OPTIMAL TAXATION WITH ENDOGENOUS DEFAULT UNDER INCOMPLETE MARKETS

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Abstract. I analyze a dynamic optimal taxation problem in a closed economy under incomplete markets allowing for default on the debt. If the government defaults, it will go to temporary financial autarky and it can only exit by paying a given fraction of the defaulted debt. The possibility of paying may not arrive immediately; thus, in the meantime, households trade the defaulted debt in secondary markets. The equilibrium price in this market is used to price the debt during the default period. Households predict the possibility of default, and this generates endogenous debt limits, which hinder the government's ability to smooth shocks using debt. I characterize the optimal default decision, optimal government policy, and the set of implementable allocations. Quantitative exercises match various qualitative features observed in the data for emerging economies.

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1. Introduction

For many governments, debt and tax policies are conditioned by the possibility of sovereign default. For emerging economies, sovereign default is a recurrent event, and is typically followed by a lengthy debt restructuring process, where the government and bond holders engage on a renegotiation process that concludes with the government paying a fraction of the defaulted debt.[1]

Emerging economies exhibit lower levels of indebtness and higher volatility of the government tax policy than industrialized economies – where, contrary to emerging economies, default is not observed in the dataset –.[2] Also, emerging economies, exhibit higher interest rate spreads, especially for high levels of domestic debt-to-output ratios, than industrialized economies. In fact, industrialized economies exhibit interest rate spreads that are low and roughly constant for different levels of domestic debt-to-output ratios. Moreover, for emerging economies, the highest interest rate spreads are observed after default and during the debt restructuring period.[3]

These empirical facts indicate that economies that are more prone to default display different government tax policy, and also different prices of government debt – before default and during the debt restructuring period –. Therefore, the option to default, and actual default event, will affect the utility of the residents of the economy; indirectly by affecting the tax policy and debt prices, but also directly by not servicing the debt in the hands of the residents of the economy during the default event.[4]

My main objective is to understand how the possibility of default and the actual default event affect the optimal tax policy, debt prices – before and during default –, and welfare of the economy. For this purpose, I analyze the dynamic optimal taxation problem of a benevolent government in a closed economy under incomplete markets. The government chooses

[1]See Pitchford and Wright (2008).

[2]To measure "indebtness" I am using government domestic debt-to-output ratios, where domestic debt is the debt issued under domestic law (see Panizza (2008)). I am using domestic and not total government debt because my model will be a closed economy. As a proxy of tax policy I am using government revenue-to-output ratio and inflation tax.

[3]Throughout this paper I will also refer to the restructuring period as the default period.

[4]For Argentina's default in 2001, almost 50% of the face value of debt to be restructured (about 53% of the total owed debt from 2001) is estimated to be in the hands of Argentinean residents; Local pension funds alone held almost 20% of the total defaulted debt (see Sturzenegger and Zettelmeyer (2006)).

distortionary labor taxes, non state-contingent debt, and whether to default, so as to maximize the representative household's life-time expected utility, and subject to the equilibrium restrictions imposed by the households' optimal decisions, market clearing conditions and feasibility. If the government defaults, the economy enters temporary financial autarky and faces exogenous offers to pay a fraction of the defaulted debt that arrive at an exogenous rate.[5] The government has the option to accept the offer – and thus exit financial autarky – or to stay in financial autarky until a new offer comes. Since these offers may not arrive immediately, during temporary financial autarky the defaulted debt still has positive value because it is going to be paid in the future with positive probability. Hence, households can trade the defaulted debt in a secondary market from which the government is excluded; the equilibrium price in this market is used to price the debt during period of default.

In the model, the government has three policy instruments: (1) distortionary taxes, (2) government debt, and (3) default decisions that consist of: (a) whether to default on the outstanding debt and (b) whether to accept the offer to exit temporary financial autarky. In order to keep the model as simple as possible, I assume that the government has the ability to make binding policy choices. Hence, since households are forward looking in this model, I need to keep track of the household's past beliefs about government's present actions to write the government's problem recursively. This recursive formulation renders the problem amenable to analytical and numerical analysis.

The government faces a trade-off between levying distortionary taxes to finance the stochastic process of expenditures and not defaulting, or issuing debt and thereby increasing the exposure to default risk. The option to default introduces some degree of state contingency on the payoff of the debt since the financial instrument available to the government becomes an option, rather than a non state-contingent bond. This option, however, does not come for free: infinitely lived households accurately predict the possibility of default, and the equilibrium incorporates it in the pricing of the bond. This mechanism hinders the ability of the government to smooth shocks using debt, renders tax policy more volatile, and implies higher interest rate spreads. Hence the possibility of default introduces a trade-off between the cost of the lack of commitment to repay the debt, reflected on the price of the debt, and

[5] In this model, financial autarky is understood as the period during which the government is precluded of issuing new debt/savings.

the flexibility that comes from the option to default and partial payments, reflected on the pay-off of the debt.

In a benchmark case, with quasi-linear utility, i.i.d. process for the government expenditure, I characterize, analytically, the determinants of the optimal default decision, and its effects on the optimal taxes, debt and allocations. For this purpose, I assume financial autarky forever after default. First, I show that default is more likely when the government's expenditure or debt are higher. Second, I show how the law of motion of the optimal government policy is affected, on the one hand, by the benefit from having "more state-contingency" on the payoff of the bond; but, on the other hand, by the cost of having the option to default. Finally, since the "cost" of exercising the option to default is in terms of allocations (i.e., autarky forever), I study how the option to default affects the allocations implementable to the planner with respect to an economy without this option. In particular, I show that, for positive initial debt, none of the allocations implementable to the planner in a risk-free debt economy can be implemented in mine.

Finally, I calibrate a more complete model, with an auto-correlated process for the government expenditure and a exogenous process for the arrival of offers of partial payments to exit financial autarky; the model is qualitatively consistent with the differences observed in the data between emerging and industrialized economies. In terms of welfare policy, the numerical simulations suggest a nonlinear relationship between welfare and the probability of receiving an offer of partial payments. In particular, increasing the probability of receiving offers for exiting autarky decreases welfare when this probability is low/medium to begin with, but increases it when the probability is high.

The paper is organized as follows. I first present the related literature. Section 2 presents some stylized facts. Section 3 introduces the model. Section 4 presents the recursive equilibrium and section 5 presents the Ramsey problem. Section 6 derives analytical results that characterize the government policy for a simple example. Section 7 contains some numerical exercises, and finally section 8 briefly concludes. All proofs are gathered in the appendices.

1.1. **Related Literature.** My paper builds on and contributes to two main strands in the literature: optimal taxation and endogenous default.

Regarding the first strand, I based my paper on Aiyagari et al. (2002), where in a closed economy the benevolent infinitely lived government chooses distortionary labor taxes and non state-contingent *risk-free* debt, taking into account restrictions from the competitive equilibria, to maximize the households' life-time expected utility. By imposing non state-contingent debt, the authors reconciled the behavior of optimal taxes and debt observed in the data with the theory developed by the seminal paper of Lucas and Stokey (1983), in which the government had access to state-contingent debt. These papers assume full commitment on taxes and risk-free debt. My paper relaxes this last assumption and endows the government with a third policy instrument: the option to default on its debt; this new instrument creates endogenous debt limits, reflected in the equilibrium prices. That is, the option to default is a way of endogeneizing the *exogenous* debt limits presented in Aiyagari et al. (2002).

Regarding the second strand, I model the strategic default decision of the government as in Arellano (2008), which in turn is based on the seminal paper by Eaton and Gersovitz (1981).[6] My model, however, differs from theirs in several ways. First, I consider distortionary taxation; Arellano (2008) and references therein implicitly assume lump-sum taxes. Second, my economy is closed, i.e., "creditors" are the representative household; Arellano (2008) and references therein, assumes open economy with foreign creditors. Note that under the closed economy assumption, the default decision has a direct effect on the households' wealth, and thus welfare, because the government does not honor the debt in the hands of the households. Third, in my model the government must pay at least a positive fraction of the defaulted debt to exit financial autarky through a "debt restructuring process"; in Arellano (2008) and references therein the government is exempt of paying the totality of the defaulted debt upon exit of autarky. Note that in my economy, these payments of defaulted debt might not occur immediately; thus households trade claims of defaulted debt during the period of default in a secondary market from which the government is excluded. This yields an equilibrium price of the defaulted debt and allows me to price the debt during default. I model this "debt restructuring" process exogenously, indexing it by two parameters, because I am only interested in studying the consequences of this process on the optimal fiscal policy and welfare. As explained below, these parameters are chosen to reflect the results in Yue

[6] See also Aguiar and Gopinath (2006).

(2005), and Pitchford and Wright (2008): debt restructuring is time consuming but at the end a positive fraction of the defaulted debt is paid.

Finally, in recent independent papers, Doda (2007) and Cuadra and Sapriza (2008), study the procyclicality of fiscal policy in developing countries by solving an optimal fiscal policy problem. Their work differs from this paper in two main aspects. First, they assume an open small economy (i.e., foreign lenders) and more crucially, no secondary markets. Second, in their model the household's problem is static in the sense that the household does not have access to any savings technology.[7]

2. Stylized Facts

In this section, I present stylized facts regarding the domestic government debt-to-output ratio and central government revenue-to-output ratio of several countries for industrialized economies (IND, henceforth), emerging economies (EME, henceforth) and a subset of these: Latin American (LAC, henceforth).[8]

As shown below, my theory predicts that endogenous borrowing limits are more active for high level of indebtedness. That is, when the government debt is high (relative to output), the probability of default is higher, thus implying tighter borrowing limits, higher spreads and higher volatility of taxes. But when this variable is low, default is an unlikely event, thereby implying slacker borrowing limits, lower spreads and lower volatility in the taxes. Hence, implications in the upper tail of the domestic debt-to-output ratio distribution can be different from those in the "central part" of it. Therefore, the mean or even the variance of the distribution are not too informative as they are affected by the central part of the distribution; quantiles are better suited for recovering the information in the tails of the distribution.[9]

Figure G.2 presents quantile-quantile plot (QQplot) of the domestic government debt-to-output ratio and the real spread for three groups: IND (black diamond shape), EME

[7]Aguiar et al. (2008) also allow for default in a small open economy with capital where households do not have access neither to financial markets nor to capital and provide labor inelastically. The authors' main focus is on the capital taxation and the debt "overhang" effect.
[8]For the latter ratios I used the data in Kaminsky et al. (2004), and for the first ratio I used the data in Panizza (2008). See appendix C for a detailed description of the data.
[9]I refer the reader to Koenker (2005) for a thorough treatment of quantiles and quantile-based econometric models.

(blue box shape) and LAC (red triangle shape).[10][11] The X-axis plots the values of the time series average domestic government debt-to-output ratio, and the Y-axis plots the values of the real spread. For each group, the last point on the right correspond to the 95% quantile, the second to last to the 90% quantile and so on; these are comparable between groups as all of them represent a quantile of the corresponding distribution. EME and LAC have lower domestic debt-to-output ratio levels than IND, in fact the domestic debt-to-output ratio value that amounts for the 95% quantile for EME and LAC, only amounts for (approx.) 80% quantile for IND.[12] Additionally, the graph exhibits a "cone pattern"; i.e., for lower/mid values of domestic debt-to-output ratio (e.g. 50% quantile and below) the spread corresponding to EME and LAC is comparable to the one corresponding to IND, but for higher level of domestic debt-to-output ratio EME and LAC display higher levels, than those corresponding to IND.

Figure G.3 (top) compares the standard deviation of the central government revenue-to-output ratio across different quantiles, between IND (black diamond shape), EME (blue box shape) and LAC (red triangle shape); for all the quantiles, especially for the mid and upper ones, the two latter show higher values than the former.[13]

Figure G.3 (bottom) shows the interest rate spread (computed using the EMBI+) for three defaulters during the period 1997-2006: Argentina (defaulted in 2001), Ecuador (defaulted in 1999) and Russia (defaulted in 1998). We can see that the levels of spread during the period of default (denoted by the darker portions of the lines) are much higher than for the rest of the sample.

Finally, figure G.2 shows that the interest rate spread for IND is low and almost constant for different levels of debt-to-output ratios. Thus, throughout this paper, I assume that the IND group has access to risk-free debt, and the EME and LAC groups have not.

[10] I constructed the spread using the EMBI+ real index for countries that is available and using the 3-7 year real government bond yield for the rest.I also studied the domestic debt net of foreign reserves; the effects present in figure G.2 are the same or are even enhanced.

[11] This type of graphs is not the conventional QQplot as the axis have the value of the random variable which achieves a certain quantile and not the quantile itself. For my purposes, this representation is more convenient.

[12] I obtain this by projecting the 95% quantile point of the EME and LAC onto the X-axis and comparing with the 80-85% quantile point of IND.

[13] I looked also the inflation tax as a proxy for tax policy; results are qualitatively the same.

3. The Model

3.1. **The Setting.** Let time be indexed as $t = 0, 1, \ldots$. The government expenditure process $(g_t)_t$ is an exogenous stochastic process such that $g_t \in \mathbb{G}$ with $\mathbb{G}$ a compact and convex subset of $\mathbb{R}$. Let $g^t \equiv (g_0, \ldots, g_t) \in \mathbb{G} \times \ldots \times \mathbb{G} \equiv \mathbb{G}^{t+1}$ be the history of government expenditures until time t. Let $\mathcal{G} \equiv \mathcal{F}(g)$ be the σ-algebra generated by g, and similarly let $\mathcal{G}^t \equiv \mathcal{F}(g^t)$ be the σ-algebra generated by g^t. Let $\pi_t(g_{t+1}|g^t)$ be the conditional probability of $g_{t+1} \in \mathbb{G}$, conditioned on $g^t \in \mathbb{G}^{t+1}$. Finally, let $\pi_0(g_0)$ be the unconditional probability of g_0; this probability can be degenerate at a point.

At each time t, the government can levy distortionary labor taxes, τ_t^n, or allocate one period, non state-contingent bonds to the households to cover the expenses g_t. I denote $B^G \in \mathbb{B}$ as the government bonds, where the set $\mathbb{B}$ is a compact interval on $\mathbb{R}$. A quantity $B_t^G > 0$ means that the government has to pay to the households B_t^G units of consumption at time t. The government, after observing the present government expenditure and the outstanding debt to be paid this period, has the option to default on 100% of this debt, i.e., the government has the option to refuse to pay the totality of the maturing debt.

As shown in figure D, in case the government opts to exercise the option to default on 100% the debt (node (A) in figure D), nature plays immediately and with probability $1 - \lambda$ sends the government to temporary financial autarky, where the government is precluded from issuing bonds that period. With probability λ the government enters a stage in which nature draws a fraction $1 - \delta$ (with δ distributed according to the probability function $\pi_\delta(\delta)$) of debt to be repaid and the government has the option to accept or reject this offer. If the government accepts, it pays the new amount (the outstanding debt times the fraction that nature chose), and it is able to issue new bonds for the following period. If the government rejects, it goes to temporary financial autarky (bottom branch in figure D).

The parameters $(\lambda, \pi_\delta(\delta))$ define the "debt restructuring process". These parameters capture the fact that debt restructuring is time consuming but, generally, at the end a positive fraction of the defaulted debt is honored (see Yue (2005) and Pitchford and Wright (2008)).[14]

[14]The exogenous probabilities π_δ and λ are set to be constant but I can also allow for probabilities that depend on the state. For instance I can have $\pi_\delta \equiv \pi_\delta(b_t, d_t)$ denoting that possible partial payments depend on the credit history and level of debt. See Reinhart et al. (2003); Reinhart and Rogoff (2008) and Yue (2005) for an intuition behind this structure. Numerical simulations allowing for this structure are qualitatively the same as those shown in this paper and are available upon request.

Finally, if the government is not in financial autarky – because it either chooses not to default, or it accepts the partial payment offer – then next period it has the option to default, with new values of outstanding debt and government expenditure. If the government is in temporary financial autarky, then the next period it will face a new offer for partial payments with probability λ.

Remark 3.1. *I also consider an alternative option for the government to exit financial autarky. At the end of the period of financial autarky, with probability α, the government receives the option to leave autarky by paying 100 % outstanding debt (this is depicted in the bottom branch of figure D).*[15]

The parameter α conveys the idea that the government should be able to exit financial autarky by paying 100% of the defaulted debt at any time, but there are transaction costs or other type of financial frictions that only allow the government to exercise this option occasionally.[16]

Households are price takers and homogeneous; at each period t, given their initial financial wealth z_t, they decide how much to consume c_t, how much to allocate to leisure $l_t = 1 - n_t$ (which yields an after tax labor income $(1-\tau_t^n)n_t$) and how much to save b_{t+1}^G (if the economy is not in financial autarky) or how many shares, L_t, of defaulted debt to trade (if the economy is in financial autarky).

Let $d_t \in \mathbb{D} \cup \{1\} \equiv \{0\} \cup \Delta \cup \{1\}$ be a state variable that, at each time t, indicates whether the government has paid 100%, a part or 0% of the debt. That is, $d_t = 0$ means that the government is not in default and fully honored its outstanding debt, $d_t = 1$ means that the government defaulted in the totally of the debt, and finally $\Delta \equiv \{\delta_1, \ldots, \delta_\Delta\}$ with $\delta_i \in (0, 1]$ is a set of all possible fractions of debt that the government could (partially) default. For instance $d_t \equiv \delta$ implies that the government partially defaulted upon a fraction δ of the

[15]In the numerical simulations I studied both options separately, and their consequences in optimal policies, allocations and welfare.

[16]How to model this process of partial payments explicitly, is outside the scope of this paper. See Pitchford and Wright (2008) and Yue (2005) for two alternative ways of modelling this process as renegotiation between the government and the holders of the debt.

outstanding debt. I refer to d_t as the "default indicator". Finally, let $\mathbb{X} \equiv \mathbb{G} \times \{\mathbb{D} \cup \{1\}\}$, $x_t \equiv (g_t, d_t) \in \mathbb{X}$.[17]

Finally, throughout the paper I assume that g_t is a Markov process. This is required to write the problem recursively. That is,

ASSUMPTION 3.1. *(Markov)* $\pi_t(G|g^t) = \pi(G|g_t)$, $\forall G \in \mathcal{G}$.

3.2. **The Household Problem.** The bellman equation of the household is

$$V(z_t, \Theta_t) = \max_{c_t, n_t, b_{t+1}} \{U(c_t, 1 - n_t) + \beta E_t [V(z_{t+1}, \Theta_{t+1})]\} \tag{1}$$

$$\textit{with } \Theta_t \equiv (x_t, B_t^G). \tag{2}$$

Where z_t is the initial financial wealth at the beginning of time t. The value function is also a function of the perceived law of motion of the households for the government expenditure, "default indicator" and debt: $\Theta_t \equiv (g_t, d_t, B_t^G)$.

I summarize some standard conditions for $U(c_t, 1 - n_t)$ in the assumption below

ASSUMPTION 3.2. *(i)* $U : \mathbb{R}_+ \times [0, 1] \to \mathbb{R}$ *is twice continuously differentiable; (ii)* $\nabla_c U > 0$, $\nabla_c^2 U \leq 0$, $\nabla_l U \geq 0$, $\nabla_l^2 U \leq 0$ *and* $\lim_{n \to 1} \nabla_l U = \infty$.

Due to the asymmetry between the financial assets described in section 3.1 I write the constraints for the household problem for the cases $d \in \mathbb{D}$, and $d = 1$ separately.

3.2.1. *Household's budget constraint for the case of no default and partial default:* $d_t \in \mathbb{D}$. For this case the agents solves the problem in equation 1 subject to

$$c_t + p_t^b b_{t+1}^G - (1 - \tau_t^n) n_t \leq z_t \tag{3}$$

$$z_{t+1}(d_{t+1}) = (1 - d_{t+1}) b_{t+1}^G, \ \forall d_{t+1} \in \mathbb{D} \tag{4}$$

where p_t^b is the price of the government bonds and $z_{t+1}(d)$ is defined as the financial wealth of household at the beginning of $t + 1$ when $d_{t+1} = d$. If $d_{t+1} = 1$

$$z_{t+1}(1) = q_{t+1} b_{t+1}^G,$$

[17] The model could also alow for a "credit history", i.e., $\widetilde{d}_t \equiv (d_{t-K}, \ldots, d_t)$ where $\widetilde{d}_t$ is the "credit history" of the last K periods of the economy. For simplicity in this version I set $K = 0$.

where q_{t+1} is the secondary market price of defaulted government debt.

If $d_{t+1} = \delta$ then the household acknowledges that it receives only a part of their asset and if $d_{t+1} = 1$ the initial financial wealth of the household at $t+1$ is whatever value the household can get out of their assets in the secondary market, i.e., $q_{t+1}b^G_{t+1}$.[18]

3.2.2. *Household's budget constraint for the case of total default: $d_t = 1$*. Under this node, the government is in temporary financial autarky, i.e., does not honor the outstanding debt today and is also precluded from issuing new debt. Although the government is excluded from the financial markets the households can trade the debt that the government owes them but is not honored today. Even though the households are homogeneous and thus no trade takes place in equilibrium, this secondary market yields an equilibrium price which reflects the fact that some fraction of the defaulted debt is going to be paid with positive probability at some point in the future. If the probability of the government repaying the debt in the future is naught, then the value of this secondary markets asset is also naught; in this case I can, without loss of generality, close this market, e.g. Arellano (2008).

I assume that households cannot issue debt. Thus, denoting L_t as the shares of defaulted debt the household can trade in the secondary markets, it follows

$$L_t \leq 1. \tag{5}$$

Therefore the budget constraint is given by

$$c_t + q_t L_t B^G_t - (1 - \tau^n_t) n_t \leq z_t, \tag{6}$$

Note that, B^G_t and not b^G_t is in the budget constraint, because under $d_t = 1$ the defaulted debt is exogenous for the household, and the only variable the household controls is the shares they trade.[19]

[18] The household also faces borrowing limits; but I assume that the exogenous borrowing limits for the household are always less stringent than those for the government and thus in equilibrium the household problem is always an interior solution regarding their choice of assets.

[19] The model could also encompass the case where, during financial autarky, the defaulted debt evolves according to a function $\psi : \mathbb{B} \to \mathbb{B}$, i.e., $B^G_{t+1} = \psi(B^G_t)$. See Yue (2005) where $\psi(\cdot) = 1 + r$ with r being an exogenous risk free rate.

At $t+1$ the initial financial wealth of the household is given by

$$\begin{aligned} z_{t+1}(d_{t+1}) &= (1-d_{t+1})L_t B_t^G, \ \forall d_{t+1} \in \mathbb{D}, \\ z_{t+1}(1) &= q_{t+1} L_t B_t^G. \end{aligned}$$

3.3. **The Government Problem.** The government finances its stream of expenditure $(g_t)_t$ by levying time-varying taxes on labor, τ_t^n and issuing government debt B_{t+1}^G in $d_t \in \mathbb{D}$ such that they satisfy its budget constraint for $d_t \in \mathbb{D}$

$$g_t + Z_t = \tau_t^n n_t + p_t^b B_{t+1}^G, \tag{7}$$

and the budget constraint for $d_t = 1$,

$$g_t = \tau_t^n n_t, \tag{8}$$

where

$$Z_{t+1}(d_{t+1}) = (1-d_{t+1})B_{t+1}^G, \ \forall d_{t+1} \in \mathbb{D} \tag{9}$$

$$Z_{t+1}(1) = 0. \tag{10}$$

Finally, as in Aiyagari et al. (2002), I assume that the government is subject to exogenous borrowing constraints,

$$\underline{M}_t^G \leq B_{t+1}^G \leq \overline{M}_t^G, \ \forall t. \tag{11}$$

Remark 3.2. *The upper bound in this model is not important, because – as shown below – the option to default generates* endogenous *debt limits. The lower bound does not affect the results qualitatively, insofar as it is above the natural limit, otherwise combined with lump sum subsidies the economy could build a "war chest" and finance all future expenditures with that; see Aiyagari et al. (2002).*

4. The Recursive Competitive Equilibrium

The main goal of this section is to define a (recursive) equilibrium for this model. In order to achieve this goal, some intermediate definitions are needed. First, let $b_0 = B_0^G = b_0^G$ the initial debt of this economy.

Definition 4.1. *A government policy is a pair of sequences* $(h_t, B^G_{t+1})_t$ *such that for each* t $h_t \equiv (g_t, \tau^n_t, d_t)$, *where* $\tau^n_t : \{b_0\} \times \mathbb{G}^t \to [0,1]$ *is* $\mathcal{G}^t$*-measurable;* $d_t : \{b_0\} \times \mathbb{G}^t \to \mathbb{D} \cup \{1\}$ *is* $\mathcal{G}^t$*-measurable; and* $B^G_{t+1} : \{b_0\} \times \mathbb{G}^t \to \mathbb{B} \subseteq \mathbb{R}$ *is* $\mathcal{G}^t$*-measurable with* $\mathbb{B}$ *a compact interval in* $\mathbb{R}$*. And finally* $\{b_0, (h_t, B^G_{t+1})_t\}$ *satisfies the government budget constraint in equations 7-11 for each* t*.*

Henceforth let $\mathbb{H}_t \equiv \mathbb{G} \times [0,1] \times \{\mathbb{D} \cup \{1\}\}$ and $\mathbb{H}^t \equiv \prod_{\tau=0}^t \mathbb{H}_\tau$.

Definition 4.2. *A feasible allocation is a sequence vector* $(c_t, n_t, g_t)_t$ *such that*

$$c_t + g_t = \frac{n_t}{1+\kappa}, \tag{12}$$

with $c_t : \{b_0\} \times \mathbb{H}^t \to \mathbb{R}_+$ *is* $\mathcal{G}^t$*-measurable;* $n_t : \{b_0\} \times \mathbb{H}^t \to [0,1]$ *is* $\mathcal{G}^t$*-measurable.*

The government policy only depends on the exogenous history of shocks and the initial government debt; but, in the definition of feasible allocation I define household consumption and labor as functions of the exogenous government policy. This asymmetry arises from the assumption that, in my model, household's behavior is non-strategic; their behavior does not affect the aggregate quantities and prices. Therefore, is not necessary to keep track of their actions. The government, however, is modelled as an agent that behaves strategically and can affect prices through its decisions of default and debt; thus I need to keep track of the past history of government actions.[20] Finally, the parameter κ represents direct cost of defaulting, e.g. $\kappa \geq 0$ if the government decides to default and zero otherwise. For simplicity, I take $\kappa \equiv 0$ and only consider a different scheme in the numerical simulations.[21]

I now present the definition of recursive competitive equilibrium in this economy.

Definition 4.3. *In this economy a (recursive) competitive equilibrium is: an initial* b_0*; a set of value functions* $V(\cdot)$*; a set of policy functions* $(c(\cdot), n(\cdot), b^G_{t+1}(\cdot), L(\cdot))$*; government policies; prices* $(p^b(\cdot), q(\cdot))$*; a perceived law of motion and actual law of motion for* $\Theta = (g, d, B^G)$*; such that*

a. *Given the initial tuple, prices, government policies and perceived laws of motion; the policy functions and value functions solve the household's problem.*

[20]See Phelan and Stacchetti (2001) for a detailed discussion.
[21]See Arellano (2008), Aguiar and Gopinath (2006), and Mendoza and Yue (2008) for a discussion about κ.

b. *Prices are such that the allocation is feasible and*

$$b^G = B^G \equiv b \ , \ for \ d \in \mathbb{D}, \tag{13}$$

$$L = 1, \ for \ d = 1. \tag{14}$$

c. *Given a. and b. the actual and perceived laws of motion coincide.*

Henceforth, I will continue to use sequence notation (indexing variable by t) for simplicity.

4.1. **Equilibrium Taxes and Price of Government Debt.** I can obtain expressions for the equilibrium price of the government debt b_{t+1}^G, the equilibrium price of one share of defaulted debt (L_t) traded in the secondary market, and for the labor taxes by first solving the household problem presented above and then substituting the equilibrium conditions in definition 4.2 and the market clearing conditions in equation 13. I am going to impinge the "correct" or actual law of motion for the Θ_t. In order to do this, I introduce two new objects $D_t[\mathbb{G}] \equiv D(g_t, d_t, B_t^G)[\mathbb{G}] \subseteq \mathbb{G}$ and $D_t[\Delta] \equiv D(g_t, d_t, B_t^G)[\mathbb{G}] \subseteq \Delta$. The first one is the set of government expenditures at time t such that the government does not pay the outstanding debt, i.e., $\{g \in \mathbb{G} : d_t(g^{t-1}, g) \neq 0\}$. The second object can be described as a set function that takes values (g_t, d_t, B_t^G) and maps into a subset of Δ of rejected offers, i.e., if $\delta \in D_t[\Delta]$ the government rejects such offer.

The expression for the taxes directly comes from the ratio of the first order conditions for c_t and n_t,

$$1 - \tau_t^n = \frac{\nabla_l U(c_t, 1-n_t)}{\nabla_c U(c_t, 1-n_t)}. \tag{15}$$

Let Γ_t be the lagrange multiplier associated to equation 3. Then, from the first order conditions of the household problem with respect to b_{t+1}^G, it follows

$$\begin{aligned} 0 =& p_t^b \Gamma_t + \beta E_t \Big[\{1 - \mathbb{I}\{D_{t+1}[\mathbb{G}]\}\} \nabla_z V(b_{t+1}^G, \Theta_{t+1}) \\ &+ \sum_{\delta \in \Delta} \{\mathbb{I}\{D_{t+1}[\mathbb{G}]\} \lambda (1-\delta)(1 - \mathbb{I}\{D_{t+1}[\Delta]\}) \pi_\delta(\delta)\} \nabla_z V((1-\delta) b_{t+1}^G, \Theta_{t+1}) \\ &+ \left\{ \mathbb{I}\{D_{t+1}[\mathbb{G}]\} \left((1-\lambda) + \lambda \sum_{\delta \in \Delta} \mathbb{I}\{D_{t+1}[\Delta]\} \pi_\delta(\delta) \right) q_{t+1} \right\} \nabla_z V(q_{t+1} b_{t+1}^G, \Theta_{t+1}) \Big] \end{aligned} \tag{16}$$

where $\mathbb{I}\{A\}$ is an indicator function that takes value one if the set A occurs.

By the envelope condition it follows that

$$\nabla_z V(z_t, \Theta_t) = -\Gamma_t. \tag{17}$$

Let $\mathcal{P} : \mathbb{G} \times \{\mathbb{D} \cup \{1\}\} \times \mathbb{B} \to \mathbb{R}_+$ such that $\mathcal{P}_t \equiv \mathcal{P}(\Theta_t) \equiv p_t^b U_{c,t}$. From equations 16 - 17, the first order condition with respect to c_t (which implies that $\nabla_c U_t(c_t, 1-n_t) \equiv U_{c,t} = \Gamma_t$), the aggregate equilibrium conditions imply that

(18)

$$p_t^b \equiv \frac{\mathcal{P}_t}{U_{c,t}} = \beta E_t \left[\{1 - \mathbb{I}\{D_{t+1}[\mathbb{G}]\}\} \frac{U_{c,t+1}(0)}{U_{c,t}} \right]$$

$$+ \beta E_t \left[\sum_{\delta \in \Delta} \{(1-\delta)\mathbb{I}\{D_{t+1}[\mathbb{G}]\}\lambda(1 - \mathbb{I}\{D_{t+1}[\Delta]\})\pi_\delta(\delta)\} \frac{U_{c,t+1}(\delta)}{U_{c,t}} \right] \tag{19}$$

$$+ \beta E_t \left[\left\{ \mathbb{I}\{D_{t+1}[\mathbb{G}]\} \left((1-\lambda) + \lambda \sum_{\delta \in \Delta} \mathbb{I}\{D_{t+1}[\Delta]\}\pi_\delta(\delta) \right) q_{t+1} \right\} \frac{U_{c,t+1}(1)}{U_{c,t}} \right] \tag{20}$$

where $U_{c,t+1}(d)$ denotes the marginal utility of consumption at time $t+1$ when $d_{t+1} = d$.

A few noteworthy remarks are in order. First, each term in the equation above corresponds to a "branch" of the tree depicted in figure D. The first line represents the value of one unit of debt in the case the planner chooses to honor the totality of the debt. The second line represents the value of the debt if the planner decides not to pay the debt, but ends up in partial defaults. The third line captures the value of the debt when the planner default in 100 % of the debt but the households can sell it in the secondary markets. Second, if $\lambda = \alpha = 0$ and $U_{c,t} = 1$ then the last two terms vanish and the price is analogous to the one obtained in Arellano (2008).

I now compute the expression for q_t. First let $\mathcal{Q} : \mathbb{G} \times \{\mathbb{D} \cup \{1\}\} \times \mathbb{B} \to \mathbb{R}_+$ be such that $\mathcal{Q}_t \equiv \mathcal{Q}(\Theta_t) = q_t U_{c,t}$. The first order condition and envelope conditions are basically the same as before, the difference lies in the law of motion for d_{t+1}. Following the same steps as before but replacing for the "correct" law of motion for d_{t+1}, it follows that the secondary market price is

$$
\begin{aligned}
q_t \equiv \frac{\mathcal{Q}_t}{U_{c,t}} =& \beta E_t \left[\sum_{\delta \in \Delta} \{(1-\delta)\lambda(1-\mathbb{I}\{D_{t+1}[\Delta]\})\pi_\delta(\delta)\} \frac{U_{c,t+1}(\delta)}{U_{c,t}} \right] \\
&+ \beta E_t \left[\left\{ \left((1-\lambda) + \lambda \sum_{\delta \in \Delta} \mathbb{I}\{D_{t+1}[\Delta]\}\pi_\delta(\delta) \right) q_{t+1} \right\} \frac{U_{c,t+1}(1)}{U_{c,t}} \right].
\end{aligned}
$$

If autarky is an absorbing state, i.e., $\lambda = \alpha = 0$ it follows that

$$
q_t = \beta E_t \left[q_{t+1} \frac{U_{c,t+1}(1)}{U_{c,t}} \right].
$$

Which by substituting forward and standard transversality conditions it yields $q_t = 0$.

Remark 4.1. *If I also allow for $\alpha > 0$ (see remark 5.1) then the price q_t is given by*

$$
\begin{aligned}
q_t \equiv \frac{\mathcal{Q}_t}{U_{c,t}} =& \beta E_t \left[\{\alpha(1-\mathbb{I}\{D_{t+1}[\mathbb{G}]\})\} \frac{U_{c,t+1}(0)}{U_{c,t}} \right] \\
&+ \beta E_t \left[\sum_{\delta \in \Delta} \{(1-\delta)(1-\alpha+\alpha\mathbb{I}\{D_{t+1}[\mathbb{G}]\})\lambda(1-\mathbb{I}\{D_{t+1}[\Delta]\})\pi_\delta(\delta)\} \frac{U_{c,t+1}(\delta)}{U_{c,t}} \right] \\
&+ \beta E_t \left[\left\{ (1-\alpha+\alpha\mathbb{I}\{D_{t+1}[\mathbb{G}]\}) \left((1-\lambda) + \lambda \sum_{\delta \in \Delta} \mathbb{I}\{D_{t+1}[\Delta]\}\pi_\delta(\delta) \right) q_{t+1} \right\} \frac{U_{c,t+1}(1)}{U_{c,t}} \right].
\end{aligned}
$$

5. The Ramsey Problem

I define the Ramsey problem as

Definition 5.1. *Given an initial $b_0^G = B_0^G \equiv b_0$ the Ramsey problem is to choose the (recursive) competitive equilibrium with the highest: $V(b_0, g_0, d_0, b_0)$ with $d_0 = 0$.*

5.1. Primal Approach.

As pointed out by Kydland and Prescott (1980) in order to write the Ramsey problem recursively, the addition of a new (co)state variable is needed. The authors noted that the policy functions in the Ramsey problem are not continuous on the "usual state" because the households current decision are based upon beliefs of government *future* actions, and the government has to validate these beliefs. Hence the new (co)state variable must convey this information. By inspecting the first order conditions of the households, it is sufficient to set the (co)state variable, denoted as μ_t, to be the marginal utility of consumption of the household at time t. That is, at time t the planner needs to keep track of the "promised" marginal utility of consumption at time $t+1$; the planner, however, only has to do this when the forward looking constraints of the households, embedded in the pricing equations, are at play, i.e., μ_t does not change when the government is in autarky.[22]

Denote $\mathcal{U}(b, g, \mu, d)$ as the value function of the economy (i.e., the planner who is solving the primal approach) with financial wealth b, government expenditure g, a (co)state variable μ (which is defined below) and default indicator d (i.e., either no default, partial default or autarky).

In the case $d_t = 1$ then the government's budget constraint is given by $g_t = \tau_t^n n_t$; from this equation, equation 15 and the feasibility constraint it follows

$$U_c(n_t - g_t, 1 - n_t)(n_t - g_t) - U_l(n_t - g_t, 1 - n_t)n_t = 0, \tag{21}$$

where $\mu = U_c(n_t - g_t, 1 - n_t)$ and $U_l \equiv \nabla_l U$. I can solve for n_t, and then plug this solution in the household's value function, thereby obtaining

$$\mathcal{U}(b_t, g_t, \mu_t, 1) = U(c_t, 1 - n_t) + E_t\left[\mathcal{U}^B(b_t, g_{t+1}, \mu_t)\right] \tag{22}$$

where

$$\mathcal{U}^B(b, g, \mu) = \lambda \sum_{\delta \in \Delta} \max\left\{\mathcal{U}((1-\delta)b, g, \mu, \delta), \mathcal{U}(b, g, \mu, 1)\right\} \pi_\delta(\delta) + (1-\lambda)\mathcal{U}(b, g, \mu, 1),$$

and $\mathcal{U}^o(b, g, \mu) = \max\left\{\mathcal{U}(b, g, \mu, 0), \mathcal{U}^B(b, g, \mu)\right\}$.

The function $\mathcal{U}^B(b, g, \mu)$ is the value function of the planner before nature plays and send him to autarky with probability $1 - \lambda$ or to the offer of partial payment (node(B) in figure D) with expenditure g, outstanding debt b, and (co)state variable μ. The function $\mathcal{U}^o(b, g, \mu)$ is the value function of the planner which has the option to default (node (A) in figure D) with expenditure g, outstanding debt b, and (co)state variable μ.

Remark 5.1. *If we allow for $\alpha > 0$ (see remark), equation 22*

$$\mathcal{U}(b_t, g_t, \mu_t, 1) = U(c_t, 1 - n_t) + \alpha E_t\left[\mathcal{U}^o(b_t, g_{t+1}, \mu_t)\right] + (1-\alpha)E_t\left[\mathcal{U}^B(b_t, g_{t+1}, \mu_t)\right]$$

[22] See also, Werning (2001), Phelan and Stacchetti (2001), and Farhi (2007), amongst others. An alternative approach is by using the recursive contract approach in Marcet and Marimon (1998) and Aiyagari et al. (2002).

In the above equations, the government default decisions are constructed using the "max" operator. The intuition behind this construction stems from the assumption that the government is benevolent; it only opts to pay the debt inasmuch as it is in the best interest of the representative household.[23] So, the sets $D_t[\mathbb{G}]$ and $D_t[\Delta]$, which characterize the default decisions, are constructed as follows

$$(23) \qquad D_t[\mathbb{G}] \equiv D(b_t, \mu_t)[\mathbb{G}] = \left\{ g \in \mathbb{G} : \mathcal{U}(b_t, g, \mu_t, 0) < \mathcal{U}^B(b_t, g, \mu_t) \right\},$$

$$(24) \qquad D_t[\Delta] \equiv D(g_t, b_t, \mu_t)[\Delta] = \left\{ \delta \in \Delta : \mathcal{U}((1-\delta)b_t, g_t, \mu_t, \delta) < \mathcal{U}(b_t, g_t, \mu_t, 1) \right\}.$$

It now remains to construct $\mathcal{U}(b_t, g_t, \mu_t, d_t)$, $d_t \in \mathbb{D}$. From the first order conditions of the household with respect to consumption and labor (equation 15), the expression for the prices derived in section 4.1, the government budget constraint and feasibility constraint the implementability condition at time t is

$$(25) \qquad U_{c,t}(n_t - g_t) - U_{c,t} b_t = U_{l,t} n_t - \mathcal{P}_t b_{t+1}, \ \textit{with} \ \mu_t = U_{c,t};$$

note that under equilibrium the beliefs embedded in $\mathcal{P}_t$ must be exactly those coming from the exogenous laws, π, π_δ, λ, α, and the endogenous government policies.

The value function $\mathcal{U}(b_t, g_t, \mu_t, d_t)$ for $d_t \in \mathbb{D}$ is thus given by

$$\mathcal{U}(b_t, g_t, \mu_t, d_t) = \max_{\{n_t, b_{t+1}, \mu_{t+1}\}} \left\{ U(n_t - g_t, 1 - n_t) + \beta E_t \left[\mathcal{U}^o(b_{t+1}, g_{t+1}, \mu_{t+1}) \right] \right\},$$

subject to $\{n_t, b_{t+1}, \mu_{t+1}\} \in \{(n_t, b_{t+1}, \mu_{t+1}) \in [0,1] \times \mathbb{S}_g : \mu_t = U_c(n_t - g_t, 1 - n_t)\}$ and the exogenous debt limits 11. The set $\mathbb{S}_g$ is defined as a fixed point, of the operator $\mathcal{S}_g$:

$$\mathcal{S}_g(Q) = \{(b_t, \mu_t) \in \mathbb{B} \times \mathbb{R}_+ : \exists (b_{t+1}, \mu_{t+1}) \in Q \ \textit{such that eqn.} \ (25) \ \textit{holds with} \ g_t = g\}$$

and has to be computed recursively.[24]

6. Analytical Results

In this section I define a set of assumptions that constitutes the benchmark case; I characterize analytically the default sets, policy and pricing implications of the model, and implementable allocations.

[23]This functional form is analogous to Eaton and Gersovitz (1981), Arellano (2008) and references therein.
[24]See Kydland and Prescott (1980), Chang (1998), Werning (2001) and Phelan and Stacchetti (2001).

Let the following hold

Assumption 6.1. *(i)* $\lambda = 0$; *(ii)* $U_{c,t} \equiv 1$.

Part (i) states that offers of partial payments do not occur. Part (ii) implies that prices do not depend on marginal utilities. Aiyagari et al. (2002) argue that by setting $U_{c,t} \equiv 1$ they are impinging a competitive behavior on the planner as it is unable to control the (implied) prices; thereby drawing an analogy between this problem and the standard incomplete markets consumption-smoothing problem.[25] In my case, the planner is still able to affect prices through the probability of default, thus the analogy to the (competitive) representative agent in the consumption-smoothing problem does not hold anymore.

6.1. **Characterization of Default Sets.** The results obtained in this section show that the decision to default follows a debt-dependent threshold rule; these results are similar to the one obtained in Chatterjee et al. (2007) and Arellano (2008) without distortionary taxes.

Assumption 6.2. *(i)* $\tau_t^n \in [0, 1]$.

Proposition 6.1. *Under assumptions 3.2-6.1(ii) and 6.2(i), if* $D[\mathbb{G}](b_t) \neq \emptyset$ *then there does not exists* $b_{t+1} : b_t - \mathcal{P}(b_{t+1})b_{t+1} \leq 0$.

The proposition above implies that if default occurs (with positive probability) then it must be true that the government is unable to roll over the debt, otherwise it would simply keep the option to default this period, and default tomorrow on a higher debt; thus default never occurs today.

The next proposition states that under additional assumptions the decision of default is equivalent to a threshold rule that, i.e., the government defaults if g is above some $\overline{g}(b)$ given a level of debt b

Assumption 6.3. *(i)* $\alpha = 0$; *(ii)* $g_t \sim$ *i.i.d.*; *(iii)* $U_{ll}(1 - n_t) - U_{lll}(1 - n_t)n_t \leq 0$.

Proposition 6.2. *Under assumptions 3.2 - 6.2 and 6.3, it follows that: if* $g_1 \in D_t[\mathbb{G}]$ *then for* $g_1 \leq g_2$, $g_2 \in D_t[\mathbb{G}]$.

[25] It is clear that the planner's problem prices do not show up, but they are implicit on budget constraint. These "implied prices" are the ones I am referring to here.

Remark 6.1. *Under assumption 3.2 a sufficient condition for assumption 6.3(iii) is* $U_{lll}(\cdot) \geq 0$.

Assumption 6.3(ii) is also imposed by Arellano (2008) and Yue (2005). This assumption is crucial for characterizing the default sets. If g_t is positively correlated with g_{t+1} then low expenditure today implies (probably) low expenditure tomorrow and in the future, therefore autarky looks better now. In fact, intuitively, the impact of a low expenditure today has a relatively larger effect under autarky than under the no-default regime because in the latter regime you have debt/savings to smooth them; thus, the government might have incentives to default when g_t is *low*, contradicting the aforementioned results.

Proposition 6.2 implies that if $D_t[\mathbb{G}] \neq \{\emptyset\}$ then $d_t = \mathbb{I}\{g \in \mathbb{G} : g > \overline{g}(b_t)\}$, where $\overline{g}(b_t)$: $\mathcal{U}(b_t, \overline{g}(b_t), 0) = \mathcal{U}(\overline{g}(b_t), 1)$.

The next proposition establishes that default sets are increasing in the debt level, or given my previous proposition, that $\overline{g}(\cdot)$ is a decreasing function.

PROPOSITION 6.3. *Under assumptions 6.1, 6.2 and 6.3(i) it follows that if* $b_{1,t} \leq b_{2,t}$ *then* $D[\mathbb{G}](b_{1,t}) \subseteq D[\mathbb{G}](b_{2,t})$.[26]

Remark 6.2. *In appendix A.1 I discuss the consequences of relaxing the aforementioned assumptions on the characterization of the default sets. I already noted that propositions 6.1 and 6.3 do not depend on the assumption of i.i.d. expenditure process, thus I focus on the latter two assumptions: marginal utility of consumption equal to unity and taking autarky as an absorbing state.*

6.2. **Implications on the optimal government policies and allocations.** By the results in sections 4.1 and 6.1 it follows that

$$\mathcal{P}(b_{t+1}) = \beta E\left[\mathbb{I}\{g \leq \overline{g}(b_{t+1})\}\right] = \beta\Pi(\overline{g}(b_{t+1})), \ \textit{and}\ q_t = 0$$

where $\Pi(G) \equiv \int_{g \leq G} \pi(dg)$.

The debt value such that $\nabla_b[\mathcal{P}(b_t^*)b_t^*] = 0$ is given by

$$b^* = -\frac{\Pi(\overline{g}(b^*))}{\pi(\overline{g}(b^*))\nabla_b \overline{g}(b^*)}.$$

[26]Note that I do not impose $g_t \sim i.i.d.$ or any other restriction over g_t other than the Markovian one.

Defining $b_* \equiv \arg\sup\{b \in \mathbb{B} : \Pi(\overline{g}(b)) = 1\}$, i.e., the maximum debt level such that default never occurs, it follows that the region $[b_*, b^*]$, which can be empty, is the region where risky borrowing takes place.[27]

I can now give a sharp characterization for the law of motion of the optimal taxes and debt. In order to achieve this, following Aiyagari et al. (2002), I characterize the law of motion of the lagrange multiplier associated to equation 25. I denote this lagrange multiplier as γ_t. First, by the envelope condition, it follows

$$\gamma_t = -\nabla_b \mathcal{U}(b_t, g_t, 0)$$

i.e., γ_t is the marginal cost of debt in terms present value utility. Thus, by studying the law of motion of γ_t, I can study the law of motion of the optimal debt by inverting the previous equation. Moreover as the first order condition with respect to n_t is given by

$$(1 - U_{l,t})(1 + \gamma_t) = -\gamma_t U_{ll,t} n_t;$$

the tax, τ_t^n, is also a nonlinear increasing function of γ_t. Therefore, by studying the law of motion of γ_t, I can also study the law of motion of the optimal taxes.

Under assumptions 6.1 and 6.3, $D_{t+1}[\mathbb{G}]$ is characterized by all the $g \in \mathbb{G}$ such that $g > \overline{g}_{t+1}$ thus, assuming natural debt limits (i.e., interior solution for the debt), the first order condition with respect to b_{t+1} is given by[28]

$$\begin{aligned}&\gamma_t \left(\nabla_b[\mathcal{P}(b_{t+1})]b_{t+1} + \mathcal{P}(b_{t+1})\right) + \beta E\left[(1 - \mathbb{I}\{D_{t+1}[\mathbb{G}]\})\nabla_b \mathcal{U}(b_{t+1}, g_{t+1}, 0)\right] \\ &= \beta E\left[\nabla_b \mathbb{I}\{D_{t+1}[\mathbb{G}]\}(\mathcal{U}(g_{t+1}, 1) - \mathcal{U}(b_{t+1}, g_{t+1}, 0))\right].\end{aligned}$$

The first expectation equals $-E\left[(1 - \mathbb{I}\{D_{t+1}[\mathbb{G}]\})\gamma_{t+1}\right]$ by the envelope condition. The derivative in the second expression is taken in the *weak* sense; the expression is basically $\mathcal{U}(g_{t+1}, 1) - \mathcal{U}(b_{t+1}, g_{t+1}, 0)$ evaluated at $g_{t+1} \in \partial D_{t+1}[\mathbb{G}]$ (i.e., the boundary of $D_{t+1}[\mathbb{G}]$)

[27] See Arellano (2008) for sufficient conditions that ensure this region is not empty. In this section I assume that $[b_*, b^*] \neq \emptyset$

[28] Differentiability of $\mathcal{P}_t$ with respect to b_{t+1} follows from applying the implicit function theorem. I am assuming though that $\mathcal{U}(b, g, 0)$ is differentiable. This is neither necessary for my general analysis nor for computing the solution in the numerical analysis; but provides better intuition for understanding the problem.

which consists of a singleton such that $\mathcal{U}(g_{t+1},1)-\mathcal{U}(b_{t+1},g_{t+1},0)=0$. Thus I obtain

$$\gamma_t\left(\nabla_b[\mathcal{P}(b_{t+1})]b_{t+1}+\mathcal{P}(b_{t+1})\right)=E\left[(1-\mathbb{I}\{D_{t+1}[\mathbb{G}]\})\gamma_{t+1}\right].$$

Finally note that $\nabla_b\mathcal{P}(b_{t+1})=\beta\nabla_b[E\left[\mathbb{I}\{g:g\leq\overline{g}_{t+1}\}\right]]\equiv\nabla_b[\Pi(\overline{g}_{t+1})]=\beta\pi(\overline{g}_{t+1})\nabla_b[\overline{g}(b_{t+1})]$, where the last term is well defined by a direct application of implicit derivative theorem, insofar the value function is differentiable. Therefore

$$\gamma_t=\frac{\Pi(\overline{g}_{t+1})}{\pi(\overline{g}_{t+1})\nabla_b[\overline{g}(b_{t+1})]b_{t+1}+\Pi(\overline{g}_{t+1})}\mathbb{E}\left[\gamma_{t+1}\right], \tag{26}$$

with $\mathbb{E}$ being the expectation with respect to the default-adjusted measure $\frac{\mathbb{I}\{g:g\leq\overline{g}_{t+1}\}}{\Pi(\overline{g}_{t+1})}\pi(dg)$; i.e., the possibility of default inserts a wedge that slants the probability measure $\pi(dg)$. Henceforth, I denote the first term in the right hand side as $\mathcal{M}(b_{t+1})\equiv\frac{1}{1-\zeta(b_{t+1})}$, with $\zeta(b_{t+1})\equiv-\nabla_b[\mathcal{P}(b_{t+1})]\frac{b_{t+1}}{\mathcal{P}(b_{t+1})}$.[29]

The lagrange multiplier associated to the implementability condition is constant in Lucas and Stokey (1983), and thus trivially a martingale. In Aiyagari et al. (2002) the lagrange multiplier associated to the implementability condition is a martingale with respect to the probability measure π.[30] Equation 26 implies that the law of motion of the lagrange multiplier differs in two important aspects. First, the expectation is computed under the default-adjusted measure; this stems from the fact that the option to default adds ”some” degree of state-contingency to the payoff of the government debt. Second, the aforementioned expectation is multiplied by $\mathcal{M}(b_{t+1})$ which can be interpreted as the “markup” that the planner has to pay for having this option to default. Of course this only holds when the government is taking debt (in the case of savings the price of equals β).

The proposition below, first provides conditions to establish which of these opposite terms dominates, and second it explores how the law of motion for the lagrange multiplier γ_t relates to the one presented in Aiyagari et al. (2002) and their *exogenous* borrowing limits. I also provide sufficient conditions such that, conditional on not defaulting, $(\gamma_t)_t$ converges with positive probability.

[29] Proposition A.1 in the Appendix summarizes some properties of $\mathcal{M}$.

[30] The martingale property is also preserved if capital is added to the economy, see Farhi (2007). This property, however, changes if I allow for ad-hoc borrowing limits (see Aiyagari et al. (2002)). The proposition below shows how the results in Aiyagari et al. (2002) – with exogenous debt limits –, relate to the results in my model – with only the option to default –.

Proposition 6.4. *Under assumptions 6.1 - 6.2 and 6.3,*

(1) *if*

$$\nabla_g[\nabla_b[\mathcal{U}(b,g,0)]] \geq 0, \tag{27}$$

then $\gamma_t > E[\gamma_{t+1}]$ *a.s., conditional on no defaulting. Moreover if* $\Pi(\overline{g}(b_t)) \geq \exp\{-C/t^2\}$ *then* $\gamma_t \to \gamma_\infty$ *w.p.p.*

(2) *if*

$$\nabla_g[\nabla_b[\mathcal{U}(b,g,0)]] \leq 0, \tag{28}$$

then:

$$\gamma_t = E[\gamma_{t+1}] + \{E[\gamma_{t+1}]\,(\mathcal{M}(b_{t+1}) - 1)\} - \left\{\frac{Cov\,(\mathbb{I}\{g \geq \overline{g}(b_{t+1})\}, \gamma_{t+1})}{\Pi(\overline{g}(b_{t+1}))(1-\zeta(b_{t+1}))}\right\} \tag{29}$$

with $E[\gamma_{t+1}]\,(\mathcal{M}(b_{t+1}) - 1) \geq 0$ *and* $\frac{1}{\Pi(\overline{g}(b_{t+1}))} Cov\,(\mathbb{I}\{g \geq \overline{g}(b_{t+1})\}, \gamma_{t+1}) \geq 0$.

(a) *If* $\nabla_b^2[\mathcal{U}(b,g,0)] \leq 0$ *and* $\nabla_b\,[\log\,(-\nabla_b[\log(\Pi(\overline{g}(b_{t+1})))])] + \frac{1}{b_{t+1}} \geq 0$, *then:* $E[\gamma_{t+1}]\,(\mathcal{M}(b_{t+1}) - 1)$ *is increasing in* b_{t+1}.

(b) *If* $\nabla_b^3[\mathcal{U}(b,g,0)] \geq 0$, *then:* $\frac{Cov(\mathbb{I}\{g \geq \overline{g}(b_{t+1})\}, \gamma_{t+1})}{\Pi(\overline{g}(b_{t+1}))(1-\zeta(b_{t+1}))}$ *is decreasing in* b_{t+1}.

The first part of the above proposition implies that the marginal cost of the debt (in terms of present value utility) behaves like a submartingale (provided that default does not occur); this result is analogous to the one in Aiyagari et al. (2002) but with *exogenous* borrowing limits. This result hinges on the marginal cost of the debt (in terms of present value utility) being a decreasing function of g; which in turn implies that γ_{t+1} is decreasing in g_{t+1}. Since the default adjusted measure weighs relatively more low values of g, the expectation term on equation 26 is lower than the expectation under the probability π; this effect is reinforced by the term $\mathcal{M}(b_{t+1})$.

Since the submartingale result in the proposition can only be used for paths such that this economy does not default, I need to ensure that the probability of no default is positive; the inequality $\Pi(\overline{g}(b_t)) \geq \exp\{-C/t^2\}$ provides a sufficient condition for this to hold.

Equation 27 implies that the cost of the debt is less severe as the government expenditure increases. This is counterintuitive as one would expect that having a certain level of debt becomes more costly as the government expenditure is higher due to concavity of the utility of the agents.

The second part of the proposition handles the case where the marginal cost of debt is increasing on g, i.e., $\nabla_b \mathcal{U}(b, g, 0)$ is decreasing in g. Under more regularity conditions over the behavior of the curvature of $\mathcal{U}(b, g, 0)$ and $\Pi(\overline{g}(b))$, the first term in the curly brackets of equation 29 is increasing in b_{t+1}. Hence this term can be seen as a "continuous" lagrange multiplier of a debt limit; i.e., is positive (with a negative sign in front), and increases continuously in b_{t+1}; a lagrange multiplier would be zero and then jump to positive values when the bound is active. The second term is decreasing, and thus acts as a "continuous" lagrange multiplier of a "savings" limit, provided the assumption about the third derivative of the value function is added.

Although, equations 27 and 28, and some of the rest of the assumptions, are somewhat unsatisfactory because they impose ad-hoc restrictions upon an endogenous object, they can easily be checked in numerical simulations and have a clear economic interpretation.

6.2.1. *Restrictions over the allocations.* The recursive competitive equilibrium defines a set of implementable sequence of allocations from which the government or planner selects the optimal one, given some initial conditions.[31]

The option to default allows the government to evade paying debt once, provided it pays a "cost" in terms of allocations: autarky forever. In an economy like Aiyagari et al. (2002) this technology is not available because debt has to be risk-free. The proposition below sheds light on how the possibility of default affects the implementable allocations.

I already showed that the default rule is given by $d_t = \mathbb{I}\{g : g > \overline{g}(b_t)\}$ which is a random variable measurable with respect to $\mathcal{G}^t$. I can define $T^* \equiv \inf\{t : d_t = 1\} \in \mathcal{G}^t$ such that for all $t \geq T^*$ my economy is in autarky and for all $t < T^*$ it is not.[32]

Proposition 6.5. *If* $b_0 > 0$, $pr_\infty\{T^* < \infty\} > 0$, *and* $\forall t \leq T^* : \Pi(\overline{g}_t) > 0$, *then the sets of implementable* $(c_t, n_t, g_t)_{t=0}^{\infty}$ *under my economy and Aiyagari et al. (2002) are disjoint.*

[31] Implementable allocations are those which satisfy the competitive equilibrium restrictions. For a precise definition see the proof of proposition 6.5. See also Lucas and Stokey (1983) and Aiyagari et al. (2002) for a thorough discussion of this solution approach.

[32] The probability measure pr is the one induced by π, i.e., $pr_t\{g^t : g_t \leq G_t\} \equiv \prod_{j=0}^{t} \Pi(G_t),\ \forall t$.

Remark 6.3. *The condition:* $b_0 > 0$ *rules out autarky for all* $t = 0, \ldots$. *The condition:* $pr_\infty\{T^* < \infty\} > 0$ *assumes that default occurs with positive probability, otherwise the problem is trivial. The condition:* $\forall t \leq T^* : \Pi(\overline{g}_t) > 0$ *states that default never occurs almost surely; studying optimal allocations this assumption is innocuous as* $\Pi(\overline{g}_t) = 0$ *never occurs because is not optimal to choose* b_t *such that* $\Pi(\overline{g}_t)b_t = 0$.

The intuition of this result is as follows. In Aiyagari et al. (2002), if the government has to run a balance budget for some histories for a time $\mathcal{T}$ onwards, then this allocation must imply a non-positive debt; otherwise the government has to run a Ponzi scheme. In my economy, however, this type of allocation can allow for positive debt; this is the case of default. Therefore, since debt is non-state contingent, at time $\mathcal{T}$ debt is non-positive in Aiyagari et al. (2002), but positive in my economy.

If $\mathcal{T}$ is the first time default has positive probability, then the price of debt in both economies is exactly the same for all $t < \mathcal{T}$ because default occurs with zero probability in this spell of time. Hence, if both economies share the same allocations, the sign of the debt at $\mathcal{T}$ must coincide as well. This contradicts the conclusion in the previous paragraph, and hinges on the assumption that there exists an allocation shared by both economies.

7. Numerical Simulations

Given the complexity of the model it is difficult to characterize the Ramsey policy analytically when I include options to exit autarky. Hence in this section I present a series of numerical simulations that account for these features.[33]

Throughout this section I compare my findings with an economy in which the option to default is not present; this is precisely the model considered in Aiyagari et al. (2002). I denote the variables associated with this model with a (sub)superscript "AMSS"; variables associated to my economy are denoted with a (sub)superscript "ED" (short for Economy with Default).

The utility function is given by

$$U(c, 1-n) = \frac{c^{1-\sigma_c}}{1-\sigma_c} + C_1 \frac{(1-n)^{1-\sigma}}{1-\sigma},$$

[33] See Appendix B for the details of the numerical algorithm I use to compute the model.

and $(g_t)_t$ follows a linear process

$$g_{t+1} = \mu_g(1-\rho_1) + \rho_1 g_t + \sigma_g\sqrt{1-\rho_1^2}\varepsilon_{t+1},\ \varepsilon_{t+1} \sim N(0,1).$$

I define $\underline{g}$ and $\overline{g}$ as the lower and upper bound for g_t; $\underline{b}$ and $\overline{b}$ are defined analogous for the debt.

7.1. **The i.i.d. Shocks case.** In this subsection I study the case where $\rho_1 = 0$, i.e.,

$$g_{t+1} = \mu_g + \sigma_g \varepsilon_{t+1}.$$

Table 1 presents the parameter values for the benchmark model. The goal of this section is, first, to verify the analytical results obtained in the previous sections, and second, to compare the debt policy functions for the ED and the AMSS economies.

Under this specification, the household has quasi-linear utility and in order to facilitate the comparison with Aiyagari et al. (2002) I choose the parameters associated to the utility function and discount factor equal to theirs.[34] Since, I am only interested on the dynamics when the government can default, the exogenous bounds on debt are such that the government is precluded from saving. Finally, under this parametrization autarky is assumed to be an absorbing state ($\alpha = \lambda = 0$).[35]

Parameter	Value	Parameter	Value
(Discount Factor) β	0.97	(Persistency of gov. process) ρ_1	0
(Utility of leisure) C_1	0.01	(Mean of gov. process) μ_g	0.20
(Utility of leisure) σ	2	(Vol. of gov. process) σ_g	0.05
(Utility of consumption) σ_c	0	(Range of gov. process) $(\underline{g}, \overline{g})$	$(0, 0.325)$
(Prob. of offer for partial payment) λ	0	(Range of gov. debt) $(\underline{b}, \overline{b})$	$(0, 0.3)$
(Prob. of escaping autarky) α	0		

TABLE 1. Parameter values for the benchmark model

Figure G.5 presents the policy function $b(b, g)$ for the ED model (black dots), and for the Aiyagari et al. (2002) model (red dots); default region is represented by yellow bars. Obviously, the debt policy function when the government is in default, is redundant. The

[34] In their paper $C_1 = 1$, but they assume that $n \in [0, 100]$ as opposed to my case: $n \in [0, 1]$ hence their constant C_1 should be re-scaled to $100^{1-\sigma} = 0.01$.

[35] I refer to Aiyagari et al. (2002) the implications when "natural" savings limits are imposed.

first row shows the policy function as a function of the government expenditure for low level of debt (first panel), mid level of debt (second panel) and high level of debt (third panel).[36] The second row shows the policy function as a function of the current debt level for low government expenditure (first panel), mid level government expenditure (second panel) and high level government expenditure (third panel). This last row also presents the 45 degree line (solid) for better comparison.

Although in all cases, higher level of current debt or higher level of current government expenditure imply higher level of debt tomorrow; in the ED economy the option to default generates endogenous debt limits that produce lower levels of debt. In particular, for the case of high level of debt the government decides to default at almost any value of g (first row, third panel).

Figure G.6 shows the equilibrium default set (bottom panel) and the price as a function of (b, g) (top panel). The area in blue (or lighter area) are the pairs of (g, b) for which the government opts to default. As predicted in the theoretical section the set increases as the government expenditure increases and as the debt level increases and is convex across g (i.e., for a given level of debt I look at the projection over g which is characterized by an interval).

Finally figure G.7 presents the value functions for the AMSS economy and for ED economy (continuation and autarky). For high values of (g, b) the value function of autarky in ED economy is over the value function for the AMSS economy; these are the values where the planner opts to exercise the option to default. For low (g, b) (in particular low g) either the value functions are close to each other (the cost of having the option to default is not too high) or the value functions for the AMSS economy is higher than the ones for the ED economy (bottom left panel), reflecting the cost of having the option to default.

7.2. **Impulse Response Functions.** Before looking at the Monte Carlo results, it is useful to look at the dynamics of one particular realization of $(g_t)_t$. I solve the model (see table 4

[36]Throughout this section when describing the figures the numbering of the panels increases from left to right and the numbering of the rows does it from top to bottom.

for the parameter values), then I draw a particular path for g_t given by

$$g_t = \begin{cases} \overline{g} & if\ t = T, T+1 \\ \overline{g}/2 & if\ t = T+2 \\ \overline{g} & if\ t = T+3, T+4 \end{cases} \tag{30}$$

This choice is completely arbitrary, chosen to showcase all the features of the model. I also choose $\alpha = 0.30$, and $\lambda = 0$; this choice is solely done to allow for default and secondary markets, which are novel features of this model, while keeping the model as simple as possible.

Figure G.4 presents the results. The dotted line in all the panels is the path d_t. For the first half of the sample the economy is not in default, immediately after the second government shock (upper right panel) the economy enters autarky, and then exists autarky until the end of the sample. The upper left panel depicts the spread, which should be taken as the envelope of $1/p_t^b - \beta^{-1}$ (solid) and $1/q_t - \beta^{-1}$ (dashed); the high level of $1/q_t - \beta^{-1}$ observed during financial autarky is qualitatively consistent with the spreads we see in the data (figure G.3). The middle left panel shows the debt (b_t) for both economies; the endogenous borrowing limits present in the ED economy render lower level of debt during "bad times". During autarky, since we keep track of the defaulted debt, hence we have a plateau in b_t^{ED}; then the economy exists default by paying the outstanding debt, and thus b_t^{ED} plummets to zero. The right middle panel shows the τ_t^n path for both economies; for the ED economy the path is more volatile and has an additional "spike" to cover for the payment of defaulted debt. Finally the last row presents the path for c_t and l_t respectively.

In brief, the aforementioned figures show a summary of the dynamics generated by this model: endogenous debt limits, higher volatility of taxes, and higher spreads due to default risk, in particular, during default period. Additionally, figure G.4 shows that consumption allocations of both economies are closed to each other; this is consistent with the findings in Ramsey theory under incomplete and complete markets.[37]

7.3. **Monte Carlo Simulations.**

7.3.1. *Linear utility on consumption case.* In this subsection, I run a battery of Monte Carlo (MC) simulation exercises, allowing government expenditure to be auto-correlated, and also

[37]See Aiyagari et al. (2002) and Farhi (2007).

allowing either α or λ to be nonzero. I, however, still maintain the linear utility on consumption assumption.[38]

I perform 1000 MC iterations each of them consisting of sample paths of 1000 observations for which the first 900 observations were taken out in order to eliminate the effect of the initial values. Government expenditures are assumed to be AR(1) with moments $(\rho_1, \mu_g, \sigma_g)$ chosen to match the autocorrelation, mean and volatility of the general government expenditure-to-output ratio for Argentina during the period 1993-2003. These values are (approx.) 0.30, 0.20 and 0.05, respectively.

Parameter	Value	Parameter	Value
(Discount Factor) β	0.9875	(Persistency of gov. process) ρ_1	0.30
(Utility of leisure) C_1	0.01	(Mean of gov. process) μ_g	0.20
(Utility of leisure) σ	2	(Vol. of gov. process) σ_g	0.05
(Utility of consumption) σ_c	0	(Range of gov. process) $(\underline{g}, \overline{g})$	$(0, 0.325)$
(Prob. of offer for partial payment) λ	0.08	(Range of gov. debt) $(\underline{b}, \overline{b})$	$(0, 0.3)$
(Prob. of escaping autarky) α	0.0	(Range of recovered debt) $(\underline{\delta}, \overline{\delta})$	$(0, 0.95)$
(Direct cost of default) κ	0.021		

TABLE 2. Parameter values for the case with $\lambda = 0.08$.

Table 2 presents the parametrization for the case in which offers for partial payments are allowed, i.e., $\lambda > 0$, and $\alpha = 0$. I constructed a grid of 3 elements for $(\underline{\delta}, \overline{\delta})$, with equal probability weights. I choose β, λ and $\overline{\delta}$ to match: a probability of default with range $[3, 4]$ percent, "autarky spell" in a range of $[5, 15]$ periods, and a default recovery rate of 45%.[39] The parameter κ is the direct cost of default, which yields an output of $\frac{1}{1+\kappa} n_t$ the first period in which the government decides to default, and is chosen to yield a reduction in output of 1-4% with respect to the mean.[40]

In figure G.11 we can verify the results obtained in the theory, which predicted that the acceptance region has the shape of $\{\delta \in \Delta : \delta \geq \delta(g, b)\}$.

[38]Experiments with $\sigma_c = 0.5$ present the same thresholds than in the quasi-linear case but with lower probability of default; they are available upon request.
[39]The default recovery rate is taken from Yue (2005) where 30% is the recovery rate for Argentina and 60% is the recovery rate for Ecuador. So I took the average of both.
[40]Pitchford and Wright (2008) report that, on average, for their dataset default began when output was (approx.) 1.5% below trend; Reinhart and Rogoff (2008) report a decline in output of (approx.) 4% (with respect to past levels) at the time of default.

Averages across MC simulations of some statistics for: the whole sample, the "no default" sample, and the "autarky or default" sample are presented in table 3. I constructed these latter subsamples by separating, for each MC iterations, the periods where the ED economy was in autarky from those where the economy was not.

Sample	All		No Default		Default	
	AMSS	ED	AMSS	ED	AMSS	ED
$E(b_t/n_t)$ (%)	12.65	1.870	12.43	1.196	22.70	17.19
$std(b_t/n_t)$ (%)	9.290	3.420	8.600	1.410	5.611	0.580
$E(\tau_t)$	0.218	0.218	0.219	0.218	0.218	0.235
$std(\tau_t)$	0.030	0.051	0.027	0.049	0.019	0.053
$E(\tau_t n_t)$	0.194	0.193	0.193	0.194	0.194	0.200
$std(\tau_t n_t)$	0.026	0.045	0.023	0.043	0.016	0.046
$E(spread)$ (%)		5.660		0.820		119.3
E(Default Spell)		11.12				
E(Recovery Rate) (%)		46.50				
$\Pr(Default)$ (%)		4.120				

TABLE 3. MC results for the case $\lambda = 0.08$. In the table E and std denote the mean and standard deviation across time, respectively. All quantities are averaged across MC simulations.

The average debt-to-output ratio (row 1) for the whole sample is around 12 % for the AMSS economy; in the ED economy, however, is around 2% because of the presence of the endogenous borrowing limits arising from the possibility of default. This level is low compared to what is observed in the data: a ratio of approximately 23% for Argentina (1990-2005).[41] For the default sub-sample, however, the average debt-to-output ratio in the ED economy is only 4 % lower than in the AMSS economy. Since in this part of the sample the average debt-to-output ratio is actually the *defaulted* debt-to-output ratio; higher quantities of debt-to-output ratios denote higher likelihood of default; this provides additional evidence of endogenous borrowing limits being "active" in higher levels of debt.

The volatility (as standard deviations) of the debt-to-output ratio is higher in the AMSS economy for all three samples (row 2).

Although the average tax rate (row 3) is similar in both economies across all three samples, the volatility of the tax rate (row 4) is higher in the ED economy; especially in the default

[41]For the default period (2001-2005) this ratio was (approx.) 45%.

sub-sample. This is a consequence of the endogenous borrowing limits which imply that the debt is not as good of an instrument to smooth shocks as it is in the AMSS economy. In particular, when the ED economy is in autarky, the planner is precluded from issuing debt rendering taxes more volatile than in the other sub-samples.

I compute the spread as $100(1/p_t^b - 1/\beta)$ for the "no default" sample, and as $100(1/q_t - 1/\beta)$ for the "default" sample. Note that I use the secondary market price to compute the spread during the "default" period. The spread (row 8) is around 7% for the whole sample, and around 1% for the "no default" sample. This is below the 5% (approx.) registered for Argentina (1997 - 2000 and 2005-2006) using the EMBI+. During the "default" period, the spread is around 120%.[42]

Figures G.12 and G.13 present the box-plots, which allow me to conclude whether the MC average present statistically significant differences; for all quantities except average taxes this is the case.

Parameter	Value	Parameter	Value
(Discount Factor) β	0.9825	(Persistency of gov. process) ρ_1	0.30
(Utility of leisure) C_1	0.01	(Mean of gov. process) μ_g	0.20
(Utility of leisure) σ	2	(Vol. of gov. process) σ_g	0.05
(Utility of consumption) σ_c	0	(Range of gov. process) $(\underline{g}, \overline{g})$	$(0, 0.325)$
(Prob. of offer for partial payment) λ	0	(Range of gov. debt) $(\underline{b}, \overline{b})$	$(0, 0.3)$
(Prob. of escaping autarky) α	0.30		
(Direct cost of default) κ	0.021		

TABLE 4. Parameter values for the case with $\alpha = 0.30$

Table 5 presents the case where I only have the possibility of exiting autarky by paying 100% of the debt, i.e., $\lambda = 0$ and $\alpha > 0$; table 4 contains the parameter values. I choose β and α to match: a probability of default with range $[3, 4]$ percent, "autarky spell" in a range of $[5, 15]$ periods.

The MC mean of average debt-output ratio is higher in AMSS than in my default economy (row 1). The ratio in the former economy is approximately 13%, whereas in my economy this ratio is about 6%. For the default sample, however, the average debt-to-output ratio in the ED economy is much higher, around 18 %.

[42]For Argentina during the default period (2001-2005) the spread using the EMBI+ was of about 55%.

Sample	All		No Default		Default	
	AMSS	ED	AMSS	ED	AMSS	ED
$E(b_t/n_t)$ (%)	12.64	5.820	12.41	5.520	29.70	18.20
$std(b_t/n_t)$ (%)	9.290	5.100	8.490	4.590	0.033	0.001
$E(\tau_t)$	0.219	0.218	0.219	0.218	0.264	0.247
$std(\tau_t)$	0.030	0.040	0.027	0.038	0.023	0.049
$E(\tau_t n_t)$	0.194	0.193	0.194	0.193	0.233	0.218
$std(\tau_t n_t)$	0.027	0.035	0.023	0.032	0.020	0.042
$E(spread)$ (%)		0.410		0.13		10.30
E(Default Spell)		5.694				
E(Recovery Rate) (%)		-				
Pr($Default$) (%)		3.300				

TABLE 5. MC results for the case $\alpha = 0.30$. In the table E and std denote the mean and standard deviation across time, respectively. All quantities are averaged across MC simulations.

The flip side of the endogenous borrowing limits, a higher volatility in the taxes, can be seen in rows 3-4 (tax) and 5-6 (tax revenue) across all three subsamples.

In the ED economy the average spread is defined as the spread associated to p_t^b in times of "no default" and to q_t in "default"; the value of this quantity (row 8) is 0.4%. For the default sample, however, the spread is around 11%; although still low, it is much higher than for the "no default" sample.

Figures G.9 to G.10 present box-plots of the statistics presented in the table for the whole sample case. In each panel, the first box-plot belongs to the AMSS economy; the second one belongs to the ED economy. From the first figure we can see that all the differences between both economies are significant (except for the average tax). From the second figure we can infer that this model can generate spreads as high as 5% (for the whole sample), and average default spells as long as 25 periods.

We observe that the results in the $(\alpha > 0, \lambda = 0)$ and $(\alpha = 0, \lambda > 0)$ experiments are qualitatively similar. Both cases have lower debt-to-output ratio, and more volatile taxes (or revenues). In both cases the *defaulted* debt-to-output ratio is higher than the ratio for "no default" samples, and the spread is also higher in this period. These similarities notwithstanding, there are some notable differences. First, debt-to-output ratio in the economy with $\alpha > 0$ and $\lambda = 0$ is higher on average than in the economy with $\alpha = 0$ and $\lambda > 0$.

Second, the autarky spell is higher in the ($\alpha = 0, \lambda > 0$) experiment and also the probability of default presents higher "outliers" (figure G.13 left-top panel). Finally, the spread for all three sub-samples is higher in the ($\alpha = 0, \lambda > 0$) experiment, moreover the spread can be as high as 120 % (figure G.13 left-bottom panel).

The last fact is driven by two effects. First, the probability of default is slightly higher in the ($\alpha = 0, \lambda > 0$); also this experiment exhibits higher "outliers". Second, the autarky spell is lower in the economy with $\alpha > 0$ and $\lambda = 0$. Given that p^b conveys information regarding "distant" defaults and autarky spells through the price q; a lower autarky spell translates into a higher probability of getting paid *at some point in the future.*

7.3.2. *Welfare Comparison.* In order to assess the welfare implications of my model, let Ω be the increment of labor income in the *initial* period, i.e., $c_0^\Omega \equiv (1+\Omega)n_0 - g$, such that

$$\int_{\mathbb{B}\times\mathbb{G}\times\Delta\cup\{1\}} \mathcal{U}^{AMSS}(b,g)\Pi_{bg\delta}(db,dg,d\delta) = \int_{\mathbb{B}\times\mathbb{G}\times\Delta\cup\{1\}} \mathcal{U}^{\Omega}(b,g,\delta)\Pi_{bg\delta}(db,dg,d\delta)$$

where $\mathcal{U}^{AMSS}(b,g)$ is the present value expected utility in the AMSS economy and $\mathcal{U}^\Omega$ is the value function with c^Ω instead of c, for the initial period. Doing Taylor approximation of U and doing some algebra it follows

$$\Omega \approx \frac{\int_{\mathbb{B}\times\mathbb{G}\times\Delta\cup\{1\}} \mathcal{U}^{AMSS}(b,g) - \mathcal{U}(b,g,\delta)\Pi_{bg\delta}(db,dg,d\delta)}{\int_{\mathbb{B}\times\mathbb{G}\times\Delta\cup\{1\}} U_c(n^*(b,g,\delta)-g)n^*(b,g,\delta)\Pi_{bg\delta}(db,dg,d\delta)},$$

where n^* is the policy function. The measure $\Pi_{bg\delta}$ is computed as the frequency across 1000 Monte Carlo repetitions at time $T = 1000$ so as to avoid any dependence on the initial values.[43]

I study the welfare implications for ($\alpha = 0, \lambda > 0$) and ($\alpha > 0, \lambda = 0$) separately; figures G.15 and G.14 present the results. For $\lambda \in (0,1)$ and $\alpha = 0$, as λ increases the option value of defaulting increases, prompting the Ramsey planner to default more; the planner, however, receives the repayment offer more often and thus the likelihood of repayment increases. For low values of λ the former effect dominates and the likelihood of being in autarky is high (dashed line in figure G.15). For high values of λ the latter effect dominates and the likelihood

[43]Another measure is given by

$$W(\alpha,\lambda) \equiv \int_{\mathbb{B}\times\mathbb{G}\times\Delta\cup\{1\}} \left(\mathcal{U}^{AMSS}(b,g) - \mathcal{U}(b,g,\delta)\right)\Pi_{bg\delta}(db,dg,d\delta)$$

but as it gives qualitatively the same results I do not report it.

of being in autarky is low (dashed line in figure G.15). The solid line in this figure shows Ω for $\lambda \in (0,1)$ and $\alpha = 0$. For $\lambda = 0$ default does not exist. As λ increases, default becomes more probable; activating the cost of having the option, buried in the pricing functions. There is another effect, however, for which autarky becomes more attractive; this effect dominates when λ is high.

For the case of $\alpha \in (0,1)$ and $\lambda = 0$, if α increase the value option of defaulting also increases but the government always has to spend *at least* one period under autarky; this non-vanishing cost implies that even for $\alpha = 1$, the probability of autarky is not zero (dashed line in figure G.14). The solid line in figure G.14 show Ω for $\alpha \in (0,1)$ and $\lambda = 0$; its behavior is similar to the one in the previous case.

Although these results are small in magnitude (at most, the agent is willing to give/receive 1% of consumption at the initial period), depend on the particular partial payments protocol – which is exogenous – and on implicit underlying assumptions (e.g. homogeneity of households), they offer a good guideline for policy. For low/medium probabilities of receiving the offers, the cost of default – paying higher returns – dominates; for high probabilities, however, the benefit of default – allowing for contingent payoffs in the "efficient directions" (see Zame (1993)) – dominates.[44]

8. Conclusion

First, this paper provides a plausible explanation for the lower debt-to-output ratios and more volatile tax policies observed in emerging economies, vis-à-vis industrialized economies. This stems from the fact that the benevolent government not only chooses distortionary labor taxes and one period non state-contingent debt, but it also has the option to default on its debt. This option to default does not come for free: the households in this economy – who are the holders of government debt – forecast the possibility of default, imposing endogenous

[44] I minor remark is that given the design of the timing there might be ex-post inefficiencies as the "third player" in this economy, nature, is not optimizing. That is, it might be the case that the government chooses to declare default and thus goes to the node where he awaits for nature to play (node (B) in figure D) but once there, when λ and δ are realized the government would find optimal *not* to default. In the case λ and δ were *endogenous*, these inefficiencies might not arise as these quantities will not be chosen by a non-optimizing player but will be functions of the state of the economy, which arise as outcomes of some decision problem involving the government and the households.

debt limits. These limits restrict the ability of the government to smooth shocks using debt, thus rendering taxes more volatile.

Second, this paper proposes a device to price the debt during temporary financial autarky. If the government defaults, it can only exit financial autarky by paying at least a positive fraction of the defaulted debt; the possibility of payments arrive at an exogenous rate and may not arrive immediately. Therefore, during temporary financial autarky the defaulted debt has positive value and households trade its shares in a secondary market. In contrast with existing literature, I can compute interest rate spreads during the default period, using the equilibrium price in this market. Numerical simulations show that the spread during the default period is higher than for the rest of the sample; this characteristic is consistent with data for defaulters, e.g. Argentina, Ecuador and Russia.

Third and last, the numerical simulations suggest that increasing the probability of receiving offers for exiting autarky decreases welfare when this probability is low/medium to begin with, but increases it when the probability is high. This non-monotonicity arises from the interaction of two dichotomous effects; on the one hand the positive effect of having more state-contingency on the debt, and on the other the negative effect of endogenous debt limits.

Although this model does a good job in explaining qualitatively the facts observed in the data, it does not do very well in matching the data quantitatively. A line of future research is to delve further into the production side of this economy and its driving shocks.[45] Another line of research that I am pursuing is to study how (a) household's heterogeneity and (b) endogenous debt renegotiation schemes, affect the trade volume in the secondary markets and consequently the welfare. Finally, this model, as well as Arellano (2008) and references therein, produces "nonstandard" pricing kernels; I am currently working on estimating these pricing kernels using nonparametric methods. This is important because it quantifies the pricing implications of these models.[46]

[45] See Aguiar and Gopinath (2006) and Mendoza and Yue (2008).

[46] This project is joint with Xiaohong Chen, and it's based on our previous work (Chen and Pouzo (2008)).

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Appendix A. Proofs

Proof of proposition 6.1. First I need to show that $(1-U_l(1-n_t))n_t$ is decreasing, for optimally chosen n_t. Under assumption 3.2 the government optimal tax revenue is given by $A(n) \equiv (1-U_l(1-n_t))n_t$; which $A(0)=0$ and $A(1)=-\infty$ (by assumption 3.2). Moreover it follows that

$$\nabla_n[A(n^*)] = 0 \iff (1-U_l(1-n^*)) = -U_{ll}(1-n^*)n^*$$

thereby implying $A(n^*) = -U_{ll}(1-n^*)(n^*)^2 > 0$ by assumption 3.2. So by continuity $A(n)$ has to decrease for a nontrivial interval included in $[n^*, 1)$. Since under assumption 3.2 the utility: $n_t - g_t + U(1-n_t)$ is increasing over n_t, it is optimal for the government to choose n_t on the decreasing part of the tax revenue; otherwise the government achieves the same tax revenue but for a low level n_t yielding lower utility. Hence $(1-U_l(1-n_t))n_t$ is decreasing, for optimally chosen n_t.

I show the result by contradiction. Assume that there exists a $\overline{b}_{t+1}$ such that $b_t - \mathcal{P}(\overline{b}_{t+1})\overline{b}_{t+1} \leq 0$ then it follows from the implementability conditions and assumption 6.1(ii)

$$(1-U_l(1-n_t))n_t - g_t = b_t - \mathcal{P}(\overline{b}_{t+1})\overline{b}_{t+1} \leq 0;$$

hence denoting n_t^C as the optimal choice under $d_t = 0$ and n_t^A as the optimal choice under $d_t = 1$ it follows that

$$(1-U_l(1-n_t^C))n_t^C \leq (1-U_l(1-n_t^A))n_t^A.$$

Since $(1-U_l(1-n))n$ is decreasing, $n_t^C \geq n_t^A$, thus it follows that the immediate utility under $d_t = 0$ is higher than under autarky ($d_t = 1$). This result, plus the fact that the continuation utility is always higher under $d_t = 0$ (the planner has the *option* to go to autarky) yields that no default is always preferred to autarky; thus $D[\mathbb{G}](b_t) = \{\emptyset\}$, a contradiction. □

Proof of proposition 6.2. Define $BL(n_t; g_t) \equiv -U_l(1-n_t)n_t + n_t - g_t$. Notice that

$$\nabla_n BL(n_t; g_t) = 1 - U_l(1-n_t) + U_{ll}(1-n_t)n_t,$$

Moreover notice that, for $g_{1,t} \leq g_{2,t}$ [47]

$$BL(n_{1,t}^C; g_{2,t}) \leq BL(n_{1,t}^C; g_{1,t}) = b_t - \mathcal{P}(b_{1,t+1}^C)b_{1,t+1}^C$$

and

$$BL(n_{2,t}^C; g_{1,t}) \geq BL(n_{2,t}^C; g_{2,t}) = b_t - \mathcal{P}(b_{2,t+1}^C)b_{2,t+1}^C$$

with $b_{i,t+1}^C$ and $n_{i,t}^C$ being the optimal policy function under $g_{i,t}$. It thus follows that there exists a $\overline{n}_{2,t}$ such that

$$BL(\overline{n}_{2,t}; g_{1,t}) = BL(n_{2,t}^C; g_{2,t}),$$

i.e., $(\overline{c}_{2,t}, \overline{n}_{2,t}, b_{2,t+1}^C)$ is feasible under $g_{1,t}$. Given that at $g_{1,t}$ the government defaults, it must be true that the immediate utility under default exceeds the immediate utility of the bundle $(\overline{c}_{2,t}, \overline{n}_{2,t}, b_{2,t+1}^C)$, i.e.,

$$U(c_{1,t}^A, 1-n_{1,t}^A) \geq U(\overline{c}_{2,t}, 1-\overline{n}_{2,t}). \tag{31}$$

Moreover it follows that

$$U(c_{1,t}^C, 1-n_{1,t}^C) + \beta E\left[\mathcal{U}^o(g, b_{1,t+1}^C)\right] \geq U(\overline{c}_{2,t}, 1-\overline{n}_{2,t}) + \beta E\left[\mathcal{U}^o(g, b_{2,t+1}^C)\right]. \tag{32}$$

[47] Throughout this proof I implicitly use a key implication of assumption $g_t \sim i.i.d.$: prices do not depend on current g_t directly.

To show that the government chooses to default under $g_{2,t}$ I need to verify that [48]

$$U(c^A_{2,t}, 1-n^A_{2,t}) + \beta E\left[\mathcal{U}(g,1)\right] \geq U(c^C_{2,t}, 1-n^C_{2,t}) + \beta E\left[\mathcal{U}^o(g, b^C_{2,t+1})\right].$$

Invoking equations 32, and that the government defaults under $g_{1,t}$; it suffices to show that

$$U(c^A_{1,t}, 1-n^A_{1,t}) - U(c^A_{2,t}, 1-n^A_{2,t}) < U(\overline{c}_{2,t}, 1-\overline{n}_{2,t}) - U(c^C_{2,t}, 1-n^C_{2,t}). \tag{33}$$

Thus I need to show that the difference on the right is "less negative", than the one on the left.

I already showed in the proof of proposition 6.1 that $BL(n;g)$ is decreasing in n; thus since $g_{1,t} \leq g_{2,t}$ it follows that $n^A_{1,t} \geq n^A_{2,t}$, and by the same argument $n^C_{2,t} \leq \overline{n}_{2,t}$. By proposition 6.1 I know that $b_t - \mathcal{P}(b_{t+1})b_{t+1} \geq 0$ thus $g_{i,t} + b_t - \mathcal{P}(b_{t+1})b_{t+1} \geq g_{i,t}$ implying

$$BL(\overline{n}_{2,t}; g_{1,t}) \geq BL(n^A_{1,t}; g_{1,t}), \text{ and } BL(n^C_{2,t}; g_{2,t}) \geq BL(n^A_{2,t}; g_{2,t})$$

so $\overline{n}_{2,t} \leq n^A_{1,t}$ and $n^C_{2,t} \leq n^A_{2,t}$.

I need to analyze two cases. First case, let $n^C_{2,t} \leq n^A_{2,t} \leq \overline{n}_{2,t} \leq n^A_{1,t}$ and define $\mathcal{V}(n) = BL(n;g)+g$ then

$$\mathcal{V}(n^C_{2,t}) - \mathcal{V}(n^A_{2,t}) = b_t - \mathcal{P}(b^C_{2,t+1})b^C_{2,t+1} = \mathcal{V}(\overline{n}_{2,t}) - \mathcal{V}(n^A_{1,t}).$$

Since I am under the case $n^C_{2,t} \leq n^A_{2,t} \leq \overline{n}_{2,t} \leq n^A_{1,t}$ and $\mathcal{V}(n_t)$ is decreasing and concave (assumption 6.3), the difference $n^A_{2,t} - n^C_{2,t}$ is greater than $n^A_{1,t} - \overline{n}_{2,t}$.[49]

By my assumptions U is increasing and concave, thus it follows that

$$\frac{U(n^A_{2,t} - g_t, 1-n^A_{2,t}) - U(n^C_{2,t} - g_t, 1-n^C_{2,t})}{n^A_{2,t} - n^C_{2,t}} \geq \frac{U(n^A_{1,t} - g_t, 1-n^A_{1,t}) - U(\overline{n}_{2,t} - g_t, 1-\overline{n}_{2,t})}{n^A_{1,t} - \overline{n}_{2,t}}.$$

which implies that $U(n^A_{2,t} - g_t, 1-n^A_{2,t}) - U(n^C_{2,t} - g_t, 1-n^C_{2,t})$ is greater than $U(n^A_{1,t} - g_t, 1-n^A_{1,t}) - U(\overline{n}_{2,t} - g_t, 1-\overline{n}_{2,t})$ and doing some algebra it is easy to verify that this implies equation 33.

In the case $n^C_2 \leq \overline{n}_{2,t} \leq n^A_{2,t} \leq n^A_{1,t}$ I can show the desired result using an analogous argument and is not be repeated here. □

Proof of proposition 6.3. Take $g_t \in D[\mathbb{G}](b_{1,t})$ then it must be true that

$$U(c^C_{1,t}, 1-n^C_{1,t}) + \beta E\left[\mathcal{U}^o(b^C_{1,t+1}, g)|g_t\right] < U(c^A_t, 1-n^A_t) + \beta E\left[\mathcal{U}(g,1)|g_t\right]$$

where $n^i_{1,t}, c^i_{1,t}, b^i_{1,t+1}$ with $i \in \{C, A\}$ denotes the policy functions given $(b_{1,t}, g_t)$ for "continuation" and "autarky", respectively. First note that the right hand side is constant as a function of b_t. Second note that, as $b_{1,t} \leq b_{2,t}$, then (using the notation in the proof of proposition 6.2)

$$BL(n^C_{1,t}; g_t) + \mathcal{P}(b_{t+1})b_{t+1} = b_{1,t} \leq b_{2,t} = BL(n^C_{2,t}; g_t) + \mathcal{P}(b_{t+1})b_{t+1}, \ \forall b_{t+1} \in \mathbb{B}.$$

Given that $BL(n;g_t)$ is decreasing as a function of n by proof of proposition 6.1, then it must follow that $n^C_{1,t} \geq n^C_{2,t}$. Given that $U(n-g_t, 1-n)$ is increasing it must follow that

$$U(n^C_{1,t} - g_t, 1-n^C_{1,t}) + \beta E\left[\mathcal{U}^o(b_{t+1}, g)|g_t\right] \geq U(n^C_{2,t} - g_t, 1-n^C_{2,t}) + \beta E\left[\mathcal{U}^o(b_{t+1}, g)|g_t\right].$$

Given that $U(c^A_t, 1-n^A_t) + \beta E\left[\mathcal{U}(g,1)|g_t\right]$ is constant as a function of the debt it must follow that

$$U(c^A_t, 1-n^A_t) + \beta E\left[\mathcal{U}(g,1)|g_t\right] > U(n^C_{2,t} - g_t, 1-n^C_{2,t}) + \beta E\left[\mathcal{U}^o(b_{t+1}, g)|g_t\right]$$

and thus $g_t \in D[\mathbb{G}](b_{2,t})$. □

Proposition A.1. *$\mathcal{M}(b_{t+1}) \equiv \frac{1}{1-\zeta(b_{t+1})}$ is such that:*

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[48]Abusing notation I excluded b from the value function of the planner under autarky as it is not needed anymore and I denote it as $\mathcal{U}(\overline{g}(b_t), 1)$.

[49]For concave and decreasing function $f(x)$ with $a < b < c < d$, it follows $\frac{f(a)-f(b)}{a-b} \geq \frac{f(c)-f(d)}{c-d}$. So, if $f(a) - f(b) = f(c) - f(d)$ it must hold that $b - a \geq d - c$.

(1) $\mathcal{M}(b_{t+1}) \geq 1$ *for all* $b_{t+1} \in [b_*, b^*]$ *and* $\mathcal{M}(0) = 1$.
(2) $\mathcal{M}(b_{t+1})$ *is increasing (decreasing) iff* $\zeta(b_{t+1})$ *is increasing (decreasing).*
(3) $\zeta(b_{t+1})$ *is increasing (decreasing) iff* $\nabla_b \left[\log\left(-\nabla_b[\log(\Pi(\overline{g}(b_{t+1})))]\right)\right] + \frac{1}{b_{t+1}} \geq (\leq) 0$

Proof of Proposition A.1. (1) It suffices to show that $\zeta(b_{t+1}) \in [0,1]$. By definition

$$\zeta(b) = -\frac{\pi(\overline{g}(b))\nabla_b[\overline{g}(b)]b}{\Pi(\overline{g}(b))}$$

and I already showed that $\nabla_b[\overline{g}(b)]$ is decreasing, thus $\zeta(b) \geq 0$. Also note that $\zeta(b) \leq 1$ if and only if

$$0 \leq \Pi(\overline{g}(b)) + \pi(\overline{g}(b))\nabla_b[\overline{g}(b)]b$$

which has the same sign as the derivative of $\mathcal{P}(b)b$ with respect to $b \in [b_*, b^*]$. The latter expression is non-negative by optimality of the choice of b, otherwise the planner could perceive higher debt income by lowering the debt, and that contradicts optimality of the debt.

(2) It is easy to see that

$$\nabla_b[\mathcal{M}(b)] = \frac{\nabla_b[\zeta(b)]}{(1-\zeta(b))^2}$$

and thus the desired result follows.

(3) I show this for the increasing part, the decreasing is analogous.[50] Note that

$$\begin{aligned}
\nabla_b[\zeta(b)] &= -\frac{\nabla_b[\nabla_b[\Pi(b)]b]}{\Pi(b)} + \left(\frac{\nabla_b[\Pi(b)]}{\Pi(b)}\right)^2 b \\
&= -\frac{\nabla_b^2[\Pi(b)]}{\Pi(b)}b - \frac{\nabla_b[\Pi(b)]}{\Pi(b)} + \left(\frac{\nabla_b[\Pi(b)]}{\Pi(b)}\right)^2 b \\
&= \frac{\nabla_b[\Pi(b)]}{\Pi(b)}\left[-\frac{\nabla_b^2[\Pi(b)]}{\nabla_b[\Pi(b)]}b - 1 + \frac{\nabla_b[\Pi(b)]}{\Pi(b)}b\right].
\end{aligned}$$

Given that $\frac{\nabla_b[\Pi(b)]}{\Pi(b)} \leq 0$, then $\nabla_b[\zeta(b)] \leq 0$ iff

$$\begin{aligned}
&\frac{\nabla_b[\Pi(b)]}{\Pi(b)} - \frac{\nabla_b^2[\Pi(b)]}{\nabla_b[\Pi(b)]} \geq \frac{1}{b} \\
\iff\ & \nabla_b[\log(\Pi(b))] - \nabla_b[\log(-\nabla_b[\Pi(b)])] \geq \frac{1}{b} \\
\iff\ & \nabla_b\left[\log\left(-\frac{\nabla_b[\Pi(b)]}{\Pi(b)}\right)\right] \leq -\frac{1}{b} \\
\iff\ & \nabla_b\left[\log\left(-\nabla_b[\log(\Pi(b))]\right)\right] \leq -\frac{1}{b}.
\end{aligned}$$

□

The expression in part (3) imposes restrictions on $\Pi(\cdot)$ and $\overline{g}(\cdot)$. For instance, if $\Pi(g) = \frac{g-\underline{g}}{\overline{g}-\underline{g}}$ and $\overline{g}(b) = C_1 \exp\{-C_2 b^2\}$ then $\nabla_b\left[\log\left(-\nabla_b[\log(\Pi(\overline{g}(b_{t+1})))]\right)\right] + \frac{1}{b_{t+1}} = 0$.[51]

Proof of Proposition 6.4. (1) First note that under equation 27 and the envelope condition γ is a decreasing function of g. Second, note that the default adjusted measure, $\frac{\int_{g \leq g_1} \mathbb{I}\{g : g \leq \overline{g}_{t+1}\}}{\Pi(\overline{g}_{t+1})}\pi(dg)$ is

[50] I omit the $t+1$ subscript and denote $F(b) \equiv F(\overline{g}(b))$ for generic function F, for the sake of keeping the notational burden low.

[51] Where the constants $C_i,\ i = 1,2$ are derived from integrating the PDE $\nabla_b\left[\log\left(-\nabla_b[\log(\Pi(b))]\right)\right] + \frac{1}{b} = 0$.

first order dominated by $\int_{g \leq g_1} \pi(dg)$. Thus putting both results together it follows that $\mathbb{E}[\gamma_{t+1}] \geq E[\gamma_{t+1}]$.

The expression for γ_t can be written as

$$\mathbb{E}[\gamma_{t+1}]\mathcal{M}(b_{t+1}) - E[\gamma_{t+1}].$$

As $\mathcal{M}(b_{t+1}) \geq 1$ it follows that is bounded from below by

$$\mathbb{E}[\gamma_{t+1}] - E[\gamma_{t+1}]$$

which by my previous result I know it is positive; hence $\gamma_t > E[\gamma_{t+1}]$. This implies that γ_t converges almost surely to some limit, γ_∞ (see Billingsley (1995)) for all the histories g^∞ such that the planner does not default. Therefore in order to prove that $\gamma_t \to \gamma_\infty$ w.p.p. it suffices to show that $\Pr(No\ default) \geq c > 0$. It is easy to see that

$$\Pr(No\ default\ until\ t) = \Pr(No\ default\ at\ t) \Pr(No\ default\ until\ t-1),$$

and by my previous results $\Pr(No\ default\ at\ t) = \Pi(\overline{g}(b_t))$. Therefore iterating, it follows

$$\log(\Pr(No\ default)) = \sum_t -\log(1/\Pi(\overline{g}(b_t))).$$

So a sufficient condition for $\Pr(No\ default) \geq c > 0$ is that $-\log(1/\Pi(\overline{g}(b_t)))$ decays faster than C/t^2. The condition in the proposition ensures that this holds.

(2) First note that $\frac{1}{\Pi(\overline{g}(b_{t+1}))(1-\zeta(b_{t+1}))} = \frac{\mathcal{M}(b_{t+1})}{\Pi(\overline{g}(b_{t+1}))} \geq 0$, and

$$\begin{aligned}
\frac{1}{\Pi(\overline{g}(b_{t+1}))} Cov\left(\mathbb{I}\{g \geq \overline{g}(b_{t+1})\}, \gamma_{t+1}\right) &= \frac{1}{\Pi(\overline{g}(b_{t+1}))} Cov\left(\mathbb{I}\{g \geq \overline{g}(b_{t+1})\} - 1, \gamma_{t+1}\right) \\
&= \frac{1}{\Pi(\overline{g}(b_{t+1}))} Cov\left(-\mathbb{I}\{g \leq \overline{g}(b_{t+1})\}, \gamma_{t+1}\right) \\
&= \frac{1}{\Pi(\overline{g}(b_{t+1}))} E\left[\left(\Pi(\overline{g}(b_{t+1})) - \mathbb{I}\{g \leq \overline{g}(b_{t+1})\}\right)\gamma_{t+1}\right] \\
&= \left(E\left[\gamma_{t+1}\right] - \mathbb{E}\left[\gamma_{t+1}\right]\right)
\end{aligned}$$

Where the first equality holds because the covariance of random variable and a constant is zero, and the second equality follows from $1 = \mathbb{I}\{g \leq \overline{g}(b_{t+1})\} + \mathbb{I}\{g \geq \overline{g}(b_{t+1})\}$. The third equality is true because, for generic random variables (X,Y), $Cov(X,Y) = E[(X - E[X])(Y - E[Y])] = E[(X - E[X])Y]$. By equation 28 and the envelope condition γ is a increasing function of g, thus the last term is positive. The term $E[\gamma_{t+1}](\mathcal{M}(b_{t+1}) - 1)$ is positive by the properties presented in proposition A.1.

(a) $\nabla_b[E[\gamma_{t+1}(\mathcal{M}(b_{t+1}) - 1)]]$ is given by

$$\nabla_b[\mathcal{M}(b_{t+1})]E[\gamma_{t+1}] + \nabla_b[E[\gamma_{t+1}]](\mathcal{M}(b_{t+1}) - 1).$$

By proposition A.1(3) and my assumptions the first term is positive. Invoking the envelope conditions, the second terms is given by

$$\left\{\int \nabla_b[\gamma(b_{t+1}, g)]\Pi(dg)\right\}(\mathcal{M}(b_{t+1}) - 1).$$

By assumption $\nabla_b[\gamma(b_{t+1}, g)] = -\nabla_b^2[\mathcal{U}(b_{t+1}, g, 0)] \geq 0$, and thus the first term is positive; implying that $\nabla_b[E[\gamma_{t+1}]] \geq 0$.

(b) Defining $\pi_{b_{t+1}}(dg)$ as the "default-adjusted" pdf, it follows

$$\nabla_b[E[\gamma_{t+1}] - \mathbb{E}[\gamma_{t+1}]] = \nabla_b\left[\int_{\mathbb{G}} \gamma(b_{t+1},g)\Pi(dg)\right] - \nabla_b\left[\int \gamma(b_{t+1},g)\Pi_{b_{t+1}}(dg)\right]$$

$$= \int \nabla_b \gamma(b_{t+1},g)(\pi(g) - \pi_{b_{t+1}}(g))dg - \gamma(b_{t+1},\overline{g}(b_{t+1}))\frac{\pi(\overline{g}(b_{t+1}))}{\Pi(\overline{g}(b_{t+1}))}$$

$$- \int_{\{g:g\leq\overline{g}(b_{t+1})\}} \gamma(b_{t+1},g)\left\{-\frac{\pi(\overline{g}(b_{t+1}))}{(\Pi(\overline{g}(b_{t+1})))^2}\nabla_b[\overline{g}(b_{t+1})]\right\}\pi(g)dg$$

Since $\gamma(b_{t+1},g)$ is positive, and $\nabla_b[\overline{g}(b_{t+1})] \leq 0$ it follows that third term is positive. Similarly, the second term is also positive. I already showed that the default probability measure is first order dominated by $\Pi(g)$. Moreover, $\nabla_b^2[\gamma(b_{t+1},g)] = -\nabla_b^3[\mathcal{U}(b_{t+1},g,0)]$, and by assumption is negative. Therefore $\nabla_b[\gamma(b_{t+1},g)]$ is a decreasing function of g, which implies that the first term of the equation above is also negative. Therefore $\nabla_b[E[\gamma_{t+1}] - \mathbb{E}[\gamma_{t+1}]]$ is negative; automatically implying that $\frac{Cov(\mathbb{I}\{g\geq\overline{g}(b_{t+1})\},\gamma_{t+1})}{\Pi(\overline{g}(b_{t+1}))(1-\zeta(b_{t+1}))}$ is decreasing. $\square$

Proof of Proposition 6.5. First, I define $\{b_0,(n_t,g_t)_{t=0}^\infty\}$ as being implementable in the Aiyagari et al. (2002) (AMSS, hereinafter) economy if

$$\{b_0,(n_t,g_t)_{t=0}^\infty\} \in \mathcal{AMSS} \equiv \left\{\{b_0,(n_t,g_t)_{t=0}^\infty\} : s_0 + \sum_{t<\mathcal{T}}\int_{g^t}\beta^t s_t pr_t(dg^t) + \int_{g^\mathcal{T}}\beta^\mathcal{T} b_\mathcal{T} pr_\mathcal{T}(dg^\mathcal{T}) = b_0, \forall\mathcal{T};\right.$$

$$\left. and\ s_i + \sum_{t=i}^{\infty}\int_{g^t}\beta^t s_t pr_t(dg^t) = b_i \in \mathcal{G}^{i-1}\right\},$$

where $s_t = (n_t - U_l(1-n_t)) - g_t$. Similarly, I define $\{b_0,(n_t,g_t)_{t=0}^\infty\}$ as being implementable in the economy with default (ED, hereinafter) if

$$\{b_0,(n_t,g_t)_{t=0}^\infty\} \in \mathcal{ED} \equiv \left\{\{b_0,(n_t,g_t)_{t=0}^\infty\} : s_0 + \sum_{t<\mathcal{T}}\int_{g^t}\beta^t \Lambda_t s_t pr_t(dg^t) + \int_{g^\mathcal{T}}\beta^\mathcal{T}\Lambda_\mathcal{T} b_\mathcal{T} pr_\mathcal{T}(dg^\mathcal{T}) = b_0, \forall\mathcal{T};\right.$$

$$\left. s_t = 0\ \forall\{t : d_t = 1\}\ and\ s_i + \sum_{t=i}^{\infty}\int_{g^t}\beta^t\Lambda_t s_t pr_t(dg^t) = b_i \in \mathcal{G}^{i-1}\right\},$$

where $\Lambda_t \equiv \prod_{j=0}^t(1-d_j)$. That is, a sequence $(n_t,g_t)_{t=0}^\infty$ and an initial debt b_0 are implementable in ED if they satisfy the measurability constraints of Aiyagari et al. (2002), the Lucas and Stokey (1983) implementability condition, and the fact that in default the debt is not honored and after that $s_t = 0$.[52]

Second, I define $\mathcal{T}$ such that $\mathcal{T} = \inf\{t > 0 : d_t = 1,\ w.p.p.\}$; the assumption $pr_\infty\{T^* < \infty\} > 0$ implies that I consider only the sequences $(g^t)_t$ such that $\mathcal{T} < \infty$ because otherwise $\mathcal{T} = \infty \Rightarrow d_t = 0,\ a.s. - \mathcal{G}^\infty \Rightarrow pr_\infty\{T^* < \infty\} = 0$

I show the proposition by contradiction; I take any $\{b_0,(n_t^{ED},g_t)_t\} \in \mathcal{ED}$ and assume that $s_t^{AMSS} = s_t^{ED}$, i.e., $\{b_0,(n_t^{ED},g_t)_t\} \in \mathcal{AMSS}$, and then I arrive to a contradiction.[53]

It follows that $(s_t^{ED})_{t\geq\mathcal{T}} = 0$ for all $(g^t)_t$ such that $g^\mathcal{T} : \{d_\mathcal{T} = 1\}$. If $s_t^{AMSS} = s_t^{ED}$ it must be true that $b_\mathcal{T}^{AMSS} \leq 0$ because (a) $\{s_t^{AMSS} = 0,\ \forall(g_t)_{t\geq\mathcal{T}} : g_\mathcal{T} - \overline{g}_\mathcal{T} > 0\}$, and (b) the government cannot roll over debt forever, i.e., $b_\mathcal{T}^{AMSS} \leq 0$ for $g_\mathcal{T} : g_\mathcal{T} - \overline{g}_\mathcal{T} > 0$, but as $b_\mathcal{T}^{AMSS} \in \mathcal{G}^{\mathcal{T}-1}$ it must

[52]For simplicity, I leave the exogenous debt limits implicit; all the results follow by adding the corresponding restrictions.

[53]For a generic variable x_t, x_t^{ED} and x_t^{AMSS} denote the value of such variable in the ED and AMSS economies, respectively.

hold $b_{\mathcal{T}}^{AMSS} \leq 0$ for all $g_{\mathcal{T}}$ (not only the ones the government defaults). Similarly $b_{\mathcal{T}}^{ED} > 0$, because if the government defaults with some probability, by construction it implies that it owes positive debt, and since $b_{\mathcal{T}}^{ED} \in \mathcal{G}^{\mathcal{T}-1}$ the aforementioned inequality must hold. Thus, by definition of the set $\mathcal{AMSS}$, it must hold that

$$
\begin{aligned}
0 \geq b_0 - s_0^{AMSS} - \sum_{t<\mathcal{T}} \int_{g^t} \beta^t s_t^{AMSS} pr_t(dg^t) =& b_0 - s_0^{ED} - \sum_{t<\mathcal{T}} \left[\int_{g^t} \beta^t s_t^{ED} pr_t(dg^t) \right] \\
0 \geq & b_0 - s_0^{ED} - \sum_{t<\mathcal{T}} \left[\int_{g^t} \beta^t \Lambda_t s_t^{ED} pr_t(dg^t) \right] \\
0 \geq & \int_{g^{\mathcal{T}}} \beta^{\mathcal{T}} \Lambda_{\mathcal{T}} b_{\mathcal{T}}^{ED} pr_{\mathcal{T}}(dg^{\mathcal{T}}),
\end{aligned}
$$

where the first inequality follows from $b_{\mathcal{T}}^{AMSS} \leq 0$; the first equality follows from $s_t^{AMSS} = s_t^{ED}$; the second equality follows from the fact that $\Lambda_t = 1$ because, since $\mathcal{T}$ is defined as the first time that $d_t > 0$ with positive probability, it must be true that $d_t = 0,\ a.s.$, for all $g^t,\ t < \mathcal{T}$; the last inequality follows from the restrictions over $\mathcal{ED}$. Moreover, it follows

$$
\begin{aligned}
0 \geq & \int_{g^{\mathcal{T}}} \beta^{\mathcal{T}} \Lambda_{\mathcal{T}} b_{\mathcal{T}}^{ED} pr_{\mathcal{T}}(dg^{\mathcal{T}}) = \int_{g^{\mathcal{T}-1}} \beta^{\mathcal{T}} \left\{ \int_{g^{\mathcal{T}}} (1 - d_{\mathcal{T}}) pr_{\mathcal{T}}(dg^{\mathcal{T}}) \right\} \Lambda_{\mathcal{T}-1} b_{\mathcal{T}}^{ED} pr_{\mathcal{T}-1}(dg^{\mathcal{T}-1}) \\
0 \geq & \int_{g^{\mathcal{T}-1}} \beta^{\mathcal{T}} \left\{ \Pi(\overline{g}_{\mathcal{T}}) \right\} b_{\mathcal{T}}^{ED} pr_{\mathcal{T}-1}(dg^{\mathcal{T}-1}) > 0,
\end{aligned}
$$

where the first equality follows from the fact that $b_t \in \mathcal{G}^{t-1}$ for all t; the last inequality follows from $\Pi(\overline{g}_{\mathcal{T}}) > 0$ and $b_{\mathcal{T}}^{ED} > 0$. This last equation yields a contradiction thereby implying that $\mathcal{ED} \cap \mathcal{AMSS} = \{\emptyset\}$. $\square$

A.1. **Default Sets Characterization: Extensions.** If I allow for $U_c(c) \neq 1$, then a version of proposition 3.2 still holds,

PROPOSITION A.2. *Under assumption 6.1(ii) and*

ASSUMPTION A.1. *(i) $U_c(n_t - g_t)(n_t - g_t) - U_l(1 - n_t)n_t$ is decreasing,*

if $D[\mathbb{G}](b_t) \neq \emptyset$ then there does not exists $b_{t+1} : U_c(n_t - g_t)b_t - \mathcal{P}(b_{t+1})b_{t+1} \leq 0$ for all b_{t+1}.

Proof of Proposition A.2. I show the result by contradiction, as I did for proposition 6.1. Assume that there exits $\overline{b}_{t+1}$ such that $U_c(n_t - g_t)b_t - \mathcal{P}(\overline{b}_{t+1})\overline{b}_{t+1} \leq 0$. This implies that

$$
(U_c(n_t^C - g_t) - U_l(1 - n_t^C))n_t^C - U_c(n_t - g_t)g_t = U_c(n_t^C - g_t)b_t - \mathcal{P}(\overline{b}_{t+1})\overline{b}_{t+1} \leq 0,
$$

which, by assumption A.2, this implies that $n_t^C \geq n_t^A$.

The current utility under $d_t = 0$ is given by $U(n_t^C - g_t, 1 - n_t^C)$, with derivative $U_c(n_t^C - g_t) - U_l(1 - n_t^C)$. This last expression is positive (as it has the same sign as $(1 - \tau_t^n)$ which is positive), therefore the current utility under $d_t = 0$ is higher than the one under $d_t = 1$. The continuation value of the former case is always higher than in the latter case as the option to default has not been exercised. This implies that default is never chosen, and thus I arrived to a contradiction. $\square$

Remark A.1. *If $U_c(c) = c^{-\sigma_c}$ and $U_l(1-n) = (1-n)^{-\sigma}$ then a sufficient condition for assumption A.1 to hold is $\sigma_c > 1$.*

The result in proposition A.2 is in some sense related to the results in Lizarazo (2007). In that paper the author allows for risk averse investors and extends the results in Arellano (2008), conditional on the investors level of wealth. In my paper, investors are the households and the planner has to incorporate the consumption level, or rather the marginal utility of consumption, in the pricing of the debt. Unfortunately I was unable to characterize the default sets any further because, as opposed to the results in Lizarazo (2007), lenders marginal utility of consumption is endogenous for the planner.

Now I study the case that autarky is not absorbing, i.e., I allow for future repayments of the defaulted debt. First lets take, $\alpha > 0$, which implies that the secondary market price, q_t is not zero. The characterization of the default sets also changes. The intuition behind proposition 6.1 was that if the government had the option to roll over the debt then it simply keeps the option to default this period, default tomorrow on a higher debt, and thus default never occurs today. This results holds because the value function for autarky (or default) remains unchanged with the level of debt; fact that does not necessarily hold anymore. I can see this by decomposing the difference $\mathcal{U}(b_t, g_t, 1) - \mathcal{U}(b_t, g_t, 0)$ as[54]

$$\begin{aligned} &\left\{U(n_t^A - g_t, 1 - n_t^A) - U(n_t^C - g_t, 1 - n_t^C)\right\} \\ &\left\{E\left[\mathcal{U}(b_t, g, 1) - \mathcal{U}(b_{t+1}^C, g, 1)\right]\right\} + \left\{(1-\alpha)\beta E\left[\mathcal{U}(b_{t+1}^C, g, 1) - \mathcal{U}^o(b_{t+1}^C, g)\right]\right\}, \end{aligned} \tag{34}$$

in which the superscripts "A" and "C" denote the optimal policy functions under autarky and no-default, respectively.

Tax income is decreasing on b_{t+1} (the more debt the government takes, the less tax income he needs to cover expenses), which given my assumptions over $(1 - U_l(1-n))n$ implies that the policy function n_t^C is increasing on b_{t+1}. This result, monotonicity of $U(n-g, 1-n)$ on n, and that $U(n_t^A - g_t, 1 - n_t^A)$ is constant (as a function of b_t and b_{t+1}) imply that the first term inside the curly brackets is decreasing on b_{t+1}, and it is naught if $\mathcal{P}(b_{t+1})b_{t+1} = b_t$.

The second term in curly brackets, which was not present when $\alpha = 0$ and is basically the term that prevents me to extrapolate the previous results to this case, is increasing on b_{t+1} because $\mathcal{U}(b, g, 1)$ is decreasing in b and is naught if $b_{t+1} = b_t$.

The third term in curly brackets is always negative, and intuitively as b_{t+1} increases is going to be eventually zero as default will be optimal.

Consequently, for levels of debt such that $\mathcal{P}(b_{t+1})b_{t+1} = b_t + \epsilon$ for a small $\epsilon > 0$ the negative effect of $U(n_t^A - g_t, 1 - n_t^A) - U(n_t^C - g_t, 1 - n_t^C)$ might be offset by the positive effect of $E\left[\mathcal{U}(b_t, g, 1) - \mathcal{U}(b_{t+1}^C, g, 1)\right]$ and moreover if $(1-\alpha)\beta E\left[\mathcal{U}(b_{t+1}^C, g, 1) - \mathcal{U}^o(b_{t+1}^C, g)\right]$ is negligible, $\mathcal{U}(b_t, g_t, 1) - \mathcal{U}(b_t, g_t, 0)$ can be positive for levels of debt such that $\mathcal{P}(b_{t+1})b_{t+1} > b_t$.

Finally, for the case with $\lambda > 0$ and $\alpha = 0$ the government can receive offers for partial defaults and has to decide whether to accept such offers. The following proposition is stated without proof (the proof is completely analogous to the proof of proposition 6.3) and characterizes the decision of rejecting/accepting the offer for partial defaults.

PROPOSITION A.3. *Under assumptions 6.1(ii), 6.2 and 6.3(i) it follows that if for δ_1: $\mathcal{U}((1-\delta_1)b, g, \delta_1) < \mathcal{U}(b, g, 1)$, then for all $\delta_2 \geq \delta_1$ it $\mathcal{U}((1-\delta_2)b, g, \delta_2) < \mathcal{U}(b, g, 1)$, given $(b, g) \in \mathbb{B} \times \mathbb{G}$.*

Appendix B. Numerical Simulations: Description of the algorithm

I construct a grid for the government expenditure, $\mathbb{G} \equiv \{g_1, \ldots, g_{\#\mathbb{G}}\}$; and for the bonds, $\mathbb{B} \equiv \{b_1, \ldots, b_{\#\mathbb{B}}\}$. I set $\#\mathbb{G} = \#\mathbb{B} = 35$. Finally I construct the transition matrix $\Pi_{ij} \equiv \pi(g_{t+1} = g_i | g_t = g_j)$ using an $AR(1)$ specification $g_{t+1} = \rho_0 + \rho_1 g_t + \varepsilon_{t+1}$, $\varepsilon_{t+1} \sim N(0, \Sigma)$ and Tauchen (1986) discretization technique.

With this at hand I solve the model as follows

[54]This result follows from: $\mathcal{U}(b_1, g, 1) - \mathcal{U}(b_2, g, 1) = \alpha\beta E\left[\mathcal{U}^o(b_1, g) - \mathcal{U}^o(b_2, g)\right] + (1-\alpha)\beta E\left[\mathcal{U}(b_1, g, 1) - \mathcal{U}(b_2, g, 1)\right]$, and by taking expectations I can solve for the difference of $\mathcal{U}^o$ in terms of the difference of $\mathcal{U}(\cdot, 1)$.

(1) Propose an initial guess for $(p^b)^{(0)}$ and $q^{(0)}$. Both prices are $\#\mathbb{G} \times \#\mathbb{B}$ matrices. Define also $\mathcal{U}^{(0)}(\cdot,\cdot,i) \in \mathbb{G} \times \mathbb{B}$ for $i = 0, 1$.
(2) Given the initial guesses in 1. solve the autarky problem for each element $(b, g) \in \mathbb{G} \times \mathbb{B}$. The maximization problem is casted as a finite dimensional optimization problem on the grids $\mathbb{G} \times \mathbb{B}$.
(3) Given the initial guesses in 1. and 2. solve the "continuation" problem for each element $(b, g) \in \mathbb{G} \times \mathbb{B}$. The maximization problem is casted as a finite dimensional optimization problem on the grids $\mathbb{G} \times \mathbb{B}$.
(4) Iterate over 2-3 and solve for the fixed point for the value functions, denote this as $\mathcal{U}^{(1)}(\cdot,\cdot,i) \in \mathbb{G} \times \mathbb{B}$ for $i = 0, 1$.
(5) Given $\mathcal{U}^{(1)}(\cdot,\cdot,i) \in \mathbb{G} \times \mathbb{B}$ for $i = 0, 1$ obtained in 4. Compute the new prices, $(p^b)^{(1)}$ and $q^{(1)}$.
(6) If $\max\{|(p^b)^{(0)} - (p^b)^{(1)}|, |(q)^{(0)} - (q)^{(1)}|\} \leq \delta_{TOL}$ then stop. If not set $(p^b)^{(0)} = (p^b)^{(1)}$ and $q^{(0)} = q^{(1)}$ and $\mathcal{U}^{(0)}(\cdot,\cdot,i) = \mathcal{U}^{(1)}(\cdot,\cdot,i)$ for $i = 0, 1$.

So as to initialize the iteration I set $(p^b)^{(0)} = \beta$ and $q^{(0)} = 0$ and $\mathcal{U}^{(0)}(\cdot,\cdot,i) = 0$. I also set δ_{TOL} as $1e^{-7}$ and $||\cdot||_E$ as $\max_{\mathbb{G}\times\mathbb{B}} |\cdot|$.[55]

In order to compute q (given all the other things) I need to solve a fix point problem in itself as q appears on both sides of the pricing equation. Once I solve this problem I compute p^b.

Appendix C. Quantitative Part

C.1. **Description of the Data.** In this section I describe how I constructed the figures presented in section 2.

The industrialized economies group consists of AUSTRALIA (1990-1999), AUSTRIA (1990-1999), BELGIUM (1990-2001), CANADA (1990-2003), DENMARK (1990-2003), FINLAND (1994-1998), FRANCE (1990-2003), GERMANY (1990-1998), GREECE (1990-2001), IRELAND (1995-2003), ITALY (1990-2003), JAPAN (1990-1993), NETHERLANDS (1990-2001), NEW ZEALAND (1990-2003), NORWAY (1990-2003), PORTUGAL (1990-2001), SPAIN (1990-2003), SWEDEN (1990-2003), SWITZERLAND (1990-2003), UNITED KINGDOM (1990-2003) and UNITED STATES (1990-2003).

The emerging economies group consists of ARGENTINA[1] (1998-2003), BOLIVIA[1] (2001-2003), BRAZIL[1] (1997-2003), CHILE[1] (1993-2003), COLOMBIA[1] (1999-2003), ECUADOR[1] (1998-2003), EL SALVADOR[1] (2000-2003), HONDURAS[1] (1990-2003), JAMAICA[1] (1990-2003), MEXICO[1] (1990-2003), PANAMA[1] (1997-2003), PERU[1] (1998-2003), VENEZUELA[1] (1997-2003), ALBANIA (1995-2003), BULGARIA (1991-2003), CYPRUS (1990-2003), CZECH REPUBLIC (1993-2003), HUNGARY (1991-2003), LATVIA (1990-2003), POLAND (1990-2003), RUSSIA (1993-2003), TURKEY (1998-2003), ALGERIA (1990-2003), CHINA (1997-2003), EGYPT (1993-2003), JORDAN (1990-2003), KOREA (1990-2003), MALAYSIA (1990-2003), MAURITIUS (1990-2003), MOROCCO (1997-2003), PAKISTAN (1990-2003), PHILIPPINES (1997-2003), SOUTH AFRICA (1990-2003), THAILAND (1999-2003) and TUNISIA (1994-2003). The LAC group is conformed by the countries with "[1]".

For section 2 I constructed the data as follows. First, for each country, I computed time average, or time standard deviations or any quantity of interest (in parenthesis is the number of observations use to construct these). Second, once I computed these averages, I group the countries in IND, EME and LAC. I do this procedure for (a) central government domestic debt (as % of output) ; (b) central government expenditure (as % of output) ; (c) central government revenue (as % of output) , and (d) Real Risk Measure. The data for (a) is taken from Panizza (2008) ; the data for (b-c) is taken from

[55] I perform sensitivity analysis on δ_{TOL} and result remained unchanged.

Kaminsky et al. (2004) ; finally the data for (d) is taken from `www.globalfinancialdata.com`.[56] [57] [58]

Appendix D. Figures

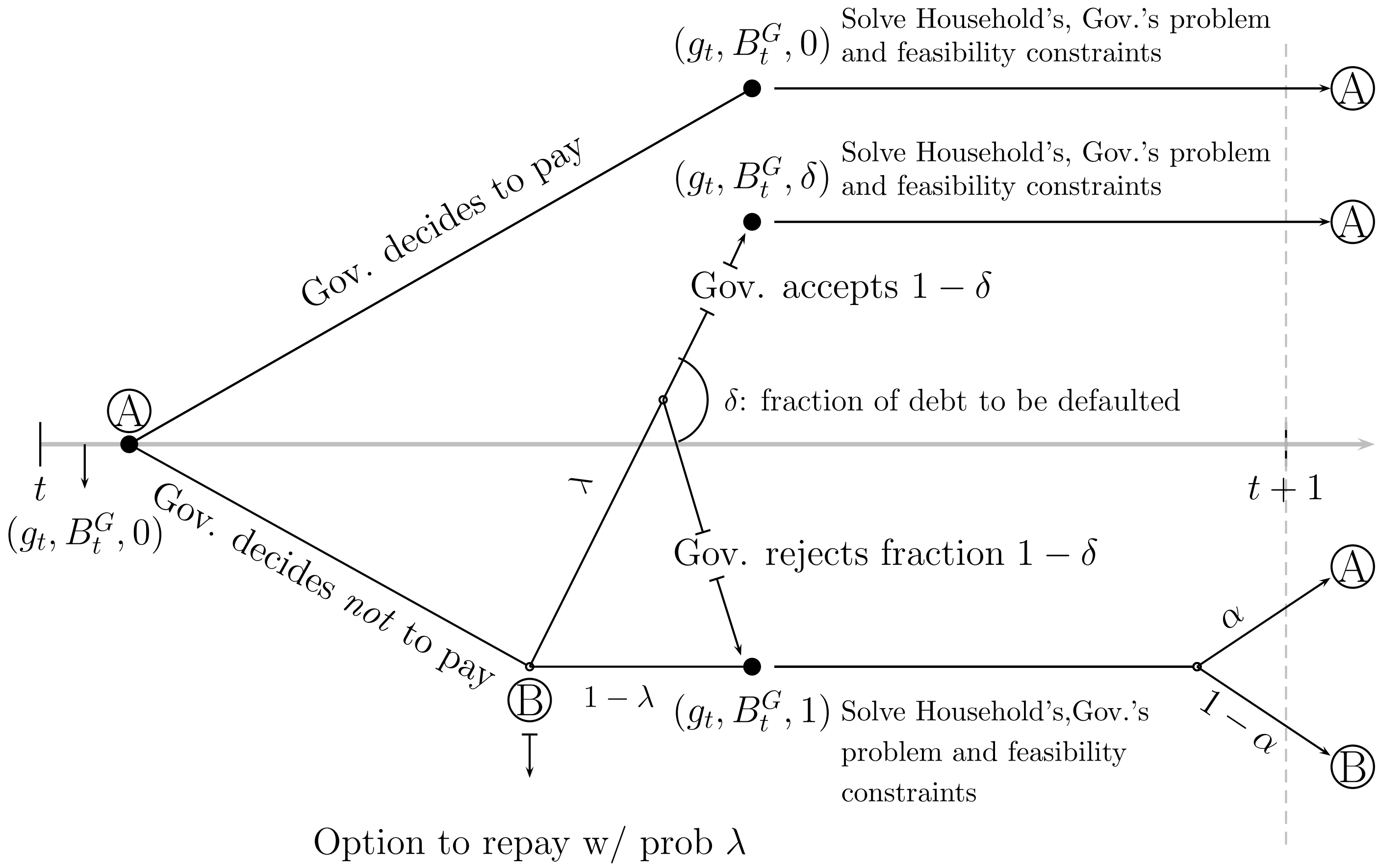

Figure G.1. Timing of the Model. g_t: government expenditure; b_t: government equilibrium debt; λ: Prob. of receiving an offer; $\delta \in \Delta$: Fraction of defaulted government debt; α: Prob. of having the option to leave autarky.

[56] For Greece and Portugal I use central government public debt because central government domestic debt was not available. For Sweden, Ecuador and Thailand I use general government expenditure because central government expenditure was not available. For Albania, Bulgaria, Cyprus, Czech Rep., Hungary, Latvia, Poland and Russia no measure of government expenditure was available and thus were excluded from the sample for the calculations of this variable. The same caveats apply to the central government revenue sample.

[57] I gratefully acknowledge that Kaminsky et al. (2004) and Panizza (2008) kindly shared the dataset used in their respective papers (see references).

[58] For Argentina, Brazil, Colombia, Ecuador, Egypt, Mexico, Morocco, Panama, Peru, Philippines, Poland, Russia, Turkey and Venezuela I used the real EMBI+ as a measure of real risk. For the rest of the countries I used government note yields of 1-5 years maturity, depending on availability.

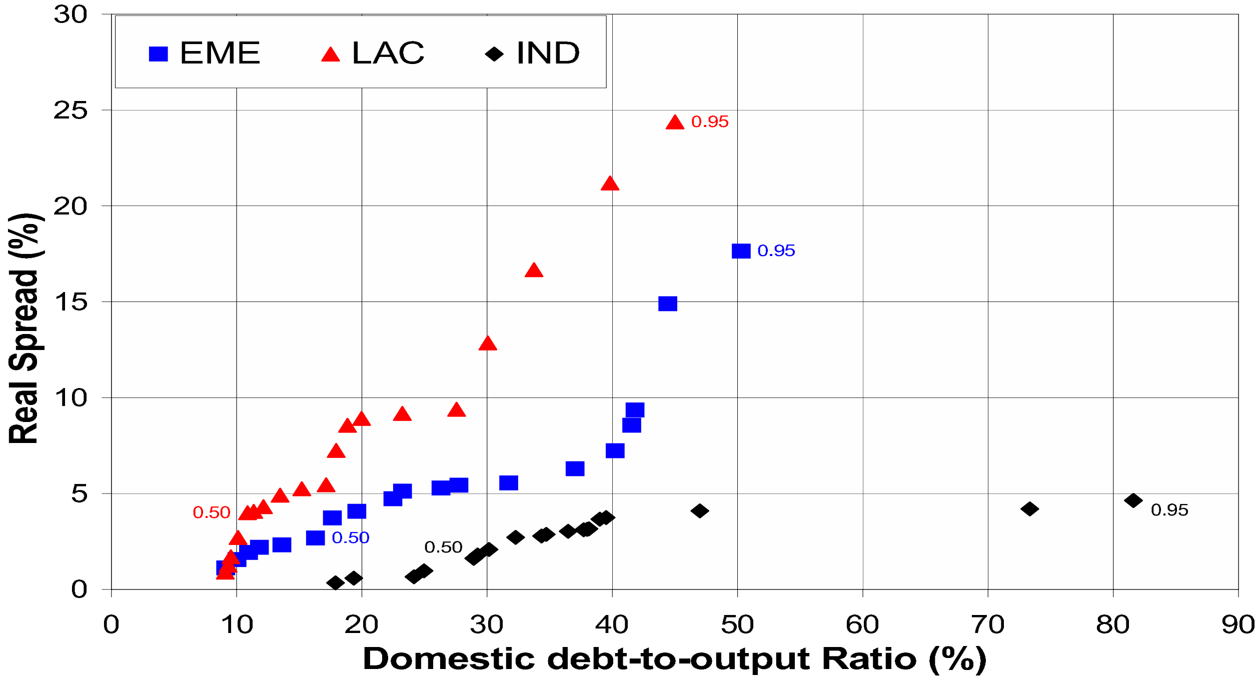

FIGURE G.2. QQplot of avg. debt-to-output ratio and Real Spread.

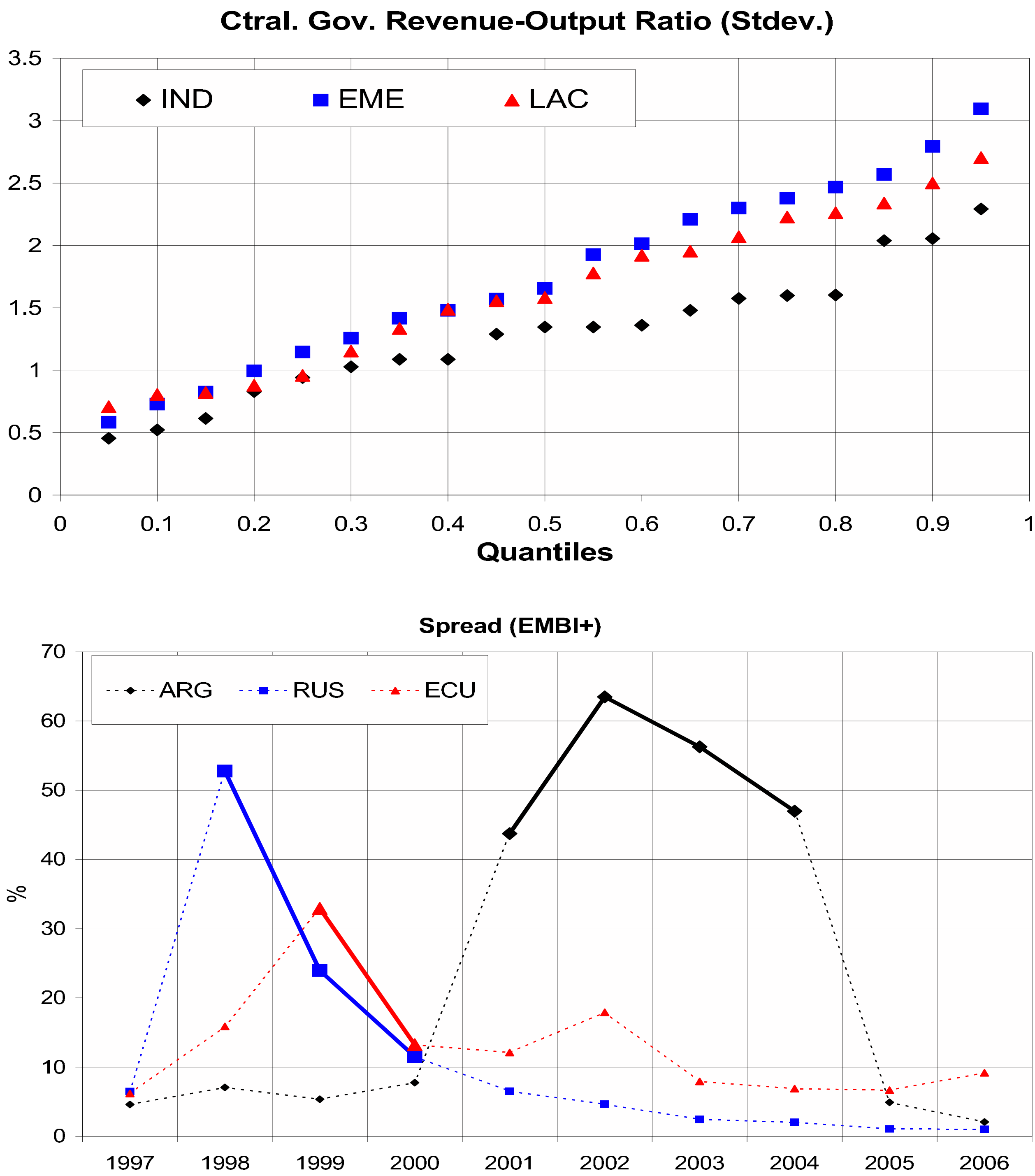

FIGURE G.3. (Top Panel) Quantiles of stdev. Ctral. gov. revenue-to-output ratio; (Bottom Panel) Spread for three defaulters (Argentina, Russia and Ecuador) during the period 1997-2006.

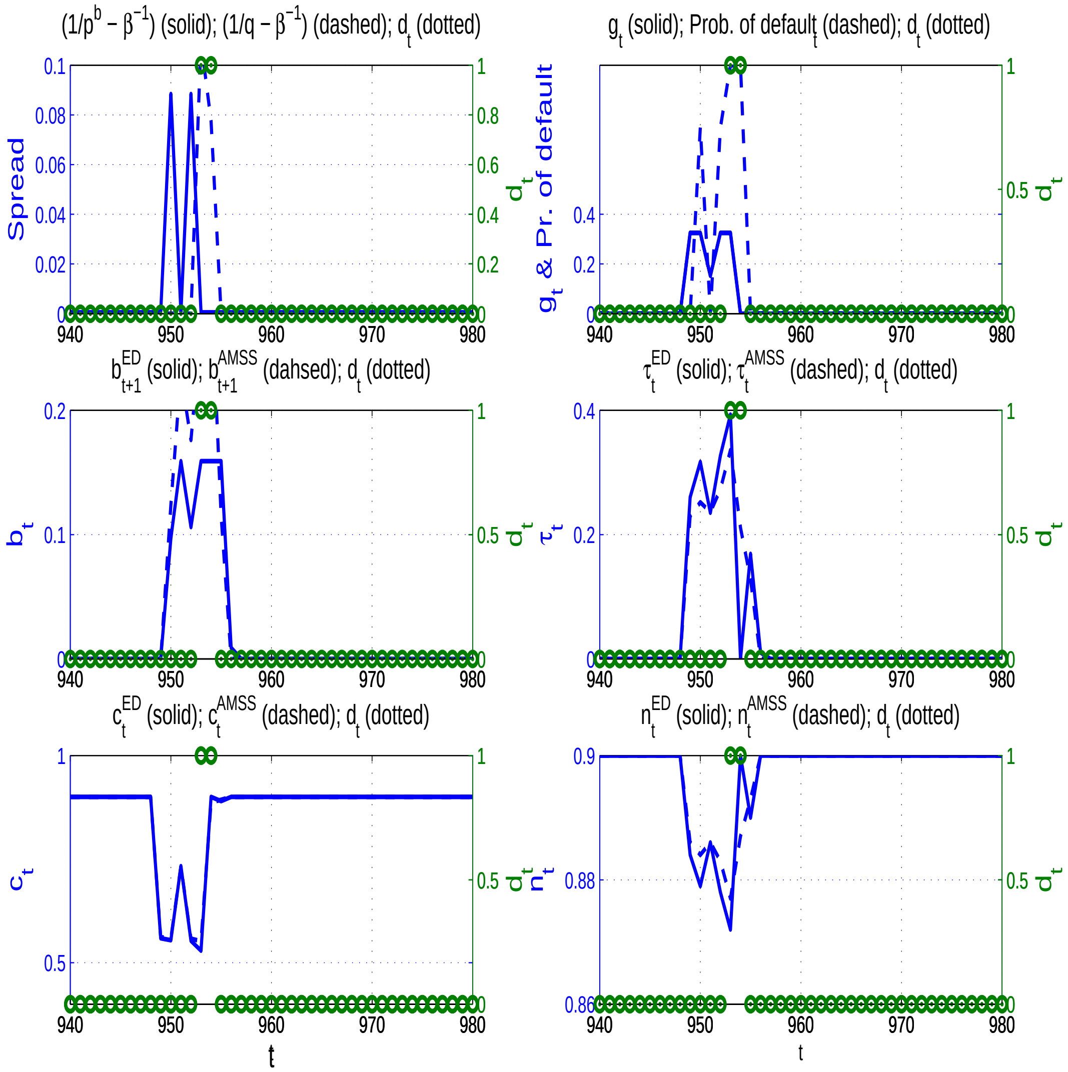

FIGURE G.4. Impulse Response functions.

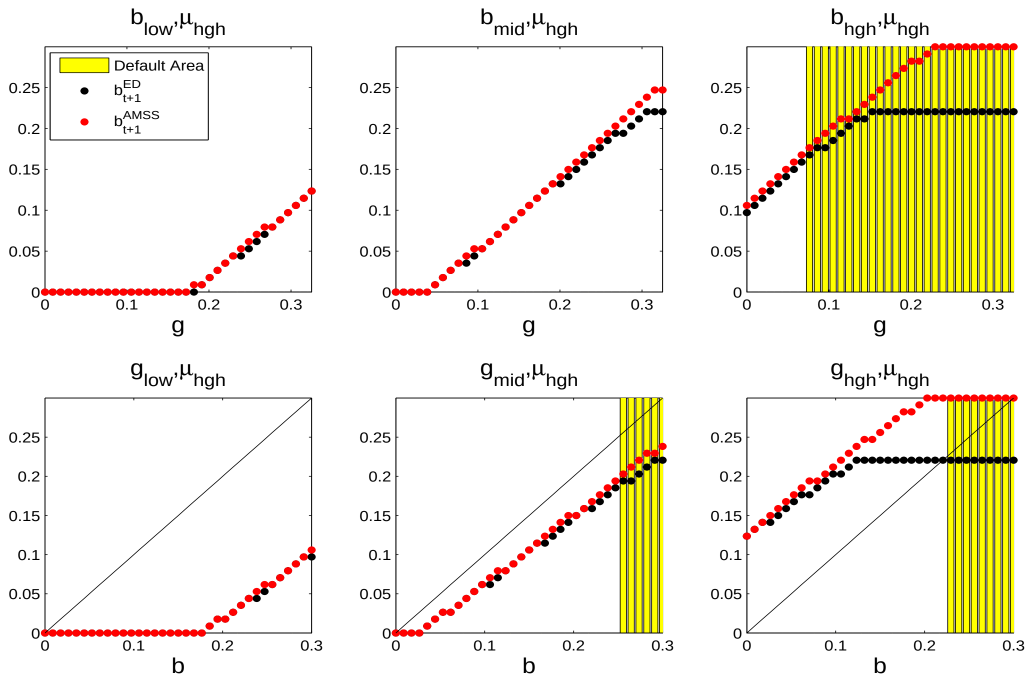

FIGURE G.5. Policy function of debt for ED Economy (black dot), AMSS (red dot) and default region (yellow area).

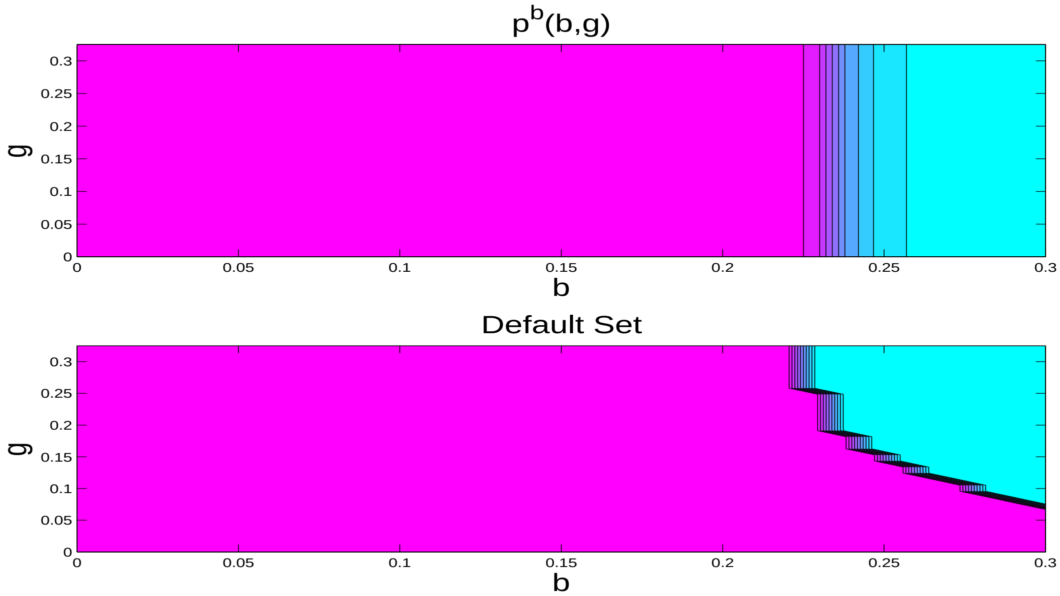

FIGURE G.6. Top Panel: Price function $p^b(b, g)$ for ED Economy. Bottom Panel: Default set for ED Economy. I.i.d. Case and $\lambda = \alpha = 0$.

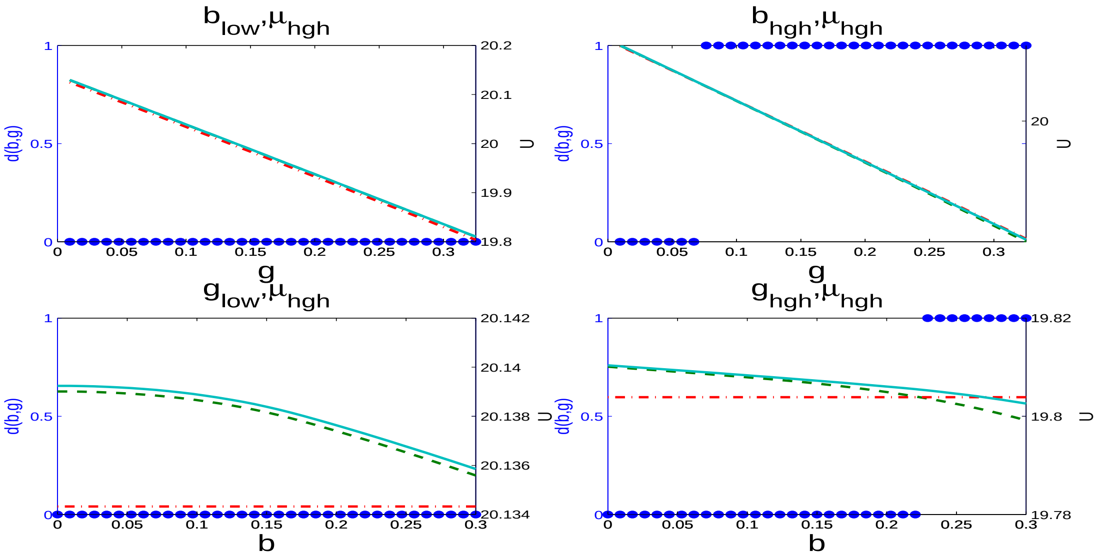

FIGURE G.7. Value functions for i.i.d. Case and $\lambda = \alpha = 0$. $\mathcal{U}^{AMSS}(b,g)$ (solid), $\mathcal{U}(b,g,0)$ (dashed), $\mathcal{U}(g,1)$ (dot-dashed) and $d(b,g)$ (dotted)

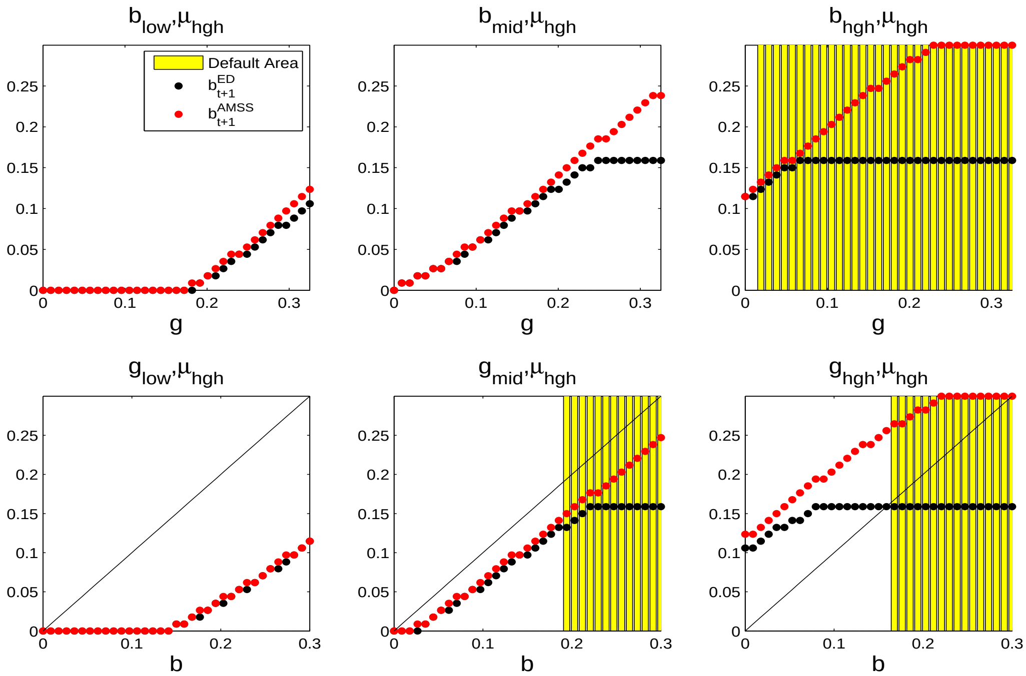

FIGURE G.8. Policy function of debt for ED Economy (black dot), AMSS (red dot) and default region (yellow area). MC Experiment: AR(1) (with $\rho = 0.30$) Case and $\lambda = \alpha = 0$.

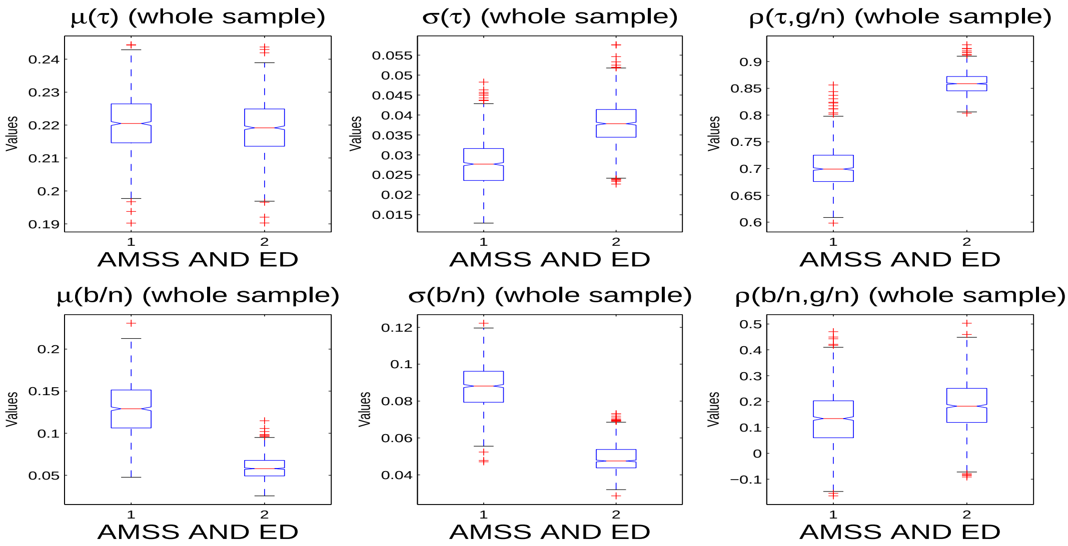

FIGURE G.9. Box-plots of government policy for the whole sample.

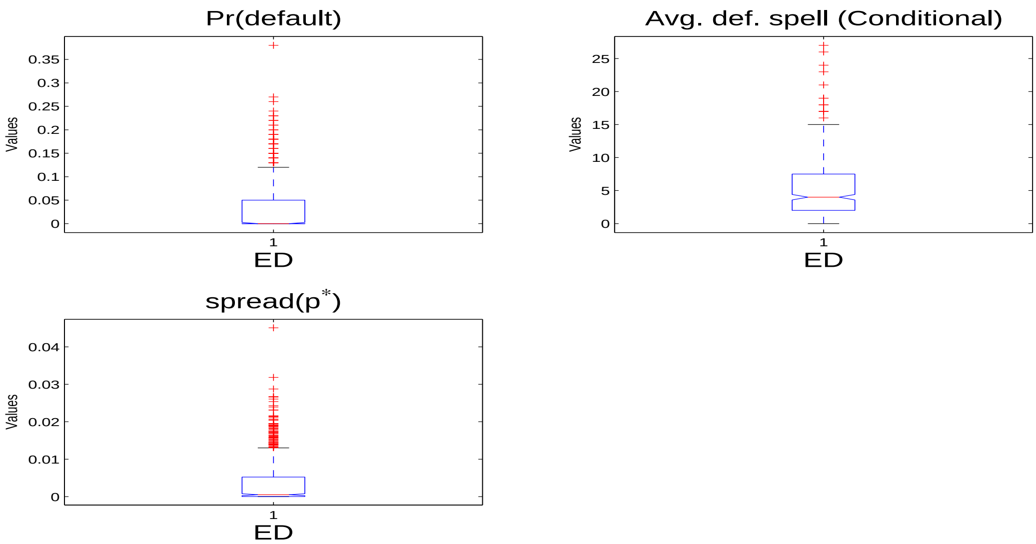

FIGURE G.10. Box-plots of prob. of default (top-left), Avg. default spell (top-right) and spread of "envelope" price (bottom-left) for the whole sample. MC experiment: AR(1) (with $\rho = 0.30$) Case and $\lambda = 0$ and $\alpha = 0.30$.

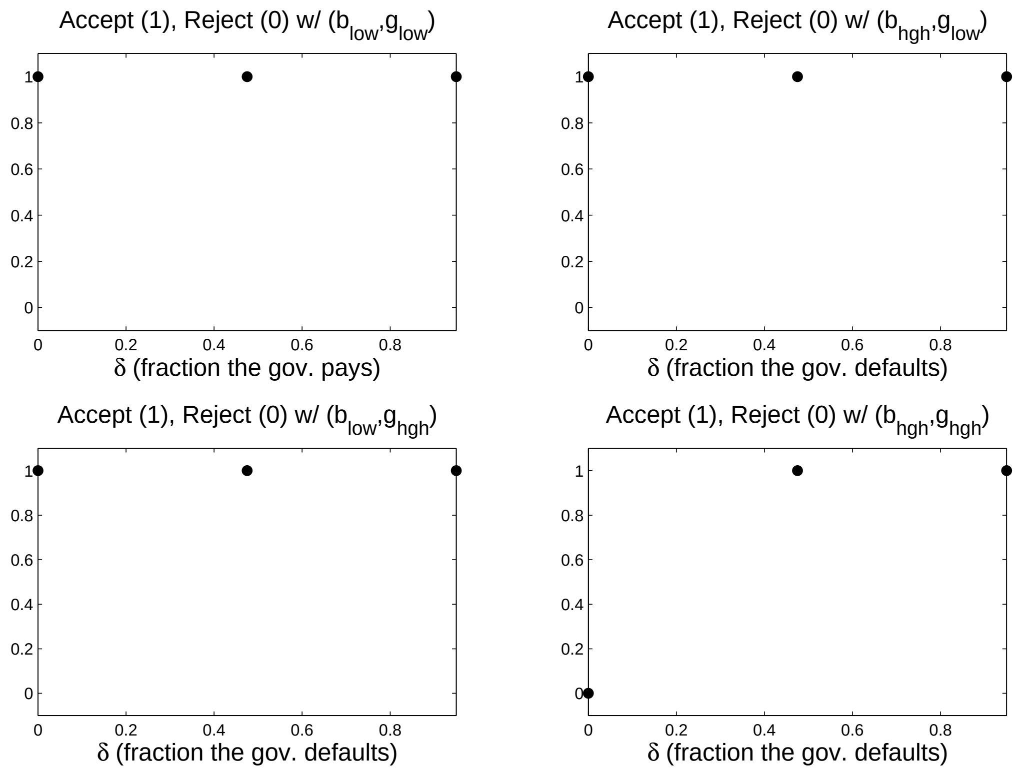

FIGURE G.11. Accept/Reject the partial payments offers. AR(1) (with $\rho = 0.30$) Case and $\lambda = 0.08$ and $\alpha = 0$.

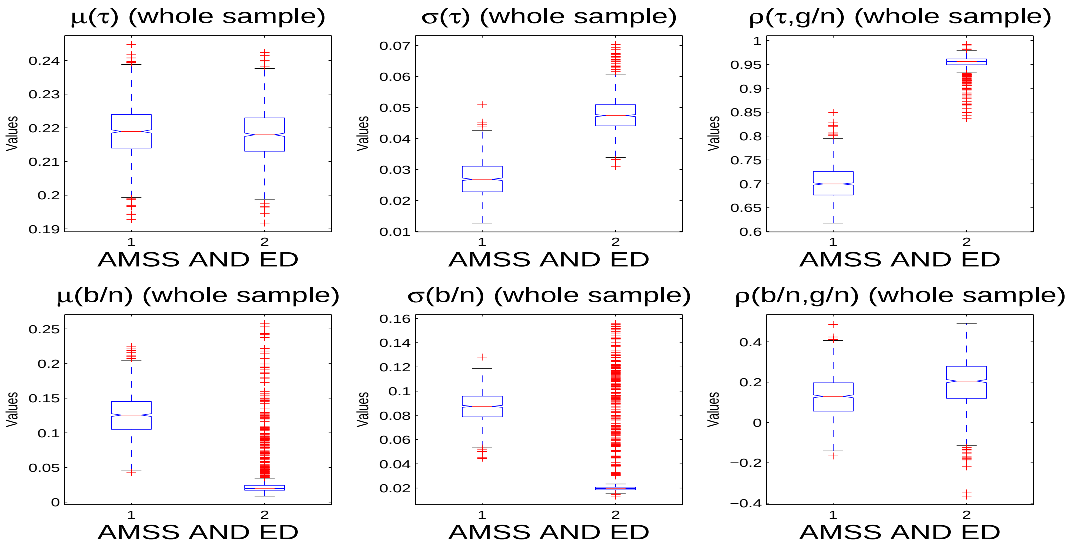

FIGURE G.12. Box-plots of government policy for the whole sample.

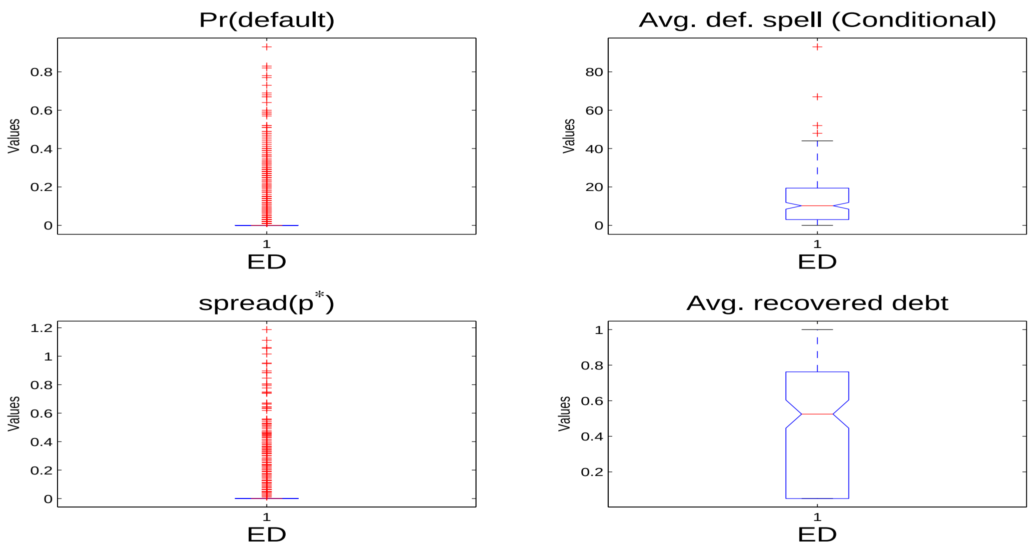

FIGURE G.13. Box-plots of prob. of default (top-left), Avg. default spell (top-right) and spread of ”envelope” price (bottom-left) for the whole sample. MC experiment: AR(1) (with $\rho = 0.30$) Case and $\lambda = 0.08$ and $\alpha = 0$.

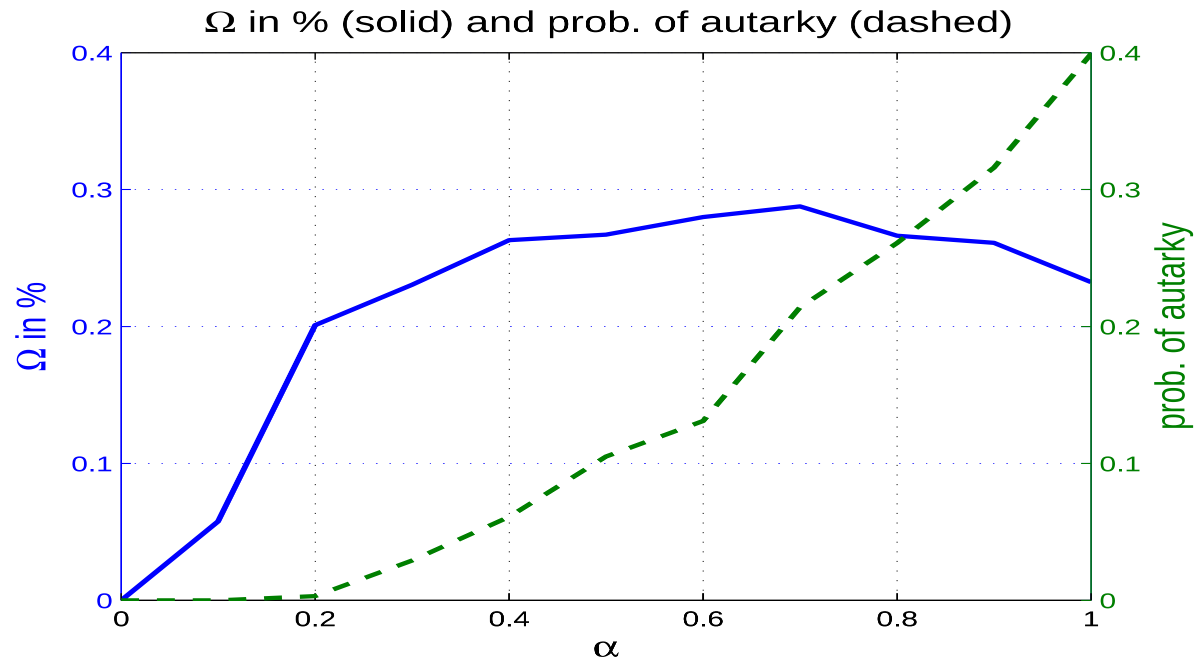

FIGURE G.14. Ω (solid) and probability of autarky (dashed).

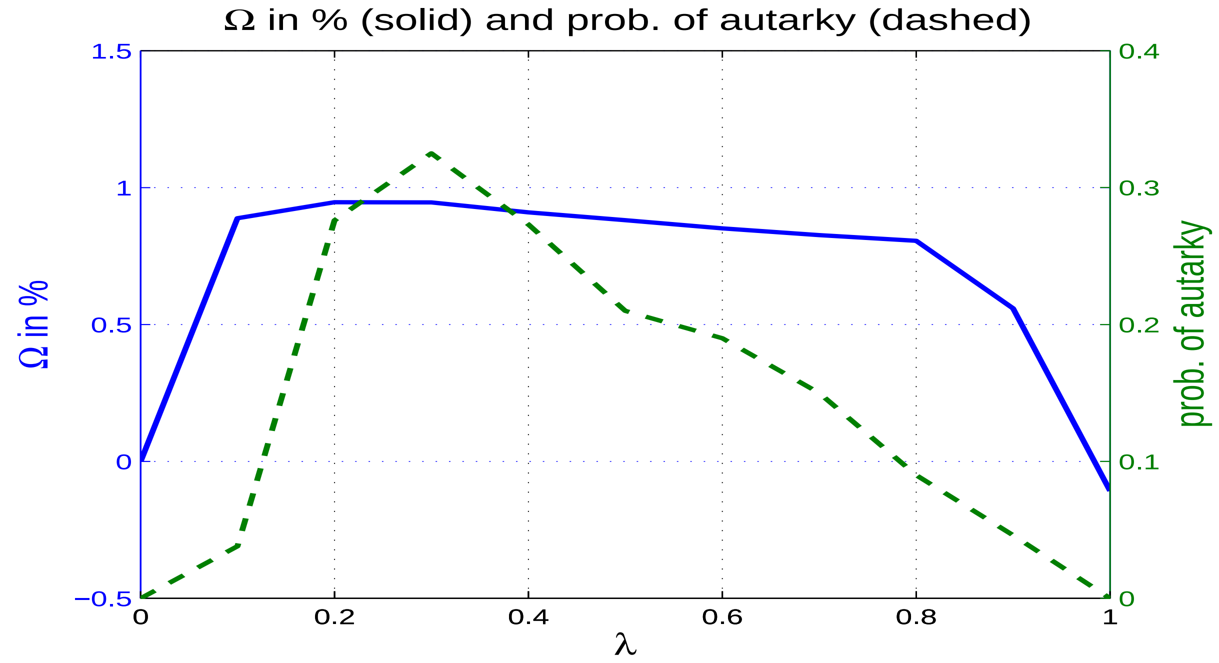

FIGURE G.15. Ω (solid) and probability of autarky (dashed).