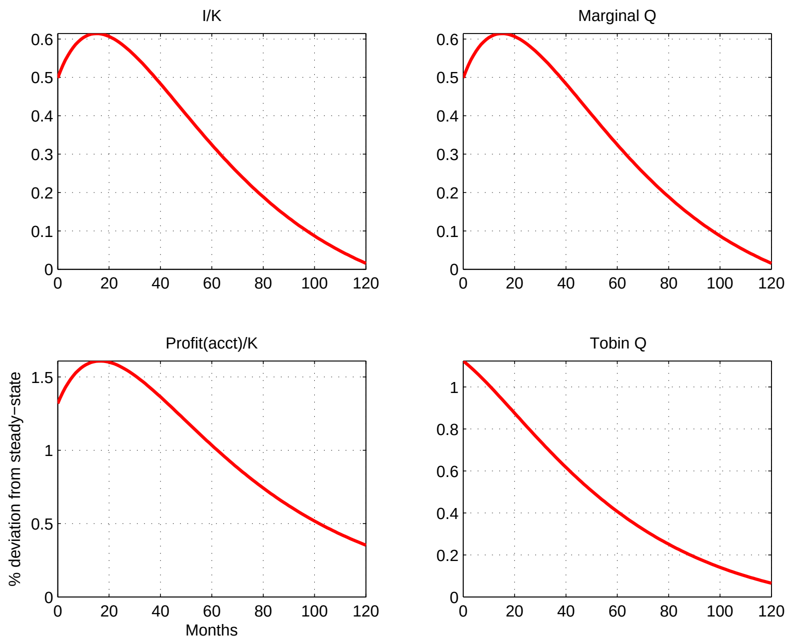

Figure 3: Responses to tfp-shock with benchmark frictions

Notes: Impulse responses in the benchmark calibration. The variables are all in % deviation from the non-stochastic steady-state and the period is a month.

where $Q_{it} = \beta E_t V_{it+1}/K_{it+1}$ is Tobin's Q, and CF_{it} the cash flow, measured as output net of labor costs and sales costs.[21]

Figure 4 presents the coefficient estimates from regression (14) as the friction increases. In the frictionless limit, the Q-theory regression works perfectly in that the coefficient on Q is consistent with the physical adjustment cost and the R^2 is 100 percent. As the friction increases, however, both the coefficient on Q and the R^2 fall. Figure 4 presents the coefficient estimates from the cash flow augmented regression (15). In the frictionless limit cash flow does not matter, and investment is explained by Q just as the theory predicts. But as the frictions increase, not only do both the coefficient on Q and the R^2 fall, but cash flow becomes significant.

These figures make two important points: First, note that the benchmark calibration implies a clearly lower coefficient on Q than the neoclassical model does, so if one were to infer the magnitude of the physical adjustment cost from the regression coefficient on Q, one would be overestimating the adjustment costs to be 2-3 times greater than the true value. Second, Figure 5 shows that under frictional markets firm cash flow has predictive power for investment – without implying that the firm faces financial constraints.

These findings are particularly interesting because in extensions of the adjustment cost model specifically designed to break the direct relationship between investment and To-

[21] Without the cashflow term, this would be the exact regression specified by the Hayashi model. Note that the timing is driven by the one-period time-to-build, hence the time-t investment is related to the expected value next period.

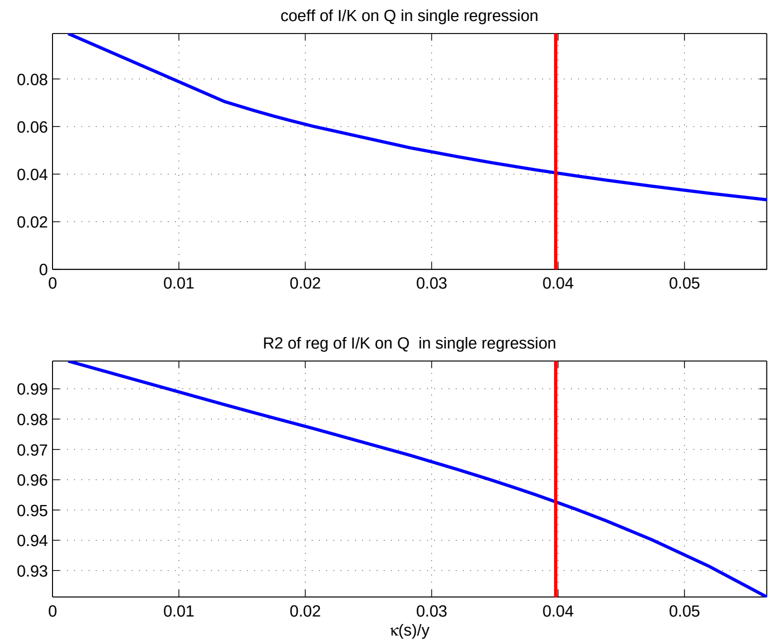

Figure 4: Impact of friction on Q-regression with cash flow

Notes: This figure reports the coefficient on Q (b_1) as well as the R^2 from regression (15) on simulated data from the model. The vertical line indicates the benchmark calibration.

bin's Q, investment regressions nevertheless tend to work well quantitatively (Gomes 2001). To generate these effects, studies appeal to measurement error in Q (e.g. Erickson and Whited 2000, Eberly, Rebelo, and Vincent 2008). In contrast, we provide an economically intuitive mechanism for the failure of Q-theory.

3 Investment and Product Market Frictions: Evidence

While product markets are likely to be somewhat frictional for all goods, some markets are more frictional than others. This section uses Compustat data to study whether firms facing more frictional markets appear to behave in ways consistent with our model. We study two predictions of the model: i) whether investment appears to lag Tobin's Q more in high friction industries, and ii) whether Q-regressions work less well in high friction industries.

3.1 Measuring Frictions

Perfect measures of the degree of product market friction are difficult to come by, but Compustat does provide a variable which seems informative about this issue, measuring how much firms spend on marketing and selling their products: total selling, general and administrative (SGA) expenses. For each 2-digit industry in Compustat, we calculate the average selling expenses i.e. the time series average of the ratio of total industry SGA

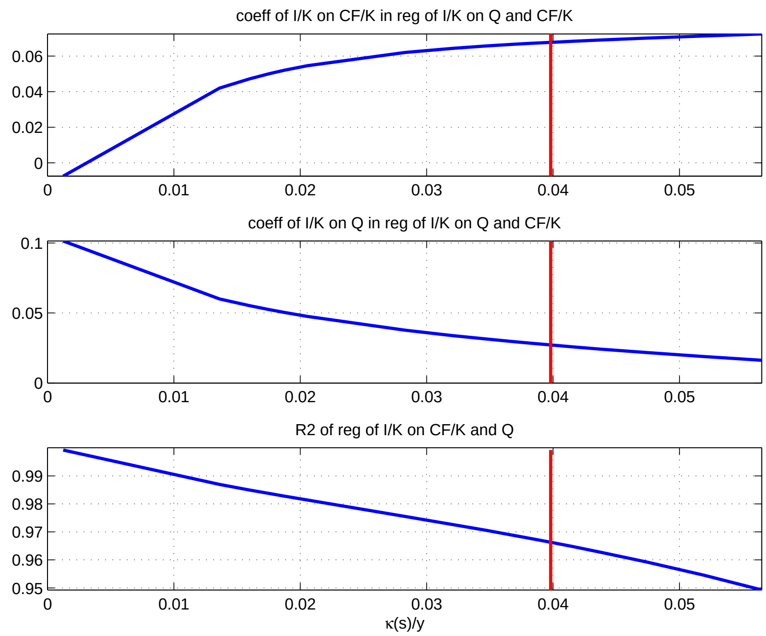

Figure 5: Impact of friction on Q-regression with cash flow
Notes: This figure reports the coefficients on Q (b_1) and cash flow (b_2) as well as the R^2 from regression (15) on simulated data from the model. The vertical line indicates the benchmark calibration.

expenses to total industry sales. We then split the sample into industries with low (below median) and high (above median) values of this ratio, and use the annual firm level data on Tobin's Q, investment and cash flow to study the predictions of our model in the two samples.

Table 2 provides summary statistics for the two samples. Consistent with our model, Tobin's Q and the profit rate are higher in the high selling expense industries. The physical investment rates on the other hand are similar, suggesting that the sort is not leading to substantial differences in production technologies across the samples.

Table 2: Summary statistics for low and high selling expense samples

Median	Low selling expenses	High selling expenses
Q	1.19	2.07
PI/K	0.19	0.26
I/K	0.12	0.12

3.2 The Lagged Response of Investment

The model predicts that frictions in product markets cause investment to lag Tobin's Q. Figure 6 examines the corresponding relationship in the data. For the figure we first calculate firm level time series correlations of the investment rate with lags of Tobin's Q. The figure plots the medians of these correlations in the high and low selling expense samples. The figure shows that: i) the correlation is clearly higher for the one period lag of Q than the current Q, and ii) the response of investment is more lagged in the higher selling expense sample. It is interesting to note that despite this evident lag-pattern, the correlations of the investment rate with the one period lag of Q are relatively similar. This suggests that the R^2 on the typical Q-regression run on the one period lag of Q may be relatively similar in the two samples, despite the lag-pattern.

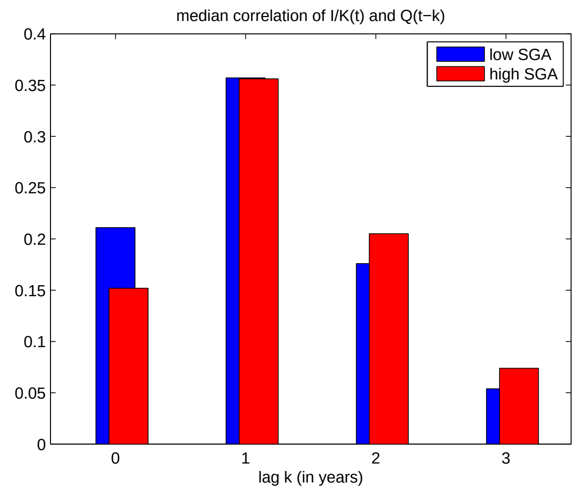

Figure 6: Correlation of investment rate with lagged Tobin's Q

3.3 Q-regressions

The model predicts that in industries with higher product market frictions, Q-regressions should work less well: Q-regressions should have a smaller coefficient on Q and larger coefficient on cash flow, as well as a lower R^2. Table 3 presents the results from running the regressions

$$\frac{I_{i,t}}{K_{i,t-1}} = b_0 + b_1 Q_{i,t-1} + f_t + d_i + \varepsilon_{i,t}, \tag{16}$$

$$\frac{I_{i,t}}{K_{i,t-1}} = b_0 + b_1 Q_{i,t-1} + b_2 \frac{CF_{i,t-1}}{K_{i,t-1}} + f_t + d_i + \varepsilon_{i,t}, \tag{17}$$

in each of the two samples, with and without firm fixed effects.[22] The results line up with our theory: in industries with higher selling expenses, the coefficient on Q is significantly lower, the coefficient on cash flow significantly higher, and the R^2 somewhat lower. Moreover,

[22]Note that we follow the standard timing of investment regressions in the empirical literature, i.e. we use the lagged values of the profit rate and Tobin Q. This is in contrast to the model regressions which were with the timing which is correct in the model.

these results appear robust to changes in the definition of Q, changes in the timing and specification of the regressions (levels vs. logs), as well as the exclusion of firm fixed effects. The appendix presents results for some alternative specifications.

Table 3: Q-regressions

	Low selling expense sample				High selling expense sample			
Coeff.	1a	2a	3a	4a	1b	2b	3b	4b
Q	.044	.041	.033	.031	.028	.025	.021	.019
s.e.	.001	.002	.001	.001	.001	.001	.001	.001
CF/K	–	.030	–	.039	–	.043	–	.047
s.e.	–	.006	–	.004	–	.003	–	.002
R^2	0.159	0.169	0.091	0.099	0.153	0.162	0.087	0.115
Fixed effects	y	y	n	n	y	y	n	n

One potential concern with this cross industry test is that the two samples may contain very different types of firms, and that these differences may be contaminating the results. One way to gauge this potential concern is to restrict the sample to manufacturing firms only. Table 4 presents the regression results for this restricted sample, showing that the results continue to hold also within manufacturing.

Table 4: Q-regressions for manufacturing

	Low selling expense sample				High selling expense sample			
Coeff.	1a	2a	3a	4a	1b	2b	3b	4b
Q	.038	.034	.032	.029	.028	.024	.022	.020
s.e.	.002	.002	.001	.001	.001	.001	.001	.001
CF/K	–	.040	–	.050	–	.043	–	.043
s.e.	–	.008	–	.006	–	.004	–	.003
R^2	0.193	0.194	0.137	0.154	0.153	0.169	0.100	0.125
Fixed effects	y	y	n	n	y	y	n	n

4 Related Literature

Our work is related to several strands of literature. Technically, our model of frictional product markets uses tools that were created to study the labor market: we utilize the tractability of the competitive search equilibrium concept proposed by Moen (1997), embedding it in a standard neoclassical model of investment (e.g. Hayashi 1982, Gomes 2001). Substantially, the idea of customer capital or customer markets has a long tradition in the macroeconomics literature, but most studies concentrate on its implications for pricing and markups (e.g. Bils 1989, Menzio 2007, Nakamura and Steinsson 2008, Ravn, Schmitt-Grohe, and Uribe 2008). Only recently have researchers started building more detailed dynamic models of customer capital (e.g. Kleshchelski and Vincent 2009, Hall 2008). Examples of recent applications include Drozd and Nosal (2008), who show that product market frictions can help explain the behavior of international prices, as well as Arkolakis (2008) who shows they can help explain international trade quantities.

A large literature in IO also focuses on the customer base, but generally as a result of switching costs rather than frictional markets (e.g. Klemperer 1995). The focus of these studies is typically on the interaction with imperfect competition and most applied studies concentrate on a single industry. Compared to most of the IO literature, our model is stylized, but we believe our results would hold in a more general class of models.

More broadly, there has been substantial interest in the macroeconomic implications of intangible capital. Robert Hall (2001a, 2001b) suggests that intangible capital accumulation can explain movements in stock market values. Atkeson and Kehoe (2005) use data on the life-cycle of plants to measure the accumulation and contribution of intangible capital to output. In a series of papers, McGrattan and Prescott (2010b, 2010a) use the notion of intangible capital to explain several puzzling macroeconomic observations. Eisfeldt and Papanikolaou (2009) study the implications of organizational capital for the cross-section of stock returns. While we focus specifically on the customer base as a form of intangible capital, these studies generally consider a fairly general form of unmeasured capital.

5 Concluding Remarks

We have argued that product market frictions can help explain observed firm investment behavior. While we have focused on an environment without aggregate shocks here, the model is parsimonious enough to easily extend itself to also studying time series variation in the aggregate. Preliminary results suggest that the model has interesting properties in that setting as well, and we plan to study them further.

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Table 5: Summary statistics: low SGA sample

	mean	median	sd
ik	0.194	0.115	0.268
pik	0.210	0.186	0.590
tobinq	1.913	1.186	1.958
manuf	0.426	0	0.495
sales	1312.1	113.4	6670.6
assets	1426.4	101.1	9787.2
employment	8.253	0.991	33.21
mkt	1093.4	59.24	8069.0
debtasset	0.232	0.193	0.232
debtcap	0.500	0.271	1.078
divasset	0.0114	0	0.0573
divearnings	0.0615	0	1.963
ispayingdiv	0.454	0	0.498

Table 6: Summary statistics: high SGA sample

	mean	median	sd
ik	0.197	0.123	0.256
pik	0.173	0.256	0.775
tobinq	2.864	2.070	2.400
manuf	0.687	1	0.464
sales	809.6	81.04	3253.4
assets	750.5	67.44	3476.3
employment	5.912	0.669	23.01
mkt	891.8	54.41	4968.9
debtasset	0.171	0.112	0.233
debtcap	0.438	0.224	0.706
divasset	0.0129	0	0.0760
divearnings	0.0576	0	1.367
ispayingdiv	0.398	0	0.489

Table 7: Regression of I/K on Q

	Simple regression		Time effects		Fixed effects		Both effects	
	Low SGA	High SGA	Low SGA	High SGA	Low SGA	High SGA	Low SGA	High SGA
b_1	0.0324	0.0200	0.0331	0.0213	0.0452	0.0294	0.0435	0.0283
	(0.000953)	(0.000668)	(0.000963)	(0.000676)	(0.00146)	(0.001000)	(0.00145)	(0.000997)
R^2	0.080	0.060	0.091	0.087	0.155	0.129	0.159	0.153

Table 8: Regression of I/K on Q and PI/K

	Simple regression		Time effects		Fixed effects		Both effects	
	Low SGA	High SGA	Low SGA	High SGA	Low SGA	High SGA	Low SGA	High SGA
b_1	0.0298	0.0181	0.0305	0.0193	0.0425	0.0259	0.0412	0.0249
	(0.00101)	(0.000664)	(0.00102)	(0.000674)	(0.00158)	(0.00105)	(0.00158)	(0.00106)
b_2	0.0398	0.0500	0.0385	0.0469	0.0341	0.0454	0.0302	0.0431
	(0.00387)	(0.00231)	(0.00386)	(0.00229)	(0.00567)	(0.00344)	(0.00571)	(0.00342)
R^2	0.089	0.093	0.099	0.115	0.159	0.141	0.169	0.162

Table 9: Regression of I/K on Q: manufacturing

	Simple regression		Time effects		Fixed effects		Both effects	
	Low SGA	High SGA	Low SGA	High SGA	Low SGA	High SGA	Low SGA	High SGA
b_1	0.0312	0.0210	0.0321	0.0222	0.0397	0.0280	0.0379	0.0275
	(0.00125)	(0.000858)	(0.00126)	(0.000870)	(0.00185)	(0.00127)	(0.00183)	(0.00128)
R^2	0.111	0.070	0.137	0.100	0.180	0.125	0.193	0.153

Table 10: Regression of I/K on Q and PI/K: manufacturing

	Simple regression		Time effects		Fixed effects		Both effects	
	Low SGA	High SGA	Low SGA	High SGA	Low SGA	High SGA	Low SGA	High SGA
b_1	0.0278	0.0195	0.0289	0.0206	0.0353	0.0248	0.0341	0.0243
	(0.00134)	(0.000846)	(0.00135)	(0.000861)	(0.00206)	(0.00134)	(0.00205)	(0.00136)
b_2	0.0532	0.0456	0.0499	0.0427	0.0465	0.0444	0.0400	0.0432
	(0.00565)	(0.00275)	(0.00557)	(0.00272)	(0.00757)	(0.00426)	(0.00756)	(0.00430)
R^2	0.131	0.099	0.154	0.125	0.182	0.138	0.194	0.169