A Theory of Housing Prices and the Homeownership Rate: Evidence from the Housing Markets in Japan and the U.S.*

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February 18, 2009

Abstract

This paper attempts to account for similarities and differences in housing markets in the last two decades in Japan and in the last decade in the United States by examining the role of interest rates, the growth rate of total factor productivity (TFP), and the change in downpayment requirements. The similarities that existed were huge fluctuations in housing prices and the housing price-to-rent ratio in both countries. The difference is that in the U.S. the increase in the homeownership rate coincided with the housing price increase, while the opposite was true in Japan. To investigate this issue, a general equilibrium overlapping generation model is constructed where housing prices and rents are determined endogenously. The model is carefully calibrated and estimated to the data so that it captures some economic features as well as the differences in housing market characteristics in both countries. I find three results. First, the model shows that the decrease in the interest rates and the increase in the TFP growth rate account for most of the increase in housing prices and the housing price-to-rent ratio in the late 80's in Japan and in the last decade in the U.S. Second, the model also predicts the subsequent decrease in housing prices and the housing price-to-rent ratio in the 90's in Japan. This result implies that in the 90's in Japan, the negative effect of the decline in the TFP growth rate on housing prices and the housing price-to-rent ratio exceeded the positive effect of the further interest rate decline. Third, I find that while the change in downpayment requirements has little effect on housing prices and the housing price-to-rent ratio, the difference in the homeownership rate change between Japan and the U.S. is likely to be accounted for by the downpayment requirement change.

*I especially thank Dirk Krueger for his kind and passionate guidance. I also thank Jesus Fernández-Villaverde, Iourii Manovskii, and Jennifer Amyx for useful comments and advice. All errors are mine.

1 Introduction

The large fluctuations in the U.S. housing prices over the last decade have attracted the attention of an increasing number of macroeconomists. The OFHEO Housing Price Index published by the US Office of Federal Housing Enterprise Oversight (OFHEO) indicates an 51.7% increase in housing prices from 1995 to 2006 in real terms. The share of U.S. residential fixed investment in GDP had grown from 4.09% in 1995 to 6.20% in 2005. However, this is not the largest housing price increase. According to the Real Estate Economic Institute (the REEI hereafter), there existed similar and even sharper housing price fluctuations in Japan in the last two decades.Average newly sold condominium prices rose almost 70% from 1986 to 1990 and has since fallen more than 30% in real terms.

To the best of my knowledge, this paper is the first attempt to conduct a comparative study between the housing markets in the last two decades in Japan and in the last decade in the U.S. with a general equilibrium framework. Comparative analysis between Japan and the U.S. is important for three reasons. First, it helps us to understand common and non country-specific factors and mechanisms of housing price fluctuations. Second, analysis of Japan's experience will cast light on the future housing market situations in the U.S. Japan has already experienced the aftermath of a sharp price increase: a continuous housing price fall with a long recession. Lastly, the role of financial innovations in U.S. housing markets in the last decade can be highlighted. Although housing prices moved similarly in both countries, there were no signs of prevalent financial innovations in housing markets in Japan while financial innovations was one of the important characteristics in U.S. housing market at that period.

A comparison between Japan and the U.S. clarifies similarities and differences in their housing markets. First, they share certain similarities in the evolution of housing prices and rents. As I mentioned above, there existed a huge rise in housing prices in the early 2000s in the U.S. and in the late 80's in Japan, followed by a sharp decrease in housing prices. Throughout the period, however, rents have increased by a relatively small amount. Consumer Price Index (CPI) for rents in the U.S. shows only a 10.5% increase from 1995 to 2006 and in Japan indicates only a 8.1% increase from 1985 to 1990. On the other hand, the changes in the homeownership rate appear to be different. An increase in the homeownership rate in the U.S. was contemporaneous with the housing price increase, while the homeownership rate moved opposite to housing prices in Japan. The Statistical Abstract of U.S. published by the U.S. Census Bureau indicates that the average homeownership rate had increased by 5.0 percentage points from 1994 to 2004. In contrast, the homeownership rate in Japan computed from the National Survey of Family Income and Expenditure (hereafter the NSFIE) shows a 0.8 percentage point decline from 1984 to 1994 and a 2.1 percentage point increase from 1994 to 2004. Moreover, while the homeownership rate of all age cohorts increased in the U.S., there was a decrease in the homeownership rate among the young and no change or a small increase in the homeownership rate among the old in Japan.

In light of these empirical observations, this paper answers the following two questions: (i) How much do changes in interest rates and productivity growth in both countries account for fluctuations of housing prices and the housing price-to-rent ratio? (ii) To what extent can differences in homeownership rate changes be explained by the reduction in downpayment requirements due to financial innovations?

To answer these questions, a general equilibrium overlapping generation model with incomplete markets is constructed. A housing production sector and a rental housing sector are explicitly modelled and housing prices and rents are determined endogenously. Each household faces a mortality risk and an idiosyncratic income risk and chooses a housing tenure choice (own a house or rent a house), a house size, risk free savings, and consumption. Assuming that the baseline economy is in a balanced growth path, I calibrate and estimate the model to replicate some important economic features in the U.S. and Japan. Some distinctive differences between housing markets in the two countries are also incorporated.

I then proceed to conduct transition path analysis where I compute transitional paths of the economy from the baseline balanced growth path to the new balanced growth path generated by a new set of exogenous variables: interest rates, the TFP growth rate, and downpayment requirements. All the variable changes come as a surprise to all households. To incorporate the timings of these changes, several transition paths are computed. Namely, the first transition path is computed from the baseline balanced growth path to the (first) new balanced growth path, and the second transition path is computed from the middle of the first transition path to the (second) new balanced growth path and so forth.

Using the model, I find three implications. First, the model shows that the decrease in the interest rates and the increase in the TFP growth rate account for most of the increase in housing prices and the housing price-to-rent ratio in the late 80's in Japan and in the last decade in the U.S. Quantitatively, the model produces a 46.0% housing price increase and a 22.8% housing price-to-rent ratio increase from 1983 to 1990 in Japan. On the other hand, in U.S., the model produces a 32.4% increase and a 42.7% increase from 1996 to 2006 respectively. Second, the model also predicts the subsequent decrease in housing prices and the housing price-to-rent ratio in the 90's in Japan. This result implies that the negative effect of the decline in the TFP growth rate in the 90's in Japan exceeded the positive effect of the further interest rate decline over the same period. Third, I find that while the change in downpayment requirements has little effect on housing prices and the housing price-to-rent ratio, the difference in the homeownership rate change between Japan and the U.S. is likely to be accounted for by the downpayment requirement change. In particular, the homeownership rate of the young in Japan decreased because they were constrained by downpayment requirements due to high housing prices. On the other hand, the homeownership rate of the young in the U.S. increased because the decrease in downpayment requirements loosened the constraints.

1.1 Related literature

The main features of my model are the following: (i) multi-production sectors where housing prices and rents are endogenously determined, (ii) incomplete markets with heterogeneous agents to accout for life cycle behavior of households, (iii) and transition analysis to examine the changes of interest rates, the TFP growth rate, and downpayment reqirements.

First, I document three studies which investigate the changes in housing prices and the homeownership rate in the U.S. by a general equilibrium model. The closest one is Kiyotaki et al. (2007) who show that an increase in productivity and a decrease in interest rates can result in a large housing price increase. It is, however, a quantitatively-oriented theoretical study not an empirical one. Kahn (2008) shows the importance of changes in the trend of productivity growth in explaining the housing price increase. Though his model incorporates aggregate shocks, the effects of the changes of interest rates on housing prices and the change of the homeownership rate are not examined. Chambers et al. (2007) attempt to account for the increase in the homeownership rate with a general equilibrium overlapping generation model and clarify the role of various factors such as financial innovations and demographic changes in the homeownership rate change. They model the heterogeneity of households with incomplete markets but ignore aggregate housing price changes.

There is another line of studies which focus on the existence of the "bubble" in the early 2000s in the U.S. such as Case and Shiller (2003) and Himmelberg et al. (2005). Their approach is totally different in that they look at the cross sectional variation of housing prices and try to determine the applicability of the "bubble" hypothesis from empirical observations.

Next, the studies that have been done regarding Japan's asset price fluctuations in the last two decades should be mentioned. Broadly speaking there are three lines of research. The first line of research is descriptive analysis of the asset markets from the late 80's to the early 90's in Japan. For example, Stone and Ziemba (1993) document the explanation of asset price fluctuations and remark that there is some evidence which supports the "bubble" hypothesis. Okina et al. (2001) give detalied description on not only the asset markets but also the Japanese economic background from the late 80's to the early 90's. They suggest five major factors which might have affected the asset price increase; aggressive bank behaviors caused by gradual financial deregulation, protracted monetary easing and credit expansion, taxation and regulations that tend to induce higher land prices, weak mechanisms to impose discipline (low corporate governance), and abundant self-confidence in Japan's future.

The second line of research is to attempt to explain stock and land price increases by the reduced form asset pricing theory. For instance, French and Porerba (1991) and Ito and Iwaisako (1995) find that the asset price increases in Japan cannot be fully accounted for by fundamentals alone. Their conclusion is the decrease in the interest rates in the late 80's was not large enough to explain the phenomena. Nishimura et al. (1999) show the importance of distortionary inheritance and capital gain tax to explain Japanese residential land prices.[1]

The last line of research is to use a general equilibrium model to account for asset price changes. Alpanda (2007) uses a calibrated general equilibrium model and finds that changes in a persistent expectation of productivity growth can account for a land price increase at the cost of counter predicting other macro economic variables. He also points out the magnification effect of the land taxation policy on land prices. Nakajima (2006) assumes adaptive learning on the growth rate of productivity and shows it can replicate Japanese land price fluctuations. My research is related to these two studies in that I use a general equilibrium model to clarify the effect of productivity growth. However, their focus is on commercial land prices not residential land prices.

Lastly, since housing is a single important asset in household life, there are number of studies regarding housing markets in and outside of Japan. Regarding the Japanese housing markets, Ohtake and Shintani (1996) use the same method as Mankiw and Weil (1989) to analyze the demographic effect on the Japanese housing market. Hyashi et al. (1988) analyze the effect of downpayment requirements of housing assets and taxation on saving rates. Outside of Japan, there are many macro oriented studies regarding housing markets which are related to mine: Davis and Heathcote (2005), Díaz and Luengo-Prado (2006), Nakajima (2005) and Ortalo-Magne and Rady (2006), to name a few.

The rest of this paper is organized as follows. Section 2 documents empirical observations of housing markets and the economic background in the last two to three decades in Japan and the U.S. In Section 3, I present the model settings. Section 4 explains the calibration and estimation strategy of the model. Section 5 provides the properties of the model and its evaluations. Section 6 shows the main results of this paper. In Section 7, the conclusion is provided as well as recomendations for future research.

2 Empirical observations

2.1 Stylized facts in housing markets

Similarities and differences exist in housing markets over the last two decades in Japan and the last decade in the U.S. The first similarity is that both countries experienced a huge increase in housing prices. The second similarity is that while housing prices increased sharply, rents showed only a small increase. The significant difference is that in the U.S. the average homeownership rate increased in every age cohort contemporaneously with the housing price increase. In Japan, however, the average homeownership rate didn't increase and even decreased among young households in contrast to the housing price increase.

[1]Nishimura et al. (1999) consider the non-Walrasian structure of the land market in the latter half of the paper. So this is not completely a reduce form asset pricing paper.

2.1.1 Housing prices and rents

Housing prices and rents: Japan Figure 1 shows three kinds of housing price indices in Japan in real terms, which are normalized to 100 in 1985. It can be observed that there exists a sharp increase in housing prices after 1986 to 1990 followed by a long decreasing period. On the other hand, rents kept increasing gradually and steadily. Brief explanations of each housing price index appear below.

The first housing index is the average condominium prices that is published by the REEI. This index is the simple average of all the new condominium prices in Japan. Since the original index doesn't control a size of condominiums, I adjust it according to the size and show the price per square meter index in the Figure.[2] This index shows that the average condominium prices increased 67% from 1985 to 1990 and fell more than 30% in the subsequent 10 years.

The second housing index is the average unit house prices which is also published by REEI. This index is based on the average price of unit houses which were released for sales in the Tokyo metropolitan district. The original index is not adjusted for a size of unit houses, so I adjust it by the size and show the price per square index in the Figure. [3] This index indicates a 52.2% increase in unit house prices from 1985 to 1990 and a 11.8% decrease from 1990 to 2000.

The third index is Consumer Price Index(CPI) for rents, excluding imputed rents. This index is based on *Kouri Bukka Toukei Chousa* (Retail Price Survey), which interviews to around 25 thousand households who live in rental houses. It can be shown that rents increased 8.1% from 1985 to 1990 and increased another 6.2% from 1990 to 2000.

Housing prices and rents: U.S. Figure 2 shows five kinds of housing price indices in the U.S. in real terms, which are normalized to 100 in 1995. It can be observed that there exists a large increase in housing prices after 1995 and a relatively small change in rent. Brief explanations of each housing price index appear below.

The first index is the Freddie Mac's Conventional Mortgage Home Price Index (hereafter the CMHPI). This index is based on mortgages for the single unit residential houses which were purchased or securitized by Freddie Mac or Fannie Mae. A construction method of the CMHPI is based on "repeat transactions," which measures the average price changes in repeat sales or refinancing of the same property. In Figure 2, the purchase-only index, which excludes data of refinancing properties, is shown to eliminate possible selection bias from refinancing decision. This index indicates 59.6% rise of housing prices from 1995 to 2006.

[2]Due to the lack of the data of the national average of condominium sizes, I use the quantity-weighted average of condominium sizes in the Tokyo and Osaka metropolitan districts.

[3]It is difficult to adjust by the size since the composition of land and structure varies from houses to houses. Unfortunately only the average land size data and the average housing structure size data are available. Thus I divide the original index by the weighted average of the average land size and the average housing structure size.

The second index is the OFHEO HPI. This index is also based on the single-family residential house mortgages which were purchased or securitized by Freddie Mac or Fannie Mae. This index uses almost the same data source of CMHPI. This index shows 51.7% of a housing price increase from 1995 to 2006.

The third index is the Constant-Quality House Price Index (hereafter the CQHPI) for New Homes, which is published by the U.S. Commerce Department through the Census Bureau. This index is based on a sample of about 14,000 transactions annually and covers sales of new homes and homes for sale. The major difference between this index and the OFHEO HPI or the CMHPI is that this index targets on new homes while the OFHEO HPI and the CMHPI focuses on the samples of "repeat transactions." This index shows 26.6% of a housing price increase from 1995 to 2006.

The fourth index is the S&P/Case-Shiller index. This index is also based on repeated transactions of single family homes but different from the OFHEO HPI and the CMHPI mainly in three aspects. First, the data of the S&P/Case-Shiller index come from county records, while above two index data are based on the conforming loans which were purchased or securitized by Freddie Mac or Fannie Mae. Second, the geographic coverage of the S&P/Case-Shiller index is smaller in that it doesn't use data from 13 states while these two indices cover all states. Third, the S&P/Case-Shiller index uses the value-weighting method, while the two indices use simple equal-weighting. This value-weighting method allows us to adjust for the quality of houses sold. This index indicates 90% increase in housing prices from 1995 to 2006. Since the coverage of this index is small and focuses on metropolitan areas, I will not use this index as a benchmark.

The last index is Consumer Price Index (CPI) for rents of primary residence (rent). This index is based on the data reported by sampled households in the Consumer Expenditure Interview Survey. It shows rents only increased 10.5% from 1995 to 2006.

2.1.2 Homeownership rate

Homeownership rate: Japan Figure 3 shows the homeownership rate from 1968 to 2003. The solid line represents the average homeownership rate from the Housing and Land Survey (hereafter the HLS).[4] It can be seen that the average homeownership rate has moved around 60% throughout the period. From 1983 to 1993, the average homeownership rate decreased from 62.4% to 59.8% and after 1993 it recovered slightly to 61.2% in 2003. The dashed line represents the average homeownership rate of households with two or more members based on the NSFIE.[5] The dashed line indicates that from 1984 to 1994 the average homeownership rate

[4]The Housing and Land Survey is a national survey which is conducted every five years to clarify housing conditions and land holdings of households. See http://www.stat.go.jp/ english/data/jyutaku/1501.htm for detailed descriptions of the survey.

[5]The National Survey of Family Income and Expenditure (the NSFIE) is a survey conducted every five years. The targets of the survey are households who live alone and households with two or more members.

had increased from 74.3% to 75.4% and it kept increasing and reached 80.3% in 2004. These two statistics seem to show different trends of the average homeownership rate.

One needs to consider the effects of the demographic changes on the average homeownership rate to determine the actual trend of the homeownership rate. Japan has experienced two large demographic changes in the last 25 years. First, the population of households who live alone has increased significantly. According to the National Census, the share of one member households in total households was 19.8% in 1980. It started to increase after 1980 and it reached 30% in 2005. The demographic change occurred mostly among young households. A young household who lives alone prefers to live in a rental house than in an ownership house because of convenience of higher mobility or lack of wealth. Consequently, this demographic change decreases the average number. Second, old age population has increased dramatically. Japan is one of the fastest aging society in the world. According to the National Census, the population with age 65 and more consists 10.3% of total population in 1985, but it has increased rapidly since then and reached 20.2% in 2005. Since old people tend to live in ownership houses, this effect increases the average number.

To eliminate these demographic effects, the age distribution of the homeownership rate should be examined. The best way is to control all the demographic effects. However, due to the lack of the micro data, I can only fix the demographic structure of each age group at some point in time. To be more precise, in each age group, the NSFIE has statistics of the homeownership rate of single households and households with two or more members. When computing the average homeownership rate of each age group in each year (from 1984 to 2004), I use the fix composition of single households and households with two or more members in all years, namely, the demographic structure of the National Census (1985) data. Table 1 shows the adjusted homeownership rate by age cohorts from the NSFIE. It shows that the changes in the homeownership rate differ by age cohorts. The homeownership rate of the young decreased from 1984 to 1994 by 2 to 5 percentage points and shows a recovery from 1994 to 2004. On the other hand, the homeownership rate of the old appears to have remained unchanged or even shows a slight increase throughout the period.

Homeownership rate: U.S. Figure 4 shows the evolution of the average homeownership rate in the U.S. since 1965. The data is taken from the U.S. Census Bureau, "Housing Vacancies and Home Ownership." As Chambers et al.(2007) point out, the homeownership rate shows relatively constant trend until 1994, followed by a large increase from 1994 to 2004. The homeownership rate was 64.0% in 1994 and it became 69.0% in 2004.

Next, the homeownership rate by age cohorts is examined to separate off demographic effects. Table 2 shows the homeownership rate by age cohorts in 1994 and in 2004 taken from the

The NSFIE gives detail statistics of these two kinds of households. The NSFIE statistics are provided at http://www.stat.go.jp/english/data/zensho/index.htm. Apart from some exceptional cases, the micro data of the NSFIE is not publicly available. In what follows, all the calculations regarding the NSFIE are done by using published table statistics.

Table 1: Homeownership rate by age of householder (Japan, %)

	Total	under 30	30 to 39	40 to 49	50 to 59	60 ot 69	70 and over
1984	65.6	10.9	51.8	74.5	82.4	84.0	82.8
1989	65.9	10.4	50.5	75.6	82.7	86.2	83.6
1994	64.8	9.5	46.0	74.8	82.5	87.2	84.6
1999	64.8	11.9	46.9	73.9	82.2	84.9	85.3
2004	66.9	11.9	50.7	75.8	83.9	87.3	87.1
Diff. 94-84	-0.8	-1.4	-5.8	0.3	0.1	3.2	1.8
Diff. 04-94	2.1	2.4	4.7	1.0	1.4	0.1	2.5

Statistical Abstract of the United States(http://www.census.gov/compendia/statab/). The homeownership rate appears to have increased in every age cohort. The homeownership rate of the youngest age cohort shows the largest increase among others by 5.8 percentage points from 1994 to 2004.[6]

Table 2: Homeownership rate by age of householder (U.S., %)

	Total	Less than 35	35 to 44	45 to 54	55 to 64	65 and over
1994	64.0	37.3	64.5	75.2	79.3	77.4
2004	69.0	43.1	69.2	77.2	81.9	81.1
Difference	5.0	5.8	4.7	2.0	2.6	3.7

2.2 Economic background

I choose three candidates for explaining the changes in housing prices, rents, and the homeownership rate: interest rates, the total factor productivity (TFP) growth rate, and downpayment requirements.[7] In sum, the decrease in interest rates and the increase in the TFP growth rate can be observed contemporaneously with the housing price increase in both countries. Furthermore, a decline of downpayment rates occurred in U.S. housing markets over the same period as the homeownership rate increase, while opposite was true in Japan in the late 80's.

[6] In the U.S. the share of the households living alone in total households is 25% in 1995 and 26.6% in 2006 according to the Statistical Abstract of the United States. Since this is a relatively small change compared to the demographic change in Japan, I show the homeownership rate distributions without controlling the demographic effect. For the robustness check, I compute the homeownership rate of households with two or more members only, and it shows almost the same changes of the homeownership rate distribution.

[7] Tax and subsidy policy can be another candidate. Tax systems, however, totally differ between two countries. Since finding the non country specific factors is one of the main focus of this study, tax and subsidy policy is beyond the scope of this research.

2.2.1 Interest rates

Interest rates: Japan Figure 5 shows long term real interest rates and short term real interest rates in Japan from 1980 to 2002. The dashed black line represents the baseline long term mortgage interest rates from then Government Housing Loan Corporation (hereafter GHLC). [8]The dotted gray line represents the yields of the 10 year government bond. The bold solid line represents the collateralized overnight call rates. Call rates are the interest rates which are applied to interbank money lending. All the series are taken from Bank of Japan (BOJ).

I use an annual CPI growth rate as a substitute for the inflation expectation rate in Japan, since there is no widely accepted such series in Japan as Michigan inflation expectation series in the U.S. Hori and Terai (2004) attempt to construct inflation expectation series by using the Carlson-Parkin (1975) method. Their estimate of inflation expectation series is almost the same as original CPI growth rate series. Thus I use the ex-post annual CPI change as the inflation expectation rate of the previous year. In other words, households have rational expectations on future inflation. All the interest rates are adjusted by the annual CPI change.

Two large declines of the short term interest rates were observed. First, the short term interest rates declined from 5.0% in 1986 to 1.3% in 1988. This corresponds to the fact that BOJ lowered the official discount rate from 5.0% at the beginning of 1986 to 2.5% in February, 1987 and left it 2.5% for more than 2 years until May, 1989. Second, after the temporary high level in 1991 the short term interest rates decreased again and remained almost zero after 1995. This reflects the so-called Zero Interest Policy adopted by BOJ to deal with the recession. In addition, the long term real interest rates appear to have moved similarly to the short term interest rates but more moderately. The 10 year government bond yield and the baseline mortgage rates of the GHLC declined in the late 80's, followed by the short high period, and decreased again in the middle of the 90's. [9]

[8]GHLC was a so-called special public corporation and it was reorganized to an incorporated administrative agency, Japan Housing Finance Agency (JHF) as of April 1, 2007. Their role in housing markets had changed from loan origination to secondary market operation such as securitization of loans purchased from private banks.

GHLC adopted the interest rate schedule where low fixed interest rates are applied in the first 10 years and higher fixed interest rates are applied from the 11th year on. Moreover, the interest rates in the fist 10 years can be classified into three categories depending on the size and characteristics of houses: the baseline interest rates, the medium size interest rates, the large size interest rates. The reason why I use the GHLC's baseline interest rates as long term mortgage rates is partly because GHLC was the single largest entities of mortgage loan origination, which provided 30-40% of outstanding mortgage loans from the late 1980's to the late 90's. It is also because GHLC was the only entity which kept fixed interest rate mortgages as its main products, while private banks eventually shifted their products to adjusted interest mortgage loans, which makes it difficult to track the trend of mortgage rates.

[9]It appears that at the beginning of 1980's, the baseline mortgage rates of GHLC were much lower than the short term interest rates and government bond yields. This is due to the regulation which put the ceiling of (nominal) baseline interest rates of GHLC to 5.5%.

Interest rates: U.S. Figure 6 shows the short term real interest rates and the long term real interest rates in the U.S. The dashed line represents the interest rates of Three-Month U.S. Treasury bill taken from Federal Reserve Bank (FRB). The bold solid line represents the interest rates of conventional 30 year Fixed Rate Mortgages of Freddie Mac. The short term interest rates and the long term interest rates are adjusted by one year ahead inflation expectations and five year ahead inflation expectations respectively. The inflation expectation data are taken from Survey Research Center, University of Michigan.

The downward trend can be observed in both interest rates since 1985. Moreover, after 2001 there existed a large decrease both in the long term interest rates and in the short term interest rates. The interest rates of conventional 30 years mortgages decreased from 4.56% in 2000 to 2.46% in 2005 in real terms. The drop in the short term interest rates is more dramatic. The real interest rates of Three-Month T-Bill declined from 2.82% in 2000 to -1.63% in 2004 as a result of the monetary policy of FRB where the (nominal) federal funds target rate was set as low as 1% in 2003 and 2004.

2.2.2 TFP growth rate

Figure 7 and Figure 8 show the TFP growth rate of the non housing sector in Japan and in the U.S. respectively. Output of the non housing sector is defined as the output from all private industry excluding the output from the residential construction sector and the real estate sector. Capital and labor of the non housing sector are defined similarly. I exclude residential housing related sectors so that the effects of housing price changes on the TFP estimates are eliminated. It is senseless to explain housing price changes by the economic growth if the only driver of the economic growth was the housing price increase. Indeed, the TFP growth rate of the housing sector was high at that time.[10] Calculation of TFP follows Cooley and Prescott(1995).

TFP growth rate: Japan Figure 7 shows that the average non housing sector TFP growth was relatively low from 1974 to 1983. After 1983, however, the TFP grew faster continuously until 1990, followed by a low growth period again. The average non housing sector TFP growth was 1.41% from 1974 to 1990, 2.59% from 1983 to 1991, and 0.365% from 1991 to 1998. This result is consistent with Hayashi and Prescott (2002) who show the high TFP growth period (1983-1991) comes between the low TFP growth periods. They calculate the TFP of the total economy including the housing sector and obtain 0.8% as the TFP growth rate in the 1973-1983 period, 3.7% in the 1983-1991 period and 0.5% in the 1991-2000 period.

TFP growth rate: U.S. Figure 8 shows the non housing sector TFP growth was 1.23% from 1974 to 1995 and 2.02% from 1995 to 2006 in the U.S. This is consistent with what Jorgenson

[10] Admitting that it is impossible to eliminate all the effects of housing price changes on the TFP estimates due to the positive externality of the housing sector on other sectors, it is still better to take the non housing sector growth as a substitution of the total economic growth at that period by the above reasons.

et al.(2004) find. They find that the TFP growth rate in the 1973-1995 period is 0.34 while the TFP growth rate in the 1995-2003 period is 1.14.

2.2.3 Downpayment rate

Downpayment rate: Japan Financial innovations in housing markets have a short history in Japan. Onozawa (2003) documents that private banks started the securitization of mortgages in Japan for the first time in 1999 with fifty billion yens (approximately five hundred million dollars). After that GHLC stared its securitization in 2001 and since then the volume of securitized mortgages grew steadily. According to Ministry of Land, Infrastructure, Transport and Tourism, the amount of the real estate which were securitized in 2006 (fiscal year) is 7.8 trillion yens (approximately 78 billion dollars). It is still small compared to the U.S. Thus it is not surprising that the downpayment rate of the purchasers of newly constructed houses didn't increase in the late 80's in Japan. Seko (1995) documents that the average self-financing ratio increased from 38.6% in 1984 to 44.2% in 1989.

Downpayment rate: U.S. From the late 90's, on the advent of financial innovations, many new mortgage contracts were introduced to the housing markets. One of the most popular products was the loans with little or no downpayment requirement. Due to this new product, households who are otherwise constrained by downpayment requirements can purchase ownership houses. Table 4 shows the downpayment rate of first time buyers in the U.S. from 1995 to 2003 taken from Chambers et al.(2007). It can be observed that the downpayment rate of first time buyers decreased from 1995 to 2003 in both types of loans.

Table 3: U.S.: Downpayment rate of First-Time Buyers (U.S.)

	FHA Loan[11]	Other Loans
1995	21.6%	29.8%
1999	13.8%	22.1%
2001	18.1%	24.5%
2003	16.3%	24.1%

3 Model

I use a dynamic overlapping generations general equilibrium model. Each household has a finite horizon and faces uninsurable idiosyncratic income uncertainty and mortality risk. The

[11]FHA loans are the mortgage loans which are assisted by Federal Housing Administration(FHA). The FHA loans are established to help low income households to purchase their homes by providing insurance to lenders.

distinctive feature of the model is that households choose not only non housing consumption, but also the housing tenure (rent a house or own a house) and the size of houses. In addition to purchasing housing assets and consuming non housing consumption goods, households can save their money as riskless capital with the rate of return r or they can borrow money up to a certain percentage of their housing assets, if any, with the same rate r.

There are four production sectors in the economy. The first sector is a non housing consumption production sector, the second sector is a residential structure production sector, the third sector is a housing production sector and the last sector is a real estate (rental housing) sector. In this model the only role of the government is to collect income taxes and distribute them as social security benefits.

3.1 Technology

I assume there are four production sectors in the economy: the non housing consumption production sector, the residential structure production sector, the housing production sector, and the real estate sector.

- **Non housing consumption production sector:** A representative non housing consumption production firm uses labor N_c and capital K_c as inputs and produces non housing consumption goods Y_c. It is assumed to have a Cobb-Douglas form of production function. Thus, its static problem is,

$$\max_{K_{c,t},N_{c,t}} Y_{c,t} - (\delta_k + r_t)K_{c,t} - w_t N_{c,t} \tag{1}$$

$$\text{s.t. } Y_{c,t} = z_{c,t} K_{c,t}^{\alpha_c} N_{c,t}^{1-\alpha_c},\ 0 < \alpha_c < 1 \tag{2}$$

  where δ_k is the capital depreciation rate, w_t is wage, r_t is the rate of return on capital, and $z_{c,t}$ is the TFP of the non housing production sector at period t. Here the price of non housing consumption goods is normalized to one

- **Residential structure production sector:** A representative residential structure production firm also has a constant return to scale production technology and produces residential structure goods Y_s from capital K_s and labor N_s.

$$\max_{K_{s,t},N_{s,t}} p_{s,t} Y_{s,t} - (\delta_k + r_t)K_{s,t} - w_t N_{s,t} \tag{3}$$

$$\text{s.t. } Y_{s,t} = z_{s,t} K_{s,t}^{\alpha_s} N_{s,t}^{1-\alpha_s},\ 0 < \alpha_s < 1 \tag{4}$$

  where $p_{s,t}$ is the price of the residential structure goods and $z_{s,t}$ is the TFP of the residential structure production sector at period t .

- **Housing production sector:** A representative housing production firm combines the residential structure goods X_s from the residential structure production firm and land

L_h to build new houses Y_h. Houses are available in the markets m period after the construction starts. This incorporates the fact that it takes time to build houses.

$$\max_{X_{s,t},L_{s,t}} p_{h,t+m}Y_{h,t+m} - p_{s,t}X_{s,t} - p_{l,t}L_{h,t} \tag{5}$$

$$\text{s.t. } Y_{h,t+m} = X_{s,t}^{1-\alpha_h} L_{h,t}^{\alpha_h}, \; 0 < \alpha_h < 1 \tag{6}$$

where $p_{h,t}, p_{l,t}$ is the price of houses and the price of land at period t respectively.

- **Real estate (rental housing) sector:** A representative real estate (rental housing) firm purchases houses and rents them out to households. The firm has a stock of rental houses from the previous period $H_{r,t-1}$ and the firm adjusts its stock in housing markets by paying $p_{h,t}(H_{r,t-1} - H_{r,t})$ at the beginning of this period. $H_{r,t}$ is the amount of stock which the firm has after the transaction at period t. There assumed to be no cost associated with the transaction. It is also assumed that rental houses and ownership houses are freely interchangeable. Moreover, the housing stock purchased this period is assumed be available for renting at the same period. Thus, operation surplus of the firm at period t is rents $p_{r,t}H_{r,t}$ minus maintenance cost of rental houses $MC_r(H_{r,t})$. A rental housing sector's dynamic problem can be written as,

$$\Gamma(H_{r,t-1}) = \max_{H_{r,t}} p_{h,t}(H_{r,t-1} - H_{r,t}) + p_{r,t}H_{r,t} - MC_r(H_{r,t}) + \frac{1}{1+r_{t+1}}\Gamma(H_{r,t}) \tag{7}$$

  The maintenance cost of rental houses is defined as follows. Let δ_r be the depreciation rate of rental houses. Then the depreciation allowance of rental houses is $\delta_r p_{h,t} H_{r,t}$. However, since land does not depreciate, the firm can sell the land part of its housing stock to the housing production firm. At equilibrium the supply of land should be equal to the demand of land from the housing production firm, therefore total supply is $p_{l_h,t}L_{h,t}$. In this model all houses have the same proportion of land areas regardless of ownership. Thus the share of land sold by the rental sector in total land supply at period t is the same as its share of housing stock $H_{r,t}$ in total housing stock H_t. Therefore the maintenance cost of rental housing $MC_{r,t}(H_{r,t})$ is,

$$MC_{r,t}(H_{r,t}) = \delta_r p_{h,t} H_{r,t} - \frac{H_{r,t}}{H_t} p_{l_h,t} L_{h,t} \tag{8}$$

3.2 Household problem

3.2.1 Demographics

Every period, measure x_1 of households is born. Each household lives at most I periods. All households of age i face possibility of death with probability $1 - \psi_i \in [0, 1]$ from age i to age $i + 1$. Thus the measure of the households who are alive at the beginning of period i is

$$x_i = \psi_{i-1}x_{i-1} = \sum_{j=1}^{i-1} \psi_j x_1 \tag{9}$$

For convenience, the sum of all households is normalized to one.

3.2.2 Annuity market

There exists complete annuity market and households can perfectly insure against mortality risk. Specifically, the household with age i holding asset a_i at the end of the period will receive a_i/ψ_i at the beginning of the next period if he survives. Since no one survives at the last period, without any assumption all households consume everything. In order to avoid this end effect, households are assumed to have bequest motive at the last period and gain utility from saving assets.

3.2.3 Preference

Household preferences are represented by function $u(c,h)$, where c is non housing consumption and h is housing service consumption. The utility function is assumed to be strictly increasing, strictly concave, and satisfy Inada conditions in both arguments. In addition, bequest preferences are represented by function $bf(a_t)$, where a_t is the amount of asset which the heirs of households receive at period t.

Households who are born at period T maximize their expected discounted utility,

$$E_{T-1}\left(\sum_{t=T}^{T+I-1}\beta^{t-T}u(c_t,h_t)+\beta^{I}bf(a_{T+I})\right) \tag{10}$$

where β is the common discount factor.

3.2.4 Income process

Households receive labor income until they retire at age I_{ret} and after the retirement they receive social security benefits as income. The labor income can be divided into three components: an average income per efficiency unit w_t, a time invariant lifecycle profile of income v_i and a labor income shock ϵ_t. The labor income shock is assumed to have two components: a persistent component η_t and a transitory component ε_t. The persistent component follows a first-order Markov process and its persistence parameter is ρ_η and its disturbance ζ_t follows a mean zero normal distribution with variance σ_ζ^2. The transitory component also follows a mean zero normal distribution with variance σ_ε^2.

After the retirement households receive benefits from social security. Regardless of their working age income shock realizations, they receive the same amount of social security benefits proportional to w_t. Let θ be the replacement rate which is defined as social security benefits divided by the average income of working households and let $m(\epsilon_t,t,i)$ be the measure of households with age i whose income shock is ϵ_t at period t. In addition, let $\tau_{l,t}$ be the labor income tax at period t, then income process of household at age i with income shock ϵ_t $y(i,\epsilon_t)$ is given by, [12]

$$y(i,\epsilon_t) \;=\; \begin{cases} (1-\tau_{l,t})w_t\epsilon_t v_i & \text{if } i\le I_{ret} \\ \theta\frac{\int_{i\le I_{ret}} w_t\epsilon_t v_i\, dm(\epsilon_t,t,i)}{\int_{i\le I_{ret}} dm(\epsilon_t,t,i)} & \text{if } i> I_{ret} \end{cases} \tag{11}$$

$$\log(\epsilon_t) = \eta_t + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2) \tag{12}$$

$$\eta_t = \rho_\eta \eta_{t-1} + \zeta_t \qquad \zeta_t \sim \mathcal{N}(0, \sigma_\zeta^2) \tag{13}$$

3.2.5 Government

The only role of the government in this model is to finance the social security benefits from the labor income taxes. The government chooses the level of the labor income tax $\tau_{l,t}$ such that a following equation holds.

$$\int_{i\le I_{ret}} \tau_{l,t} w_t \epsilon_t v_i \; dm(\epsilon_t, t, i) = \int_{i>I_{ret}} \theta \frac{\int_{i\le I_{ret}} w_t \epsilon_t v_i \; dm(\epsilon_t, t, i)}{\int_{i\le I_{ret}} dm(\epsilon_t, t, i)} dm(\epsilon_t, t, i) \tag{14}$$

$$\Leftrightarrow \tau_{l,t} = \theta \frac{\int_{i>I_{ret}} dm(\epsilon_t, t, i)}{\int_{i\le I_{ret}} dm(\epsilon_t, t, i)}$$

$$= \theta \frac{\text{mass of retired households}}{\text{mass of working households}} \tag{15}$$

It can be seen that $\tau_{l.t}$ only depends on demographic structure of the economy, not on the specification of the income process or the level of the average income.

3.2.6 Housing characteristics

In this paper housing assets are modelled as investment goods as well as consumption goods. Housing assets have the following features.

- **Housing consumption:** Housing assets are both investment goods and consumption goods. The houses of size h generate a flow of housing services every period which is denoted by $g(h)$. I assume that the amount of housing services that households can extract are proportional to housing assets that households have. Without loss of generality, coefficient is set to one. Thus, a size h house generates h amount of housing services every period.

- **Housing tenure choice and house size:** The first decision of each household is to choose whether to live in a rental house or in an ownership house. In other words, each household needs to choose to rent or buy his house. I denote the decision at period t by $b_t \in \{r, o\}$. The second decision is to decide the size of his house $h_t \in \mathcal{H}$ where $\mathcal{H} = \{h_{min}, \ldots, h_{max}\}$. Since a house is a lumpy and indivisible object, I assume there exists a minimum house size h_{min}. The minimum size is the same for both ownership houses and rental houses. This might prevent some poor households from purchasing their houses. h_{max} is set only for computational purposes.

[12] It should be noted that the average income per efficiency unit w_t is different from the average income of all working households. The actual before tax average income of all working households is $\frac{\int_{i\le I_{ret}} w_t \epsilon_t v_i \; dm(\epsilon_t,t,i)}{\int_{i\le I_{ret}} dm(\epsilon_t,t,i)}$.

- **Housing transaction cost:** A household who lives in an ownership house faces an exogenous event which requires him to move out from his house. This event is assumed to happen with probability $\pi(\xi)$ where $\xi \in \Xi = \{move, notmove\}$. When he is required to move, the household has to pay transaction cost $AC(h_{t-1})$ which is assumed to be a function of his house size at the previous period. The transaction cost consists of two components: the cost which is proportional to the housing asset value $\phi_s p_h h$ and the fixed cost which is proportional to average income per efficiency unit $\phi_f w$. The former cost can be interpreted as fees for real estate agents and the latter cost can be interpreted as moving cost or cost for lawyers for the contract. Thus the transaction cost can be written as,

$$AC(h_{t-1}) = \phi_s p_{h,t} h_{t-1} + \phi_f w_t \tag{16}$$

- **Housing maintenance:** When a household owns his house, he has to pay maintenance cost $MC_o(b_t, h_t)$ each period in order to maintain the quality of the house. A house consists of structure and land and structure does depreciate but land doesn't. Thus the household can sell his land part of the house where depreciated structure is located. Therefore, as we have seen at the rental housing production firm's problem, the maintenance cost for owning a size h_t house can be written as,

$$MC_o(h_t) = \delta_o p_{h,t} h_t - \frac{h_t}{H_t} p_{l,t} L_{h,t} \tag{17}$$

  where δ_o is the depreciation rate of ownership houses which could be different from the depreciation rate of rental houses δ_r.

  As in Chambers et al.(2007), a landlord of rental houses faces a possible asymmetric information problem where the landlord cannot monitor the action of his tenants. Suppose there exits two effort levels $\{e_h, e_l\}$ which tenants can put to maintain the quality of the houses. The tenants know their effort level cannot be monitored, thus without any punishment they always put lower level of effort on keeping the house quality. Due to this moral hazard problem the landlord has to pay more as maintenance cost. Thus, it is assumed that $\delta_r > \delta_o$.

  To avoid computational complications, define the depreciation rate of total housing stock $\delta_h = \delta_o$. Thus an extra depreciation rate for rental houses $\delta_r - \delta_o$ can be interpreted as the additional non housing cost the landlord has to pay to maintain the quality of houses.

- **Housing finance:** Housing assets can be used as a collateral for financing. A household is allowed to borrow up to $(1-\lambda)p_h h_t$ where $\lambda \in [0, 1]$ when he owns a house with a size h_t. λ can be considered as downpayment requirements which every household needs to satisfy for his housing purchase. On the other hand, a household is not allowed to borrow when he chooses to live in a rental house.

3.2.7 Household problem

The Household's lifetime utility maximization problem can be written as the following form of the dynamic problem. Each household faces a binary housing tenure choice every period, a choice between renting a house or owning a house. Each household maximizes his utility by choosing the best housing tenure choice. Let $v_t(i, a_t, \epsilon_t)$ be expected discounted utility of the household with age i, asset a_t, and a labor income shock ϵ_t at period t. Let $V_t(\cdot)$, $W_t(\cdot)$ be the expected discounted utility of the household at period t who chooses to rent a house and who chooses to own a house, respectively. The household's decision problem is,

$$v_t(i, a_t, \epsilon_t) = \max_{b_t \in \{r,o\}} \{V_t(i, a_t, \epsilon_t), W_t(i, a_t, \epsilon_t)\} \tag{18}$$

A household faces different problems depending on his housing tenure choice. Each household with age i, asset a_t, and a labor income shock ϵ_t at period t solves his problem, given a set of prices $r_t, w_t, p_{r,t}, p_{h,t}, , p_{l,t}$ and exogenous shock $\psi_i, \epsilon_t, \xi_t$.

Renter today ($b_t = r$) A households who chooses to rent a house has asset a_t/ψ_{i-1} at the beginning of the period and receives labor income $y(i, \epsilon_t)$ according to his age i and his labor income shock ϵ_t. Then he allocates his asset and income into consumption c_t, savings s_t and rent $p_{r,t}h_t$ subject to his budget constraint and the other constraints for renters. The dynamic problem of renters can be written as,

$$\begin{aligned} V_t(i, a_t, \epsilon_t) \quad &= \max_{(c_t, s_{t+1}, h_t)} u(c_t, h_t) + \beta\psi_i \sum_{\epsilon_{t+1} \in \mathcal{E}} \pi(\epsilon_t, \epsilon_{t+1}) v_{t+1}(i+1, a_{t+1}, \epsilon_{t+1}) &(19)\\ &\qquad + \mathcal{I}_{i=I}\beta b f(a_{t+1}) \\ \text{s.t.} \quad & c_t + s_{t+1} + p_{r,t}h_t = a_t/\psi_{i-1} + y(i, \epsilon_t) &(20)\\ & c_t \geq 0 \qquad h_t \geq h_{min} \qquad s_{t+1} \geq 0 &(21)\\ & a_{t+1} = (1 + r_{t+1})s_{t+1} &(22) \end{aligned}$$

where $\mathcal{I}_{i=I}$ is a indicator function which takes value one if $i = I$ and takes value zero if $i \neq I$.

Homeowner today ($b_t = o$) Similarly, a household who chooses to own a house has asset a_t/ψ_{i-1} and labor income $y(i, \epsilon_t)$. Then he allocates his asset and income into consumption c_t, savings s_t, and housing asset $p_{h,t}h_t$ subject to the budget constraint and the other constraints for homeowners. The dynamic problem of homeowners can be written as,

$$\begin{aligned} W_t(i, a_t, \epsilon_t) = \max_{(c_t, s_{t+1}, h_t)} \; & u(c_t, h_t) \\ & + \beta\psi_i \sum_{\epsilon_{t+1}\in\mathcal{E}, \xi_{t+1}\in\Xi} \pi(\epsilon_t, \epsilon_{t+1})\pi(\xi_{t+1}) v_{t+1}(i+1, a_{t+1}(\xi_{t+1}), \epsilon_{t+1}) \\ & + \mathcal{I}_{i=I}\beta b f(a_{t+1}) \end{aligned} \tag{23}$$

s.t.

$$c_t + s_{t+1} + p_{h,t}h_t + MCo(h_t) = a_t/\psi_{i-1} + y(i, \epsilon_t) \tag{24}$$

$$c_t \geq 0 \qquad h_t \geq h_{min} \qquad s_{t+1} \geq -(1-\lambda)p_{h,t}h_t \tag{25}$$

$$a_{t+1}(\xi_{t+1}) = (1 + r_{t+1})s_{t+1} + p_{h,t+1}h_t - \mathcal{I}_{\xi_{t+1}=move}AC(h_t) \tag{26}$$

where $\mathcal{I}_{\xi_{t+1}=move}$ is a indicator function which is equal to one if $\xi_{t+1} = move$ and equal to zero if $\xi_{t+1} \neq move$.

3.3 Definition of equilibrium

Let $g_{b,t}(i, a_t, \epsilon_t)$, $g_{h,t}(i, a_t, \epsilon_t)$, $g_{s,t}(i, a_t, \epsilon_t)$, and $g_{c,t}(i, a_t, \epsilon_t)$ be policy fuctions of a household with age i, asset a_t, and a labor income shock ϵ_t at period t regarding a housing tenure choice, a housing size choice, a savings choice, and a non housing consumption choice respectively. Then an equilibrium is defined as follows.

Definition 1 An equilibrium is a set of prices $\{r_t, w_t, p_{r,t}, p_{s,t}, p_{h,t}, p_{l,t}\}$ for all possible states and for all $t \geq 0$ such that given the set of prices households solve their optimization problems with decision rules $\{g_{b,t}(i, a_t, \epsilon_t), g_{h,t}(i, a_t, \epsilon_t), g_{s,t}(i, a_t, \epsilon_t), g_{c,t}(i, a_t, \epsilon_t)\}$, and firms maximize their profits with aggregates $\{K_{s,t}, K_{c,t}, N_{s,t}, N_{c,t}, X_{s,t}, L_{h,t}, H_{r,t}\}$ and all markets clear.

Let Ω_t be the set of state variables (i, a_t, ϵ_t) at period t and $\Phi(\Omega_t)$ be the measure of households for each possible state at period t. Market clearing conditions imply that
for all t the following conditions are met.

- Labor markets clear (labor $\bar{N}_t$ is inelastically supplied)

$$N_{c,t} + N_{s,t} = \int \epsilon_t v_i \, d\Phi(\Omega_t) = \bar{N}_t \tag{27}$$

- Land markets clear (land $\bar{L}_t$ is inelastically supplied)

$$L_{h,t} = \bar{L}_t \tag{28}$$

- Residential structure markets clear

$$X_{s,t} = Y_{s,t} \tag{29}$$

- Housing markets clear

$$
\begin{aligned}
Y_{h,t} &= \delta_h H_t + (H_t - H_{t-1}) && (30)\\
\Leftrightarrow Y_{h,t} &= \delta_h \int g_{h,t}(i, a_t, \epsilon_t)\; d\Phi(\Omega_t)\\
&\quad + \left(\int g_{h,t}(i, a_t, \epsilon_t)\; d\Phi(\Omega_t) - \int g_{h,t-1}(i, a_t, \epsilon_t)\; d\Phi(\Omega_{t-1}) \right) && (31)
\end{aligned}
$$

- Rental markets clear

$$
H_{r,t} = \int_{g_{b,t}(i,a_t,\epsilon_t)=r} g_{h,t}(i, a_t, \epsilon_t)\; d\Phi(\Omega_t) \tag{32}
$$

- Financial asset markets clear (households hold the share of the real estate firm)

$$
\int g_{s,t}(i, a_t, \epsilon_t)\; d\Phi(\Omega_t) = K_{c,t+1} + K_{s,t+1} + \Gamma(H_{r,t}) \tag{33}
$$

- Non housing consumption goods markets clear (by Walras law)

$$
\begin{aligned}
&\int g_{c,t}(i, a_t, \epsilon_t)\; d\Phi(\Omega_t) + K_{c,t+1} + K_{s,t+1} + \int AC(h_{t-1})\; d\Phi(\Omega_t)\\
&\quad + (\delta_r - \delta_h) p_{h,t} H_{r,t} = z_{c,t} K_{c,t}^{\alpha_c} N_{c,t}^{1-\alpha_c} + (1-\delta_k)(K_{c,t} + K_{s,t}) \qquad (34)
\end{aligned}
$$

3.4 Price characterizations

Now we can characterize equilibrium prices. Note that there is no aggregate shock in this economy, thus all prices are known to all households. In order to gain some intuitions as to how parameters affect housing prices, it is useful to solve $p_{h,t}, w_t$ for $Y_{h,t}, r_t$. Other variables are characterized in Appendix A. With some algebras we have,

$$
\begin{aligned}
\log p_{h,t+m} &= \frac{\alpha_h}{1-\alpha_h} \log Y_{h,t+m} + \left(1 - \frac{1-\alpha_s}{1-\alpha_c}\right) \log(\delta_k + r_t) - \log z_{s,t} + \frac{1-\alpha_s}{1-\alpha_c} \log z_{c,t}\\
&\quad + O(\alpha_s, \alpha_c, \alpha_h) && (35)\\
\log w_t &= -\frac{1}{1-\alpha_c} \log(\delta_k + r_t) + \frac{1}{1-\alpha_c} \log z_{c,t}\\
&\quad + \log\left(\frac{1-\alpha_c}{\alpha_c} \alpha_c^{\frac{1}{1-\alpha_c}} \right) && (36)
\end{aligned}
$$

where $O(\alpha_s, \alpha_c, \alpha_h)$ is a constant term which is a function of $\alpha_s, \alpha_c, \alpha_h$. For simplicity, open economy is considered where $r_t = R_t$ where R_t is the world interest rates. The forth term of the first equation tells us that the TFP of the non housing consumption sector z_c has a direct positive effect on housing prices. Also it has an indirect positive effect through the first term because the second equation tells us that z_c has a positive effect on the average wage which leads to have larger demand for houses $Y_{h,t}$ ceteris paribus. This implies the faster productivity growth of the non housing consumption sector induces the faster housing price growth. Furthermore, the larger the capital share of the non housing consumption production sector is, the larger the positive effect on housing prices is. Also the smaller the capital share of the residential structure production is, the smaller the positive effect on housing prices is.

On the other hand the TFP of the residential structure production sector z_s has only a direct negative effect on housing prices and it doesn't affect the average wage. The effect of the reduction in the world interest rates is difficult to examine. I can only say it has a positive effect on housing prices through the average wages, but a direct effect depends on parameter values.

3.5 Technology

3.5.1 Utility function

In order to have a balanced growth path, the utility function is assumed to be of Cobb-Douglas form.

$$u(c,h) = \frac{\left(c^{\theta_u} h^{1-\theta_u}\right)^{1-\gamma_u}}{1-\gamma_u} \tag{37}$$

where θ_u is a share of consumption, γ_u is a coefficient of risk aversion.

3.5.2 Bequest function

In order to have a balanced growth path, the bequest function needs to have the same form as the utility function. Heirs are assumed to gain utility by consuming all the assets given by households at the last period. Then, the problem of heirs can be written as,

$$bf(a) = \max_{c,h} \bar{\chi} \frac{\left(c^{\theta_u} h^{1-\theta_u}\right)^{1-\gamma_u}}{1-\gamma_u} \tag{38}$$

$$\text{s.t. } c + p_r h = a \tag{39}$$

where $\bar{\chi}$ controls an incentive of bequest. This problem can be solved analytically and solutions are $c = \theta_u a, h = (1-\theta_u)a/p_r$. Thus bequest function at time t can be written as,

$$bf(a_t) = \chi \frac{\left(a_t (1/p_{r,t})^{1-\theta_u}\right)^{1-\gamma_u}}{1-\gamma_u} \tag{40}$$

$$\text{where } \chi = \bar{\chi}(\theta_u^{\theta_u}(1-\theta_u)^{1-\theta_u})^{1-\gamma_u} \tag{41}$$

3.6 Balanced growth path

Before calibrating the model, it is useful to define a balanced growth path of this economy. Since all the production functions and preferences are of Cobb-Douglas form, there exists a balanced growth path. The growth rates of different variables are documented in Table 4.

The growth rate of housing prices is given by

$$\begin{aligned}\log g_{ph} &= \log g_k - \log g_h \\ &= \frac{1-\alpha_s(1-\alpha_h)}{1-\alpha_c}\log g_{z_c} + \alpha_h \log g_n - (1-\alpha_h)\log g_{z_s} - \alpha_h \log g_l \\ &= \frac{1-\alpha_s(1-\alpha_h)}{1-\alpha_c}\log g_{z_c} - (1-\alpha_h)\log g_{z_s} - \alpha_h \log(g_l/g_n) \qquad (42)\end{aligned}$$

The way to read this equation is the following. The first term shows that the TFP growth of the non housing production sector has a positive effect on the housing price growth because it stimulates the economy and the housing constructions. The second term tells us the TFP growth of the residential structure production sector has a negative effect on the housing price growth since it reduces cost of residential investments. The last term means the growth rate of land per head g_l/g_n contributes to the housing price growth negatively because land becomes less scarce.

Table 4: Growth rates on balance growth path

Variables (exogenously determined growth rate):		
Labor	$N_c, N_s, \bar{N}$	g_n
Land	$L_h, \bar{L}_h$	g_l
TFP(non housing consumption production sector)	z_c	g_{z_c}
TFP(residential structure production sector)	z_s	g_{z_s}
Variables (endougenously determined) :		
Output/Capital	Y_c, K_c, K_s	$g_k = g_{z_c}^{\frac{1}{1-\alpha_c}} g_n$
Residential structures	Y_s	$g_s = g_{z_s} g_k^{\alpha_s} g_n^{1-\alpha_s}$
Price of structures	p_s	$g_{ps} = g_k/g_s$
Housing flow/stock	Y_h, H, H_r, h_{min}	$g_h = {g_s}^{1-\alpha_h} {g_l}^{\alpha_h}$
Housing price	p_h	$g_{ph} = g_k/g_h$
Land price	p_l	$g_{pl} = g_k/g_l$
Wage	w	$g_w = g_k/g_n$
Interest rate	r	1

I can rewrite the household problem by detrending growth. Define $\hat{g_x} = g_x/g_n$ and make variable transformation $\hat{x}_t = x_t/\hat{g_x}^t$ by detrending all the variables of its growth. Also balanced growth path equilibrium where transformed variables are time invariant can be defined as follows. Detrended household's problems and firm's problems are documented in Appendix B.

Definition 2 ***A balanced growth path equilibrium*** is a set of prices $\{r, \hat{w}, \hat{p}_r, \hat{p}_s, \hat{p}_h, \hat{p}_l\}$ such that given the set of prices households solve their optimization problems with stationary policy functions $\{\hat{g}_b(i,a,\epsilon), \hat{g}_h(i,a,\epsilon), \hat{g}_s(i,a,\epsilon), \hat{g}_c(i,a,\epsilon)\}$, and firms maximize their profits with stationary aggregates $\{\hat{K}_c, \hat{K}_s, \hat{N}_c, \hat{N}_s, \hat{L}_h, \hat{X}_s, \hat{H}_r\}$ and the following conditions are met.

- (detrended) Labor supply is fixed

$$\hat{N}_c + \hat{N}_s = 1 \tag{43}$$

- (detrended) Land supply is fixed

$$\hat{L}_h = 1 \tag{44}$$

- Residential structure markets clear

$$\hat{X}_s = \hat{Y}_s \tag{45}$$

- Housing markets clear

$$\hat{Y}_h = (1 + \delta_h - \frac{1}{g_h})\hat{H}_t = (1 + \delta_h - \frac{1}{g_h}) \int \hat{g}_h(i, a, \epsilon)\, d\Phi(\Omega) \tag{46}$$

- Rental market clears

$$\hat{H}_r = \int_{g_b(i,a,\epsilon)=r} \hat{g}_h(i, a, \epsilon)\, d\Phi(\Omega) \tag{47}$$

- Financial asset markets clear (households hold the share of the real estate firm)

$$\int \hat{g}_s(i, a, \epsilon)\, d\Phi(\Omega) = \hat{K}_c + \hat{K}_s + \Gamma(\hat{H}_r) \tag{48}$$

- Ω is stationary distribution

$$\Omega = \Pi(\Omega) \tag{49}$$

  where Π is the update operator of the distribution.

4 Parameterization of the model

I assume that the changes of the TFP growth rate, interest rates, and downpayment requirements occurred after 1983 in Japan and after 1995 in the U.S. I further assume that the economy had been in the balanced growth path in both countries until the changes happened. I calibrate the parameter values so that some economic features at that time are replicated.[13]

[13] The Japan's data which don't have any source specification are those taken from the National Accounts of Japan (the SNA). Likewise, non source specified U.S. data are taken from the National Income and Product Account (the NIPA). All rates are annual.

4.1 Calibration

Measure of Output/Capital/Labor: First, I construct a measure of the output. The measure of the total output $Y_t = Y_{c,t} + p_{s,t}Y_{s,t}$ is simply GDP minus housing services.[14] I define total labor input $N_t = N_{c,t} + N_{s,t}$ as hours worked by employee in private sectors. I define total capital input $K_t = K_{c,t}+K_{s,t}$ as the stock of non-residential private fixed asset plus the stock of consumer durables and the stock of private inventories. Lastly, GDP is defined consistently with the NIPA. Since cost of raw land is not considered the part of GDP in the NIPA, I need to include the value of the residential structure instead of the value of the housing stock. Thus, GDP is the sum of non housing output, residential structure output and housing services, which is given by,

$$GDP_t = Y_{c,t} + p_{s,t}Y_{s,t} + p_{r,t}H_t \tag{50}$$

Share of capital: In this model I have two sectors which produce output from labor and capital: the non housing consumption production sector and the residential structure production sector. I need to divide total output, total labor, total capital and associated data into two sectors in order to calculate the share of capital in each sector. The detailed calibration strategy is described in Appendix C. Following Cooley and Prescott (1995), In Japan, I find the capital share of the residential structure production sector α_s is 0.292 and that of the non housing consumption production sector α_c is 0.332 by taking the sample average from 1955 to 1998. Similarly, in the U.S., I find the capital share of the residential structure production sector is 0.211 and that of the non housing consumption production sector is 0.291 by taking sample average from 1948 to 1997.

Depreciation rate: The depreciation rate of capital is computed as consumption of fixed capital divided by the total capital stock. In Japan, it is found that the private non residential capital depreciation rate δ_k is 0.123 and the private residential capital depreciation rate δ_h is 0.0884 by taking the sample average from 1956 to 1998. The SNA classifies consumption of fixed capital not by its purpose but by its owner. Thus I assume all the capital owned by the households sector and the private non-profit institutions serving households is residential capital and estimate the residential capital depreciation rate as the depreciation rate of the capital owned by the households sector and the non-profit institutions. Likewise, in the U.S., the private non residential capital depreciation rate δ_k is estimated as 0.1030 and the private residential capital depreciation rate δ_h is estimated as 0.0159 by taking sample average from 1948 to 2007. The estimate of capital in Japan is obtained from the SNA. The estimate of capital in the U.S.is obtained from the NIPA Fixed asset table.

[14] I add imputed durables services to GDP. Imputation follows Cooley and Prescott (1995). I don't impute services from government capital since this model doesn't have a government sector. All the government purchase is expensed and included in consumption. For consistency I exclude fixed capital consumption of government capital from GDP.

Land share estimate: Since the housing production sector has the Cobb-Douglas production function, the land share of housing production function is equal to the share of land value in total housing value. Using the sample average from 1975 to 1978, I set the land share to be 0.450 in Japan.[15]The data are taken from the Annual Economic Report (1980) published by then Economic Planning Agency in Japan (now Cabinet Office). For the land share of the U.S. housing production sector, I use the same value 0.106 as Davis and Heathcote (2005) due to the lack of alternative.

Exogenous growth rate: I need to determine three kinds of exogenous growth rates for the balanced growth path: the population growth rate, the land growth rate, and the TFP growth rate.

- **Population growth** The population growth rate in Japan is set to 0.657% which is the average total labor input growth rate from 1955 to 2002. The population growth rate in U.S. is set to 1.64% which is the average total labor input growth rate from 1950 to 2006.

- **Land growth** I set the land growth rate in Japan to 2.26% which is the average growth rate of the area of land for building from 1970 to 1998. The data are taken from "Private Land Area by Land Category," the Historical Statistics of Japan(http://www.stat.go.jp/english/data/ch Similarly, I set the land growth rate in the U.S. to 2.79% which is the average growth rate of land in urban areas from 1974 to 1997 from Economics Research Service Data at United States Department of Agriculture (USDA).

- **TFP growth** By following Cooley and Prescott(1995), the TFP growth rates of each sector are calculated. In Japan, the TFP growth rate of the residential structure production sector is set to -0.365% by taking the sample average from 1970 to 1998, while that of the non housing consumption production sector is set to 1.61% by taking the sample average from 1970 to 1983. Likewise, in the U.S. the TFP growth rate of the residential structure production sector is set to 0.648% by taking the sample average from 1974 to 2006, while that of the non housing consumption production sector is set to 1.23% by taking the sample average from 1974 to 1995.

Time to build: Houses are assumed to be ready for living in the following period ($m = 1$), as in Davis and Heathcote (2005). Since houses are one of the lumpiest objects, it takes time to build. According to the Statistical Abstract of the United States, the average length of time from start to completion of new privately owned one-unit residential buildings is six to seven

[15]The other available statistics regarding the land share are summary statistics of the characteristics of houses financed by GHLC loans. The land share estimate from the GHLC data from 2003 to 2007 is 0.397. Since this is not far from the above estimate, I choose to use the Annual Economic Report estimate since the calibration target is the economy before 1984.

months. Some might think it is not rare to take more than one year to build larger houses such as condominiums. However, according to the statistics from Ministry of Land, Infrastructure, Transport and Tourism (MILT) in Japan, though the average duration from start to completion of the condominium construction in Tokyo metropolitan area is approximately 15 months, it takes only 6 months from start of construction to start of sales of those condominiums (the sample data from 1995 to 2001). Thus I assume housing output is available in the markets in the following period in both countries. The housing production sector has a lag of one period between purchasing input and selling output.

Downpayment requirements/Transaction cost: Downpayment requirements λ are set to 20% in the U.S. and 25% in Japan. First, many literatures set the downpayment requirement to 20% in the U.S. (for example, Nakajima (2005), Chambers et al. (2007)). Second, Hayashi et al.(1988) explains that the average downpayment rate in the 80s is higher in Japan than in the U.S. They show that the average downpayment rate is about 25 to 30% for the conventional mortgages in the U.S., and it is about 35% in Japan. Therefore, consolidating above two facts together, I set the baseline downpayment requirements to 20% in the U.S. and set a little higher rate, 25%, as the baseline downpayment requirements in Japan.

Transaction cost consists of the part proportional to the housing value and the fixed cost part. Haurin and Gill (2002) estimate that housing transaction cost is the sum of 3% of the housing asset value and 4% of the household earnings. Thus I set $\phi_s = 0.03$ and $\phi_f = 0.04/\frac{\int_{i\leq I_{ret}} \epsilon_t v_i \, dm(\epsilon_t,t,i)}{\int_{i\leq I_{ret}} dm(\epsilon_t,t,i)}$ in the U.S.[16] Moreover, the same value can be applied to Japan. The regulation which has existed since 1970 specifies the maximum amount of transaction cost which real estate agents can impose at the time of the transaction.[17] It says if a house is sold more than four million yens, which is about forty thousand dollar, the maximum transaction cost is 3% of housing price and six million yens (about sixty thousand dollars)[18]. Though this number is merely an upper limit, it is a widespread rough standard of transaction cost of real estate intermediary. Since it is difficult to calculate actual transaction cost, I use this number as a substitute. There is no specific evidence which suggests otherwise, I use the same number as the U.S. for the fixed cost part. Therefore I set $\phi_s = 0.03$ and $\phi_f = 0.04/\frac{\int_{i\leq I_{ret}} \epsilon_t v_i \, dm(\epsilon_t,t,i)}{\int_{i\leq I_{ret}} dm(\epsilon_t,t,i)}$.

The probability of paying transaction cost is defined as the average probability of changing houses. The average ratio of households who move out from their ownership houses in a year

[16]The fixed cost is $\phi_f w_t$ but w_t is the average income of all household. The average wage of working households is defined as $\frac{\int_{i\leq I_{ret}} w_t \epsilon_t v_i \, dm(\epsilon_t,t,i)}{\int_{i\leq I_{ret}} dm(\epsilon_t,t,i)}$. This number coincide with the conventional view that the proportional part of seller's transaction cost in the U.S. is 3%. Normally real estate commission is 6% of housing value and is often split between buyers and sellers into a half. The fixed cost can be interpreted as a fee for lawyer, home inspection fee and moving cost

[17]Ministry of Construction instruction (Kensetsusyou Kokuji) no. 1552, October 23rd, 1970

[18]Specifically, the regulation says that marginal transaction cost is 5% if sales price is two million yens or less (about twenty thousand dollars), 4% if four million yens or less and more than two million yens, and 3% if more than four million yens. These numbers exclude sales taxes.

is 7.19% in the U.S. which is calculated by Panel Study of Income Dynamics (PSID) from 1968 to 1992. Likewise, the average ratio of people who move out from their ownership houses is 3.11% in Japan computed from the HLS. [19]

Demographics and preference: This is a finite horizon model and all households are born at the age of 21 and die at the age of 80. Also all households retire at the age of 60 in Japan and 65 in the U.S. I set 65 as the U.S. retirement age since households can get unreduced benefits of social security benefits after this age. On the other hand, it was common for the Japanese companies to set the mandatory retirement age to 60. According to the Koyou Kanri Chosa (Employment Management Survey) published from Ministry of Health, Labour and Welfare, the share of companies which set the mandatory retirement age was around 80% in the late 80s in Japan. In addition, the companies which set the mandatory retirement age to 60 is around 50% and which set it to less than 60 is around 40% of those companies, meaning more than 70% of all companies have the mandatory retirement before 60. Considering this fact, I set the mandatory retirement age as 60 in Japan.

To compute conditional survival rates, I use the male death probability table from Ministry of Health, Labour and Welfare (MHLW) in Japan and the period life table 2004 of Social Security Administration in the U.S. The series appear in Figure 9 and Figure 10. Also, I assume $\gamma_u = 1.5$.

Social Security: All households receive social security benefits after they retire. The estimate from MHLW in Japan shows that the replacement rate of the social security is about 69% in 1985 and 59% in 2002. MHLW has changed the way to calculate replacement from 2002 on, so these numbers are not exactly comparable. The replacement rate until 2002 is defined by the annual social security benefits divided by the then current average annual income of male over periods of enrollment in the social security system. On the other hand, the replacement rate in 2002 is defined by the annual social security benefits divided by the then current average annual after tax income of all male workers. I use 55% as the social security replacement rate in Japan, which gives roughly 70% of the replacement rate of the average after tax income.

Mitchell and Phillips (2006) find that a U.S. worker with medium scale income profile receives from 48 % to 55% of their lifetime average earnings depending on benchmarks. Also they document that using the population measure instead of the individual earning measure gives 35% to 46% as replacement rate. Note that everyone receives the same amount of social security benefits after his retirement in this model. In order to reconcile two measures, I set

[19] The average probability of moving can be different by age. According to PSID the moving probability is decreasing as households get older. However, it is also presumable that it is easier for younger households to move due to physical conditions or weakness of bond to the neighborhood. Ideally, these statistics should be produced by the model, but due to computational limitations, this model cannot keep track of mobility decisions of households. Thus for simplicity I assume the exogenous risk which households face is the same across all ages.

45% as the replacement rate of the social security so that the replacement of the population average (before tax) income is 45%. This gives roughly 50% as the replacement rate of the average after tax income. This also means a hypothetical worker who receives mean income every period receives 50% of their lifetime income as social security benefits.

Initial asset distribution: Households are assumed to be born with some initial assets in order to incorporate the fact that some young households have financial assets as well as real assets which are sometimes given by their parents, as in Chambers et al.(2007). I estimate the distribution of the assets to earnings ratio of initial households by fitting Pareto distributions to the data distribution. In Japan, the households samples of under 25 years old from the NSFIE (2004) are used for the estimation. In the U.S., the households samples of 18 to 22 years old from Survey of Consumer Finance (SCF) are used. The details of the estimation are documented in Appendix D.

Income process: Since the household labor decision is not endogenous, I estimate the exogenous income process to feed into the model.

- **Japan:** The income process is estimated from the NSFIE (2004). Since I only know the aggregate income distribution of households, I take the following steps to estimate it. Due to the lack of micro level panel data, here I need to assume that there exists no time or cohort effect on household income.

  First, I estimate the life cycle profile of income. Since the NSFIE has the statistics of the average income of each five year age group from age 20 to age 70, I can estimate the life cycle profile by fitting a fourth order polynomial. Figure 11 shows the estimated results of the life cycle profile of income and also the tax and social security adjusted life cycle profile of income.

  Next, I estimate persistence and variance of income shocks. The target data is the household income distribution of eight age groups (under 25, 25-29, ..., 55-59) and nine income level groups. Based on the life cycle profile and the specification of the income shock process, I can generate a sample life cycle income profile of each household. Using the Monte Carlo simulation method, I generate ten thousand samples and count the number of samples which fall into each age and income group. Then I can estimate the parameters by minimizing the distance between the target distribution and the simulated distribution. The identity matrix is used as weighting matrix due to the lack of alternative.

  As a result, I obtain $\sigma_\epsilon = 0.3161, \sigma_\eta = 0.0549, \rho = 0.9932$. For computational purposes, I discritize the income process using the method proposed by Tauchen (1986). The transitory component is discritized into two grids $(-\sigma_\epsilon, \sigma_\epsilon)$ and the persistent component is discritized into five grids $(-0.34, -0.17, 0.0, 0.17, 0.34)$, so I have ten grids for the income process. Transition matrix is,

$$\pi = \begin{bmatrix} 0.9161 & 0.0771 & 0.0000 & 0.0000 & 0.0000 \\ 0.0839 & 0.8413 & 0.0793 & 0.0000 & 0.0000 \\ 0.0000 & 0.0816 & 0.8413 & 0.0816 & 0.0000 \\ 0.0000 & 0.0000 & 0.0793 & 0.8413 & 0.0839 \\ 0.0000 & 0.0000 & 0.0000 & 0.0771 & 0.9161 \end{bmatrix} \tag{51}$$

- **U.S.**

The income process is estimated from the PSID, following Nakajima (2006) who uses the method developed by Heathcote et al. (2003). I document the major procedure of the estimation here.(Details are in Appendix E.)

The first step is a sample selection. I use the similar criteria to the previous studies (Heathcote et al.(2003), Nakajima (2006), Guvenen(2007)). I use PSID data from 1968 to 1993 with households who have the following characteristics: (i) white male, (ii) head of households, (iii) age from 20 to 65, (iv) report positive but not top coded income, (v) working hours from 520 to 5096 per year, (vi) hourly income between minimum wage and maximum wage, (vii) in a sample more than two consecutive years, (viii) and not poverty (SEO) subsample.

The second step is to compute the time invariant life cycle income profile. I pool all the samples from 1968 to 1993 and regress their hourly income on a fourth order polynomial in age. Figure 12 shows the estimation results and the tax adjusted income profile with social security benefits.

The final step is to estimate the parameters of the income process by minimizing the distance between variance covariance matrix calculated by the samples and that calculated theoretically from the model parameters.

Since the focus of this model is not the change in the income process, I use the average parameter values as the income shocks. Thus, I obtain $\sigma_\epsilon = 0.2447, \sigma_\eta = 0.1441, \rho = 0.9783$. For computational purposes, I discritize the income process using the method proposed by Tauchen (1986). The transitory component is discritized into two grids into $(-\sigma_\epsilon, \sigma_\epsilon)$ and the persistent component is discritized into five grids $(-0.56, -0.28, 0.0, 0.28, 0.56)$, so I have ten grids for the income process. Transition matrix is,[20]

$$\pi = \begin{bmatrix} 0.8125 & 0.1553 & 0.0018 & 0.0000 & 0.0000 \\ 0.1852 & 0.6684 & 0.1638 & 0.0020 & 0.0000 \\ 0.0023 & 0.1743 & 0.6688 & 0.1743 & 0.0023 \\ 0.0000 & 0.0020 & 0.1638 & 0.6684 & 0.1852 \\ 0.0000 & 0.0000 & 0.0018 & 0.1553 & 0.8125 \end{bmatrix} \tag{52}$$

4.2 Summary of Calibration

Calibration results are summarized in Table 5. It would be helpful to make a few comments regarding calibrated parameter values. There are three significant points which reflect clear differences between the housing markets in Japan and the U.S.

First, the residential structure depreciation rate in Japan is much higher than that in the U.S. This matches with the conventional view that Japanese houses are less durable than the houses in other countries. *Kensetsu Hakusho* (Construction report) in 1996 published from MLIT estimates that the average life span of Japanese houses is about 26 years while that of U.S. houses is about 44 years. It also points out that its reasons could be due to (i) the rapid increase in the housing construction to overcome the shortage of the housing stock after the Second World War, (resulting in low housing quality) or (ii)the tendency of people to dispose houses when they move or die.

Third, the land share of the housing production sector is much higher in Japan than that in the U.S. This reflects the fact that land is more scares in Japan and land prices are much higher in Japan than in the U.S. Stone and Ziemba (1993) provide the rational explanations for high land prices in Japan following Boone and Sachs (1989). They point out four factors: low average property taxes, a high intensity of land use, a high growth rate of Japanese economy, and a low rate of time preference.

Last, the average moving probability in Japan is mush lower than that in the U.S. It is well known that the Japanese secondhand housing markets are thinner than that in other countries. According to *Pocket Housing Statistics*(Housing Statistics Abstract) published from *Jutaku Kinyu Hukyu Kyokai*(Housing Loan Promotion Association), the secondhand housing trading volume to total housing stock ratio in 2004 is 0.3% in Japan while 5.6% in the U.S. Kanemoto (1997) points out three factors as to why Japanese secondary housing markets are so unpopular in addition to the possible difference in preference towards new houses: high transaction taxes, high capital gain taxes, and the GHLC subsidy favorable to a new house purchase.

4.3 Estimation

I need to estimate five parameters: the discount rate β, the share of consumption in utility function θ_u, the additonal depreciation rate of the rental housing capital δ_r, the minimum house size h_{min} and the incentive to bequeath χ. I use the exactly-indentified Method of Moments to determine the values of these parameters.

[20]Due to the rounding for documenting here, some rows of transition matrix don't add up to unity.

Table 5: Calibrated Parameters (baseline economy)

			Japan	U.S.
Growth rate:	TFP (non housing sector)	g_{z_c}	1.61%	1.23%
	TFP (residential structure sector)	g_{z_h}	-0.365%	0.648%
	Labor	g_l	0.657%	1.64%
	Land	g_l	2.26%	2.79%
Demographics:	Maximum age	I	80	80
	Retirement age	I_{ret}	60	65
Preference:	Coef. of risk aversion of utility f.	γ_u	1.5	1.5
Technology:	Capital share (non housing sector)	α_c	0.332	0.291
	Labor share (non housing sector)	$1-\alpha_c$	0.668	0.709
	Capital share (residential structure sector)	α_s	0.292	0.211
	Labor share (residential structure sector)	$1-\alpha_s$	0.708	0.789
	Land share (housing sector)	α_h	0.450	0.106
	Capital depreciation rate	δ_k	0.123	0.103
	residential structure depreciation rate	δ_h	0.0884	0.0159
	Time to build	m	1	1
Housing:	Downpayment rate	λ	0.25	0.20
	Adjustment cost (% of housing asset)	ϕ_s	0.03	0.03
	Fixed adjustment cost/average wage	ϕ_b	0.04	0.04
	Moving probability	$\pi(move)$	3.11%	7.19%
Government:	Social security replacement rate	θ	0.55	0.45
Income process:	Persistence	ρ	0.9932	0.9783
	S.D. (persistent shock)	σ_ξ	0.0549	0.1441
	S.D. (transitory shock)	σ_ϵ	0.3161	0.2447

4.3.1 U.S.

To estimate by the exactly indentified Method of Moments, five targets are chosen, namely, the non housing capital to output ratio, the average homeownership rate, the housing consumption to non housing consumption ratio, the net wealth of the oldest age cohort to the average income ratio, and the average ownership house size to the average rental house size ratio.

The first target is the non housing capital to output ratio estimated from the NIPA. Non housing capital is defined as the sum of private non residential fixed capital, private inventories and durable stocks attributed to the non housing consumption production sector. (See Calibration section and Appendix C as to how to attribute capital input.) Output is defined as the output of the non housing consumption production sector, namely, GDP with imputed durable services less the sum of government consumption of fixed capital, residential fixed investments, and housing consumption expenditure of households. Taking the sample average from 1951 to 1995, the ratio is estimated as 1.594. The second target is the average homeownership rate. The average homeownership rate is calculated as 0.6448 which is the sample average from 1974 to 1995 according to the U.S. census bureau. The third target is the housing consumption to non housing consumption ratio estimated from the NIPA. The housing consumption is defined as housing consumption expenditure of households and the non housing consumption is defined as private consumption expenditure and imputed durable services less durable investments and housing consumption expenditure of households. Taking the sample average from 1951 to 1995, the ratio is estimated as 0.1795. The fourth target is the net wealth of households with age over 70 to the average income ratio. I choose the average income of total households as denominator because the average income of households with age over 70 years old in this model is equal to the social security benefits, which is not true in reality. The net wealth is defined as all the assets they have less all the debts they have. For example, automobiles, jewelry, and stock holdings are included in assets and credit card loan is included in debts. The number is estimated as 6.43 from SCF (1995). The last target is the average ownership house size to the average rental house size ratio. The ratio is calculated as 1.814 from American Housing Survey (AHS hereafter) (1995). Due to highly nonlinear nature of the problem, it is not clear that there exist parameter values which produce five target values. Fortunately, I find a set of parameter values which generates close enough numbers to the target values. Table 6 summarizes the estimation results.

4.3.2 Japan

I pick the same moments as in the U.S. to estimate parameters for Japanese economy. The first target is the non housing capital to output ratio estimated from the SNA. Non housing capital is defined by the sum of private non residential fixed capital, private inventories and durable stocks attributed to non housing consumption production sector. Output is defined as the output from non housing consumption production sector, namely, GDP with imputed

Table 6: Estimation results: U.S.

Targets:		Data	Model	Error (%)
Non housing capital/ output ratio	K_c/Y_c	1.594	1.5939	0.0006
Housing /Non housing consumption ratio	$p^r H/C$	0.1795	0.17949	0.00702
Average homeownership rate		0.6448	0.64484	0.00534
Net wealth(Old)/Average income ratio	a_{old}/w	6.43	6.4301	0.0023
Ownership House/Rental House size ratio		1.814	1.81395	0.0028

Variable	Parameters	Value
Discount rate	β	0.961160
Share of consumption in utility function	θ_u	0.854888
Additonal dep. rate of rental house	$\delta_r/\delta_h - 1$	0.253598
Incentive to bequest	χ	0.138043
Minimum house size (/average wage)	$\frac{p^h h_{min}}{w/\int_{i<i^*} d\Phi\Omega}$	1.444984

durable services less government consumption of fixed capital, residential fixed investment, housing consumption expenditure of households (Gross rent, water, fuel and power). Taking the sample average from 1970 to 1998, the ratio is estimated as 1.468. The second target is the average homeownership rate. The average homeownership rate is 0.656 computed from the NSFIE (1984) using the demographic of the National Census (1985). The third target is the housing consumption to non housing consumption ratio estimated from the SNA. The housing consumption is defined as housing consumption expenditure of households (Gross rent, water, fuel and power). The non housing consumption is defined as private consumption expenditure and imputed durable services less durable investments and housing consumption expenditure of households. Taking the sample average from 1970 to 1998, the ratio is estimated as 0.218. The last target is the net wealth of households with age over 70 years old to the average income ratio estimated from the NSFIE (1984). Income is total income before tax. The ratio is estimated as 7.43.

I don't estimate the minimum house size in Japan. The model cannot get the moment, the average ownership house size to the average rental house size ratio, correct with the reasonable values of the other parameters. The minimum house size is supposed to be determined mainly by the average ownership house size to the average rental house size ratio. Because the existence of the fixed transaction cost, it is relatively more costly to live in a small ownership house. Thus people who live in minimum houses are those who choose rental houses. This implies if the minimum house size goes down the average ownership house size to the average rental house size ratio is likely to go up. However, the value in Japan is so large that even if I assume no minimum housing size I cannot generate it. (No minimum housing size is suppose to generate the maximum value.) Therefore I assumed that the minimum house size is about 0.05 of the average household income. This is consistent with what many literatures point out. One of the distinctive characteristics of the Japanese housing markets is the short of ”family size” rental houses. According to Kanemoto(1997), the ownership house size to rental house ratio is 2.63 in 1989 which is much higher than other developed countries such as 1.37(ratio of median) in the U.S. in 1985 and 1.17. un the U.K. in 1986.

Again, due to highly nonlinear nature of the problem, it is not clear that there exists parameter values which produce four target values. Fortunately, I find a set of parameter values which generates close enough numbers to the target values. Table 7 summarizes the estimation results.

Table 7: Estimation results: Japan

Targets:		Data	Model	% Error
Non housing capital/ output ratio	K_c/Y_c	1.468	1.46764	0.0038
Average homeownership rate		0.656	0.65599	0.0047
Housing /Non housing consumption ratio	$p^r H/C$	0.218	0.21799	0.0014
Net wealth(Old)/Average income	a_{old}/w	7.43	7.4305	0.0065
Ownership House/Rental House size ratio		(2.63)	(1.92)	
Variable	Parameters	Value		
Discount rate	β	0.952139		
Share of consumption in utility function	θ_u	0.822090		
Additonal dep. rate of rental house	$\delta_r/\delta_h - 1$	0.022241		
Incentive to bequest	χ	0.392160		
Minimum house size (/average wage)	$\frac{p^h h_{min}}{w/\int_{i<i^*} d\Phi\Omega}$	(0.05329)		

5 Model evaluation

Now I need to evaluate the model to see whether it successfully replicates some important features of the economy in both countries. It should be noted that aggregate moments are used for the estimation except one moment, thus it is necessary to see if the model generates reasonable statistics of the age distribution to evaluate the validity of the model. [21]

[21] One non aggregate moment used in estimation is the housing asset of households with over 70 years old to average income ratio.

5.1 Homeownership rate

First, the homeownership distribution across age groups is examined. Figure 13 and Figure 14 show life cycle patterns of the homeownership rate in Japan and in the U.S. respectively. Each dashed line represents the data and each solid line represents the results produced by the model. The Japanese data are taken from the NSFIE (1984) and the U.S. data are taken from the AHS (1995). The data show the increasing and concave trend of the homeownership rate over age in both countries. The homeownership distributions of two countries look similar in general, but one difference is that the homeownership rate of households under 30 is significantly lower in Japan than in the U.S.

The model fits the data quite well. It successfully generates the increasing and concave life cycle pattern of the homeownership distribution. Moreover, the model generates the difference in steepness of the homeownership rate increase among young households between two countries. On the other hand, the model overstates the homeownership rate of old households in Japan.[22]

The key factor behind this life cycle pattern is the fixed transaction cost. While the additional depreciation cost to live in a rental house is proportional to housing value, the transaction cost per housing value is decreasing in housing value due to the existence of fixed transaction cost. Thus, wealthy and high consumption households who live in large houses tend to live in ownership houses and poor and low consumption households tend to live in rental houses. Therefore, the homeownership rate shows a similar life cycle pattern to the consumption profile which in this case is increasing and concave.

The difference in steepness of the homeownership rate increase between two countries comes partly from the fact that the life cycle profile of income is steeper in Japan than that in the U.S. and partly from the fact that Japanese households are more patient than U.S. households.[23]Due to the low income of households, the frictions in the housing markets such as downpayment requirements, moving risk and fixed transaction cost, prevent more young households in Japan than in U.S. from purchasing their houses. In addition, when households are more patient, they consume less when they are young and more when they are old. Consequently, the homeownership rate of young households in Japan becomes lower. As they get older, the homeownership rate eventually catches up the rate in the U.S.

[22]This is partly due to the demographic effect. The demographic composition of the model and the actual demographic composition are different in that there are more old people in the actual world. Thus the average homeownership is higher in the real world than the model, even if I have exactly the same homeownership distribution. Since I target to match the aggregate statistics, the homeownership rate can be overstated in whole.

[23]Patience here means $\beta(1+r)$ is higher in Japan than in the U.S.

5.2 Size of houses

Second, the housing size distribution by age cohorts is examined. Figure 15 and Figure 16 show life cycle patterns of the housing size as the deviation from the mean. Each dashed line represents the data and each solid line represents the results from the model. The Japanese data are taken from the HLS (1989) and the U.S. data are taken from the AHS (1995). In my model, the housing size in absolute term cannot be generated. So I compare the deviation from the mean of the housing size over age cohorts. Also in order to compare the data to the model, I adjusted the data by dividing by the family equivalence scale based on the family size of each age cohort.[24]For simplicity, the family equivalence scale is taken to be square root of the number of family members. So the equivalence scale of the family with four members is two. A life cycle profile of the household size in each country adjusted by the family equivalence scale is shown in Figure 17 and Figure 18.

Comparing the two dotted lines, the first thing to notice is their increasing pattern over age in a broad way. Moreover, while a steadily increasing pattern is observed in the U.S., a small U-shape of the housing size distribution is observed in Japan. This implies that Japanese young to middle age households live in relatively small houses considering the number of family members.

The model matches with data well in both countries. The results of both countries show increasing patterns of the housing size over age. The model even succeeds in predicting a difference in shape between two countries. First, the increasing pattern of the housing size over age is because of the increasing pattern of the housing service consumption profile. Second, the reason why the housing size distribution in Japan shows U-shape is due to the steepness of the hump shape of the homeownership rate. Although each homeowner's housing consumption is increasing over age, the average housing consumption depends on the distribution (or composition) of homeowners. When they are young, only rich people can afford living in ownership houses. When reaching to the middle age, more households becomes homeowners. Households who purchase homes in the middle age are those who become just rich enough to afford houses. Consequently the compositional effect decreases the average. This effect is large in Japan where a steep increase in the homeownership rate can be seen in the middle age.

5.3 Asset portfolio

Third, the age distribution of the asset portfolio of households should be examined. Since this model doesn't differentiate mortgage loans from risk free assets, analyzable statistics are the average gross housing asset to average net wealth ratio. Figure 19 and Figure 20 show the housing asset to net wealth ratio of households who own their houses in Japan and in the U.S. respectively. The dashed lines represent the data, the solid black lines represent the results

[24]The housing size in the U.S. is the mean of housing size per households, while that in Japan is the mean of housing size divided by mean of household size.

from the model and the gray line represents the adjusted results of the model which I will discuss later. The U.S. data are taken from the Survey of Consumer Finance (1995) and the Japan's data are taken from the NSFIE (1984).[25]Net wealth is defined as all assets such as financial assets and housing assets net of all debts such as credit card debts and mortgage debts.

The data show the decreasing and convex shape over the life cycle in both countries. This implies that younger households tend to allocate more assets into housing assets. Some differences between two countries should be noted. Though the housing asset share of young households are similar in both countries, that of old households are larger in Japan than in the U.S. This means Japanese old households allocate more of their wealth into the housing assets than U.S. old households.

First, Figure 20 is examined. The model successfully generates the decreasing shape of housing asset to net wealth ratio with small overstating among the oldest age cohort. This shape comes from two factors. First, households want to smooth their consumption over the life cycle not their asset position. So young households buy housing assets by borrowing money from their future income. Second, the bequest motive encourages old households to save. Because of the incentive to save until the last period, old households reduce their savings but not that much. Consequently, the housing asset to net wealth ratio has a decreasing shape in whole and a small rise at the oldest age cohort.

Second, Figure 19 shows that though the model successfully generates the overall trend of the housing asset share over the life cycle, it understates the housing share except households with age 30 to 39. This is mainly because of the difficulty for this model to capture market housing asset value in Japan due to the high land share and the high depreciation rate. All the houses in this model consist of the same composition of structure and land as the new houses. The housing value documented in the NSFIE is the sum of depreciated structure value and (non depreciated) land value. As a result, used houses show a larger land share than new houses. This difference becomes larger if the land share is larger and the housing depreciation rate is higher. In the estimation, I target the total housing consumption not the total housing asset, thus it is likely that the model misses capturing land part which is not consumable.

To take the compositional change in houses into account, I adjust the model results so that the structure share of houses owned by households in the model is equal to that in the market. Adjusted results are shown in the solid gray line in Figure 19. Computing the market composition, I simply assume that all houses are the houses with the average age in Japan.

[25]The NSFIE in 1984 surveyed only financial assets held by households and no survey regarding real asset was existed in 1984. I use the statistics estimated by Takayama et al.(1989). They estimate the real assets in household portfolio by using the NSFIE (1984) data for households with two or more members. Moreover, the NSFIE (1984) doesn't have statistics of household living alone conditional on owning houses. Thus, the housing asset to net wealth ratio in Japan is housing assets divided by housing assets plus net financial assets of homeowners with two or more members. It should be noted that durables and idle land are not included in denominator. From 1989, the NSFIE started to include questions regarding real assets in interviews.

Kensetsu Hakusyo 1996 (the Construction Report 1996) estimates that the average age of houses is 16 years old. Combining with housing depreciation rate 0.0884 gives the land share of the average age houses is 78%. Then, I divide the structure value of the model results by the structure to total housing value ratio of the average age houses and obtain the adjusted results. I can be seen the improvements of statistics.

However, the model still understating the ratio of old households. This could be due to the inheritance tax. As pointed out in Kanemoto (1997), Iwaisako (2003) and many others, Japanese inheritance tax is favorable to house owners in that assessed value of land for the tax purpose is historically evaluated much lower than the market price. Thus there is an incentive for old households to buy houses by borrowing money. Since this model doesn't incorporate taxes, that effect in the general equilibrium context remains to be studied in the future research.

5.4 Housing asset to income ratio

Last, I examine the housing asset to income ratio. Figure 21 and Figure 22 show the housing asset to income ratio over the life cycle. The dashed lines represent the data, the solid lines represent the results of the model and the gray line represents the adjusted results of the model. The ratio is calculated as the mean of housing asset of homeowners divided by the mean of income of all households in each cohort. The U.S. data are taken from the SCF (1995) and the Japan's data are taken from the estimate of Takayama et al. (1989) and the NSFIE (1984).[26] An U-shape can be observed in the data of both countries. This U-shape of the housing asset to average income ratio indicates that young people buy relatively larger houses considering their income and old people still keep the size of the house even after they retire. Also, it can be noticed that average housing asset to average income ratio is twice as large in Japan as in the U.S., though the housing consumption to non housing consumption ratio is similar.

The first thing to notice is that the model successfully generates an U-shape of the life cycle pattern in both countries. As argued above, the young households borrow money from their future income to finance their optimal size of houses and old people don't downsize their houses much even when their income becomes lower. Another thing to notice is that the model fails to generate the level of the ratio in Japan, while it matches quite well with U.S. data. Since this is likely due to the housing composition issue, I adjust the result by taking into account the change in housing composition of used houses. The adjusted results are shown as the gray solid line in Figure 21. It can be seen that the level of the ratio has improved. So the reason why the housing asset to income ratio is so high in Japan is because Japanese houses have the higher land share and land does not depreciates. The last comment is that the model overstates the ratio of the old age cohort a little in the U.S. This might be because in this

[26] As documented before, the estimate of Takayama et al. (1989) is only for households with two or more members. Thus I multiply their estimate by average housing asset of all households to average housing asset of household with two or more members ratio from 1989 to 2004 in each age cohort.

model only source of income for retired households is social security but it is not always the case in reality.

Since the model seems to do well in replicating the characteristics of the economy of each countries, it can now be said that the model is a valid instrument in examining the influence of macroeconomic changes. I move on to the next step to consider how macroeconomic changes impact on housing prices and household behavior using this model.

6 Results

Now I am ready to examine the effects caused by the changes in the following key factors: interest rates, the growth rate of the TFP of the non housing consumption production (hereafter the TFP) and downpayment requirements. To understand the direct effects on the behavior of each household, the transition analysis is conducted as well as the balanced growth path analysis. The balanced growth path comparison is suitable for clarifying the differences between the two long run equilibria. However, it is not able to capture the direct effects on the specific economy. I start out from analyzing the long run effects of the changes in key factors by computing balanced growth paths under different sets of key factor values. Then I proceed to compute the transition path from one economy to another to examine its actual impact on the baseline economy. In order to incorporate the order the changes occur, I compute several transition paths. The first one is from the baseline economy to the first balanced growth path, the second one is from the first transition path to the second balanced growth path, and so forth. All variables change will come as a surprise to households.[27]

Table 8 shows the sets of key factor values in each period in each country. The sets of values in Japan before 1983 and the U.S. before 1995 are the baseline economy values used for the estimation. Japan experienced an increase in the TFP growth rate by 0.98 percentage points in 1984 and a drop of the TFP growth rate by 2.225 percentage points in 1991. The interest rates declined 0.235 percentage points in 1987 and another 0.07 percentage points in 1991. Likewise, an increase in the TFP growth rate occurred in the U.S. in 1996 by 0.79 percentage points and the reduction in the interest rates occurred in 2002 by 2.73 percentage points. It is difficult to know the exact date of when the financial innovations became popular in the housing markets. Thus I assume the downpayment requirements decreased gradually in 1996 and in 2002 by 2.5% in the U.S.[28]

[27]It should be noted that although interest rates is endogenously determined in the baseline model where I assume closed economy, to examine direct effects of the interest rate change, interest rates are exogenously determined in transition paths. When economy has low interest rates without having low growth, there should be capital inflow from the rest of the world. And it is the case in Japan and in the U.S. U.S. capital account debt more than tripled from 1995 to 2006. Japan's capital account surplus decrease from 4.7% of GDP in 1986 to 1.2% in 1990.

[28]The TFP growth rate of the non housing consumption production sector in Japan from 1983 to 1991 is set to 2.590% which is the average growth rate from 1983 to 1991 calculated from the SNA. Similarly, the TFP growth

Table 8: Sets of key factor values

Variable name		Japan -1983	1984-1986	1987-1990	1991-
TFP (non housing consumption production)	z_c	1.61	2.59	2.59	0.365
Interest rate	r_s	0.103	0.1032	0.0797	0.0727
downpayment rate	λ	0.25	0.25	0.25	0.25
		U.S.			
Variable name		-1995	1996-2001	2002-	-
TFP (non housing consumption production)	z_c	1.230	2.020	2.020	-
Interest rate	r_s	0.0795	0.0795	0.0522	-
downpayment rate	λ	0.20	0.175	0.15	-

6.1 Balanced growth path analysis

First I document balanced growth path statistics to analyze the long run property of the economy under a new set of key factor values. The results are shown in Table 9-12. In Table 10-12, to make the role of the downpayment requirements explicit, I document the results of balanced growth paths without the downpayment requirement change, shown in parentheses in the U.S. section.

6.1.1 Growth rates

First, I examine the effects on economic growth. The increase in the TFP growth rate boosts up the total economy resulting in a rise of all the macroeconomic aggregates (Table 9). On the other hand, the decline in the TFP growth rate lessens their growth rates. In terms of housing prices, a 0.979 percentage point increase in the TFP growth rate results in a 1.231 percentage point increase in the housing price growth rate in Japan. On the other hand, a 1.246 percentage point decrease in the TFP growth rate from the benchmark results in a 1.56

rate of non housing consumption production sector after 1991 is set to 0.365% which is the average growth rate from 1991 to 1998. The rate of decline of interest rates is calculated as the difference of the collateralized overnight call rate between the periods. Specifically, the difference between the 1981-1986 period and the 1987-1990 period is -0.235 percentage points and between the 1987-1990 period and the 1991-1998 period is -0.07 percentage points. Since there exists no evidence that the downpayment rate decreased in 80's, I leave the downpayment rate unchanged.

Likewise, the TFP growth rate of the non housing consumption production sector in the U.S. is set to be 2.020 which is the average growth rate from 1996 to 2006. I assume the interest rates lowered by 2.73 percentage points in 2001 which is the difference between the average interest rates of three month Treasury bill from 1986-2001 period to 2002-2006 period.

percentage point decrease in the housing price growth rate. In the U.S., when the TFP growth rate rises by 0.79 percentage points, housing prices rise by 0.94 percentage points. As shown above, the housing price growth rate is positively correlated with the TFP growth rate of the non housing consumption production sector. This is because technological progress in the non housing consumption production sector makes housing service consumption more and more valuable relative to non housing consumption.

In addition, the housing price growth rate in Japan are a little more sensitive than that in the U.S. to the change of the TFP growth rate. According to the equation (42), the contribution of log of the TFP growth rate to log of the housing price growth is $\frac{1-\alpha_s(1-\alpha_h)}{1-\alpha_c}$. This implies that the smaller the share of capital input in the housing production relative to that of the non housing consumption production, the more sensitive the housing price growth rate becomes to the TFP growth rate change. Although the share of capital in the non housing consumption production and the residential structure production is similar in the two countries, due to the large share of land in housing assets in Japan, this coefficient is larger in Japan than in the U.S. Consequently, the housing price growth rate is more sensitive to the TFP growth rate in Japan.

Table 9: Growth rates of balanced growth paths

		Japan			
Variable name		-1983	1984-1986	1987-1990	1991-
Output (non housing consumption)	g_c	3.092	4.582	4.582	1.207
Output (residential structure)	g_h	0.992	1.416	1.416	0.449
Housing price	g_{p_h}	1.506	2.737	2.737	-0.054
Land price	g_{p_l}	0.810	2.267	2.267	-1.033
		U.S.			
Variable name		-1995	1996-2001	2002-	-
Output (non housing consumption)	g_c	3.407	4.547	4.547	-
Output (residential structure)	g_h	2.672	2.910	2.910	-
Housing price	g_{p_h}	0.704	1.603	1.603	-
Land price	g_{p_l}	0.602	1.711	1.711	-

6.1.2 Macroeconomics Aggregates (Prices)

Next, I examine the effects on detrended prices. Table 10 shows the changes in detrened prices in each balanced growth path. The first thing to notice is little or no effect of the downpayment requirement change. Comparing the numbers in and outside the parentheses in the U.S. section shows the pure effect of the downpayment requirement change. Income and the housing price-to-rent ratio are not affected at all. Also, there exists only a 0.0-0.35% increase in land prices and a 0.0-0.04% increase in housing prices. These results are consistent with Kiyotaki et al. (2007) who find little effect of the downpayment requirement change on housing prices. This is not surprising considering the assumption that rental houses and ownership houses are interchangeable. Regardless of being constrained by the downpayment requirements, households choose the optimal size of their house.

Second, the change in the TFP growth rate affects not only the trend of prices but also the detrended level of prices in the following ways: (Compare the first and the second column in each countries.) (i) no effect on income, (ii) increase in land prices, (iii) increase in housing prices,[29](iv) and increase in the housing price-to-rent ratio. The last effect comes from the fact that higher expectations on future housing prices induce relatively lower rents. (ii) and (iii) come from (iv) since lower rents stimulate the demand of housing assets.

Third, the interest rate decline also has positive effects on all prices and the housing price-to-rent ratio. Comparing the second column and the third column in Japan and in the U.S. shows the pure interest rate reduction effect. The reduction in interest rates induces the capital inflow from outside the economy, which boosts up the total economy. A 2.35 percentage point reduction in interest rates in Japan results in a 9.37% increase in housing prices, while a 2.73% reduction in interest rates in the U.S. results in a 5.61% increase in housing prices. Housing prices are more sensitive in the Japanese economy due to the high land share (see the equation (35)). Moreover, the housing price-to-rent ratio becomes higher because the reduction in interest rates reduces the user cost of housing services. The interest rate reduction causes a 22.8% increase in the housing price-to-rent ratio in Japan and it causes a 49.4% increase in the U.S. The housing price-to-rent ratio in the U.S. is more sensitive than in Japan to the interest rate change because the depreciation of housing assets is lower in the U.S.

It is interesting to see that land prices, housing prices and the housing price-to-rent ratio decrease in the fourth column in the Japan's section. This implies that the effect of the decrease in the TFP growth rate exceeds that of the further interest rate reduction.

6.1.3 Macroeconomics Aggregates (Output/Input)

Next, Table 11 shows the effects on the allocation of input/output in the economy. The first thing to notice is that again the reduction in downpayment requirements affects little to the macroeconomic allocation of output/input.

Second, both the increase in the TFP growth rate and the decline in interest rates cause the macroeconomic reallocations in a similar way: (i) reallocation of capital and labor more

[29]At first glance, it might look odd to have an increase in land prices and a small decrease in housing prices in the U.S. However, what should be compared is current land prices and one period ahead housing prices due to the existence of the time to build effect. If the change in the housing price growth is taken into account, it should be noticed that the housing prices have actually increased.

Table 10: Changes of macroeconomic aggregates (Prices)
(Deviation from the trend)

		Japan			
Variable name		-1983	1984-1986	1987-1990	1991-
Income	w	1.0000	1.0000	1.0560	1.0747
Land price	p_{h_l}	1.0000	1.0408	1.2599	1.0781
Housing price	p_h	1.0000	1.0058	1.1000	1.0554
Housing price-to-rent ratio	p_h/p_r	8.1973	9.1630	11.2533	8.8763

		U.S.		
Variable name		-1995	1996-2001	2002-
Income	w	1.0000	1.0000(1.0000)	1.0687(1.0687)
Land price	p_{h_l}	1.0000	1.0697(1.0697)	1.5348(1.5294)
Housing price	p_h	1.0000	0.9982(0.9982)	1.0543(1.0539)
Housing price-to-rent ratio	p_h/p_r	12.122	13.535(13.535)	20.216(20.216)

into the residential construction sector. (ii)and an increase of both types of capital stocks. The reason why the reallocation occurs is that households allocate their money more into housing consumption than non housing consumption. The rent, the user cost of housing services, becomes lower in both cases meaning housing consumption becomes cheaper than non housing consumption. Thus, due to the substitution effect, households allocate their consumption more on housing consumption. In addition, the reason why there is an increase in both kinds of capital stocks in the economy is that the higher TFP growth rate and the lower interest rates cause capital inflow from outside world due to the open economy assumption.

6.1.4 Homeownership rate

Lastly, Table 12 shows the effects on the homeownership distribution by age groups. First, I examine the effects of the downpayment requirement change. A comparison between the second and the third row in the U.S. section presents the pure downpayment reduction effect under the high TFP growth rate and a comparison between the fourth and the fifth row shows the pure downpayment reduction effect under the high TFP growth rate and low interest rates. Downpayment requirements are constraints which prevent some poor households from purchasing ownership houses. Thus, the reduction in downpayment requirements should have a positive effect on the homeownership rate. Indeed, it increases the homeownership rate especially among young households.[30] Moreover, the effect of the reduction in downpayment

[30]This results are consistent with the result of Kiyotaki et al.(2007) who find the large effect of downpayment requirement change on homeownership rate and small effect on housing prices. This results seem to be opposite to the results of Chambers et al.(2006) who find the small effect of sole downpayment requirements change. It

Table 11: Changes of macroeconomic aggregates (Input/Output)

Variable name		Japan -1983	1984-1986	1987-1990	1991-
Residential production to Non housing production ratio					
Labor input	l_s/l_c	0.0814	0.0850	0.0986	0.0817
Capital input	k_s/k_c	0.0676	0.0705	0.0818	0.0678
Capital stock to (non housing) Output ratio					
Non housing Capital	$(k_s+k_c)/Y_c$	1.5672	1.5715	1.7723	1.8118
Housing stock	p_hH/Y_c	1.3055	1.3145	1.5256	1.3724

Variable name		U.S. -1995	1996-2001	2002-
Residential production to Non housing production ratio				
Labor input	l_s/l_c	0.0756	0.0813(0.0813)	0.1122(0.1118)
Capital input	k_s/k_c	0.0492	0.0530(0.0530)	0.0731(0.0728)
Capital stock to (non housing) Output ratio				
Non housing Capital	$(k_s+k_c)/Y_c$	1.6725	1.6784(1.6784)	2.0114(2.0108)
Housing stock	p_hH/Y_c	1.7476	1.7739(1.7739)	2.4497(2.4401)

requirements is larger if interest rates are lower. For example, the homeownership rate of the households with age 30 to 39 increases by 1.3% when interest rates are 7.95%, while it increases by 14.0% when interest rates are 5.22%. This is because when interest rates are low, households would like to increase their housing consumption. However, due to the high housing price-to-rent ratio and high housing prices, they have to spend more money to increase housing consumption by purchasing ownership houses. Consequently, more households are constrained and this amplifies the effect of the downpayment requirement reduction.

Also the increase in the TFP growth rate increases the homeownership rate of young to middle age households and decreases that of old households in both countries. (Compare the first row and the second row in each country.) If the economy is growing fast, the households consume more in their early age and consequently the homeownership rate among young households rises. On the contrary, old households don't have enough assets in the end, thus their homeownership rate decreases.

The effect of the reduction in interest rates is quite interesting. Comparing the second and the third column in Japan and comparing the third and fifth column in the U.S. shows the pure interest rate reduction effect. The decrease in interest rates has complicated effects on

should be noted that Chambers et al.(2006) two compare general equilibria with the closed economy assumption but this model assumes an open economy. As they mention, this might be due to clouding out effects of the endogenous interest rates.

the distribution of the homeownership rate. For example, the Japan section shows that the homeownership rate increases among households with under 30 years old and 60 to 69 years old, while the U.S. section shows that the homeownership rate increases among households with 50 to 69 years old. In the economy with low interest rates, households tend to consume more in their early age, which induces the higher homeownership rate among young households and the lower homeownership rate among old households. This is the same effect as the TFP growth rate increase. However, due to high housing prices and the high price-to-rent ratio, young households are constrained and cannot buy houses. As a result, they save more money and buy ownership houses later in their life. This second effect causes the decrease in the homeownership rate among young households.[31]

Lastly, the fourth column in Japan section shows that the TFP growth rate decline and the further reduction in interest rates offset each other, resulting in similar patterns to the baseline distribution.

Table 12: Changes of the homeownership rate distribution

	Japan						
	Total	under 30	30 to 39	40 to 49	50 to 59	60 ot 69	70 and over
1983-	65.6	5.7	43.5	77.9	89.2	95.1	98.6
1984-1986	64.1	8.9	44.5	78.4	93.2	88.6	82.5
1987-1990	54.9	12.5	28.6	52.7	89.3	92.9	63.4
1991-	69.0	7.8	49.5	83.0	92.7	97.1	99.1
	U.S.						
	Total	under 30	30 to 39	40 to 49	50 to 59	60 ot 69	70 and over
-1995	64.5	23.6	58.0	71.1	78.3	83.3	86.9
1996-2001	64.8	29.6	60.0	76.9	78.9	77.3	74.0
(w/o down)	(64.2)	(28.0)	(58.7)	(76.7)	(78.8)	(77.3)	(74.0)
2002-	64.9	32.8	47.8	69.6	85.1	92.3	72.4
(w/o down)	(56.9)	(20.1)	(33.8)	(59.5)	(79.6)	(91.2)	(72.5)

6.2 Transition path analysis

To examine the effects of the macroeconomic changes chronologically, the transition path analysis is conducted. While the balanced growth path analysis only allows us to compare two economies that have reached in the balanced growth path, the transition analysis can clarify

[31] The increase in the homeownership rate among the youngest age cohort in Japan is due to the fact that they are not constrained as much as other cohorts in this model. Figure 19 shows that the housing asset to total net wealth ratio of households with 20-29 is quite small in the model compared to the older age cohort.

what happens in the transitional state from one balanced growth path to another.

6.2.1 Housing prices

Figure 23 shows the time series of housing prices in Japan. The bold solid line represents the simulation results. The dashed line represents the condominium price index and the dashed-two dotted line represents the unit house price index. The model successfully generates the trend and the amplitude of the fluctuations in unit house prices and condominium prices. The model predicts a 46.0% increase in housing prices from 1983 to 1990, which accounts for 82.6% of the average condominium price increase and 107.2% of the average unit house price increase. Moreover, the model predicts a 20.5% drop from 1990 to 1991 when interest rates went down further contemporaneously with the decline in the TFP growth rate. This indicates that the effect of the large drop in the TFP growth rate exceeds the the effect of the further interest rate reduction. This is a clear answer to the question of Okina et al. (2001), "Why hasn't a bubble emerged under such extreme monetary easing conditions as created by the zero interest rate policy since February 1999?" Housing prices are determined not solely by the level of interest rates but by the relation between interest rates and the TFP growth rate. Considering that I change only two exogenous parameter values, it can be said that the model successfully captures a rise and fall of housing prices.

Next, I turn now to the time series of housing prices in the U.S. in Figure 24. The bold solid line represents the simulation results. The dashed line represents the Freddie Mac HPI, the dashed-two dotted line represents the OFHEO HPI and the gray solid line represents the CQHPI from Census Bureau. The model shows a large increase in housing prices coming together with two large spikes. The model generates 32.4% of the housing price increase from 1995 to 2006, which accounts for 62.7% of the OFHEO HPI increase, 54.4% of the Freddie Mac HPI increase, more than the CQHPI increase. The model successfully generates a goodly portion of the large housing price increase in the U.S.

The housing price increase can be decomposed into factor contributions. Table 13 shows the contribution rates of each factor on the housing price increase in each country evaluated at the peak of housing prices, i.e. 1990 in Japan and 2006 in the U.S. Roughly speaking, one third of the housing price increase attributes to each factor: the baseline growth, the TFP growth rate change, the interest rate change. Also the downpayment requirement reduction has no impact on housing prices. The transition paths where I change only one factor holding other factors constant are shown in Figure 25 and Figure 26.

One characteristic of the transition paths is that some large jumps are observed. For example, the first spike of housing prices in Japan due to the TFP growth rate change shows a 10.3% increase, while the second spike due to the interest rate change shows 21.1% increase. These timings are exactly when each change is realized to households. Since all the changes come as a surprise, households adjust their behavior right after they know the changes. There

Table 13: Factor decomposition of the housing price increase

	Japan		U.S.	
Data	Condominium	55.6%	Freddie Mac	59.6%
	Unit house	42.9%	OFHEO	51.7%
			Census	26.6%
Model	Total	46.0%	Total	32.4%
	Baseline	11.0%	Baseline	8.0%
	TFP change	13.3%	TFP change	10.5%
	Interst rate	13.8%	Interst rate	9.8%
			downpayment	0.%

are two main reasons behind these large increases. The first reason is that the asset level in the economy before the changes is higher than in the economy in the new balanced growth path. Thus, the largest amount of assets exists in the economy right after the change and therefore housing demand is the largest in the transition path. Housing demand goes down eventually as the amount of asset in the economy decreases and converges to the level of the new balanced growth path. The second reason is the existence of the time to build effect. Until one period after the shock occurs, a housing stock cannot rise due to the time to build effect. This inelastisity of supply results in an even higher spike in the beginning. Admitting that my assumptions that the changes of household expectations come suddenly and permanently are strong, it clarifies the effects which cannot be analyzed by the balance growth path analysis.

6.2.2 Housing price-to-rent ratio

Figure 27 shows the time series of the housing price-to-rent ratio in Japan. The bold solid line represents the simulation results. The dashed line represents the condominium price index divided by the CPI rents and the dashed-two dotted line represents the unit house price index divided by the CPI rents. The model generates the trend of the housing price-to-rent ratio quite well. The simulation result shows that the housing price to rent ratio goes up with the TFP growth rate rise in 1984 and the interest rate fall in 1987 and goes down with the TFP growth rate fall and even lower interest rates after 1991. This implies that, again, the effect of the TFP growth rate decrease exceeds the effect of the interest rate reduction. Quantitatively, the model accounts for a 22.8% increase in the ratio which is a 77.0% increase in the ratio of unit houses and a 55.0% increase of the ratio of condominiums.

Figure 28 shows the time series of the housing price-to-rent ratio in the U.S. The bold solid line represents the simulation results. The dashed line represents the Freddie Mac HPI, the dashed-two dotted line represents the OFHEO HPI and the gray solid line represents the CQHPI from Census Bureau. All the indices are divided by the CPI rents. The model successfully generates the full amplitude of the housing price-to-rent ratio in the U.S. as well. The model generates a 42.7% increase which is 95.9% of the Freddie Mac HPI increase and exceeds the OFHEO HPI increase and the CQHPI increase.

Similar to the housing price increase, the housing price-to-rent ratio increase can be decomposed into factor contributions. Table 14 shows the contribution rates of each factor on the housing price to rent ratio increase in each country evaluated at the peak of the ratio, i.e. 1990 in Japan and 2006 in the U.S. The TFP growth rate change accounts for 7.2% of the increase in Japan and 9.6% in the U.S. Also the interest rate decline accounts for 11.9% of the increase in Japan and 25.2% in the U.S. The downpayment requirement reduction and the baseline growth don't contribute to the increase at all. The transition paths where I change only one factor holding other factors constant are shown in Figure 29 and Figure 30.

Table 14: Factor decomposition of the housing price-to-rent ratio increase

	Japan		U.S.	
Data	Condominium	40.8%	Freddie Mac	44.5%
	Unit house	29.3%	OFHEO	37.3%
			Census	14.5%
Model	Total	22.8%	Total	42.7%
	Baseline	0.0%	Baseline	0.0%
	TFP change	7.2%	TFP change	9.6%
	Interst rate	11.9%	Interst rate	25.2%
			downpayment	0.%

6.2.3 Homeownership rate

Figure 31 shows the time series of the homeownership rate in Japan. The bold solid line represents the simulation results and the dashed line represents the data. The data series are computed using the fixed demographic structure of the National Census 1985. The results capture the qualitative trend where the homeownership rate doesn't change much from 1984 to 1989 and drops in 1994 and recovers in 2004. Quantitatively, the model results are more volatile than the data. For example, two positive spikes of the homeownership rate are observed in 1984 and in 1987. These spikes occur because households adjust their decision right after the changes are notified. Nonetheless, this shows the importance of the transition path analysis, since this results are totally opposite to the results from the balance growth path analysis where the homeownership rate decreases with low interest rates and the high TFP growth rate.

Next I examine the age distribution of the homeownership rate to see if the model generates the change in the life cycle pattern of the homeownership rate. Table 15 shows that the model succeeds in generating the qualitative changes. The homeownership distribution among the young doesn't change much from 1984 to 1989, while that of the old increases. In 1989, due to the high TFP growth rate and the low interest rates, households demand more houses, which results in the increase in the homeownership rate in old age cohorts. Meanwhile, the high demand causes higher housing prices which prevent some young households from purchasing new houses due to the downpayment requirement. Consequently, the homeownership rate of the young decreases. In addition, households delay their decision to buy houses due to the sudden change in the expectation on the economy after 1991. Thus in 1994 the homeownership rate decreased in most of the age cohorts. This is roughly consistent with data where young households seem to have been mainly affected. After that households come back to the ownership housing markets gradually.

Table 15: Homeownership rate distribution by age cohort: Japan

		average	under 30	30-39	40-49	50-59	60-69	70 and over
Data	1984	65.6	10.9	51.8	74.5	82.4	84.0	82.8
	1989	65.9	10.4	50.5	75.6	82.7	86.2	83.6
	1994	64.8	9.5	46.0	74.8	82.5	87.2	84.6
	2004	66.9	11.9	50.7	75.8	83.9	87.3	87.1
Model	Baseline	65.6	4.5	41.1	80.0	91.0	95.2	98.1
	1989	69.2	11.1	42.1	79.7	98.6	99.8	99.6
	1994	64.3	7.9	47.1	76.7	83.6	89.9	95.4
	2004	68.9	9.3	53.1	86.1	92.4	92.4	95.2

Next, Figure 32 shows the time series of the average homeownership rate in the U.S. The bold solid line represents the simulation results, the dotted line represents the (counterfactual) results without the downpayment requirement reduction and the dashed line represents the data. The model quantitatively and qualitatively matches with the homeownership rate increase. The model shows a 9.6% increase in the homeownership rate due to the changes.

Table 16 shows the age distribution of the homeownership rate in the U.S. The last row is the (counterfactual) results without the downpayment rate change. The table clarifies two different mechanisms which cause the homeownership rate increase in every age cohort. First, the higher TFP growth rate and the lower interest rates encourage more households to buy houses. Consequently old households, who are less constrained, increase their ownership house purchases. On the other hand, due to the higher housing prices caused by the changes, young households, who are more likely to be constrained, cannot purchase ownership houses. Second, the reduction in downpayment requirements loosens above constraints of the young. As a result, the homeownership rate among young households increases, while that of old households remains mostly unchanged. Therefore, I conclude that the increase in the homeownership rate among the young is due to the loosened downpayment requirements and that of the old is due to the higher TFP growth rate and the lower interest rates.

Table 16: Homeownership rate distribution by age cohort: U.S.

		average	20-29	30-39	40-49	50-59	60-69	70 and over
Data	1995	65.0	27.4	57.6	71.5	77.9	81.4	76.8
	2005	68.9	34.1	62.4	73.5	79.9	82.8	79.0
Model	1995	64.5	23.6	58.0	71.1	78.3	83.3	86.9
	2005	74.1	33.7	62.0	84.1	94.0	95.7	93.4
	2005(w/o Down)	68.5	24.6	52.2	78.7	92.9	95.7	93.4

7 Conclusion

This paper accounts for the changes in housing prices, the housing price-to-rent ratio, and the homeownership rate in the last two decades in Japan and the last decade in the United States by the changes of interest rates, the growth rate of total factor productivity, and downpayment requirements. First, the model shows that the decrease in the interest rates and the increase in the TFP growth rate account for most of the increase in housing prices and the housing price-to-rent ratio in the late 80's in Japan and in the last decade in the U.S. as well as the subsequent decrease in housing prices and the housing price-to-rent ratio in the 90's in Japan. Second, I find that while the change in downpayment requirements has little effect on housing prices and the housing price-to-rent ratio, the difference in the homeownership rate change between Japan and the U.S. is likely to be accounted for by the downpayment requirement change.

There are two ways to extend this model for the future research: the introduction of aggregate shocks and endogenising the interest rate change and the downpayment requirement change. First, this model is a perfect foresight model and all changes come as a surprise. it is more natural to assume that households know the aggregate shock process and form expectations. Second, in this model, the changes of interest rates and downpayment requirements are taken to be exogenous. It is important to know why the reduction in downpayment requirements happened in the last decade in the U.S. At the same time, it is also important to examine why interest rates went down in Japan and the U.S. for assessing the effect of the globalization and the policies of central banks. Though these extensions are demanding, they are necessary step forward to take.

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8 Appendix

8.1 Appendix A: Price characterizations

Here I characterize equilibrium prices. Note that there is no aggregate shock in this economy, thus all prices are known to all households.
From rental sector's problem,

$$p_{r,t} = (1+\delta_r)p_{h,t} - \frac{p_{l,t}L_{h,t}}{H_t} - \frac{1}{1+r_{t+1}}\Big\{p_{h,t+1}\Big\} \tag{53}$$

From the residential structure production sector's problem,

$$\alpha_s \frac{p_{s,t}Y_{s,t}}{K_{s,t}} = \delta_k + r_t \tag{54}$$

$$(1-\alpha_s)\frac{p_{s,t}Y_{s,t}}{N_{s,t}} = w_t \tag{55}$$

From the housing production sector's problem,

$$\alpha_h \frac{p_{h,t+m}Y_{h,t+m}}{L_{h,t}} = p_{l,t} \tag{56}$$

$$(1-\alpha_h)\frac{p_{h,t+m}Y_{h,t+m}}{Y_{s,t}} = p_{s,t} \tag{57}$$

Lastly, from the non housing production sector's problem,

$$\frac{\alpha_c Y_{c,t}}{K_{c,t}} = \delta_k + r_t \tag{58}$$

$$\frac{(1-\alpha_c)Y_{c,t}}{N_{c,t}} = w_t \tag{59}$$

Combined with market clearing condition, it is easy to show,

$$w_t = (\delta_k + r_t)\left(\frac{1-\alpha_c}{\alpha_c}K_{c,t} + \frac{1-\alpha_s}{\alpha_s}K_{s,t}\right)\frac{1}{\bar{N}\,t} \tag{60}$$

$$p_{l,t} = \frac{\alpha_h}{(1-\alpha_h)\alpha_s}(\delta_k + r_t)K_{s,t}\frac{1}{\bar{L}\,t} \tag{61}$$

$$p_{h,t+m} = \frac{1}{\alpha_h Y_{h,t+m}} p_{l,t}\bar{L} \tag{62}$$

$$Y_{h,t+m} = z_{s,t}^{(1-\alpha_h)} K_{s,t}^{\alpha_s(1-\alpha_h)} N_{s,t}^{(1-\alpha_s)(1-\alpha_h)} \tag{63}$$

$$K_{s,t} = \frac{\alpha_s}{1-\alpha_s}\frac{1-\alpha_c}{\alpha_c}\left[-K_{c,t} + \left(\frac{\alpha_c z_{c,t}}{\delta_k + r_t}\right)^{\frac{1}{1-\alpha_c}}\bar{N}_t\right] \tag{64}$$

$$N_{s,t} = \frac{\frac{1-\alpha_s}{\alpha_s}K_s}{\frac{1-\alpha_c}{\alpha_c}K_{c,t} + \frac{1-\alpha_s}{\alpha_s}K_{s,t}}\bar{N}_t \tag{65}$$

$$N_{c,t} = \bar{N} - N_{s,t}$$

$$= \frac{\frac{1-\alpha_c}{\alpha_c}K_{c,t}}{\frac{1-\alpha_c}{\alpha_c}K_{c,t} + \frac{1-\alpha_s}{\alpha_s}K_{s,t}}\bar{N}_t \tag{66}$$

Value of the firm is equal to the price of assets.

$$\Gamma(y_{r,t-1}) \quad = \quad p_{h,t}Y_{r,t-1} \tag{67}$$

We are left with two sequence of variables $K_{c,t}, r_t$ which will be determined by the household's problem.

8.2 Appendix B: Detrended household problem/firm problem

I can rewrite the household problems and firms' problems by detrending growth. Define $\hat{g_x} = g_x/g_n$ and make variable transfromation $\hat{x}_t = x_t/\hat{g_x}^t$ by detrending all the variables of its growth.

8.2.1 detrended problem of firms

The problems of four representative firms are transformed as follows.

- **Non housing consumption production sector:**

$$\max_{\hat{K}_{c,t},\hat{N}_{c,t}} \hat{Y}_{c,t} - (\delta_k + r_t)\hat{K}_{c,t} - \hat{w}_t\hat{N}_{c,t} \tag{68}$$

$$\text{s.t. } \hat{Y}_{c,t} = \hat{z}_c\hat{K}_{c,t}^{\alpha_c}\hat{N}_{c,t}^{1-\alpha_c},\ 0 < \alpha_c < 1 \tag{69}$$

- **Residential structure production sector:**

$$\max_{\hat{K}_{s,t},\hat{N}_{s,t}} \hat{p}_{s,t}\hat{Y}_{s,t} - (\delta_k + r_t)\hat{K}_{s,t} - \hat{w}_t\hat{N}_{s,t} \tag{70}$$

$$\text{s.t. } \hat{Y}_{s,t} = \hat{z}_s\hat{K}_{s,t}^{\alpha_s}\hat{N}_{s,t}^{1-\alpha_s},\ 0 < \alpha_s < 1 \tag{71}$$

- **Housing production sector:**

$$\max_{\hat{Y}_{s,t},\hat{L}_{s,t}} g_{p_h}^m\hat{p}_{h,t+m}g_h^m\hat{Y}_{h,t+m} - \hat{p}_{s,t}\hat{Y}_{s,t} - \hat{p}_{l,t}\hat{L}_{h,t} \tag{72}$$

$$\text{s.t. } g_h^m\hat{Y}_{h,t+m} = \hat{Y}_{s,t}^{1-\alpha_h}\hat{L}_{h,t}^{\alpha_h},\ 0 < \alpha_h < 1 \tag{73}$$

- **Real estate (rental housing) sector:**

$$\begin{aligned}\hat{\Gamma}(\hat{H}_{r,t-1})/g_h \quad = \quad & \max_{\hat{H}_{r,t}} \hat{p}_{h,t}(\hat{H}_{r,t-1}/g_h - \hat{H}_{r,t}) + \hat{p}_{r,t}\hat{H}_{r,t} \\ & -MC_r(\hat{H}_{r,t}) + \frac{1}{1+r_{t+1}}\hat{\Gamma}(\hat{H}_{r,t})\end{aligned} \tag{74}$$

8.2.2 detrended problem of households

Simlary, the household problem can be rewritten as,

$$\hat{v}(i,\hat{a},\epsilon) = \max_{b\in\{r,o\}}\{\hat{V}(i,\hat{a},\epsilon), \hat{W}(i,\hat{a}',\epsilon')\} \tag{75}$$

where

$$\begin{aligned}
\hat{V}(i,\hat{a},\epsilon) &= \max_{(\hat{c},\hat{s}',\hat{h})} u(\hat{c},\hat{h}) + \hat{\beta}\psi_i \sum_{\epsilon'\in\mathcal{E}} \pi(\epsilon,\epsilon')\hat{v}(i+1,\hat{a},\epsilon) && (76)\\
&\qquad + \mathcal{I}_{i=I}\hat{\beta} b f(\hat{a}') \\
\text{s.t.} \quad & \hat{c} + \hat{g}_k\hat{s}' + \hat{p}_{r,t}\hat{h} = \hat{a}/\psi_{i-1} + \hat{y}(i,\epsilon) && (77)\\
& \hat{c}\geq 0 \qquad \hat{h}\geq \hat{h}_{min} \qquad \hat{s}'\geq 0 && (78)\\
& \hat{a}' = (1+r)\hat{s}' && (79)\\
& \text{where } \hat{\beta} = \beta\hat{g}_k^{\theta_u(1-\gamma_u)}\hat{g}_h^{(1-\theta_u)(1-\gamma_u)} && (80)\\
W(i,\hat{a},\epsilon) &= \max_{(\hat{c},\hat{s}',\hat{h})} u(\hat{c},\hat{h}) + \hat{\beta}\psi_i \sum_{\epsilon'\in\mathcal{E},\xi'\in\Xi} \pi(\epsilon,\epsilon')\pi(\xi')v(i+1,\hat{a}'(\xi'),\epsilon') && (81)\\
&\qquad + \mathcal{I}_{i=I}\hat{\beta} b f(a') \\
\text{s.t.} \quad & \hat{c} + \hat{g}_k\hat{s}' + \hat{p}_h\hat{h} + MCo(\hat{h}) = \hat{a}/\psi_{i-1} + \hat{y}(i,\epsilon) && (82)\\
& \hat{c}\geq 0 \qquad \hat{h}\geq \hat{h}_{min} \qquad \hat{g}_k\hat{s}' \geq -(1-\lambda)\hat{p}_h\hat{h} && (83)\\
& \hat{a}'(\xi) = (1+r)\hat{s}' + \hat{p}_h\hat{h}/\hat{g}_h - \mathcal{I}_{\xi'=move}AC(\hat{h})/\hat{g}_h && (84)
\end{aligned}$$

8.3 Appendix C technology calibration

Here I explain how capital shares of two sectors, the non housing consumption sector and the residential structure production sector, are calibrated. Basic idea is to attribute total output Y, total labor N, total capital K and other related data into two sectors. In what follows I call non housing consumption production sector as COP and residential structure production sector as RSP.

Output from RSP $p_{s,t}Y_{s,t}$ is measured as private residential fixed investment from NIPA and output from COP $Y_{c,t}$ is measured as total output Y minus private residential fixed investment. Here Y is GDP plus imputed durable services minus housing services minus government consumption of fixed capital.

Next I attribute capital K and labor N into two sectors. Following Davis and Heathcote (2005), I attribute residential fixed investment to sales from construction sector. Let assume the attribution rate of construction output to construction sector is κ. As Davis and Heathcote (2005) mention, there are two ways to define construction sector sales. The first way is to consider all sales of construction sector as what is produced within the construction sector. The second way is to take into account the fact that some part of output from construction sector attributes to construction sector and the other part comes from other sector as intermediate goods, then track down all the intermediate goods which are used in construction sector. I take the latter way since it captures composition of the housing structure more precisely.

I assume there exists no intermediate sales from construction sector to non construction sector and all the intermediate sales of construction sector attribute to non-construction sector.[32] As a results I find 50.8% of construction output (private fixed residential investment) attributes to construction sector and the other 49.2% to non-construction sector for the U.S. benchmark. Thus κ is 0.508 for U.S. Similarly, I find that κ is 0.444 in Japan. Let $\theta_{i,j}$ be the share of output of j sector which is counted into i sector's output. Subscript i represents either RSP sector s or COP sector c and subscript j represents either construction sector or non-construction sector. Given the information above, $\theta_{i,j}$ can be computed as,

$$\theta_{s,const,t} = \frac{p_{s,t}Y_{s,t} * \kappa}{Y_{const,t}} \tag{85}$$

$$\theta_{c,const,t} = 1 - \theta_{c,const,t} \tag{86}$$

$$\theta_{s,non-const,t} = \frac{p_{s,t}Y_{s,t} * (1-\kappa)}{Y_{non-const}} \tag{87}$$

$$\theta_{c,non-const,t} = 1 - \theta_{s,non-const,t} \tag{88}$$

I document average weight from 1947 to 1997 in the U.S. and from 1955 to 1998 in Japan in Table XXX.

Table 17: Average weight $\theta_{i,j,t}$

j \i	U.S.		Japan	
	RSP	COP	RSP	COP
construction	53.9 ($\bar{\theta}_{s,const}$)	46.1 ($\bar{\theta}_{c,const}$)	30.7 ($\bar{\theta}_{s,const}$)	69.3 ($\bar{\theta}_{c,const}$)
non-construction	2.5 ($\bar{\theta}_{s,non-const}$)	97.5 ($\bar{\theta}_{c,non-const}$)	3.6 ($\bar{\theta}_{s,non-const}$)	96.4 ($\bar{\theta}_{c,non-const}$)

Then we are able to construct labor and capita input for RSP sector and COP sector.

$$N_{c,t} = \bar{\theta}_{c,const} * N_{const} + \bar{\theta}_{c,non-const} * N_{non-const}$$
$$N_{s,t} = \bar{\theta}_{s,const} * N_{const} + \bar{\theta}_{s,non-const} * N_{non-const}$$
$$K_{c,t} = \bar{\theta}_{c,const} * K_{const} + \bar{\theta}_{c,non-const} * K_{non-const}$$
$$K_{s,t} = \bar{\theta}_{s,const} * K_{const} + \bar{\theta}_{s,non-const} * K_{non-const}$$

[32] I use IO Use table (2003) to compute intermidiate share of each sectors. From the IO Use table for U.S. data, I find that 99.9% of intermediate goods used in construction sector comes from non-construction sector and 98% of intermediate goods used in non-construction sector comes from non-construction sector. It is safe to assume construction sector output is not used from intermediate goods in both two sectors. Similary I find that there exists small rate of intermediate input from construction sector into two sectors in Japan.

Finally, I need to attribute other statistics to each sector. Following Davis and Heathcote (2005), I classify value added by industry (VA) as output of each sector, compensation for employee (COM) as unambiguous labor income and classify proprietor's income (Pro) and tax on production less subsidy (Tax) as ambiguous labor income[33]. Capital income share is defined as

$$\alpha_{i,t} = 1 - \frac{\sum_j \theta_{i,j,t} COM_{j,t}}{\sum_j \theta_{i,j,t}(VA_{j,t} - Pro_{j,t} - Tax_{j,t})}$$

I use GDP by Industry Data from Industry Economic Accounts at Bureau of Economic Analysis from 1948 to 1997 and SNA data from 1948 to 1998 in Japan. Consequently, taking a sample average I obtain 0.291 as capital share of the COP(non housing consumption production) sector and 0.211 as capital share of RSP (residential structure production) sector. Similarly, I take 0.332 and 0.292 as capital share of COP and RSP in Japan respectively.

8.4 Appendix D initial asset distribution estimation

The initial asset distribution is estimated by fitting Pareto distribution to asset to earning ratio of young households.

8.4.1 U.S.

It is well known that probability density function of Parero distribution is given with two parameters b_1 and b_2 by,

$$f(x; b_1, b_2) = b_2 \frac{{b_1}^{b_2}}{x^{b_2+1}} \tag{89}$$

and cumlative distribution is given by,

$$Pr(X > x) = \left(\frac{x}{b_1}\right)^{-b_2} \tag{90}$$

Also likelihood function for the Pareto distribution parameters b_1 and b_2, given a sample $x = (x_1, x_2, \ldots, x_n)$ is,

$$L(b_1, b_2) = \prod_{i=1}^{n} b_2 \frac{b_1^{b_2}}{x_i^{b_2+1}} = b_2^n b_1^{b_2 n} \prod_{i=1}^{n} \frac{1}{x_i^{b_2+1}} \tag{91}$$

Therefore, the logarithmic likelihood function is

$$l(b_1, b_2) = n \ln b_2 + n b_2 \ln b_1 - (b_2 + 1) \sum_{i=1}^{n} \ln x_i \tag{92}$$

[33]I use private industry minus housing sector data of VA, COM, Pro and tax for non construction sector so that I keep consistency with the measure of output in this model. In SNA doesn't have classification of "housing sector" so I subtract the part of real estate sector which is related to residential housing. Namely I attribute 78% of real estate sector output to housing sector. Since value added by private industry is not equal to our GDP thus, I rescale all the components of value added by private industry so that I match output value in this model.

It can be seen that $l(b_1, b_2)$ is monotonically increasing with b_1, and therefore estimator of $\hat{b}_1$ is,

$$\hat{b}_1 = \min_i x_i \tag{93}$$

Also estimator of b_2 is,

$$\hat{b}_2 = \frac{n}{\sum_i (\ln x_i - \ln \hat{b}_1)} \tag{94}$$

Estimation follows following steps.

1. Use household samples with age from 18 to 23 of Survey of Consumer Finance (1995)
2. Replace asset value to zero, if households have a negative asset.
3. Since Pareto distribution is defined large than zero, compute asset/income+1 by shifting all the ratio by unity.
4. Fit Pareto distribution by maximizing likelihood. $\hat{b}_1 = 1$ by construction and I obtain $\hat{b}_2 = 1.83$

   It should be noted that due to the shifting, the cumulative distribution is given by and cumulative distribution is given by,

$$Pr(x > X) = 1 - (x + 1)^{-1.83} \tag{95}$$

8.4.2 Japan

The only aggregate data is available for NSEFI. I use following method to estimate initial distribution. The first step is to estimate income distribution. Let $y_{n,30}$ be the income of household n with age under 30. I assume that income distribution can be mimicked by log normal process. Thus,

$$\log y_{n,30} \sim \mathcal{N}(\mu_{y30}, \sigma_{y30}) \tag{96}$$

Since the average income is known, so only parameter I should estimate is variance. Thus I use the following income distribution data of age under 30 to estimate variance of income distribution. Monte Carlo simulation method is used to generate one hundred thousand draws and count the number of draw which fall into each income grid. Then by minimizing the error between data and simulation in each bin I estimate the variance. When minimizing the error, identity matrix is used.

The next step is to estimate parameter value of Pareto distribution. Let the asset to income ratio be $\kappa_{n,30}$. It is assume to be drawn from Pareto distribution. Further assume that the asset to income ratio is independent from income level then we can back out asset as $w_{n,30} = \kappa_{n,30} y_{n,30}$. Following the same way I use to estimate the income distribution, I use Monte Carlo simulation to generate hundred thousand set of draws $(\kappa_{n,30}, y_{n,30})$ and minimize the error of each bin of asset distribution. Same as U.S. case, I assume $\hat{b}_1 = 1$ and given that I estimate the value $\hat{b}_2$. $\hat{b}_2$ is estimated to be 1.582.

8.5 Appendix E income process estimation (U.S.)

Earnings process is estimated from the Panel Study of Income Dynamics (PSID), following Nakajima (2006) which uses the method developed by Heathcote et al. (2003). I document the detained procedure of estimation here.

The first step is a sample selection. Criteria I use are similar to the previous studies (Heathcote et al.(2003), Nakajima (2006), Guvenen(2007)). I use PSID data from 1968 to 1993 with households who have following characteristics. (1) White male, (2) Head of households, (3) age from 20 to 65, (4) report positive but not top coded income, (5) Working hours from 520 to 5096 per year, (6)hourly income is between minimum wage and maximum wage, (7) in a sample more than two consecutive years (8) not poverty (SEO) subsample.

The second step is to compute time invariant lifecycle income profile. I pool all the samples from 1968 to 1993 and regress earnings on a fourth order polynomial in age. average lifecycle income profile $\bar{v}_i$ is estimated as

$$\bar{v}_i = c_0 + c_1 i + c_2 i^2 + c_3 i^3 + c_4 i^4 \tag{97}$$

where $c_j, 1 \leq j \leq 4$ are coefficients of regression. Figure O shows the result.

The third step is to estimate parameters associated with the labor income shock. Functional form of income process for households b with age i, at period t which I use for estimation is,

$$\hat{y}_{b,i,t} = \hat{\epsilon}_{b,i,t}\hat{v}_{i,t} \tag{98}$$

$$\log(\hat{\epsilon}_{b,i,t}) = \hat{\eta}_{b,i,t} + \pi_t \hat{\varepsilon}_{b,i,t} \qquad \hat{\varepsilon}_t \sim \mathcal{N}(0, \sigma^2_{\hat{\varepsilon}}) \tag{99}$$

$$\hat{\eta}_t = \rho_\eta \hat{\eta}_{t-1} + \tau_t \hat{\zeta}_t \qquad \hat{\zeta}_t \sim \mathcal{N}(0, \sigma^2_{\hat{\zeta}}) \tag{100}$$

The parameters which need to be estimated is $\{\tau_t\}_{t=1967}^{1992}, \{\tau_t\}_{t=1967}^{1992}, \rho$ Note $\sigma^2_{\hat{\zeta}}$ and $\sigma^2_{\hat{\varepsilon}}$ is normalized to one and PSID reports income of previous year. extract labor income shocks from process.

1. Regress data ($\log \hat{y}_{b,i,t}$) on fourth order polynomial of age with time variant coefficient and store residuals on $\log \hat{\epsilon}_{b,i,t}$.

$$\log \hat{v}_{i,t} = c_{0,t} + c_{1.t} i + c_{2,t} i^2 + c_{3,t} i^3 + c_{4,t} i^4 \tag{101}$$

$$\log \hat{\epsilon}_{b,i,t} = \log \hat{y}_{b,i,t} - \log \hat{v}_{i,t} \tag{102}$$

2. Compute theoretical variance covariance matrix from functional from.
variance of income shock of household b at age i and period i is,

$$var(\log \hat{\epsilon}_{b,i,t}) = var(\hat{\eta}_{b,i,t}) + \pi_t^2 \tag{103}$$

where

$$\begin{array}{ll} var(\hat{\eta}_{b,i,t}) = \rho^2 var(\hat{\eta}_{b,i-1,t-1}) + \tau_t^2 & \text{if } i > 20 \text{ and } t > 1967 \\ var(\hat{\eta}_{b,i,t}) = \sum_{j=20}^{i-1} \rho^{2j} \tau_t^2 & \text{if } t = 1967 \\ var(\hat{\eta}_{b,i,t}) = \tau_t^2 & \text{if } i = 20 \end{array} \tag{104}$$

Table 18: Statistics of PSDI Sample 1968-1993

year	Number of observations	average age	average hourly income	mean log income	variance log income
1968	1,307	40.40933	14.67297	2.5489	0.2805579
1969	1,388	40.67003	15.19797	2.578162	0.2859008
1970	1,507	40.38354	15.47549	2.593333	0.300674
1971	1,552	39.90851	15.89749	2.612016	0.3098458
1972	1,624	39.14409	15.91564	2.612326	0.3081035
1973	1,689	38.50266	16.34204	2.645574	0.2950386
1974	1,735	38.06744	16.42606	2.655831	0.2862024
1975	1,790	37.70894	16.28358	2.640306	0.2948624
1976	1,812	37.45254	15.73187	2.61083	0.2898247
1977	1,854	37.41909	16.20751	2.636568	0.2976124
1978	1,893	37.40412	16.56929	2.670111	0.2754476
1979	1,923	37.37546	17.01671	2.68637	0.2917599
1980	1,960	37.50663	16.73076	2.670523	0.3042286
1981	1,981	37.55528	16.39298	2.647738	0.3122219
1982	2,027	37.70893	16.14498	2.627254	0.3232315
1983	2,027	37.65762	16.27611	2.610492	0.3613806
1984	2,040	37.82304	16.24505	2.607638	0.3594112
1985	2,102	38.00809	16.85787	2.628421	0.3798994
1986	2,117	38.10392	17.04067	2.631259	0.3951045
1987	2,143	38.37984	17.35964	2.657786	0.3883097
1988	2,150	38.47302	17.32919	2.655518	0.3925347
1989	2,164	38.73152	17.66551	2.673478	0.3903341
1990	2,186	38.98536	17.34307	2.648103	0.3989014
1991	2,161	39.11476	17.24251	2.640783	0.3951588
1992	2,060	39.76214	17.91367	2.671883	0.3991032
1993	1,851	40.49163	19.28607	2.754513	0.3925266
Total	49,043	38.49501	16.68132	2.64168	0.340602

and covariance of the same household with a years before is,

$$cov(\hat{\eta}_{b,i,t}, \hat{\eta}_{b,i-a,t-a}) = \rho^a var(\hat{\eta}_{b,i-a,t-a}) \tag{105}$$

Let this theoretical variance covariance matrix be $\mathcal{V}$ and $vec(\mathcal{V})$ be vectorized version of $\mathcal{V}$.

3. It is not difficult to compute sample variance covariance matrix. In order to increase sample size, I take age i as a band from age $i-4$ to age $i+5$. For example, New sample of age "24" contains households with age 20 to 29 in original sample. Now total age bin reduce from 46 to 37. Let this sample variance covariance matrix be $\hat{\mathcal{V}}$ and vectorized version be $vec(\hat{\mathcal{V}})$

4. Lastly, estimate the parameters of income process by minimizing the distance between variance covariance matrix calculated by samples and that calculated theoretically from model parameters. Parameter results are show in Figure 0.

Since the focus of this model is not a change in earning process, I use average parameters value as income shock process. Thus I have $\sigma_\epsilon = 0.2447, \sigma_\eta = 0.1441, \rho = 0.9783$.

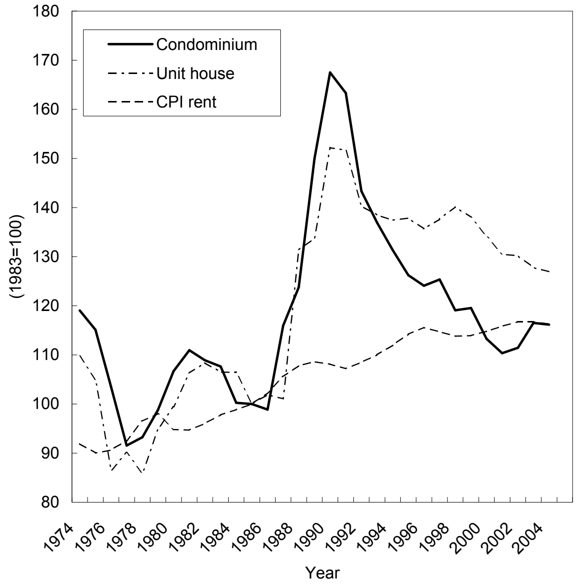

Figure 1: Housing prices and rents (JP)

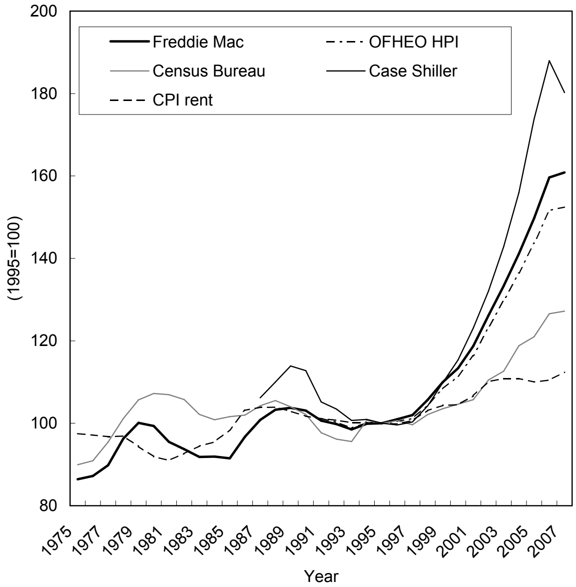

Figure 2: Housing prices and rents (US)

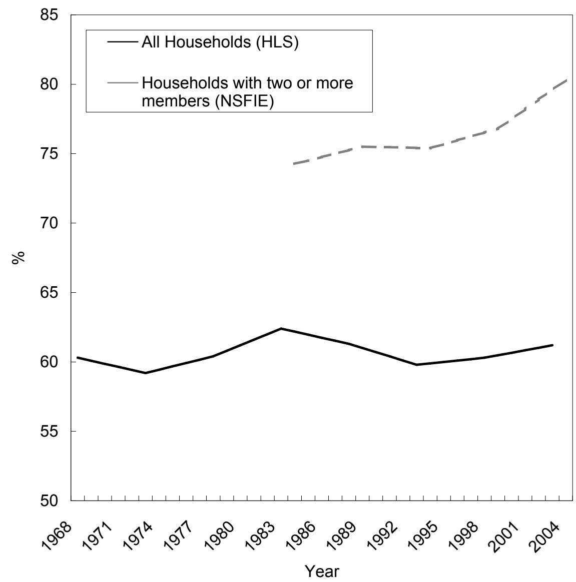

Figure 3: Homeownership rate (JP)

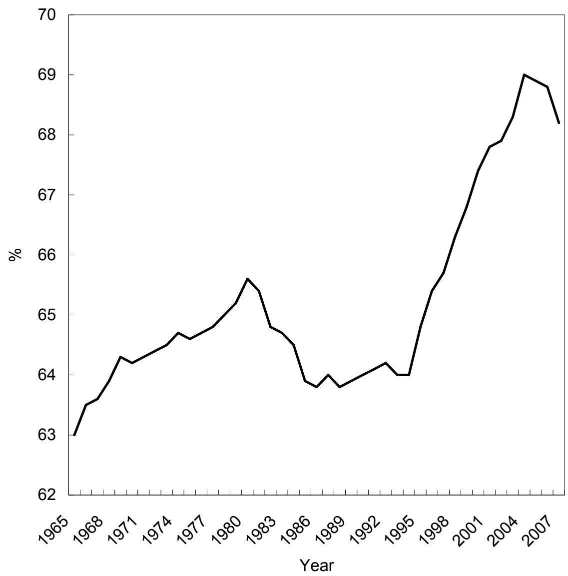

Figure 4: Homeownership rate (US)

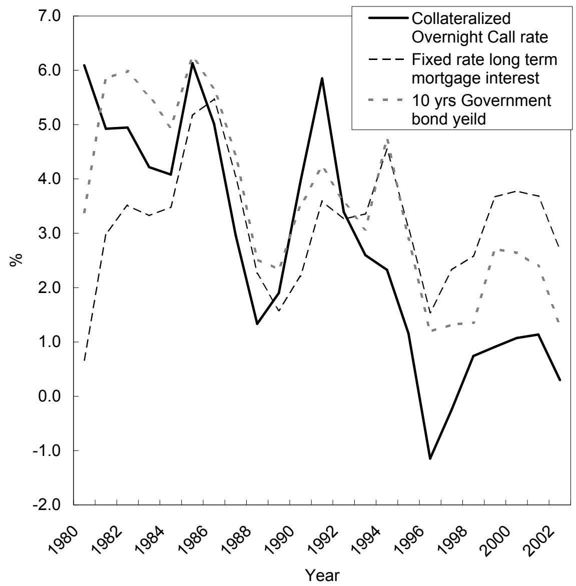

Figure 5: Interest rates (JP)

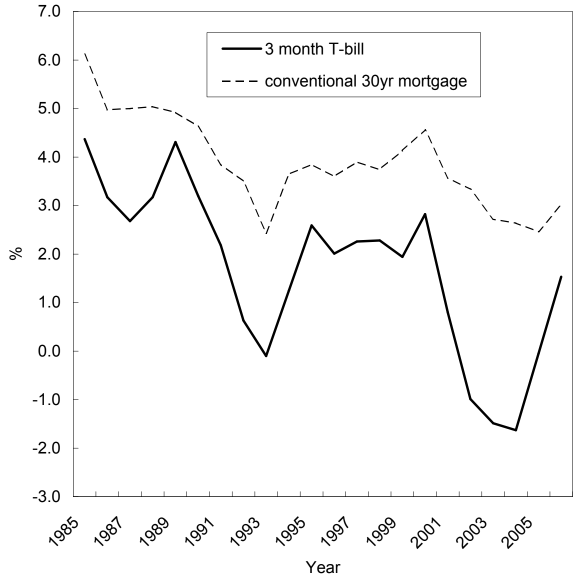

Figure 6: Interest rates (US)

Figure 7: TFP growth rate (JP)

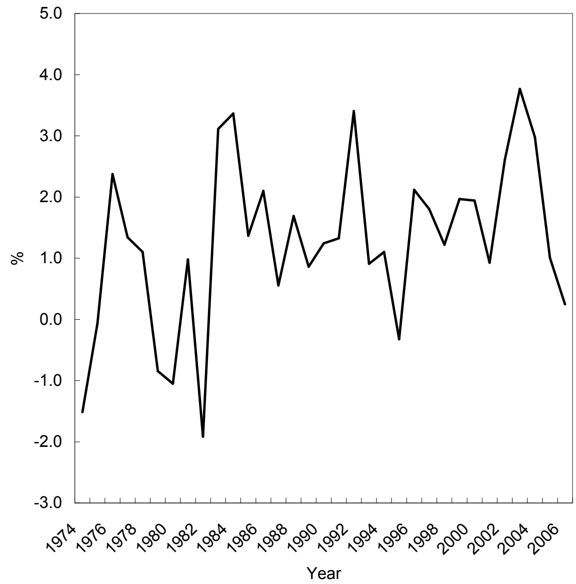

Figure 8: TFP growth rate (US)

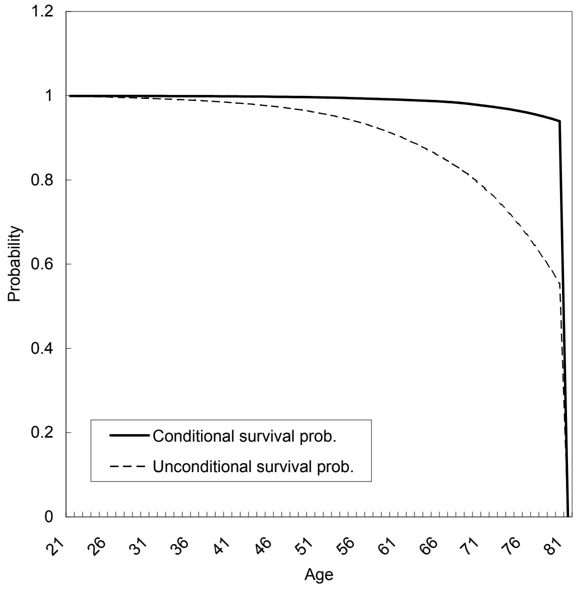

Figure 9: Survival rates (JP)

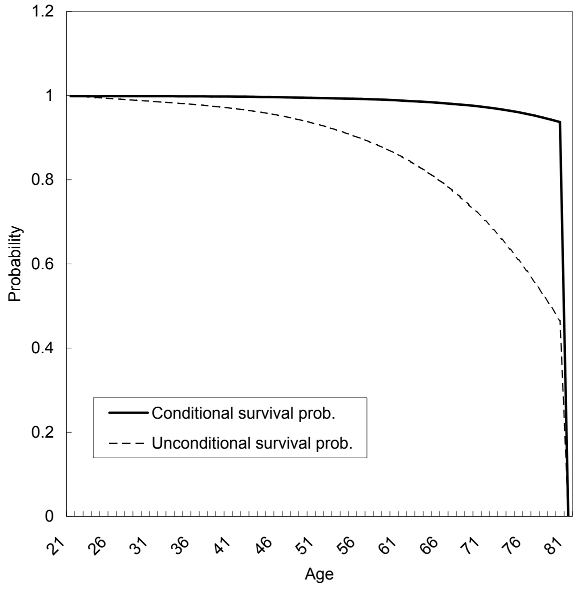

Figure 10: Survival rates (US)

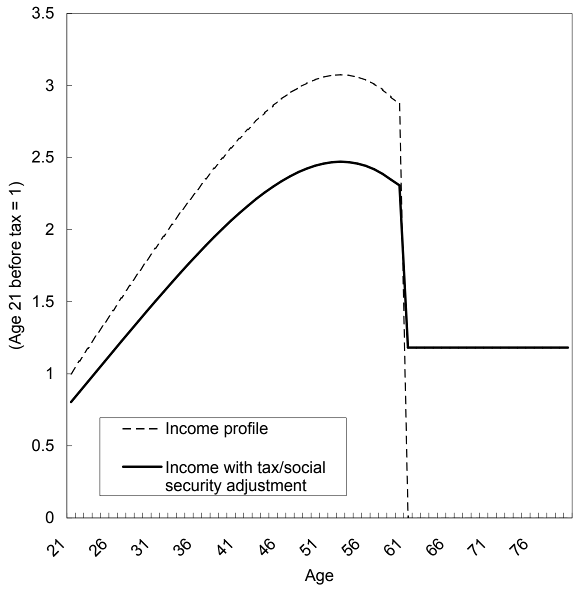

Figure 11: Life cycle income profile (JP)

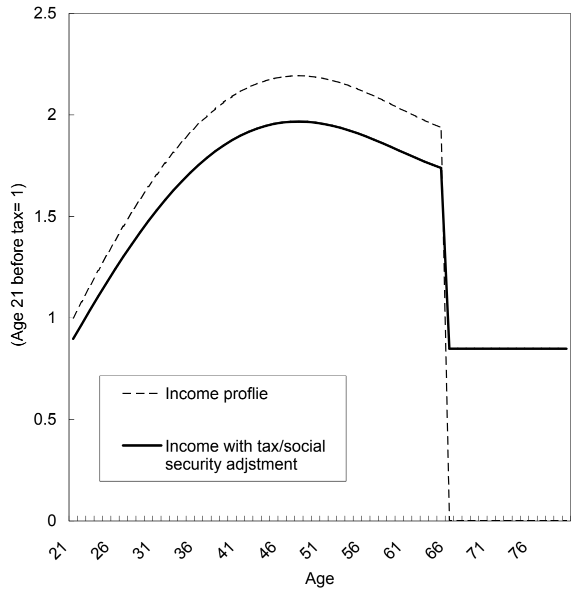

Figure 12: Life cycle income profile (US)

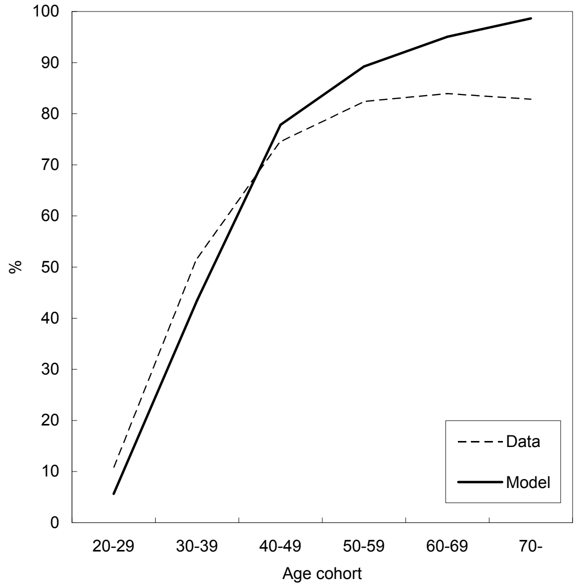

Figure 13: Homeownership rate (JP)

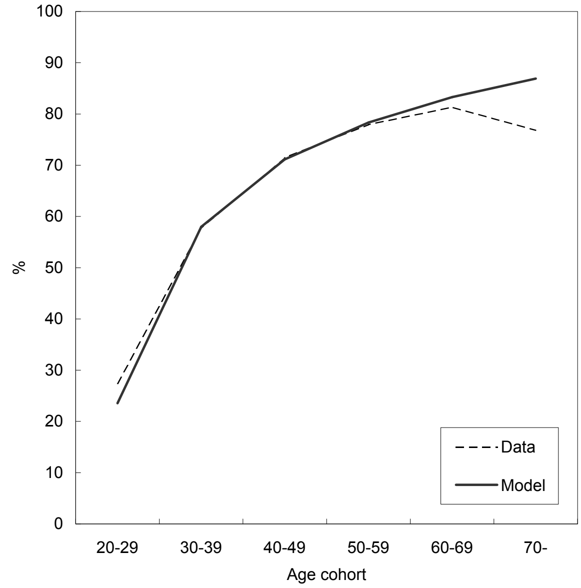

Figure 14: Homeownership rate (US)

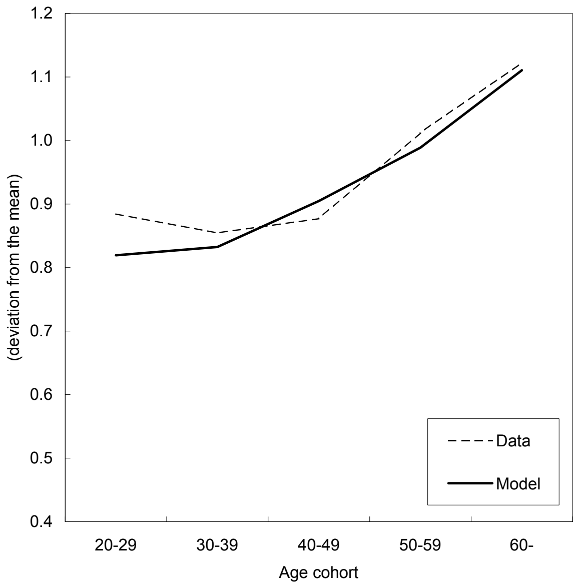

Figure 15: Housing size (JP)

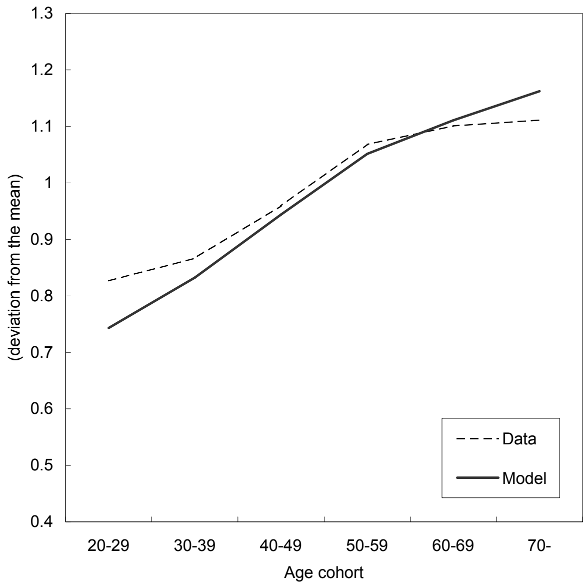

Figure 16: Housing size (US)

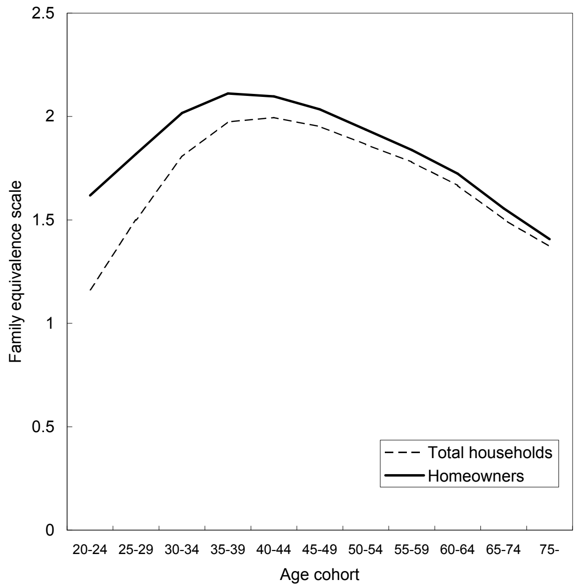

Figure 17: Household size (JP)

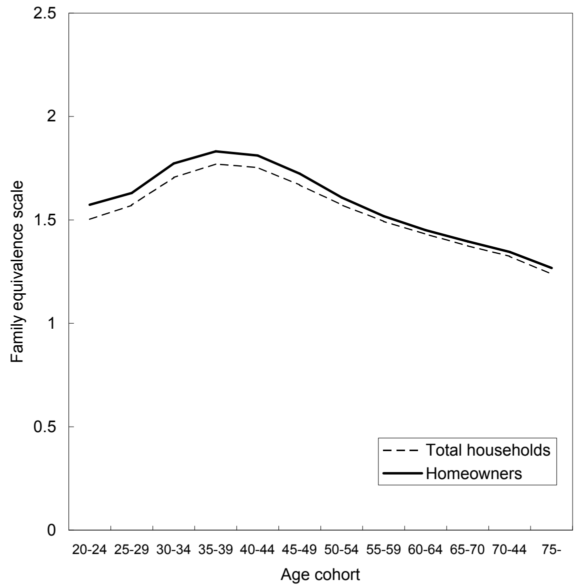

Figure 18: Household size (US)

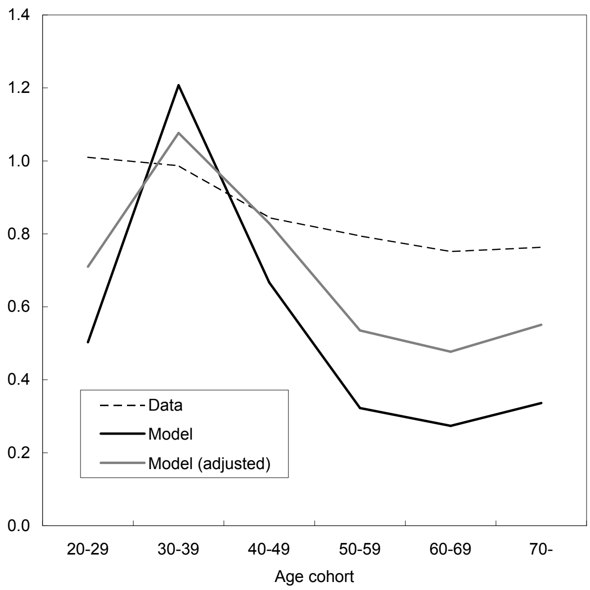

Figure 19: Housing asset to total net wealth ratio (JP)

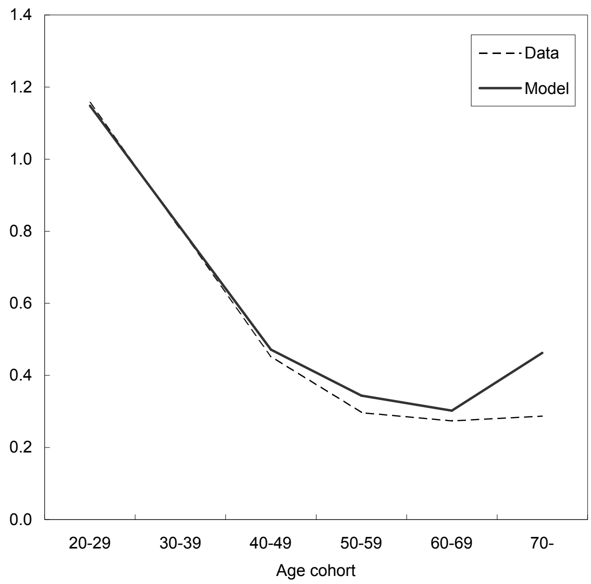

Figure 20: Housing asset to total net wealth ratio (US)

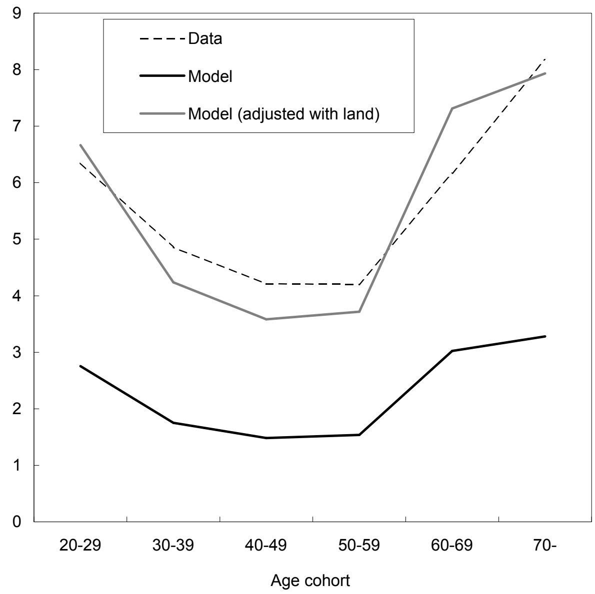

Figure 21: Housing asset to average income ratio (JP)

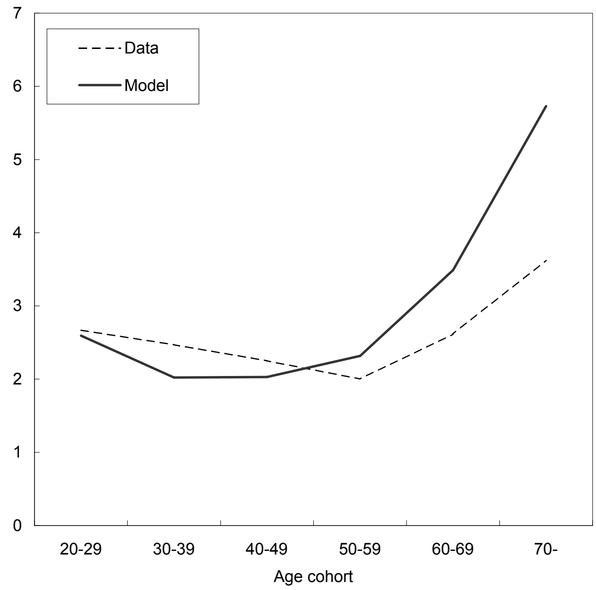

Figure 22: Housing asset to average income ratio (US)

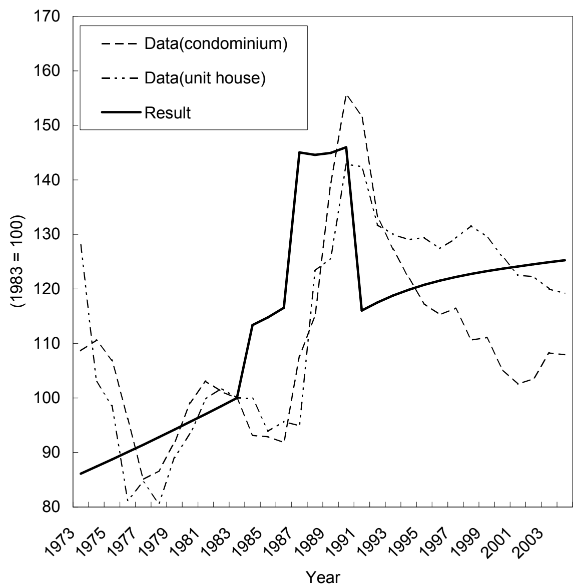

Figure 23: Housing price transition (JP)

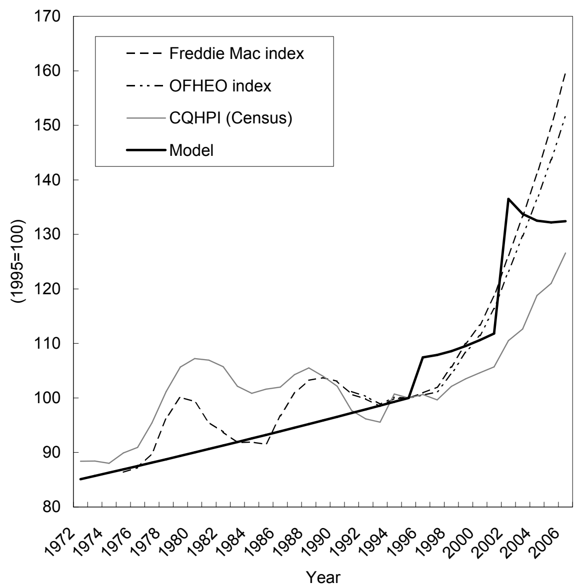

Figure 24: Housing price transition (US)

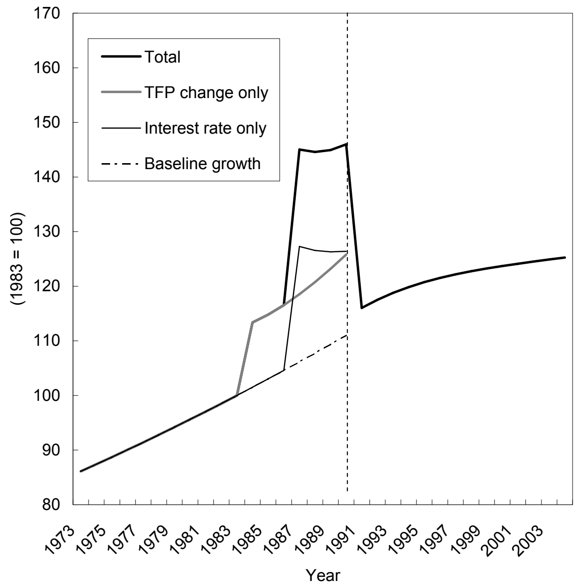

Figure 25: Decomposition (housing prices:JP)

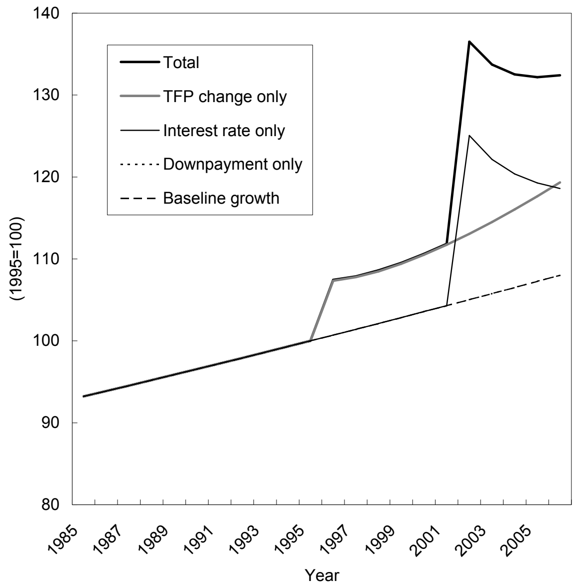

Figure 26: Decomposition (housing prices:US)

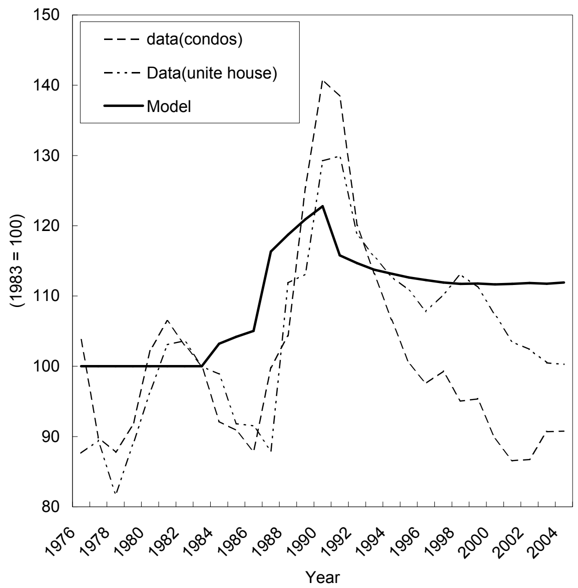

Figure 27: Housing price-to-rent ratio transition (JP)

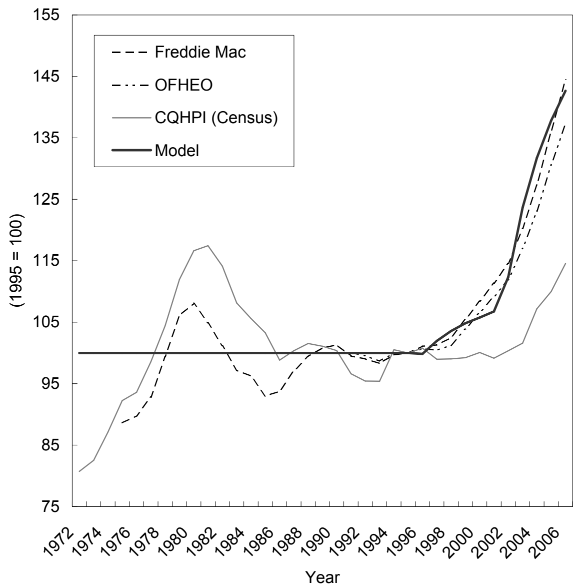

Figure 28: Housing price-to-rent ratio transition (US)

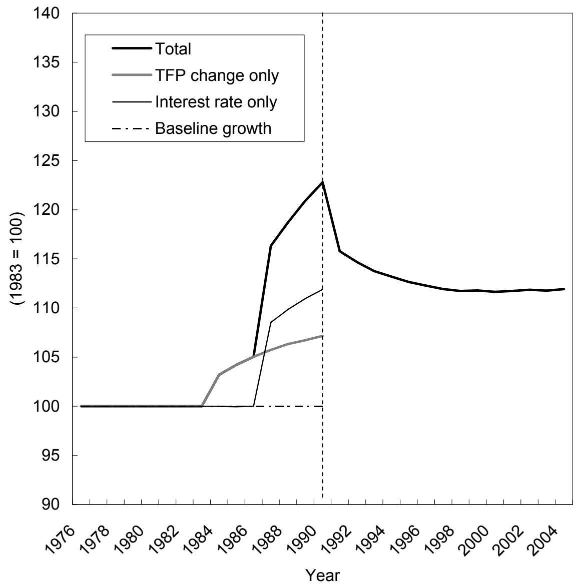

Figure 29: Decomposition (Housing price-to-rent ratio:JP)

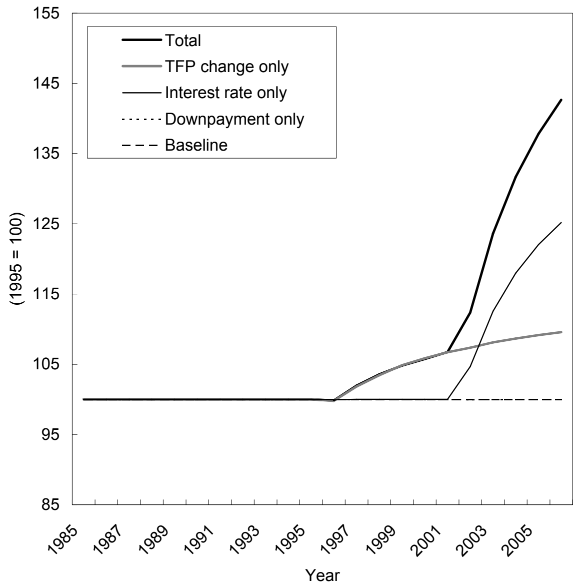

Figure 30: Decomposition (Housing price-to-rent ratio:US)

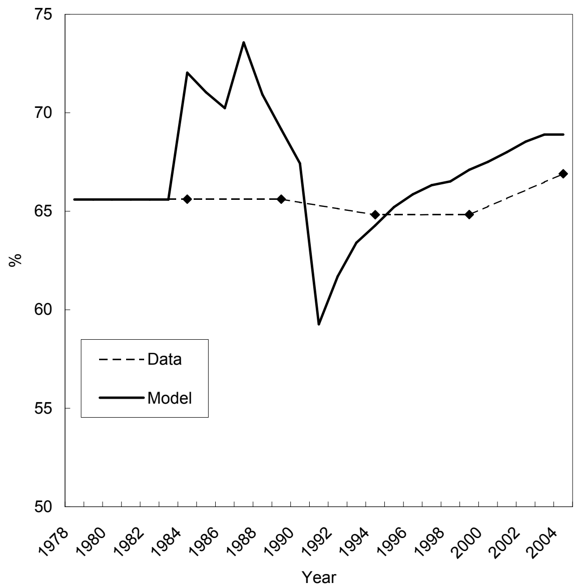

Figure 31: Homeownership rate transition (JP)

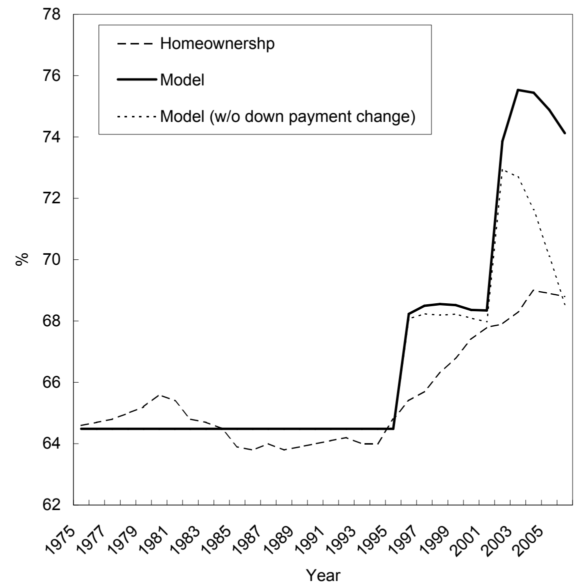

Figure 32: Homeownership rate transition (US)