Non-exclusive Dynamic Contracts, Competition, and the Limits of Insurance *

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Abstract

We study how the presence of non-exclusive contracts limits the amount of insurance provided in a decentralized economy. We consider a dynamic Mirrleesian economy in which agents are privately informed about idiosyncratic labor productivity shocks. Agents sign insurance contracts with multiple firms (i.e., they are non-exclusive), which include both labor supply and savings aspects. These contract arrangements are private information. Firms have no restriction on the contracts they can offer, interact strategically, and, as in common agency problems, might offer latent contracts to sustain equilibrium allocations. In equilibrium, contrary to the case with exclusive contracts, a standard Euler equation holds, and the marginal rate of substitution between consumption and leisure is equated to the worker's marginal productivity. Finally, each agent receives zero net present value of transfers from insurance providers. To sustain this equilibrium, more than one firm must be active in offering the equilibrium allocation. Each active firm must also offer latent contracts to deter deviations to more profitable contingent contracts. In this environment, the non-observability of contracts removes the possibility of additional insurance beyond self-insurance. To test the model, we allow firms to observe contracts at a cost. The model endogenously divides the population into agents that are not monitored and have access to non-exclusive contracts and agents that have access to exclusive contracts. We use US household data and find that high school graduates satisfy the optimality conditions implied by the non-exclusive contracts while college graduates behave according to the second group.

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1 Introduction

What type of contractual arrangements are available to workers in a decentralized economy when firms compete for the provision of social insurance? In this paper, we study how the presence of non-exclusive contracts endogenously limits the contracts offered, and hence the amount of insurance in a decentralized economy. We find that competition and non-observability of insurance contracts significantly reduce the amount of insurance provided: the equilibrium allocation in our environment is equivalent to a self-insurance economy and only linear contracts are offered. On the other hand, under observable contracts agents are able to achieve the constrained efficient allocation. The model studied can explain the presence of different insurance regimes in the US data.

Multiple credit and labor relations are an important aspect of everyday life. In the data, we observe that individuals and households receive insurance against idiosyncratic risk from a multitude of sources: publicly provided insurance (unemployment, Medicare, Medicaid, disability, food stamps, progressive income taxation); privately provided insurance (employer, between and within family transfers);[1] financial instruments in credit markets; and housing and other large durable goods. The same consideration is true for labor relationships. Paxson and Sicherman (1994) look at the number of concurrent labor relationships held by survey respondents of the Panel Study of Income Dynamics (PSID) between 1977 and 1990 and the Current Population Survey (CPS) of 1991. They find that for any given year, 20% of working males held at least a second job, and during their working life there is at least a 50% probability of holding a second job. Monitoring all the transactions an agent might engage in with other firms is very costly for an individual firm, especially if these relationships include activities in the informal labor market, hidden storage, and the ability to transfer leisure into consumption through either home production or shopping time (see Aguiar and Hurst (2005)). Motivated by these considerations, the key friction addressed in this paper is the

[1] The Panel Study of Income Dynamics continuously reports for the years 1969 to 1985 any income transfer received by households. We find that, in a given year, 24% of the households report receiving a transfer and 67% of the households received a transfer at some stage. These transfers are significant, averaging $1,930 (1983 dollars) and represent between 70% to 90% of total food expenditures.

non-exclusivity and non-observability of contractual relations. In the first part of the paper, we characterize the optimal contract under the assumption that *none* of the transactions an agent engages in can be observed by an individual firm.[2] In the second part of the paper, we allow firms to costly monitor contracts and take the model to the data.

The environment studied is a finite horizon dynamic Mirrleesian economy in which agents are privately informed about idiosyncratic labor productivity shocks that evolve over time. Agents wish to insure this risk by signing contracts with insurance providers (firms). Agents are not limited to a single insurance/labor relationship and can sign contracts with multiple firms. The contracting arrangements are private information of the contracting parties.

We impose no restriction on the contracts that firms can offer. A firm can, for example, offer a spot labor contract, a linear intertemporal borrowing and saving contract, a state contingent dynamic insurance contract, and so on. Due to the presence of non-observable contracts, in general, the communication between agent and firms cannot be limited to the exogenous private shock of agents (firms might also seek information about the other relations the agent has engaged in), as in the case with observable contracts. We extend the results in the common agency literature to our dynamic environment and characterize equilibrium using a menu game.[3] In this game, each firm offers collections of payoff relevant alternatives – menus – and delegates to the agent the choice within these menus. The choice of the agent from a menu can potentially reveal information about his type and the other contractual arrangements in which he might be involved.

In our environment, the non-observability of contracts removes the possibility of additional insurance beyond self-insurance. We fully characterize the equilibrium and find that, endogenously, only linear non-contingent contracts emerge. We find that three optimality conditions must hold in equilibrium. First, the intertemporal marginal rate of substitution between consumption at time t and consumption at $t+1$ is equal to the marginal rate of transformation (a standard Euler equation holds).[4] Second, the marginal rate of substitution

[2]The characterization under exclusive contracts is well understood, see Prescott and Townsend (1984).

[3]See Peters (2001), Martimort and Stole (2002), and Epstein and Peters (1999).

[4]If contracts are exclusive, the Euler equation does not hold and agents are savings constrained (see

between consumption and leisure is equated to the marginal productivity for any time and any history.[5] Third, the net present value of the transfers received in equilibrium is equal zero for every agent in the economy. These optimality conditions imply that the equilibrium allocation is equivalent to an economy in which agents can trade non-contingent bonds and are paid their marginal productivity and in which there is no redistribution. These results, linking side trading and linear contracts, are reminiscent of Allen (1985), Hammond (1987), Cole and Kocherlakota (2001). We contribute to this literature by explicitly modelling competition between firms.

In our environment, the only contracts that emerge endogenously are competitive contracts, with linear implicit prices. If, for example, a firm offers an intertemporal contract at an implicit rate of return lower than the marginal rate of transformation, it would provide a profitable opportunity for an entrant: it can offer a contract with a return slightly higher and make profits.[6] Such entry cannot be prevented by the first firm by also offering latent contracts because it cannot induce negative profits to the entrant.

We characterize the strategies of firms to implement the equilibrium allocation and show that an incumbent firm must offer latent contracts to deter deviations of other incumbent firms. Moreover, in equilibrium more than one firm must offer the equilibrium allocation. The intuition for this result is that the equilibrium allocation is the most profitable non-contingent contract; however some contingent contracts deliver higher profits. If there is a unique incumbent or no latent contracts, a firm will deviate and offer one of these contracts.

To derive testable implications between non-exclusivity of contracts and the availability of insurance in the data, we consider our general model, relaxing the assumption about the observability of contracts. We assume that at time 0, a firm can pay a cost for each agent which allows the firm to observe all the contracts the agent signs. We consider agents het-

Golosov, Kocherlakota, and Tsyvinski (2003)).

[5] This is also different with respect to the exclusive contracting environment (see, for example, Mirrlees (1971) and Golosov, Tsyvinski, and Werning (2006)), where this relation holds only for the highest skill type, while all of the remaining types face a distortion on the intratemporal margin that discourages consumption and hours provided.

[6] Or similarly, offering a labor contract at an implicit wage lower than marginal productivity.

erogeneous with respect to this cost. If the cost is paid, a firm offers the optimal contract under exclusivity (as in Golosov, Kocherlakota, and Tsyvinski (2003) and Albanesi and Sleet (2006)). If the cost is not paid, firms offer the contract described in this paper, which implements the self-insurance allocation. With this extension, the model endogenously generates a partition of the population into groups with different lifetime utility and that have access to different insurance contracts. Agents with a high monitoring cost receive lower lifetime utility and rely on self-insurance, while a low monitoring cost translates to higher lifetime utility and access to the constrained efficient level of insurance. We use US survey data to test whether agents' decisions, when grouped by education, satisfy the optimality conditions of the constrained efficient environment or of the linear contracts economy. We find that the consumption of college graduates evolves according to the inverse Euler equation, while for individuals with less than college, the consumption satisfies the standard Euler equation. Looking at the static consumption-leisure distortion calculated in the data, we investigate how it evolves as agents age. The model prescribes a constant distortion over age if workers have access to non-exclusive contracts while an increasing distortion in the other case. We find that also in this dimension, we cannot reject the hypothesis that high school graduates behave according to the linear contracts whereas the other group is closer to the constrained efficient contract.

Related Literature

This paper is related to a large and growing literature on optimal social insurance contracts and its implementation through taxation as in the *dynamic public finance* literature (for a review, refer to Kocherlakota (2006) and Albanesi (2008)). In general, the environment studied in these papers assumes that insurance is provided by a unique provider who perfectly controls both consumption and labor decision of the agents. With respect to this literature, this paper has two distinct implications. Our main result suggests that the constrained efficient allocation cannot be implemented in decentralized environments unless every aspect of the contracting is observable, thus making necessary the provision of insurance via taxes or a centralized institution that makes information public. However, our results also highlight that the presence of hidden and self-enforcing activities (for both consumption and labor) might undo any incentives the government provides through taxes.

Our work is also related to a growing literature on optimal contract in the presence of hidden trades.[7] In particular, Cole and Kocherlakota (2001) show that, in an private information endowment economy, equilibrium is equivalent to self-insurance when agents can secretly save in a storage technology. In an environment similar to ours, Golosov and Tsyvinski (2007) characterize equilibrium when agents can engage in hidden trades of Arrow-Debreu securities. They show a standard Euler equation holds and that the decentralized equilibrium is not efficient, since firms do not internalize the effects of the contracts offered on the market rate of return. This paper can be seen as a generalization of the previous two papers, in those the recontracting possibilities are assumed exogenously (a market with linear prices or a storage technology) while in this paper the recontracting market is a result of an equilibrium game between insurance providers.

This paper also relates to Bisin and Guaitoli (2004), who analyze a static moral hazard environment under non-exclusive contracting. Their main result shows that latent contracts are used to sustain the equilibrium. However, the nature of the moral hazard environment, differently from our environment, enables latent contracts to prevent any profitable entry by additional insurance providers, thus delivering a positive profit equilibrium to the incumbents.

The quantitative analysis in this paper is related to Townsend (1995) and Ligon (1998). These papers investigate whether the consumption patterns in villages in Thailand and India, respectively, are consistent with the predictions of a constrained efficient allocation or the full information model. Ligon (1998) estimates the inverse Euler equation and the Euler equation for three villages in India. He finds that in two villages the consumption behavior is consistent with the Euler equation while in one village it is consistent with the constrained efficient allocation. Townsend (1995) investigates the consumption in Thai villages and finds that for some the constrained efficient allocation describes accurately the fluctuations while

[7]For example Cole and Kocherlakota (2001), Golosov and Tsyvinski (2007) and Abraham and Pavoni (2005).

for others the full information model is a good benchmark. The study also emphasizes how villages differ in *information* flows between households (including assets and transactions) and how this could be responsible for the different insurance regimes observed.

The paper is organized as follows. In Section 2, we describe the environment and show that any equilibrium can be implemented by a menu game. Section 3 characterizes the equilibrium of our benchmark environment and shows that it is equivalent to self-insurance. We also show that latent contracts are necessary to implement the equilibrium allocation. Section 4 extends the model, allowing firms to observe contracts, and analysis its implications using US survey data. In Section 5 we show how the inability to prevent entry using latent contracts holds for a larger class of environments. Section 6 is the conclusion.

2 Environment

Consider an economy populated by a continuum of measure one of ex ante identical agents and I firms (insurance providers), where I is a natural number. The economy lasts for T periods, a finite number. Agents' period utility is defined over consumption c and labor l and is given by $u(c)-v(l)$. Agents discount future utility at rate $0<\beta<1$. Assume $u:\mathbb{R}_+\rightarrow\mathbb{R}$ is twice continuously differentiable, increasing and a strictly concave function and $v:\mathbb{R}_+\rightarrow\mathbb{R}$ is twice continuously differentiable, increasing and a strictly convex function. At every time t, $t=1,2,...,T$, each agent draws a privately observed productivity shock $\theta_t\in\Theta$, where Θ is a finite set. We assume the law of large numbers holds. The shock is distributed according to probability distribution $\pi(\cdot)$ and is independent and identically distributed over time and across agents. Let $\theta^t=(\theta_1,...,\theta_t)$ denote the history of uncertainty of an agent up to time t. Given a sequence of consumption and leisure $\{c,l\}=\{c_t,l_t\}_{t=1}^T$, the expected discounted utility of an agent is given by

$$U(\{c,l\})=\mathbb{E}_0\sum_{t=1}^{T}\beta^{t-1}[u(c_t)-v(l_t)]. \tag{1}$$

For a given realization of the labor productivity shock θ, an agent can produce y units of effective output according to $y = \theta l$, where l denotes his labor input. We assume the labor input is private information of the agent while output y is publicly observable to each firm for which the agent is producing output y.

Each firm $i \in \{1, ..., I\}$ offers labor and credit contracts to agents to insure against productivity shocks. A contract prescribes, at every time t, output requirement y_t^i and consumption transfer $y_t^i + b_t^i$. The period profit of firm i is given by $V^i(b^i) = -b^i$. Firms can transfer resources over time at constant rate q.[8]

An important feature of our environment is that agents can sign contracts with more than one firm simultaneously, and the terms of the contract between an agent and a firm i are not observed by other firms.[9] We do not impose any restriction on the contracts offered by each firm. For example, a firm can offer a contract for the entire time horizon $t = 1, ..., T,$, for a particular set of dates, it can be only credit contracts ($y_t = 0, \ \forall t$), only labor contracts, or both. We also do not impose any specific contingency on the contracts; in particular, we do not restrict to linear contracts.

At time 0, before any uncertainty is realized, agents commit to a contract with each firm i. To take into account the voluntary participation of agents, every firm is required to offer at time 0 a null contract that determines no output requirement and no consumption transfers in every period. The contracts offered by a firm at time 0 are contingent on the future communication with the agents.

We assume agents and firms commit to the contracts they sign at time 0. We also assume that contracts must be honored and neither firms nor agents can renege on them.[10]

[8]This fixed interest can be interpreted as the firm having access to external credit markets.

[9]In our environment, each agent is atomless and no interaction between agents is allowed.

[10]In our environment, we interpret the contracts as self-enforcing contracts in the following way. Both agents and firms have access to an enforcement mechanism ("court") upon the payment of a cost, whenever one of the parties reneges on a contract. If this cost is paid, the terms of the contract between the two parties in consideration become public, and this court can enforce a punishment to the party that reneged on the contract. If either firms or agents falsely report a breach on the contracts, they can also be punished by court. We assume this punishment can be made large enough so that in equilibrium neither firms nor agents will renege the contracts signed.

2.1 Communication and Menu Games

Communication

A communication mechanism consists of message spaces $\mathcal{R}^i$ for time 0 and message spaces $\mathcal{M}_t^i$ for each $t \in \{1, ..., T\}$, for each firm $i \in \{1, ..., I\}$. Denote the set of all possible messages that can be exchanged by an agent and firm i up to time t by $\mathcal{M}^{i,t} = \mathcal{M}_1^i \times ... \times \mathcal{M}_t^i$. For a given communication, each firm chooses allocation functions as follows. The allocation functions are $g_t^i : \mathcal{M}^{i,t} \to \mathbb{R}^2$, which specifies transfers of consumption and output at time t, and $\phi^i : \mathcal{R}^i \to G_1^i(\mathcal{M}^{i,1}) \times ... \times G_T^i(\mathcal{M}^{i,T})$, where $G_t^i(\mathcal{M}^{i,t})$ is the set of all measurable mappings from message space $\mathcal{M}^{i,t}$ to the allocation space $\mathbb{R}^2$. The function ϕ^i determines the contracts that the agent will face in all subsequent periods. An allocation function $g_t^i(m^{i,t})$ determines the consumption transfers and the output recommendation, given that message $m^{i,t} = (m_1^i, ..., m_t^i)$ is received. Let $(b(m^{i,t}), y(m^{i,t})) = g_t^i(m^{i,t})$. Denote by $G^i(\mathcal{M}^i) = G_1^i(\mathcal{M}^{i,1}) \times ... \times G_T^i(\mathcal{M}^{i,T})$ and $\mathcal{M}^i = \mathcal{M}_1^i \times ... \times \mathcal{M}_T^i$. Let $\Phi^i(\mathcal{R}^i, \mathcal{M}^i)$ be the set of all measurable mappings from message space $\mathcal{R}^i$ to the set G^i and notice that $\phi^i \in \Phi^i(\mathcal{R}^i, \mathcal{M}^i)$. Let $\mathcal{M} = \times_{i=1}^I \mathcal{M}^i$ and $\mathcal{R} = \times_{i=1}^I \mathcal{R}^i$. Denote the game associated with the communication mechanism $(\mathcal{M}, \mathcal{R})$ by $\Gamma_{\mathcal{M},\mathcal{R}}$. At time 0, before any uncertainty is realized, each firm i simultaneously offers a collection of allocation functions ϕ^i, and agents communicate with firms sending a message r^i. This message determines, through ϕ^i, the functions g_t^i at every period t. The timing of the game $\Gamma_{\mathcal{M},\mathcal{R}}$ is the following:

- At time 0:

    1. Each firm i simultaneously offers contract $\phi^i : \mathcal{R}^i \to G^i(\mathcal{M}^i)$;
    2. Agents send a report $r^i \in \mathcal{R}^i$ to each firm i.

- At time t:

    1. Agents learn his private type θ_t;
    2. Firm offers allocation rule $g_t^i : \mathcal{M}^{i,t} \to \mathbb{R}^2$ according to $\phi^i(r^i)$;

3. Agents send a message $m_t^i \in \mathcal{M}_t^i$ to each firm i;

4. Payoffs are realized.

Given messages $(\mathcal{M}, \mathcal{R})$, firms play static Nash at time 0 when choosing the contracts that are offered in future periods. Given these contracts, agents optimize choosing the report at time 0 and messages in every period $t = 1, ..., T$.

Definition 1 (**Equilibrium of Communication Game**). *A pure strategy equilibrium of $\Gamma_{\mathcal{M},\mathcal{R}}$ is (r^*, m^*, ϕ^*, g^*) such that:*[11]

1. ***Agent's message*** m_t^**:* $G_t^1 \times ... \times G_t^I \times \Theta^t \to \mathcal{M}_t$ *solves for each* $t \in \{1, ..., T\}$*:*

$$\begin{aligned} U_t\left(m^{t-1}, \theta_t | g^*\right) &= \max_{m_t \in \mathcal{M}_t} u\left(\sum_{i=1}^{I}\left(b^i(m^{i,t}) + y(m^{i,t})\right)\right) - v\left(\frac{\sum_{i=1}^{I} y(m^{i,t})}{\theta_t}\right) + \\ &\quad + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(m^t, \theta_{t+1} | g^*\right), \end{aligned}$$

$(b(m^{i,t}), y(m^{i,t})) = g_t^{*,i}(m^{i,t})$.

2. ***Agent's reporting strategy at time 0***, $r^* : G^1 \times ... \times G^I \to \mathcal{R}$ *solves:*

$$\max_{r \in \mathcal{R}} U(g)$$

$g^i = \phi^{i,*}(r_i)$, *where* $U(g) = \sum_{\theta_1} \pi(\theta_1) U_1\left(m^0, \theta_1 | g\right)$.

3. *For each* $i \in \{1, ..., I\}$, *taking as given the choices of the other firms and the agents' choices,* ***firm's*** i ***allocation function*** $\phi^{i,*}$ *solves:*

$$V^i(\phi^{i,*}, \phi_{-i}^*) \equiv \min \sum_{t=0}^{T} \sum_{\theta^t} \pi(\theta^t) q^t b_t^{i,*}(\theta^t),$$

$b_t^*(\theta^t) = b(m^{t,*}(\theta^t))$, $g^i = \phi^i\left(r^{i,*}\right)$ *and* $g_{-i}^* = \phi_{-i}^*\left(r_{-i}^*\right)$.

[11] We do not allow random strategies.

Denote the equilibrium *allocation* of a general communication game by (b^*, y^*).

Menu Games

If contracts are exclusive (or equivalently observable), the environment is a standard dynamic Mirrleesian environment as in Golosov, Kocherlakota, and Tsyvinski (2003). In this case, the revelation principle guarantees that without loss of generality, firms can restrict to direct mechanisms that are incentive compatible. However, under non-exclusive contracting, the preference ordering of the agents is influenced not only by their exogenous private information, but also by the set of contracts offered. In particular, the choice of an agent in the contracts offered by firm i depends on the contracts offered by other firms. This implies that restricting to a direct mechanism may not allow a firm to have a rich enough communication with the agent in order to obtain information on the other contracts.

In order to characterize the contracts offered by each firm, we extend the *delegation principle* proved by Peters (2001) and Martimort and Stole (2002) to our environment. This principle states that, without loss of generality, the equilibrium outcomes of any communication game can be implemented as an equilibrium of a menu game. The key idea is that any communication in the original communication mechanism can be replaced by firms offering menus of payoff-relevant alternatives and delegating to the agents the choice within this menu. To incorporate a richer communication between firms and agents, firms might offer menus with elements that are not chosen in equilibrium (latent contracts). As highlighted by Arnott and Stiglitz (1991), offering latent contracts might be necessary to sustain particular equilibria by deterring entry of additional insurance providers and by preventing deviation of the incumbent insurance providers.

Our environment differs from the previous literature along two dimensions. First, the environment is dynamic in the sense that the exogenous uncertainty is realized in every period. Second, agents choose a communication-contingent contract from each firm i *before* any uncertainty is realized. We now define menus that are compatible with a given communication mechanism $(\mathcal{M}, \mathcal{R})$. A communication mechanism induces allocation functions and, hence, distribution over allocations. This means that to prove the equivalence between the equilibrium allocation of a given communication mechanism and the equilibrium of a menu game, it is essential that the menus offered are rich enough to capture the strategies used to implement equilibrium in a communication mechanism. In our environment, a menu is a sequence of sets, where each set is a subset of the allocation space $\mathbb{R} \times \mathbb{R}_+$. For a message space $(\mathcal{M}, \mathcal{R})$, define, for each firm i, the set $C_t^i(m^{i,t-1}, \mathcal{M}_t^i | G_t^i)$ as the menu that can be implemented through a message space $\mathcal{M}_t^i$ at time t given a history of messages $m^{i,t-1}$. Formally, a menu at time t is the following set:

$$C_t^i(m^{i,t-1}, \mathcal{M}_t^i | G_t^i) \equiv \{C_t^i \subseteq \mathbb{R} \times \mathbb{R}_+ |\ \exists\ g_t^i \in G_t^i \subseteq G_t^i(\mathcal{M}^{i,t}) : C_t^i = \mathbf{Im}(g_t^i | m^{i,t-1})\} \ \ \forall t,\ \forall i \quad (2)$$

where

$$\mathbf{Im}(g_t^i | m^{i,t-1}) = \left\{x \in \mathbb{R} \times \mathbb{R}_+ |\ \exists\ m_t^i \in \mathcal{M}_t^i \ : x = g_t^i \left(m^{i,t-1}, m_t^i\right)\right\} \quad \forall t,\ \forall i. \quad (3)$$

Each set defined in (2) contains all subsets of $\mathbb{R}^2$ with cardinality at most $\mathcal{M}_t^i$.

Using the above definition of a menu, we now define the sequence of menus offered by firms at time 0. For any subset $G_t^i \subseteq G_t^i(\mathcal{M}^{i,t})$, let $G^i = G_1^i \times ... \times G_t^i$ and define a sequence of menus as follows:

$$C\left(G^i\right) = \left\{C_t^i \subseteq C_t^i(m^{i,t-1}, \mathcal{M}_t^i | G_t^i),\ t = 1, ..., T,\ \forall m^{i,t-1} \in \mathcal{M}^{i,t-1}, m_t^i \in \mathcal{M}_t^i\right\}. \quad (4)$$

At time 0, each agent chooses a sequence of menus in the collection of menus offered by firm i. Define $\mathcal{C}^i$ as the collection of menus that are consistent with a communication system $(\mathcal{M}, \mathcal{R})$.

$$\mathcal{C}^i(\mathcal{R}^i, \mathcal{M}^i) \equiv \{C^i \subseteq C^i\left(G^i\right) | \exists\ \phi^i \in \Phi^i(\mathcal{R}^i, \mathcal{M}^i) : G^i = \mathbf{Im}(\phi^i)\}. \quad (5)$$

This set contains all the collections of sets C^i with cardinality less than or equal to the cardinality of $\mathcal{R}^i$. Without explicitly writing the dependence on the message spaces, let $\mathcal{C}^i = \mathcal{C}^i(\mathcal{R}^i, \mathcal{M}^i)$ and let C^i be a generic element of $\mathcal{C}^i$. Let $C = \prod_i C^i$ and $\mathcal{C} = \prod_i \mathcal{C}^i$ be the collection of all menus. Let $\Gamma_{C,\mathcal{C}}$ be the game associated with menus $(C, \mathcal{C})$.

Definition 2 (**Equilibrium of Menu Games**). *A pure strategy equilibrium of a menu game is a collection of menus* $\hat{\mathcal{C}}$ *and agents' choices* $\hat{C} \in \hat{\mathcal{C}}$ *and* $(\hat{b}^i_t, \hat{y}^i_t) \in C^i_t(\hat{b}^{i,t-1}, \hat{y}^{i,t-1}|\hat{C}^i)\ \ \forall t \in \{1, ..., T\}\,,\ \ \forall i \in \{1, ..., I\}$*:*[12]

1. *Agents' choice at time t,* $(\hat{b}_t, \hat{y}_t) : C_t(\hat{b}^{t-1}, \hat{y}^{t-1}|\hat{C}) \times \Theta^t \to C_t(\hat{b}^{t-1}, \hat{y}^{t-1}|\hat{C})$ *solves:*

$$U_t\left(b^{t-1}, y^{t-1}, \theta_t|\hat{C}\right) = \max_{(b_t,y_t)\in C_t\left(\hat{b}^{t-1},\hat{y}^{t-1}|\hat{C}\right)} u\left(\sum_{i=1}^{I}(b^i_t + y^i_t)\right) - v\left(\frac{\sum_{i=1}^{I} y^i_t}{\theta_t}\right) + \\ +\beta\sum_{\theta_{t+1}}\pi(\theta_{t+1})U_{t+1}\left(b^t, y^t, \theta_{t+1}|\hat{C}\right),$$

   subject to $\sum_{i=1}^{I}(b^i_t + y^i_t) \geq 0\ \ \forall t.$

2. *Agents' choice at time* 0*,* $\hat{C} : \hat{\mathcal{C}} \to \hat{\mathcal{C}}$ *solves:*

$$\max_{C\in\hat{\mathcal{C}}} U(C),$$

   where $U(C) = \sum_{\theta_1}\pi(\theta_1)U_1\left(b^0, y^0, \theta_1|C\right)$*.*

3. *For each* $i \in \{1, ..., I\}$*,* $\mathcal{C}^i$ *solves, taking as given* $\hat{\mathcal{C}}_{-i}$ *chosen by firms* $-i$ *and the agents' choice* $\hat{C}_{-i}$*,* $\{\hat{b}^t(\theta^t), \hat{y}^t(\theta^t)\}_{t=1}^T$*:*

$$V^i(\hat{\mathcal{C}}^i, \hat{\mathcal{C}}^{-i}) \equiv \min\sum_{t=0}^{T}\sum_{\theta^t}\pi(\theta^t)q^t\hat{b}^i_t(\theta^t)$$

   $\hat{b}^i_t(\theta^t) \in \hat{C}^i_t(\hat{b}^{i,t-1}, \hat{y}^{i,t-1}|\hat{C}^i),\ \hat{C}^i_t(\hat{b}^{i,t-1}, \hat{y}^{i,t-1}|\hat{C}^i) \in \hat{\mathcal{C}}^i$
   $\hat{b}^{-i}_t(\theta^t) \in \hat{C}^{-i}_t(\hat{b}^{-i,t-1}, \hat{y}^{-i,t-1}|\hat{C}^{-i})$ *and* $\hat{C}^{-i}_t(\hat{b}^{-i,t-1}, \hat{y}^{-i,t-1}|\hat{C}^{-i}) \in \hat{\mathcal{C}}^{-i}$*.*

Denote the equilibrium allocation of a menu game by $(\hat{b}, \hat{y})$.

Notice that a menu might contain more alternatives than the cardinality of the type space, implying that some alternatives are not chosen in equilibrium. Similarly, at time 0 a firm might offer more than one set of contracts, also implying that some contracts are

[12] We do not allow for random menus.

offered and not chosen by agents in equilibrium. We denote a contract as **latent** if it is offered in equilibrium by a firm but is not chosen in equilibrium by any agent. As we show in this paper, latent contracts have an important role in sustaining equilibrium allocations by preventing other firms from deviating to other contracts.

The following proposition shows that an equilibrium in a general communication system can be implemented as an equilibrium of a menu game. In this menu game, the collection of menus offered by each firm must be compatible with the general communication mechanism as defined above.

Proposition 1 (Delegation Principle)**.** *Let* (b^*, y^*) *be an equilibrium allocation of a general communication game* $\Gamma_{\mathcal{M},\mathcal{R}}$*. Then there exists* $(\hat{b}, \hat{y})$ *that is an equilibrium allocation of a menu game* $\Gamma_{C,\mathcal{C}}$ *and* $(b^*, y^*) = (\hat{b}, \hat{y})$*.*

Proof. In Appendix A. ∎

Proposition 1 states that for given message spaces $(\mathcal{M}, \mathcal{R})$, there exists a menu game that implements the same equilibrium allocation. It is important to note that message spaces restrict the menus that can be offered in a menu game. From the previous result, if firms are allowed to use unrestricted message spaces, the same equilibrium can be implemented if firms can offer unrestricted menus. This result is formally stated below and is the same as Corollary 1 in Martimort and Stole (2002).

Corollary 1. *An equilibrium of a general communication game with unrestricted message spaces is an equilibrium of a menu game in which firms can use unrestricted menus.*

From Corollary 1, we can, without loss of generality, focus on menu games with unrestricted menus. For completeness, we define menus in this game. At time 0, each firm i offers a collection of menus denoted $\mathcal{C}^i$. A menu $C^i \in \mathcal{C}^i$ is a sequence of sets as follows:

$$\begin{aligned} C_1^i\left(C^i\right) &= \left\{(b_1^i, y_1^i) : (b_1^i, y_1^i) \in \mathbb{R} \times \mathbb{R}_+\right\}; \\ C_t^i\left(b^{i,t-1}, y^{i,t-1} | C^i\right) &= \left\{(b_t^i, y_t^i) : (b_t^i, y_t^i) \in \mathbb{R} \times \mathbb{R}_+\right\} \quad \forall t = 2, ..., T. \end{aligned}$$

Given the collection of menus $\mathcal{C}$ offered by firms, an agent chooses a menu $C \in \mathcal{C}$ at time 0. At every time t, the agent chooses a pair (b_t^i, y_t^i) in menu $C_t^i(b^{i,t-1}, y^{i,t-1}|C^i)$ for each firm i. This means that the agent receives transfer $b_t^i + y_t^i$ and has to deliver output y_t^i at time t and will choose from menu $C_{t+1}^i(b^{i,t}, y^{i,t}|C^i)$ at time $t+1$. The timing of a menu game is the following:

- At time 0:
  1. Each firm simultaneously offers a collection of menus $\mathcal{C}^i$;
  2. Agents choose a menu $C^i \in \mathcal{C}^i$ for each firm $i \in \{1, ..., I\}$.
- At time $t, t \in \{1, ..., T\}$:
  1. Agents learn type θ_t;
  2. Agents choose a pair $(b_t^i, y_t^i) \in C_t^i\left(b^{i,t-1}, y^{i,t-1}|C^i\right) \quad \forall i$;
  3. Payoffs are realized.

Denote the choices at time t of an agent with history θ^t from the menu offered by firm i by $b^i(\theta^t)$ and $y^i(\theta^t)$. An equilibrium of an unrestricted menu game is defined as an equilibrium of a menu game above.

As previously mentioned, latent menus (contracts) and menus with latent points might be offered in equilibrium. In our environment, given the presence of two rounds of communication (at time 0 and at every time t), we show that any time t menu that contains latent points can alternatively be replaced by a time t menu with the same number of elements as the type space and latent menus at time 0. This implies that, without loss of generality, we can restrict firms to offering time t menus that have the same cardinality of the type space, which we call minimal menus.

Definition 3 (**Minimal Menus**). *A menu $C^i \in \mathcal{C}^i$ is minimal if for all $C^{-i} \in \mathcal{C}^{-i}$ and $(b_t^i, y_t^i) \in C_t^i$, for all $C_t^i \in C^i$, there exists $\theta_t \in \Theta$, such that $(b_t^i, y_t^i) = (b^{*,i}(\theta^t), y^{*,i}(\theta^t))$.*

Intuitively, a menu is minimal if all of its elements are chosen by some agent in equilibrium.

Proposition 2. *Let $\mathcal{C} = \{\mathcal{C}^i, \mathcal{C}^{-i}\}$ be an equilibrium of a menu game. There exists a payoff equivalent equilibrium $\tilde{\mathcal{C}}$, such that every $\tilde{C}^i \in \tilde{\mathcal{C}}^i$ is a minimal menu for all i.*

Proof. In Appendix A. ∎

3 Equilibrium Characterization

In this section, we characterize properties an equilibrium allocation must satisfy.[13]

3.1 Characterization under Exclusive Contracts

Before characterizing the optimality conditions in our environment, we review two robust equilibrium conditions in an environment in which there is competition between insurance providers and *contracts are exclusive*. The seminal paper of Prescott and Townsend (1984) shows that in a general class of private information economy, the first welfare theorem holds. The decentralized economy is equivalent to a planning problem that maximizes the ex ante lifetime utility of the agents subject to feasibility and incentive compatibility constraints (in every period for every realization agents weakly prefer the allocation designed for them).

In an environment similar to ours, and in the presence of exclusive contracting, the equilibrium allocation has the following features:[14]

1. The marginal rate of substitution between consumption and leisure is equated to the marginal productivity only for the highest type;

[13]Throughout the paper, an incumbent refers to a firm that offers a menu that contains transfers and/or output recommendations other than the null contract and some agent chooses some of these contracts in equilibrium. An entrant refers to an insurance provider that, at all times, every agent chooses the null contract from the menus offered by this firm. We assume the number of firms I is large enough so that an entrant always exists.

[14]For a review of the results of constrained efficient allocation in dynamic Mirrleesian environments, refer to Golosov, Tsyvinski, and Werning (2006).

2. If preferences are separable in consumption and leisure, the marginal rate of substitution of consumption between any two periods differs from the intertemporal rate of transformation for all types (the standard Euler equation does not hold).

The first result was originally shown by Mirrlees (1971) and is formally:

$$u'(c(\bar{\theta})) = \frac{1}{\bar{\theta}} v' \left(\frac{y(\bar{\theta})}{\bar{\theta}} \right), \tag{6}$$

$$u'(c(\theta)) > \frac{1}{\theta} v' \left(\frac{y(\theta)}{\theta} \right), \quad \forall\, \theta \neq \bar{\theta}, \theta \in \Theta, \tag{7}$$

where $\bar{\theta} \equiv \max_{\theta \in \Theta} \theta$. The intuition for this result is the following: in order to separate types, it is optimal to discourage less productive agents to work. This implies that all but the most productive agents work and consume less than they would in a competitive environment.

The second distortion is formally the following equation:

$$\frac{1}{u'(c(\theta^t))} = \frac{1}{\beta R} E\left[\frac{1}{u'(c(\theta^{t+1}))} | \theta^t \right], \quad \forall\, t, \theta^t. \tag{8}$$

This distortion was derived originally by Rogerson (1985) and generalized in Golosov, Kocherlakota, and Tsyvinski (2003) . It implies that for all periods $u'(c(\theta^t)) < \beta R E\left[u'(c(\theta^{t+1})) | \theta^t \right]$, so that it is optimal to make any type of agent saving constrained in order to encourage the truthful revelation of productivity in the future period.

3.2 Optimality Conditions under Non-exclusivity

We now focus on the equilibrium conditions in our environment in the presence of non-exclusive contracting. The presence of non-observable contracts implies that the above equilibrium conditions cannot be implemented.

Proposition 3. *In any equilibrium for every $\theta^t \in \Theta^t$, for all t, the following holds:*

$$u'(c_t(\theta^t)) = \frac{\beta}{q} \sum_{\theta_{t+1}} u'(c_{t+1}(\theta^{t+1})) \pi(\theta_{t+1}), \tag{9}$$

where $c_t(\theta^t) = \sum_{i=1}^{I}\left(b_t^i(\theta^t) + y_t^i(\theta^t)\right)$.

Proof. Suppose that for some history $\hat{\theta}^t$ equation (9) does not hold.

Case 1:

$$u'(c_t(\hat{\theta}^t)) < \frac{\beta}{q}\sum_{\theta_{t+1}} u'(c_{t+1}(\hat{\theta}^t, \theta_{t+1}))\pi(\theta_{t+1}). \tag{10}$$

In this case, the agent is savings constrained. An entrant can make strictly positive profits offering a savings contract at a rate lower than $1/q$, contradicting the original allocation being an equilibrium. The first step is to construct the contract to be offered by a firm. Let $\delta^*(\varepsilon)$ be the solution of the following problem:

$$U(\varepsilon) \equiv \max_{\delta\geq 0} u(c_t(\hat{\theta}^t) - \delta) + \beta E_t u\left(c_{t+1}(\hat{\theta}^t, \theta_{t+1}) + \delta(\frac{1}{q} - \varepsilon)\right). \tag{11}$$

The first order condition for this problem is:

$$u(c_t(\hat{\theta}^t)) \geq \beta(\frac{1}{q} - \varepsilon)E_t u\left(c_{t+1}(\hat{\theta}^t, \theta_{t+1}) + \delta(\frac{1}{q} - \varepsilon)\right). \tag{12}$$

If $\varepsilon = 0$, the solution for the above problem is $\delta^*(0) > 0$ given that (10) holds. From the Theorem of the Maximum, the solution $\delta^*(\varepsilon)$ is continuous on ε. Fix $\epsilon_1 > 0$ such that $|\delta^*(0) - 0| > \epsilon_1$. $\exists\ \epsilon_2 > 0$ such that if $|\varepsilon - 0| < \epsilon_2$ then $|\delta^*(\varepsilon) - \delta^*(0)| < \epsilon_1$. Let ε be such that $0 < \varepsilon < \epsilon_2$. Consider an entrant that offers the contract $C_t = \{(\delta^*(\varepsilon), 0), (0, 0)\}$ and $C_{t+1} = \{(-\delta^*(\varepsilon)(\frac{1}{q} - \varepsilon), 0), (0, 0)\})$ and the contract $(0, 0)$ for all other periods. This firm is making strictly positive profits, proportional to $\delta^*(\varepsilon)\varepsilon$, and the agent is strictly better off keeping the original equilibrium together with this contract given that the contract increases the agent's utility in a history with positive probability and keeping the same utility in all other histories.

Hence, under the original equilibrium, a firm can offer a contract that makes strictly positive profits. This contradicts the allocation being an equilibrium.

The other case can be proved using a similar argument. ∎

The intuition for the result is the following. If the equilibrium allocation does not satisfy the Euler equation, an entrant firm can offer a savings (borrowing) contract at time t with an implicit interest rate lower (higher) than the marginal rate of transformation. As long as this contract is accepted, the entrant makes strictly positive profits and such contract can be constructed in a way that provides higher utility to the agent.

Under exclusivity, the optimal contract provides incentives to more skilled workers by discouraging less skilled agents to work. The next lemma shows that this distortion cannot be implemented when contracts are not exclusive, since agents can work an extra amount to other firms.

Lemma 1. *In any equilibrium for every $\theta^t \in \Theta^t$, for all t the following holds:*

$$u'(b(\theta^t) + y(\theta^t)) \leq v'\left(\frac{y\left(\theta^t\right)}{\theta_t}\right)\frac{1}{\theta_t}, \tag{13}$$

where $b(\theta^t) = \sum_i b^i(\theta^t)$ and $y(\theta^t) = \sum_i y^i(\theta^t)$ and where $(b^i(\theta^t), y^i(\theta^t))$ are the contracts chosen by an agent with history θ^t from firm i at time t.

Proof. Suppose that for some history θ^t equation (13) does not hold:

$$u'(b(\theta^t) + y(\theta^t)) > v'\left(\frac{y\left(\theta^t\right)}{\theta_t}\right)\frac{1}{\theta_t}. \tag{14}$$

In this case, the agent would like to consume and work more than the equilibrium contract. An entrant can make strictly positive profits offering a supplemental contract with more consumption and output. Consider an entrant that offers the contract at time t, $C_t^E = \{(-\varepsilon, \delta^*(\varepsilon)), (0,0)\}$ where δ^* and ε are constructed as follows. Let $\delta^*(\varepsilon|\theta_t)$ be the solution of the following problem:

$$U(\varepsilon|\theta_t) \equiv \max_{\delta \geq 0} u(b(\theta^t) + y(\theta^t) + \delta - \varepsilon) - v\left(\frac{y\left(\theta^t\right) + \delta}{\theta_t}\right). \tag{15}$$

The first order condition for this problem is:

$$u'(b(\theta^t) + y(\theta^t) + \delta^*(\varepsilon|\theta_t) - \varepsilon) \leq v'\left(\frac{y\left(\theta^t\right) + \delta^*(\varepsilon|\theta_t)}{\theta_t}\right)\frac{1}{\theta_t}. \tag{16}$$

If $\varepsilon = 0$, the solution for the above problem is $\delta^*(0|\theta_t) > 0$ given that (30) holds. From the Theorem of the Maximum, the solution $\delta^*(\varepsilon)$ is continuous on ε. Fix $\epsilon_1 > 0$ such that $|\delta^*(0) - 0| > \epsilon_1$. There exists $\epsilon_2 > 0$ such that if $|\varepsilon - 0| < \epsilon_2$ then $|\delta^*(\varepsilon) - \delta^*(0)| < \epsilon_1$. Let ε be such that $0 < \varepsilon < \epsilon_2$.

An entrant offering this contract makes strictly positive profits, proportional to ε, and the agent is strictly better off given that his utility is higher in some history with positive probability. This contract is always profitable for the entrant even if other type $\tilde{\theta}_t$ accepts the deviating contract. The only way to deter this deviation is to have some latent contract that makes no agent willing to choose it. However, if such a contract existed, it would have been chosen in the original equilibrium, contradicting the fact that it is a latent contract. ∎

In the next proposition, we show that in equilibrium the marginal rate of substitution (MRS) between consumption and leisure is equated to the marginal productivity for every history and also that the lifetime transfer received under any history is equal to zero, so that there is no cross-subsidization between types.

Proposition 4. *In any equilibrium the following two conditions hold:*

1. *Zero net present value of transfers:*

$$\sum_{t=1}^{T} R^{1-t} b_t(\theta^t) = 0 \qquad \forall \theta^T \in \Theta^T. \tag{17}$$

2. *MRS equal to marginal productivity:*

$$u'(b(\theta^t) + y(\theta^t)) = v'\left(\frac{y\left(\theta^t\right)}{\theta_t}\right)\frac{1}{\theta_t} \qquad \forall \theta^t, t. \tag{18}$$

Proof. Define the continuation net present value of transfers chosen in equilibrium for a continuation history as follows:

$$A_t(\theta^{t-1}, \theta_{t-1}^T) = \sum_{n=t}^{T} R^{t-n} b_n(\theta^{t-1}, \theta_{t-1}^n), \tag{19}$$

where $\theta_{t-1}^n = (\theta_t, \theta_{t+1}, \ldots, \theta_n)$ is the continuation history from time t to n and $b_n(\theta^{t-1}, \theta_{t-1}^n)$ is the equilibrium transfer chosen at time n by an agent with history θ^n. We show, using a backward induction argument, that for all t, $A_s(\theta^{s-1}, \theta_{s-1}^T)$ is independent of θ_{s-1}^T for all $s \geq t$. This implies that $A_1(\theta^T)$ is the same for all $\theta^T \in \Theta^T$. If $A_1(\theta^T) > 0$, firms make strictly negative profits in equilibrium and would be better off offering a null contract. If $A_1(\theta^T) < 0$, an entrant can offer the same sequence of transfers giving an additional transfer $\varepsilon > 0$ in the terminal period. Since the sequence of transfers is not contingent and is profitable for all types, there is no latent contract that makes it unprofitable.

1. **Statement holds for** $t = T$.

We first show that at time T, transfers are independent of realization of time T shock and then show that for time T equation (18) holds.

Transfers independent at $t = T$**:**

Suppose the statement is not true and let $b(\theta^T) = \min_{b \in C(b^{T-1}, y^{T-1})} b$ and $b(\theta^{T-1}, \hat{\theta}_T)$ the second smallest b. Denote by $\hat{\theta}^T = (\theta^{T-1}, \hat{\theta}_T)$. The contradiction argument relies on the incumbent firm deviating to an allocation that delivers higher profits. First notice that it must be true that $y(\hat{\theta}^T) + b(\hat{\theta}^T) > y(\theta^T) + b(\theta^T)$. If not, given that $b(\hat{\theta}^T) > b(\theta^T)$ then $y(\hat{\theta}^T) < y(\theta^T)$, an entrant firm can offer the following contract $\tilde{C}_T = \{(-\varepsilon, y(\theta^T) - y(\hat{\theta}^T)); (0, 0)\}$, for some ε small enough. An agent with type θ^T is better off by choosing allocation $(b(\hat{\theta}^T), y(\hat{\theta}^T))$ in menu C_T together with $(-\varepsilon, y(\theta^T) - y(\hat{\theta}^T))$. With these choices, his utility is:

$$u\left(b(\hat{\theta}^T) - \varepsilon + y(\theta^T)\right) - v\left(\frac{y(\theta^T)}{\theta_T}\right) > u\left(b(\theta^T) + y(\theta^T)\right) - v\left(\frac{y(\theta^T)}{\theta_T}\right)$$

where the inequality holds as long as $b(\hat{\theta}^T) - \varepsilon > b(\theta^T)$. No latent contracts can prevent this deviation, since it is profitable for the entrant insurance provider as long as some agent accepts it.[15]

The equilibrium allocation must satisfy the following constraints:

$$u(b(\hat{\theta}^T) + y(\hat{\theta}^T)) - v\left(\frac{y(\hat{\theta}^T)}{\hat{\theta}_T}\right) \geq u(b(\theta^T) + y(\theta^T)) - v\left(\frac{y(\theta^T)}{\hat{\theta}_T}\right), \tag{20}$$

$$u(b(\theta^T) + y(\theta^T)) - v\left(\frac{y(\theta^T)}{\theta_T}\right) \geq u(b(\hat{\theta}^T) + y(\hat{\theta}^T)) - v\left(\frac{y(\hat{\theta}^T)}{\theta_T}\right). \tag{21}$$

Case 1 If (20) holds with equality, an agent of type $\hat{\theta}_T$ is indifferent between his equilibrium choice and the choice of agent θ_T. However, the insurance providers receive strictly higher profits from the allocation θ_T, since by assumption $b(\hat{\theta}^T) > b(\theta^T)$. This incumbent insurance provider can deviate to an alternative menu that differs from the original by offering at time T only the allocation chosen by agent θ_T. No latent contract can induce lower profits to deter this deviation, since now the deviating incumbent offers a subset of the allocations that were available in the original equilibrium. The argument also holds if the equilibrium allocation is divided between multiple insurance providers.

Case 2 Suppose that (20) holds with strict inequality. Notice that, following the argument in the previous case, for any type $\bar{\theta}_T$ such that $b\left(\bar{\theta}_T\right) > b_T(\theta^T)$, it must be true that:

$$u(b(\bar{\theta}^T) + y(\bar{\theta}^T)) - v\left(\frac{y(\bar{\theta}^T)}{\bar{\theta}_T}\right) > u(b(\theta^T) + y(\theta^T)) - v\left(\frac{y(\theta^T)}{\bar{\theta}_T}\right). \tag{22}$$

Otherwise, the incumbent firm will offer only the contract containing $b_T(\theta^T)$.

Consider the following deviation by an incumbent firm $\tilde{b}(\hat{\theta}^T) = b(\hat{\theta}^T) - \varepsilon$ and $\tilde{b}(\theta^T) = b(\theta^T) + \varepsilon - \delta$ for $\varepsilon, \delta > 0$ and $\varepsilon > \delta$, to be defined explicitly below and keeping unchanged all

[15] Note that this case arises in the solution of the constrained efficient allocation: high skilled agents work more and make positive transfers to less skilled agents. The deviation $\tilde{C}_T$ makes this allocation unprofitable in our environment, since it induces skilled agents to choose the allocation designed for low skilled agents and working an additional amount with entrant.

the other allocations.[16] This deviation increases net present value profits of the incumbent by a factor proportional to δ.

To show that such deviation is profitable, thus reaching a contradiction, we need to show that there is no latent contract $\alpha \equiv (\alpha_b, \alpha_y)$ that can induce this firm to reduce its profits following this deviation. Suppose such contract exists. One possibility is to induce θ_T agents, when faced with the deviating allocation $\tilde{b}$, to choose $\tilde{b}(\hat{\theta}^T)$. This would imply a reduction of profits, since $\tilde{b}(\hat{\theta}^T) > \tilde{b}(\theta^T)$. Such latent contract has to satisfy:

$$u(\tilde{b}(\hat{\theta}^T) + y(\hat{\theta}^T) + \alpha_b + \alpha_y) - v\left(\frac{y(\hat{\theta}^T) + \alpha_y}{\theta_T}\right) > u(\tilde{b}(\theta^T) + y(\theta^T)) - v\left(\frac{y(\theta^T)}{\theta_T}\right). \tag{23}$$

Since α is not chosen in the original equilibrium, it must also be true that

$$u(b(\theta^T) + y(\theta^T)) - v\left(\frac{y(\theta^T)}{\theta_T}\right) \geq u(b(\hat{\theta}^T) + y(\hat{\theta}^T) + \alpha_b + \alpha_y) - v\left(\frac{y(\hat{\theta}^T) + \alpha_y}{\theta_T}\right). \tag{24}$$

However, $u(\tilde{b}(\theta^T) + y(\theta^T)) > u(b(\theta^T) + y(\theta^T))$ and $u(b(\hat{\theta}^T) + y(\hat{\theta}^T) + \alpha_b + \alpha_y) > u(\tilde{b}(\hat{\theta}^T) + y(\hat{\theta}^T) + \alpha_t + \alpha_y)$, which combined with (24) implies

$$u(\tilde{b}(\theta^T) + y(\theta^T)) - v\left(\frac{y(\theta^T)}{\theta_T}\right) > u(\tilde{b}(\hat{\theta}^T) + y(\hat{\theta}^T) + \alpha_b + \alpha_y) - v\left(\frac{y(\hat{\theta}^T) + \alpha_y}{\theta_T}\right), \tag{25}$$

contradicting (23).

Consider now any other type $\bar{\theta}_T \neq \theta_T$ with $b\left(\bar{\theta}_T\right) > b\left(\theta_T\right)$:

$$u(\tilde{b}(\theta^T) + y(\theta^T) + \alpha_b + \alpha_y) - v\left(\frac{y(\theta^T) + \alpha_y}{\bar{\theta}_T}\right) > u(b(\bar{\theta}^T) + y(\bar{\theta}^T)) - v\left(\frac{y(\bar{\theta}^T)}{\bar{\theta}_T}\right). \tag{26}$$

Since a latent contract is not chosen in the original equilibrium, it must also be true that

$$u(b(\bar{\theta}^T) + y(\bar{\theta}^T)) - v\left(\frac{y(\bar{\theta}^T)}{\bar{\theta}_T}\right) \geq u(b(\theta^T) + y(\theta^T) + \alpha_t + \alpha_y) - v\left(\frac{y(\theta^T) + \alpha_y}{\bar{\theta}_T}\right). \tag{27}$$

[16] If there are multiple θ with values equal to $b(\hat{\theta}^T)$ or $b(\theta^T)$, the same deviation applies to all such transfers.

The previous equation must hold with equality, otherwise in the original equilibrium the deviating firm would not offer contract $b\left(\bar{\theta}_T\right)$. Let

$$\begin{aligned}\Delta(\bar{\theta}) &= \min_{\alpha\in C_T^{-i}}\left\{u(b(\bar{\theta}^T)+y(\bar{\theta}^T))-v\left(\frac{y(\bar{\theta}^T)}{\bar{\theta}_T}\right)+\right.\\ &\qquad \left.-u(b(\theta^T)+y(\theta^T)+\alpha_b+\alpha_y)+v\left(\frac{y(\theta^T)+\alpha_y}{\bar{\theta}_T}\right)\right\}.\end{aligned} \tag{28}$$

This gives the minimum utility gain agent $\bar{\theta}^T$ receives from choosing allocation $(b(\bar{\theta}^T), y(\bar{\theta}^T))$ instead of $(b(\theta^T), y(\theta^T))$ combined with any other latent contract α. Since (27) holds with strict inequality, $\Delta\left(\bar{\theta}\right)$ is strictly positive for each $\bar{\theta}$. Let $\bar{\alpha}\equiv\arg\min\Delta(\bar{\theta})$. There exists $\varepsilon\left(\bar{\theta}\right)>0$ such that

$$\begin{aligned}&u(b(\bar{\theta}^T)+y(\bar{\theta}^T))-v\left(\frac{y(\bar{\theta}^T)}{\bar{\theta}^T}\right)\geq\\ &u(b(\theta^T)+y(\theta^T)+\bar{\alpha}_t+\bar{\alpha}_y+\varepsilon\left(\bar{\theta}\right))-v\left(\frac{y(\theta^T)+\bar{\alpha}_y}{\bar{\theta}^T}\right)>\\ &u(b(\theta^T)+y(\theta^T)+\bar{\alpha}_b+\bar{\alpha}_y+\varepsilon\left(\bar{\theta}\right)-\delta)-v\left(\frac{y(\theta^T)+\bar{\alpha}_y}{\bar{\theta}^T}\right).\end{aligned} \tag{29}$$

Let $\varepsilon=\min_{\bar{\theta}\neq\theta}\varepsilon\left(\bar{\theta}\right)$. Under this choice of ε, the above equation contradicts (27). Note that (29) also implies that for all $\bar{\theta}\neq\theta$ the choice of the agent, following the deviation, is the same as in the original equilibrium.

The last step in the proof requires checking that the time $T-1$ incentive constraints hold. This is necessary in order to leave the decision of the agents unchanged at time $T-1$. Notice that for a given $\varepsilon>0$, there exists $\delta^*>0$ that makes the utility, calculated in time $T-1$, of the modified contract the same as in the original contract. To see this, notice that if $\delta=\varepsilon$ the change in utility of the agent is negative following the proposed deviation, while if $\delta=0$ the utility change is positive, since the agent now faces a reduction in the spread of consumption at time T because $y(\hat{\theta}^T)+b(\hat{\theta}^T)>y(\theta^T)+b(\theta^T)$. This implies that there exists an intermediate value of δ^* such that $\varepsilon>\delta^*>0$ so that the change is zero. Hence, the time $T-1$ decision will be unchanged if $\delta=\delta^*$.

Equated MRS at time T:

Proposition 1 implies that there is only one case left to consider. Suppose that for some $\theta^T = (\theta^{T-1}, \theta_T)$

$$u'(b(\theta^{T-1}) + y(\theta^T)) < v'\left(\frac{y\left(\theta^T\right)}{\theta_T}\right)\frac{1}{\theta_T}. \tag{30}$$

In this case, the agent would like to consume and work less than the equilibrium contract. A deviation that reduces the total output and consumption by agent θ^T cannot be provided by an entrant insurance provider, since a worker cannot deliver negative hours. However, an incumbent firm will find it optimal to deviate from the equilibrium contract ,offering an allocation with lower consumption and lower output requirement and making strictly positive profits. Formally, it offers the original contract at all time $t < T$ and at time T, a menu that contains a null contract, the modified allocation chosen by θ^T and the original allocation chosen by the remaining types:

$$C_T\left(b(\theta^{T-1}), y(\theta^{T-1})\right) =$$
$$\left\{\left(b(\theta^T) + y(\theta^T) + \delta^*(\varepsilon|\theta_T) - \varepsilon, y(\theta^T) + \delta^*(\varepsilon|\theta_T)\right); (0,0); \left(y(\hat{\theta}^T) + b(\hat{\theta}^T), y(\hat{\theta}^T)\right)\ \hat{\theta^T} \neq \theta^T\right\}$$

where δ^* and ε are constructed in a similar fashion to the proof of Lemma 1, with the constraint $\delta \leq 0$.

With this deviation, the incumbent makes strictly positive profits, proportional to ε, and ε can be chosen large enough so that agents' utility is unchanged following this deviation. This guarantees that no deviation at time $T-1$ takes place. This contract is always profitable for the incumbent even if another type $\tilde{\theta}_T$ accepts it. If an agent with type $\tilde{\theta}^T$ is able to choose the pair $(b(\theta^T)+y(\theta^T)+\delta^*(\varepsilon|\theta_T)-\varepsilon, y(\theta^T)+\delta^*(\varepsilon|\theta_T))$ at time T, it implies that he must also have chosen the allocation sequence $\{(b(\theta^n) + y(\theta^n), y(\theta^n))\}_{n=1}^{T-1}$ in previous periods. From the previous step in the proposition, transfers from any history are independent of time T; i.e., this agent will receive transfers with the same net present value as under his original choice. Hence, the deviation is profitable as long as some agent is accepting it.

2. **Statement holds for $t < T$.**

As an inductive assumption, suppose statement holds for $t+1$. We now show it holds for period t. Rewrite the net present value of transfers as:

$$A_t(\theta^{t-1}, \theta_{t-1}^T) = \sum_{n=t}^{T} R^{t-n} b_n(\theta^{t-1}, \theta_{t-1}^n) =$$
$$b_t(\theta^{t-1}, \theta_t) + \frac{1}{R} \sum_{n=t+1}^{T} R^{t+1-n} b_n(\theta^{t-1}, \theta_{t-1}^n) = b_t(\theta^{t-1}, \theta_t) + \frac{1}{R} A_{t+1}(\theta^t, \theta_t^T).$$

Since by the inductive assumption A_{t+1} is independent of θ_t^T, without loss of generality, there exist θ_t and $\hat{\theta}_t$ following history θ^{t-1} so that

$$b_t(\theta^{t-1}, \theta_t) + \frac{1}{R} A_{t+1}(\theta^{t-1}, \theta_t) < b_t(\theta^{t-1}, \hat{\theta}_t) + \frac{1}{R} A_{t+1}(\theta^{t-1}, \hat{\theta}_t). \tag{31}$$

Let $\theta^t = (\theta^{t-1}, \theta_t)$ and $\hat{\theta}^t = (\theta^{t-1}, \hat{\theta}_t)$. As in the time T proof, the contradiction argument relies on deviations by entrants to guarantee that the MRS holds and on deviations by entrant and incumbent firms to imply that the net present value of transfers is zero.

Under the inductive assumption, the agent faces no distortion on both his intratemporal margin and intertemporal margin (recall Proposition 3) from time $t+1$ onward. This implies that the allocation of the agent, from time $t+1$ onwards, is equivalent to a self-insurance economy (this will be formally proved in Proposition 6). In particular, let $S(x)$ be the utility the agent receives from entering time $t+1$ with a level x of net present value of assets. The value function S is monotonically increasing in the level of assets. Given this, the agents' equilibrium choices at time t satisfy the following:

$$u(b(\hat{\theta}^t) + y(\hat{\theta}^t)) - v\left(\frac{y(\hat{\theta}^t)}{\hat{\theta}_t}\right) + \beta S\left(\frac{1}{R} A_{t+1}(\hat{\theta}^t) - b(\hat{\theta}^t)\right) \geq$$
$$u(b(\theta^t) + y(\theta^t)) - v\left(\frac{y(\theta^t)}{\hat{\theta}_t}\right) + \beta S\left(\frac{1}{R} A_{t+1}(\theta^t) - b(\theta^t)\right), \tag{32}$$

and

$$u(b(\theta^t) + y(\theta^t)) - v\left(\frac{y(\theta^t)}{\theta_t}\right) + \beta S\left(\frac{1}{R}A_{t+1}(\hat{\theta}^t) - b(\hat{\theta}^t)\right) \geq$$
$$u(b(\hat{\theta}^t) + y(\hat{\theta}^t)) - v\left(\frac{y(\hat{\theta}^t)}{\theta_t}\right) + \beta S\left(\frac{1}{R}A_{t+1}(\hat{\theta}^t) - b(\hat{\theta}^t)\right). \tag{33}$$

First consider the case where $y(\theta^t) \geq y(\hat{\theta}^t)$. An entrant can offer the following menu that enables the agent to work additional hours and move resources between time t and time $t+1$:

$$\tilde{C}_t = \left\{\left(b(\theta^t) - b(\hat{\theta}^t), y(\theta^t) - y(\hat{\theta}^t)\right); (0,0)\right\},$$
$$\tilde{C}_{t+1} = \left\{\left(-R[b(\theta^t) - b(\hat{\theta}^t)] - \varepsilon, 0\right)\right\}.$$

This menu generates strictly positive profits to the entrant, proportional to ε, and for the logic applied above cannot be prevented by a latent contract. In addition, if this menu is offered, agent θ^t will deviate, accepting the allocation for $\hat{\theta}^t$ together with the allocation specified in the entrant's menu. This is due to the fact that the agent can now replicate his original time t level of output and have access to a strictly higher net present value of transfers at a cost equal to ε.

Suppose now that $y(\theta^t) < y(\hat{\theta}^t)$. The first case we consider is when consumption at time t is higher for the agent with a higher net present value of transfer, $y(\theta^t) + b(\theta^t) < y(\hat{\theta}^t) + b(\hat{\theta}^t)$. As for the time T argument, inequality (32) cannot hold with equality. This enables us to reduce the time t spread of consumption between histories θ^t and $\hat{\theta}^t$. Following the same steps of time T, a contradiction can be reached.

The final case is $y(\theta^t) < y(\hat{\theta}^t)$ and $y(\theta^t) + b(\theta^t) \geq y(\hat{\theta}^t) + b(\hat{\theta}^t)$. This case violates the intertemporal Euler equation for at least one of the two types, thus contradicting Proposition 3.

To see this, suppose that the Euler equation (9) holds for agent θ^t. We have

$$\begin{aligned} u'(y(\theta^t)+b(\theta^t)) &= \frac{\beta}{q}\sum_{\theta_{t+1}}\pi(\theta_{t+1})u'(c(\theta^{t+1})) \\ \Rightarrow\ u'(y(\hat{\theta}^t)+b(\hat{\theta}^t)) &\geq \frac{\beta}{q}\sum_{\theta_{t+1}}\pi(\theta_{t+1})u'(c(\theta^{t+1})) \\ \Rightarrow u'(y(\hat{\theta}^t)+b(\hat{\theta}^t)) &> \frac{\beta}{q}\sum_{\theta_{t+1}}\pi(\theta_{t+1})u'(c(\hat{\theta}^t,\theta_{t+1})), \end{aligned}$$

where the last implication follows from the fact that an agent with higher transfer will have higher consumption at time $t+1$, thus a lower expected marginal utility of consumption.

To conclude, given that it was shown that the net present value of transfers is independent of the time t choice, we can follow the same steps as in time T to show that equation (30) holds for time t. ∎

From strict concavity of the functions $u(\cdot)$ and $-v(\cdot)$, there is a unique allocation $\{\hat{b},\hat{y}\}$ that satisfies the necessary optimality conditions derived. Let $\{\hat{b},\hat{y}\} = \{(b(\theta^t),y(\theta^t))_{t=1}^T\ \theta^t \in \Theta^t\}$ be the unique solution of optimality conditions (9), (17), and (18). The next proposition determines *strategies* of the firms (menus) that implement this allocation as an equilibrium. This implies that $\{\hat{b},\hat{y}\}$ is the unique equilibrium allocation of the menu game.

Proposition 5. *Allocation $\{\hat{b},\hat{y}\}$ is the unique equilibrium allocation of a menu game.*

Proof. We construct strategies of the firms and the agents that implement allocation $\{\hat{b},\hat{y}\}$ as an equilibrium. Let firm $i \in \{1,2\}$ offer the following menus:

$$\begin{aligned} &\hat{C}_1^i = \left\{(b_1^i,y_1^i) : b_1^i \in \mathbb{R},\ y_1^i \in \mathbb{R}_+ |\ u'(b_1^i+y_1^i) = \frac{1}{\theta}v'\left(\frac{y_1^i}{\theta}\right)\ \forall\theta\in\Theta\right\}, \\ &\hat{C}_T^i(b^{i,T-1},y^{i,T-1}) = \left\{(b_T^i,y_T^i) : b_T^i = 0,\ y_T^i \in \mathbb{R}_+ |\ u'(-Rb_{T-1}^i+y_T^i) = \frac{1}{\theta}v'\left(\frac{y_T^i}{\theta}\right)\ \forall\theta\in\Theta\right\}, \end{aligned}$$

and for all remaining periods

$$\hat{C}_t^i(b^{i,t-1},y^{i,t-1}) = \left\{(b_t^i,y_t^i) : b_t^i \in \mathbb{R},\ y_t^i \in \mathbb{R}_+ |\ u'(-Rb_{t-1}^i+b_t^i+y_t^i) = \frac{1}{\theta}v'\left(\frac{y_t^i}{\theta}\right)\ \forall\theta\in\Theta\right\}.$$

These firms also offer the following latent menus:

Dynamic Contract:

$$C_t^{i,D}(b^{i,t-1}, y^{i,t-1}) = \left\{ (b_t^i, y_t^i) : b_t^i \in \mathbb{R},\ y_t^i = 0 | b_t^i = -\frac{1}{q} b_{t-1}^i + x,\ x \in \mathbb{R} \right\}, \quad b_T^i = b_0^i = 0$$

Static Contract:

$$C_t^{i,S} = \{(0, \delta) : \delta \in \mathbb{R}_+\}$$

Remaining firms $i \in \{3, \ldots, I\}$ offer the null contract. Given these menus, the agents choose at time zero menu $\hat{C}^i$ from one of the two firms. Conditional on this choice, we derive the agents' choices by backward induction. At time T, an agent with history (θ^{T-1}, θ_T) and past choices $(\tilde{b}(\theta^{T-1}), \tilde{y}(\theta^{T-1}))$ chooses from menu $C_T^i(\tilde{b}^i(\theta^{T-1}), \tilde{y}^i(\theta^{T-1}))$ the allocation $(\tilde{b}^i(\theta^T), \tilde{y}^i(\theta^T))$ such that $u'(-R\tilde{b}^i(\theta^{T-1}) + \tilde{y}^i(\theta^T)) = \frac{1}{\theta_T} v'\left(\frac{\tilde{y}^i(\theta^T)}{\theta_T}\right)$. For time $t \in \{1, ..., T-1\}$, an agent with history θ^t and past choices $(\tilde{b}^i(\theta^{t-1}), \tilde{y}^i(\theta^{t-1}))$ chooses from menu $C_t^i(\tilde{b}^i(\theta^{t-1}), \tilde{y}^i(\theta^{t-1}))$ allocation $(\tilde{b}^i(\theta^t), \tilde{y}^i(\theta^t))$ such that

$$u'(-R\tilde{b}^i(\theta^{t-1}) + \tilde{b}^i(\theta^t) + \tilde{y}^i(\theta^t))) = \beta R \mathbb{E}_t \left[u'(-R\tilde{b}^i(\theta^t) + \tilde{b}^i(\theta^{t+1}) + \tilde{y}^i(\theta^{t+1})) \right], \quad (34)$$

$$u'(-R\tilde{b}^i(\theta^{t-1}) + \tilde{b}^i(\theta^t) + \tilde{y}^i(\theta^t))) = \frac{1}{\theta_t} v' \left(\frac{\tilde{y}^i(\theta^t))}{\theta_t} \right). \quad (35)$$

Given the choices of the agents, the profits of firm i are $\sum_{t=1}^T \sum_{\theta^t} q^t \tilde{b}^i(\theta^t) = 0$ so that firm i is indifferent between offering C^i and the null contract. We next show that such strategies can be supported in equilibrium. Note that only offering the menus $\hat{C}^i$, described above, is not an equilibrium of the menu game: both incumbent and entrants (firms currently offering the null contract) will deviate, offering a welfare increasing and profitable deviation in the shape of a contingent contract (the current allocation is the non-redistributive contract that attains the highest level of utility).[17] To prevent deviations, firms $i = 1, 2$ must also offer latent contracts $C^{i,S}$ and $C^{i,D}$. Menu $C^{i,S}$ consists of a static contract that allows agents to work additional hours. We now show that the latent contracts $C^{i,S}$ and $C^{i,D}$ are sufficient

[17] As an example, consider the following profitable deviation (motivated by Abraham and Pavoni (2005)).

to deter any potential deviations by entrants (firms $i \neq 1, 2$) and incumbents.

As a first step, we show that is not profitable for any firm to offer a contract that specifies only intertemporal transfers (without output requirements). To see this, suppose that an entrant j offers a menu C^j containing arbitrary sequences of transfers $\{b_t\}_{t=1}^T$. For each feasible sequence in this menu,[18] define the net present values of transfers: $NPV(\{b_t\}) = \sum_{i=1}^T \frac{1}{R^{t-1}} b_t$. Notice that the menu C^j is, at the same time, chosen by agents and profitable if it contains at least one sequence with $NPV > 0$ and one with $NPV < 0$.[19] Denote the feasible transfer with highest NPV by $\{\tilde{b}_t\}_{t=1}^T$. We now show that, in the presence of menu C^D, all agents choose this sequence, implying that the entrant makes negative profits. Suppose, in order to reach a contradiction, that there is some agent with history θ^T that chooses a sequence $\{b_t\}_{t=1}^T \in C^j$. This agent is better-off choosing sequence $\{\tilde{b}_t\}_{t=1}^T$ and the following strategy in the menu C^D: $\delta_t(\theta^t) = b_t - \tilde{b}_t$. This strategy enables the agent to replicate his original allocation and have extra resources, since the net present value of δ is negative:

$$\delta_0 = \sum_{t=1}^{T} \frac{1}{R^{t-1}} b_t - \sum_{t=1}^{T} \frac{1}{R^{t-1}} \tilde{b}_t < 0$$

Let $\{\tilde{b}(b^{-1}), \tilde{y}(b^{-1})\}$ be the solution to the following problem:

$$\tilde{U}(b^{-1}) = \max_{b,y} \sum_{\theta} \pi(\theta) \left[u(b(\theta) + y(\theta)) - \frac{1}{\theta} v\left(\frac{y(\theta)}{\theta}\right)\right], \tag{36}$$

$$s.t. \quad u(b(\theta) + y(\theta)) - \frac{1}{\theta} v\left(\frac{y(\theta)}{\theta}\right) \geq u(b(\hat{\theta}) + y(\hat{\theta})) - \frac{1}{\theta} v\left(\frac{y(\hat{\theta})}{\theta}\right),$$

$$\sum_{\theta} \pi(\theta) b(\theta) = b^{-1}.$$

Note that $\tilde{U}(b^{-1})$ is strictly larger than the utility of autarky with b^{-1} additional (possibly negative) resources. The firm can deviate from the set of menus C^i defined above by substituting the $T-1$ menu with the original time T menu, and by replacing the time T menu with

$$\tilde{C}^T(b^{i,T-1}, y^{i,T-1}) = \left\{ \{\tilde{b}(b^i_{T-1} - \varepsilon), \tilde{y}(b^i_{T-1} - \varepsilon)\} | \ \ solves \ \ (36). \ \ \varepsilon > 0 \right\},$$

For a sufficiently small ε, the agent prefers this contract to the original, and in addition, this deviation provides additional $\frac{\varepsilon}{R^{T-1}}$ profits.

[18] A sequence $\{b_t\}_{t=1}^T$ is feasible if $b_t \in C_t^j(b^{t-1}) \forall b_t, t$.

[19] If all sequences have NPV=0, the menu is not chosen, since the equilibrium allocation $\{\hat{b}, \hat{y}\}$ is the allocation that maximizes agents' welfare with no redistribution. Similarly, if all transfers are negative, the menu is also not chosen, while if all transfer have $NPV > 0$ the firm makes a loss.

since $\{\tilde{b}_t\}_{t=1}^T$ has highest NPV. These additional resources can be used to increase consumption in the last period, making the agent better-off.

The next step is to rule out contracts that offer jointly consumption transfers and output requirements. We do that showing that any contract that does not incur losses to the entrant, makes the agent strictly worse-off than the self-insurance contract. A contract with labor requirements and transfer can redistribute resources from productive to unproductive agents. In the last period such a redistribution cannot be implemented, since, as shown in the proof of proposition 4, the productive agents will choose the allocation designed for the unproductive agents and work additional hours using any of the latent contracts $C^{i,S}$. In appendix B we show that any contract that implies redistribution for unproductive to more productive agents reduces agents' welfare with respect to self-insurance. Thus, in the last period, a firm can either provide a contract with no redistribution or decrease welfare of the agents.[20]

For the dynamic case we focus on a two period example with two productivity types. If the firm provides negative redistribution at time 1, given appendix B, the contract will not be chosen at time zero. The remaining alternative is to provide, at time 1, some redistribution from the productive to the unproductive agent. To do this, a firm must offer transfers with higher net present value together with higher output requirement. If not, the productive agent deviates, using both $C^{i,D}$ and $C^{i,S}$, replicating his original allocation and receiving transfers with higher NPV. Suppose now that at time 1, the θ_H agent receives transfers equal to $b_1 - \Delta$ while θ_L agent receives $b_1 + \Delta$ (with $\Delta > 0$). The best case for both agents is to receive transfers at time 2 that does not depend on the realization of the type in that period. Thus we can write transfers for the high type as $b_{2,H}$ and for the low type $b_{2,L}$. These transfers are such that

$$b_1 - \Delta + \frac{1}{R} b_{2,H} > b_1 + \Delta + \frac{1}{R} b_{2,L}.$$

This implies that a lower rate of return is charged to low productivity agents relative to high

[20] In a static environment this completes the proof since it rules out the existence of a contract that is, at the same time, profitable and preferred by the agents.

productivity agents. Since the low agent has lower consumption, this interest rate differential is welfare decreasing. Hence the benefits to the high agent are offset by the utility loss of the low agent. This implies that, from an ex-ante perspective, the agent is better-off choosing the self-insurance equilibrium, which implies optimal amount worked without differential interest rate.

Finally for $\{\hat{b}, \hat{y}\}$ to be sustained as an equilibrium allocation, at least two firms must offer the equilibrium and the latent contracts. If not, the unique firm active in equilibrium will re-optimize, and offer a contract that implies some redistribution (as in footnote 17, for example) since no latent contract is preventing such deviation. ∎

Summarizing, the allocation $\{\hat{b}, \hat{y}\}$ can be sustained in equilibrium by at least two incumbents simultaneously offering the menu $\hat{C}^i$ and the latent contracts $C^{i,S}$, and $C^{i,D}$. This is necessary to prevent deviations by any firm, to a more profitable and ex ante welfare improving contract that features redistribution. This result highlights the importance of allowing firms to offer latent contracts. If offering such contracts were not allowed, as in direct mechanisms, equilibrium would fail to exist for this environment.

3.3 Equivalence to Self-Insurance

The previous propositions show that in equilibrium a standard Euler equation holds, the marginal rate of substitution between consumption and leisure is equated to marginal productivity in every period, and the net present value of transfers received under any history is equal to zero (there is no redistribution). These equilibrium conditions are the same optimality conditions in a decentralized economy in which agents can borrow and save at rate $R = 1/q$.

Let $\{c^*, y^*\} = \{c^*(\theta^t), y^*(\theta^t)\}_{t=1}^T$ be the solution to the following problem:

$$\max_{c,y} \sum_{t=1}^{T} \sum_{\theta^t} \beta^{t-1} \pi\left(\theta^t\right) \left[u\left(c(\theta^t)\right) - v\left(\frac{y(\theta^t)}{\theta_t}\right) \right] \tag{37}$$

$$s.t. \qquad \sum_{t=1}^{T} \frac{c(\theta^t) - y(\theta^t)}{R^{1-t}} = 0, \quad \forall \theta^T,$$

where R is taken as given.

Proposition 6. *Let $\{\hat{b}, \hat{y}\} = \{\hat{b}(\theta^t), \hat{y}(\theta^t)\}_{t=1}^T$ be the equilibrium allocation of a menu game. Let the agents' consumption be $\hat{c}(\theta^t) = \hat{b}(\theta^t) + \hat{y}(\theta^t)\ \forall \theta^t,\ \forall t$. If $R = 1/q$, $c^*(\theta^t) = \hat{c}(\theta^t)$ and $y^*(\theta^t) = \hat{y}(\theta^t)\ \forall \theta^t,\ \forall t$.*

Proof. The first order conditions of (37) are:

$$u'(c(\theta^t)) = \beta R \sum_{\theta_{t+1}} u'(c(\theta^{t+1}))\pi(\theta_{t+1}), \tag{38}$$

$$u'(c(\theta^t)) = \frac{1}{\theta_t} v'\left(\frac{y(\theta^t)}{\theta^t}\right), \tag{39}$$

$$\sum_{t=1}^{T} \frac{c(\theta^t) - y(\theta^t)}{R^{1-t}} = 0, \quad \forall \theta^T. \tag{40}$$

The maximization problem (37) has a strictly concave objective function and the constraint set is convex; hence, the first order conditions are necessary and sufficient for the optimum and the optimum is unique. ∎

The previous proposition summarizes how non-exclusivity and non-observability of contracts limits the ability to redistribute resources and also the type of contracts that can be offered in equilibrium. Firms only offer linear contracts for both labor and credit relationships. As a result, the original environment with firms interacting strategically and being allowed to offer any type of contracts, in equilibrium, is equivalent to an environment with competitive firms offering linear contracts.

4 Quantitative Analysis

In this section, we relax the assumption on observability of the contracts. As in a costly state verification model, we allow firms the option of paying a fixed cost, $\gamma \geq 0$, to monitor all the transactions an agent engages in.[21] We assume that agents are heterogeneous with respect to the probability distribution of the productivity shock. There are two types of agents: the first draws, at every time t, a shock $\theta_t \in \Theta$, while the second draws a shock $\lambda\theta_t \in \Theta$, for some $\lambda > 1$. We show considering this heterogeneity implies that agents will have different insurance possibilities. Using survey data from the US economy, we show that this extension can rationalize the coexistence of multiple insurance regimes in the data.

4.1 Monitoring Costs

At time 0 (and only at time 0), before offering a set of contracts to an agent of type $j \in \{1, 2\}$, each firm chooses between the following two options: pay a cost γ to observe all the contracts the agent engages in, and choose which contract to offer under full observability; or not pay the cost and offer the most profitable contract under non-exclusivity. Agents are heterogeneous with respect to the probability distribution of the productivity shock. Agents of type 1 draws, at every time t, a shock $\theta_t \in \Theta$, while agents of type 2 draw a shock $\lambda\theta_t \in \Theta$, for some $\lambda > 1$. This information is publicly available to all the firms.

If a firm monitors an agent, as shown by Prescott and Townsend (1984), the contract offered implements the constrained efficient allocation, which is the solution of problem (41) below:

$$V(w_0) = \max_{c,y} \sum_{\theta^t,t} q^t \pi(\theta^t)[y(\theta^t) - c(\theta^t)] \tag{41}$$

[21] Note that costly state verification models as in Townsend (1979) allow, upon paying the cost, the realization of uncertainty to be observable. Here we keep the realization of uncertainty private but allow the contracts of the agent to be observable.

$$\sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u(c(\theta^t)) - v\left(\frac{y(\theta^t)}{\theta_t}\right)\right] = w_0$$

$$\sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u(c(\theta^t)) - v\left(\frac{y(\theta^t)}{\theta_t}\right)\right] \geq \sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u(c(\tilde{\theta}^t)) - v\left(\frac{y(\tilde{\theta}^t)}{\theta_t}\right)\right] \quad \forall \tilde{\theta}^t$$

A necessary optimality condition in this environment is the inverse Euler equation:

$$\frac{1}{u'\left(c_t\left(\theta^t\right)\right)}\frac{\beta}{q} = E_t\left[\frac{1}{u'\left(c_{t+1}\left(\theta^{t+1}\right)\right)}\right]. \tag{42}$$

If the monitoring cost is not paid, the environment is the one studied in this paper. From Proposition 6 this environment implements the same allocation as an economy in which agents can borrow and save at fixed rate R and are paid wages equal to marginal productivity. This implies it is the solution of the following problem:

$$\Pi(w_0) = \max_{b,c,y} \sum_{\theta^t,t} q^t \pi(\theta^t)[y(\theta^t) - c(\theta^t)] \tag{43}$$

$$\sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u(c(\theta^t)) - v\left(\frac{y(\theta^t)}{\theta_t}\right)\right] = w_0$$

$$c(\theta^t) + b(\theta^t) = y(\theta^t) + Rb(\theta^{t-1}), \quad \forall\ \theta^t, t$$

$$b(\theta^T) = 0, \quad \forall\ \theta^T$$

In this case, the following optimality condition must hold:

$$u'\left(c_t\left(\theta^t\right)\right) = \frac{\beta}{q}E_t\left[u'\left(c_{t+1}\left(\theta^{t+1}\right)\right)\right]. \tag{44}$$

We assume agents have an outside option that delivers an initial promised utility equal to w_{aut}.[22] For each agent, competition between firms determines lifetime level of utility

[22] Define $\bar{w}^{aut}$ as:

$$\bar{w}^{aut} = \max_{y} \sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u(y(\theta^t)) - v\left(\frac{y(\theta^t)}{\theta_t}\right)\right].$$

delivered to the agent so that profits are zero. Define the initial utility level that delivers zero profits in problem (43) by $\bar{w}^{BS}$, such that $\Pi(\bar{w}^{BS}) = 0$. In order to determine what kind of contract an agent will have access to, we determine, for a given value of monitoring cost γ, the utility level each type of agent would receive if the cost is paid and if the cost is not paid. This implies that a firm finds profitable to pay the cost and offer the exclusive contract if agent's utility is higher in this case. If firms do not find it profitable to pay the cost, an agent will receive utility $\bar{w}^{BS}$. We show, under a particular assumption on the utility function, that agents that draw the shock from Θ distribution have access to the linear contracts, receiving a lower level of lifetime utility, whereas agents that draw from $\lambda\Theta$ distribution have access to the exclusive contracts, achieving a higher level of lifetime utility.

The next lemma shows that if $\gamma = 0$, the exclusive contract is always preferred over the linear contracts, since the level of utility under the first is higher because is cheaper to provide a given level of lifetime utility under the constrained efficient allocation.

Lemma 2. *For all feasible utility levels w, $V(w) > \Pi(w)$.*[23]

Proof. In appendix C. ∎

Assumption 1. $u(c,l) = \log c + a \log l$.

Define $\bar{w}^{BS}(\theta)$ as follows:

$$\bar{w}^{BS}(\theta) = \max_{c,y} \sum_{\theta^t, t} \beta^{t-1} \pi(\theta^t) \left[u\left(c(\theta^t)\right) + v\left(1 - \frac{y(\theta^t)}{\theta_t}\right)\right] \tag{45}$$

$$\sum_t \left[\frac{c(\theta^t) - y(\theta^t)}{R^{1-t}}\right] = 0, \quad \forall \theta^T.$$

[23] We restrict the set of feasible initial utility levels to the open interval $\left(\frac{1-\beta^{T+1}}{1-\beta}\underline{U}, \frac{1-\beta^{T+1}}{1-\beta}\bar{U}\right)$, where $\underline{U} = \inf_{c,l\geq 0} u(c) - v(l)$ and $\bar{U} = \sup_{c,l\geq} u(c) - v(l)$.

Define $w^M(\theta,\gamma)$ as follows:

$$w^M(\theta,\gamma) = \max_{c,y} \sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u\left(c(\theta^t)\right) + v\left(1 - \frac{y(\theta^t)}{\theta_t}\right)\right] \tag{46}$$

$$\sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u\left(c(\theta^t)\right) + v\left(1 - \frac{y(\theta^t)}{\theta_t}\right)\right] \geq \sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u\left(c(\tilde{\theta}^t)\right) + v\left(1 - \frac{y(\tilde{\theta}^t)}{\theta_t}\right)\right] \quad \forall\tilde{\theta}^t$$

$$\sum_t \left[\frac{c(\theta^t) - y(\theta^t)}{R^{1-t}}\right] = \gamma, \quad \forall\theta^T. \tag{47}$$

Proposition 7. *There exists $\gamma^* > 0$ such that:*

$$\begin{aligned} \bar{w}^{BS}(\theta) &= w^M(\theta,\gamma^*) \\ \bar{w}^{BS}(\lambda\theta) &< w^M(\lambda\theta,\gamma^*) \end{aligned}$$

Proof. In appendix C. ∎

The steps to show the result are the following. We first show that, under linear contracts, the indirect lifetime utility of agents with shock $\lambda\theta$ is proportional to the indirect lifetime utility of agents with shock θ (by a factor proportional to λ). The assumption on utility function is important to show this result. Second, we show that, under the constrained efficient allocation, the indirect utility is scaled by a bigger factor. This implies that, for a given λ, there is a value of the monitoring cost so that the firms can promise a higher lifetime utility under the constrained efficient contract than under the linear contracts. The same result can also be proved if $u(c,l) = \frac{\left(c^\alpha l^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}$. The general CRRA case, with $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} + a\frac{l^{1-\sigma_l}}{1-\sigma_l}$, is verified numerically.[24]

The environment described in this section, given agents' heterogeneity in the distribution of the shocks, endogenously generates a partition of the population into groups with different

[24] Another way to endogenously divide the population in two different insurance regimes is to assume agents are heterogenous with respect to the monitoring cost γ. In this case, we show that there exists a cutoff value γ^* such that if agents have cost γ, with $0 < \gamma \leq \gamma^*$, they have access to the exclusive contracts and receive lifetime utility $w(\gamma)$. If agents have cost γ, with $\gamma > \gamma^*$, they will have access to the linear contracts and receive lifetime utility equal to $\bar{w}^{BS}$.

lifetime utility. Agents with lower average productivity receive a lower level of promised utility, $\bar{w}^{BS}$, and their allocation solves the linear contracts described in previous sections. On the other hand, agents with higher average productivity receive a high level of expected future utility and have access to the contract that solves the constrained efficient problem. In the next subsection, we use US household survey data to show that this factorization implied by the model is a feature observed in the data.

4.2 Quantitative Implications

We use US household survey data to estimate two testable implications of this model dividing the population by the education level of the reference person.

Data

We use data from the Consumer Expenditure Survey (CEX) and divide the population by the education level of the reference person. In order to abstract from college and retirement decisions, we constrain our sample to households with the reference person age is between 25 and 55.[25] We only consider reference person who worked more than 520 hours and less than 5096 hours per year and with positive labor income. We exclude households with wage less than half of the minimum wage in any given year. Table 9 (in appendix D) describes the number of households in each stage of the sample selection. All the nominal data are deflated using the consumer price index calculated by the Bureau of Labor Statistics with base 1982-84=100.[26]

In Table 11 (in appendix D) we present some descriptive statistics of the sample considered. All the earnings variables and hours refer to the reference person, while the expenditure variables are total household expenditure per adult equivalent.[27]

The consumption data is from the Krueger and Perri CEX dataset for the period 1980 to 2003. Our baseline sample is limited to households who responded to all four interviews

[25]By stopping at age 55 we also minimize the disconnection between consumption expenditure and actual consumption (due to the progressive larger use of leisure in both preparation and shopping time) highlighted in Aguiar and Hurst (2005).

[26]For a more detailed description of the data and sample selection, refer to Ales and Maziero (2008).

[27]We use the Census definition of adult equivalence.

and with no missing consumption data. The consumption measure used includes the sum of expenditures on nondurable consumption goods, services, and small durable goods, plus the imputed services from housing and vehicles, as calculate by Krueger and Perri (2006).

In order to test the implications of the model, we divide the population into two groups, according to the education level of the reference person: those with less than a college degree and those who completed college or more. We estimate for each of the groups two implications of the model: an intertemporal optimality condition and an intratemporal condition. According to our model, agents with higher productivity (college graduates) satisfy the optimality conditions of the constrained efficient contracts, while agents with lower average productivity (high school graduates) satisfy behave according to the linear contracts.

Intertemporal Optimality Conditions

The first optimality condition we estimate is the intertemporal optimality conditions (42) and (44) derived in the models described in the previous section. Assuming $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, these equations imply, respectively:

$$c_t(\theta^t)^{\sigma}\frac{\beta}{q} = E_t\left[c_{t+1}(\theta^{t+1})^{\sigma}\right], \tag{48}$$

$$c_t(\theta^t)^{-\sigma} = \frac{\beta}{q}E_t\left[c_{t+1}(\theta^{t+1})^{-\sigma}\right]. \tag{49}$$

These two equations can be nested in the following equation:

$$c_t(\theta^t)^b\left(\frac{\beta}{q}\right)^{\frac{b}{|b|}} = E_t\left[c_{t+1}(\theta^{t+1})^b\right]. \tag{50}$$

If agents' decisions satisfy the inverse Euler equation (48), then $b > 0$, whereas if they satisfy a standard Euler equation (49), then $b < 0$. Taking expectation of (50) at time t, we get

$$\sum_{\theta^{t+1}}\pi(\theta^{t+1})\left[c_t(\theta^t)^b\left(\frac{\beta}{q}\right)^{\frac{b}{|b|}} - c_{t+1}(\theta^{t+1})^b\right] = 0, \quad \forall\theta^t. \tag{51}$$

In order to test whether the intertemporal decision consumption of households, for a given education group, is compatible with the constrained efficient allocation or with the linear contracts, we estimating (51) for each group. If for an education group the value of b is negative, the consumption of these agents is compatible with the predictions of (43). If the estimation of b is positive, it implies that agents' consumption satisfies the implications of (41). Our theory predicts that more educated individuals have positive b. The analysis closely follows Ligon (1998) and Kocherlakota and Pistaferri (2008).[28]

Estimation Procedure

A typical household is on the sample for a total period of four quarters. For the estimation, we construct sample averages as follows. Denote by $c_{i,t}$ the consumption for household i in the quarter that ends with month t, and let N_t be the number of observations available at time t. We de-seasonalize consumption with dummies corresponding to the month the household was interviewed. The sample analog of equation (51) is:

$$g\left(b\right)=\frac{1}{T}\sum_{t=3}^{T}\left[\left(\frac{\beta}{q}\right)^{\frac{b}{|b|}}\frac{1}{N_{t-3}}\sum_{i=1}^{N_{t-3}}c_{i,t-3}^{b}-\frac{1}{N_t}\sum_{i=1}^{N_t}c_{i,t}^{b}\right]. \tag{52}$$

As shown by Kocherlakota and Pistaferri (2008), this sample analog is still valid in the presence of multiplicative classical measurement error in the consumption data.

The main disadvantage of this sample analog is that, by taking means over the population, we do not take into account the individual changes on consumption over time. An alternative valid sample analog is the following:

$$\tilde{g}\left(b\right)=\frac{1}{T}\sum_{t=3}^{T}\left[\left(\frac{\beta}{q}\right)^{\frac{b}{|b|}}\frac{1}{N_t}\sum_{i=1}^{N_t}\left(\frac{c_{i,t-3}}{c_{i,t}}\right)^{b}\right] \tag{53}$$

where in this equation N_t is the total number of households with consumption data for time t and $t-3$. The estimation of this equation, in the presence of multiplicative classical

[28]In particular, Ligon (1998) tests whether the standard Euler or inverse Euler condition better describes the consumption behavior for three Indian villages. His results indicate that two out of three village provides evidence for the inverse Euler equation.

	Education Group		
Consumption	**Less than College**	**College**	**All Sample**
Baseline de-seasonalized	-1.128	**0.855**	-1.030
	(0.459)	**(0.716)**	(0.728)
Baseline truncated*	-1.128	**0.864**	-1.030
	(0.458)	**(0.726)**	(0.728)

Table 1: Estimation results for risk aversion from (51). A positive solution denotes the coefficient of risk aversion consistent with the household's decision under the constrained efficient contract, while a negative solution denotes the estimated risk aversion consistent with the group being under a borrowing and saving contract. *We drop households with consumption changes bigger than 5 times in absolute value.

measurement error in consumption, implies inconsistent estimation of the parameter β. A standard approach in the literature is to estimate the log-linearized version of this sample analog. Simple algebra shows that the log-linearized versions of equations (42) and (44) result in the same equation. This implies that this procedure cannot be used to test whether the consumption of a group of household satisfy (42) or (44).

Results

In table 1 we report the estimation of parameter b for the two education groups, assuming $\frac{\beta}{q} = 1$. We estimate (52) using non-linear generalized method of moments. We find that college graduates have $b = 0.855$, which is consistent with the constrained efficient allocation. Individuals with education less than college have $b = -1.128$, which corresponds to consumption evolving as predicted by the incomplete markets model. As a robustness check, we perform the same estimation by dividing the population into four education groups: those with less than a high school education, those who completed high school, those with some college, and those who completed college or more. The results are reported in table 2 and are consistent with the previous one: individuals with education less than college have a negative value of b, while for college graduates, this value is positive.

Another robustness check performed is to estimate the moment condition not only for

	Education Group			
Consumption	**Less than HS**	**HS**	**Less than College**	**College**
Baseline de-seasonalized	-0.773	-1.346	-0.962	**0.855**
	(0.355)	(0.415)	(0.865)	**(0.716)**
Baseline truncated	-0.774	-1.348	-0.962	**0.864**
	(0.355)	(0.414)	(0.864)	**(0.726)**

Table 2: Estimation results for risk aversion from (51).

	Education Group		
Consumption	**Less than College**	**College**	**All Sample**
Baseline de-seasonalized	-0.970	**1.157**	-0.898
	(0.206)	**(0.467)**	(0.341)
Baseline truncated	-1.00	**1.155**	-0.904
	(0.210)	**(0.486)**	(0.360)

Table 3: Estimation results for risk aversion from (51) with households who answered at least one interview.

the households who have answered the four interviews, but for all the households that have answered at least one of the interviews. The results are reported on table 3 and they are consistent with the results for the baseline sample. For both groups the estimation of the coefficient of risk aversion is bigger than in the benchmark case and the standard errors are smaller. In this case, both estimates are statistically different than zero.

We also perform our benchmark estimation by using the previous period interest rate as an instrument. The results are displayed in table 4 and is consistent with the benchmark results.

Intratemporal distortions

	Education Group		
Consumption	Less than College	College	All Sample
Baseline instrumented	-0.972	**1.167**	-0.890
	(0.213)	**(0.450)**	(0.365)

Table 4: Estimation results for risk aversion from (51) with using previous period interest rate as instrument.

An additional testable implication of the model discussed focuses on the intratemporal distortion of individuals over time. Consider the following labor distortion, for an individuals j:

$$\tau_{cl}^{j}(t) = 1 - \frac{1-\alpha}{\alpha} \frac{1}{w_t^j} \frac{c_t^j}{L - l_t^j}, \tag{54}$$

where w_t^j is the imputed hourly wage, c_t^j is the total consumption expenditure, l_t^j is the yearly hours worked, and L is the feasible amount of yearly working hours, set at $5,200$. Equation (54) represents the intratemporal distortion on the consumption-leisure margin faced by a household with a Cobb-Douglas utility function, where α is the share of consumption in the utility function. The advantage of using this utility specification is that the intratemporal distortion is not affected by the risk aversion parameter and α (if constant) does not affect the behavior of $\tau_{cl}(t)$ over time.

As shown in Ales and Maziero (2008), in a constrained efficient environment an individual faces an *increasing* average value of τ_{cl} over the course of his working life. On the other hand, if the individual behavior is characterized by the linear contracts, $\tau_{cl}(t)$ is constant over time. This difference between environments provides another testable implication for our environment with monitoring cost. Our model implies a low and constant τ_{cl} for less educated individuals and an increasing τ_{cl} for college graduates.

In order to estimate the dependence of τ_{cl} on age, we regress its value on age. We run the regression on the standardized values of all variables.[29] We calculate the labor distortion

[29]Precisely: $\tau_{cl}^{j}(age) = \bar{\tau}_{cl} + \beta * age + \varepsilon_{age}^{j}$.

	Education Group	
β	**Less than College**	**College**
OLS	0.011 (0.008)	**0.016 (0.009)**
Robust Regression	-0.0018 (0.005)	**0.021(0.005)**
t-statistic	-0.36	3.93

Table 5: Intra temporal distortion by age and education group.

as follows. We calculate the imputed hourly wage by dividing the total labor income by the total number of hours in a year. To abstract from changes in family composition, we restrict the sample to individuals who are single.[30] The results of this estimation are displayed in table 5. The coefficient on age for the entire sample is positive, as highlighted in Ales and Maziero (2008). We divide the sample into two education groups and estimate β using a robust regression to control for heteroscedasticity and outliers. We find that a zero coefficient (implying independence over age) cannot be rejected for individuals with education less than college, whereas higher education groups return a positive, and significant, point estimate.

We also estimate (54) assumption utility function is separable on consumption and leisure and it has constant relative risk aversion in both arguments. Table 6 shows the results assuming the coefficient of risk aversion is the one estimated in the previous section.[31] The result is the same as in the non-separable case: less educated individuals have the coefficient on age not significantly different than zero, while more educated individuals have this coefficient positive. As a robustness check, we also calculate the labor distortion for married individuals. In this case, we consider as measure of consumption the total household consumption. For the labor variables, we assume that leisure for the husband and the wife is perfect substitute and we use total household earnings and hours to compute the distortion. Due to data limitation,[32] we restrict the sample to households with two adults, the reference

[30] In our baseline sample the single individuals represent 18% of the population.

[31] We also estimate the equation for different values of risk version, within the range estimated in the literature, and the result still holds.

[32] The CEX records hours and earnings for the reference person and the spouse.

β	**Education Group**	
	Less than College	**College**
OLS	0.011 (0.008)	**0.016 (0.009)**
Robust Regression	-0.0078 (0.005)	**0.05(0.006)**
t-statistic	-1.44	7.28

Table 6: Intra temporal distortion by age and education group.

β	**Education Group**	
	Less than College	**College**
OLS	-0.0007 (0.0003)	**0.0003 (0.0005)**
Robust Regression	-0.0004 (0.0002)	**0.0011 (0.0003)**
t-statistic	-1.76	3.59

Table 7: Intra temporal distortion by age and education group.

person and the spouse. For this sample, the results are also consistent with our benchmark estimation, as reported in table 7.

5 Off-Equilibrium Contracts and Entry

In this section, we highlight the features of the studied environment that deliver the previous results on no redistribution under competition. As shown, latent contracts can be used to sustain the equilibrium of the menu game by deterring entries that are otherwise profitable. These entries can be of two kinds: an entrant can offer an additional contract that is accepted together with the equilibrium allocation, or some incumbent can find it profitable to deviate and change the equilibrium contract. The nature of competition in our environment restricts the use of latent contracts to deter profitable deviations of incumbents, but it cannot be used to prevent profitable deviations of entrants when contracts other than linear are offered.

However, as Arnott and Stiglitz (1991) highlight, latent contracts might be used only in particular circumstances. This section follows the intuition provided in Arnott and Stiglitz (1991) and provides a sufficient condition for when latent contracts are used in equilibrium to deter entry.

Condition 1 (**A**). *Consider the period profit function $V_t : \Theta \times X \rightarrow \mathbb{R}$. For any $x \in X$, if there exists $\theta \in \Theta$ so that $V_t(\theta, x) > 0$, then $V_t(\hat{\theta}, x) > 0$ for all $\hat{\theta} \in \Theta$.*

Condition **(A)** is reminiscent of the sufficient condition given by Peters (2003) and similarly in Attar, Piaser, and Porteiro (2007) for moral hazard environments. These papers provide a sufficient condition for latent contracts not being used at all in equilibrium: the action chosen by the agent has to be independent of the contracts offered by the planner. As pointed out in Attar, Piaser, and Porteiro (2007) (footnote 4), no commonly studied environment satisfies such a condition. Condition **(A)** is weaker and, as the next proposition shows, it can be used to characterize environments in which latent contracts cannot be used to deter entry if contracts other than linear are offered.

The key intuition is that if Condition **(A)** holds, incumbent insurance providers cannot induce negative profits by influencing which agents accept the contract offered by entrants; hence, entry cannot be prevented by latent contracts. Denote by $\hat{I}^e \subset I$ the subset of firms that are offering only the null contract (the potential entrants).

Proposition 8. *Consider an environment where Condition **(A)** holds. Let $(\hat{\mathcal{C}}, \hat{C}, \hat{c})$ be an equilibrium. It does not exist an incumbent $i \in I \setminus \hat{I}^e$ and a menu $C^i \in \hat{\mathcal{C}}^i$ such that: if $C^i \notin \hat{C}$, exists a $j \in \hat{I}^e$ and a best reply $\tilde{C}^j$ to $\hat{\mathcal{C}} \setminus C^i$, such that:*

$$V^j(\tilde{C}^j, \hat{\mathcal{C}}^i \setminus C^i) > 0, \tag{55}$$

$$V^j(\tilde{C}^j, \hat{\mathcal{C}}^i) \leq 0. \tag{56}$$

where $\hat{\mathcal{C}}^i \setminus C^i$ denotes the original incumbent's menu without the latent contract C^i.

Proof. Suppose such entrant does exist. First consider the case where the inequality in (55) is strict: the latent contract induces strictly negative profits. This implies that there exists a non-empty subset of $\Theta' \subseteq \Theta$ so that for every $\theta' \in \Theta'$, $V^j(\theta', c^*(\theta') \in \tilde{C}^j) < 0$. Two cases can arise: if $c^*(\theta') \in \tilde{C}^j$ is chosen only by types in Θ', then $\tilde{C}^j$ fails to be a best reply to $\hat{\mathcal{C}}^i \setminus C^i$ being dominated by $\tilde{C}^j \setminus c^*(\theta')$. If there exists $\theta \notin \Theta'$ so that $c^*(\theta')$ is also chosen by θ ($c^*(\theta') = c^*(\theta)$), then we contradict Assumption **(A)**, since there exists an allocation of the contract space that induces profits of opposite signs depending on the type choosing the allocation.

The remaining case occurs when the latent contract delivers zero profits to an entrant: equation (55) holds with equality. Since elements of $\tilde{C}^j$ that lead to negative profits are ruled out ($\Theta' = \emptyset$), one possible alternative is that C^i induces agents to not choose any element of $\tilde{C}^j$ (profits of the null contract are normalized to 0). This implies that for all $\theta \in \Theta$ that $U(\hat{C}^i, \emptyset, \theta) \geq U(\hat{C}^i, \tilde{C}^j, \theta)$, but this contradicts C^i being latent in the original equilibrium ($C^i \in \hat{C}$), since elements in the latent menu would have been chosen by all types in the original equilibrium where there was no entrant. Finally, if C^i induces agents to choose an element x of $\tilde{C}^j$ that induces zero profit from Assumption **(A)** and (55), it must be that $\tilde{C}^j$ contains at least two elements; then if $V^j(\tilde{C}^j \setminus x) > 0$ the original $\tilde{C}^j$ was not a best reply, while if $V^j(\tilde{C}^j \setminus x) = 0$, we proceed as before by removing entries from the menu until the only alternative is that no agent is choosing elements of the entrant's menu. ∎

5.1 Modified Endowment Economy

In this section, we modify the benchmark environment so that Condition (**A**) is not satisfied. We show that although latent contracts can prevent some entries, the linearity and non-contingency results also apply. The environment is modified in the spirit of an adverse selection economy, as in Rothschild and Stiglitz (1976), where the privately observed shock determines the probability of the observable income realization. In this environment, any contract has to specify payoffs for two income realizations (otherwise, it is either never profitable or never chosen by agents). This implies that the profits of the insurance providers depend on which agent accepts the contract.

Consider the following modifications to the baseline environment. Assume labor is supplied inelastically (the environment reduces to an endowment economy) and let T=1. At time zero, an agent has probability p_g of being type G (the good type) and probability p_b of being type B (the bad type), and realization of types occurs at the beginning of time 1. The law of large numbers holds, so that we also denote by p_i the fraction of agents of type i. An agent of type $i = B, G$ receives endowment ω_H with probability π_i and ω_L with probability $1-\pi_i$, with $\omega_H > \omega_L$, and realization of the endowment occurs at the end of time 1. Assume $\pi_G > \pi_B$ and that these probabilities are private information of the agent. The realization of the endowment is publicly observed. The consumer signs insurance contracts with firms. Using the delegation principle, we can restrict the analysis to menu games. In a menu game, each firm offers a menu consisting of collection of transfers pairs, with each element referring to the transfer conditional on the realization of the endowment. A menu is any C_i element of $\mathcal{P}(\mathbb{R}^2)$ (the power set of $\mathbb{R}^2$). The problem of firm i is:

$$\begin{aligned} \Pi(C_i) = & \max_{C_i \in \mathcal{P}(\mathbb{R}^2)} - \sum_{j=B,G} p_j [\pi_j \tau_H^{j*} + (1-\pi_j)\tau_L^{j*}] \\ s.t. \qquad & (\tau_L^{j*}, \tau_H^{j*}) \in C_i \end{aligned} \tag{57}$$

where $(\tau_L^{j*}, \tau_H^{j*})$ denotes the transfer pair chosen by an agent of type j. Let $U^j(C)$ be the utility of the agent of type $j = B, G$ if he accepts menus $C = C_1 \times \ldots \times C_I$:

$$U^j(C) = \max_{(\tau_L, \tau_H) \in C} \left[\pi_j u \left(\omega_H + \sum_{i=1}^{I} \tau_H^i \right) + (1-\pi_j) u \left(\omega_L + \sum_{i=1}^{I} \tau_L^i \right) \right]. \tag{58}$$

Notice that with the modifications introduced in this environment, Condition **(A0** does not hold. To see this, consider any allocation that delivers a zero profit if only G agents accept it: any (c_L, c_H) so that $\pi_G c_H + (1-\pi_G)c_L = 0$. This allocation, if chosen by the B agent, delivers negative profits.

In this environment, two distinct sources of uncertainty can be insured: the private realization of the type and the public realization of the endowment shock. If the private type is perfectly insured, both agents will receive identical contracts. Borrowing from the adverse selection literature, we call this allocation a *pooling* equilibrium. In the characterization we show that no perfect insurance for the private shock can be provided. In addition, in equilibrium only the bad type receives insurance against the public realization of the endowment.

5.1.1 Characterization of Equilibrium

The following lemma provides the necessary conditions for a pooling equilibrium.[33]

Lemma 3. *In any pooling equilibrium $c = (c_L, c_H)$, the following conditions must be satisfied:*

$$\frac{1-\pi_b}{\pi_b}\frac{u'(c_L)}{u'(c_H)} \leq \frac{1-\pi_b}{\pi_b}, \tag{59}$$

$$\frac{1-\pi_g}{\pi_g}\frac{u'(c_L)}{u'(c_H)} \geq \frac{1-\hat{p}_H}{\hat{p}_H}, \tag{60}$$

where $\hat{p}_H = p_g\pi_g + p_b\pi_b$.

The first condition implies that the marginal rate of substitution between the two states for the B agent is less than or equal to the actuarially fair price for the insurance only if B agents accept. If not, entrants can provide some additional insurance at slightly less than that price. The second condition requires that the marginal rate of substitution between states for the G agent is greater than the price for insurance when all agents accept the contract (the actuarially fair pooling price). Otherwise, entrants can profitably provide alternative insurance contracts at a slightly lower price. A direct implication of this lemma is that there is no pooling equilibrium, since there is no allocation that satisfies these two conditions at the same time.

Proposition 9. *There is no pooling equilibrium.*

Proof. Suppose there exists a pooling equilibrium $c = (c_L, c_H)$. This equilibrium has to

[33] All the proofs for this section are provided in Appendix E.

satisfy condition (59) and (60). Using (59) in (60):

$$\frac{1-\pi_g}{\pi_g}\frac{\hat{p}_H}{1-\hat{p}_H} \geq 1 \quad \Rightarrow \quad \frac{1-\pi_g}{\pi_g} \geq \frac{1-\hat{p}_H}{\hat{p}_H},$$

which is a contraction since $\pi_b < \pi_g$. ∎

We now characterize the equilibrium considering different allocations for the two types: $c^B = (c_L^B,\, c_H^B)$, $c^G = (c_L^G, c_H^G)$, denote with $C = \{c^B, c^G\}$. In this case, each agent prefers his own allocation:

$$U^B\left(c^B\right) \geq U^B\left(c^G\right), \qquad U^G\left(c^G\right) \geq U^G\left(c^B\right). \tag{61}$$

The equilibrium must also deliver positive profits: $\Pi(C) \geq 0$. The following lemma characterizes three necessary conditions an equilibrium must satisfy. The first condition is equivalent to (59) in a pooling equilibrium, requiring the equilibrium allocation for the B type to be on the region of over-insurance. The second is that the indifference curve of the G agent at the equilibrium allocation must be less steep than the average zero profit line. The last condition requires that the allocation for the G type must be on the under-insurance region.

Lemma 4. *Any equilibrium must satisfy:*

1. *For the B agent:*
$$\frac{1-\pi_b}{\pi_b}\frac{u'\left(c_L^B\right)}{u'\left(c_H^B\right)} \leq \frac{1-\pi_b}{\pi_b}, \tag{62}$$

2. *For the G agent:*
$$\frac{1-\pi_g}{\pi_g}\frac{u'\left(c_L^G\right)}{u'\left(c_H^G\right)} \leq \frac{1-\hat{p}_H}{\hat{p}_H}, \tag{63}$$

$$\frac{1-\pi_g}{\pi_g}\frac{u'\left(c_L^G\right)}{u'\left(c_H^G\right)} \geq \frac{1-\pi_g}{\pi_g}. \tag{64}$$

Using the above restrictions, we now characterize the equilibrium and show that it is unique. Let $c^B = (\omega^B, \omega^B)$, $c^G = (\omega_L, \omega_H)$, where $\omega_B = \pi_b\omega_H + (1-\pi_b)\omega_L$ is a candidate equilibrium.

This equilibrium provides no insurance to the G agent and the actuarially fair insurance to the B type. Note that this allocation satisfies conditions (61), (62), (64) and delivers zero profits. To satisfy (63), the following parameter restriction is needed:

$$\frac{1-\pi_g}{\pi_g}\frac{u'(\omega_L)}{u'(\omega_H)} \leq \frac{1-\hat{p}_H}{\hat{p}_H}. \tag{65}$$

This condition is satisfied if, for example, π_g is large relative to π_b or if the spread between ω_L and ω_H is sufficiently small. Note that there exists a non-empty set of parameter values for which it holds.[34] Also note that $\pi_g u(\omega_H) + (1-\pi_g)u(\omega_L) \geq u(\omega_B)$.[35]

Proposition 10. *Let $\{\pi_g, \pi_b, \omega_h, \omega_l, u\}$ satisfy condition (65); then an equilibrium of the menu game must satisfy:*

1. $c^B = (\omega^B, \omega^B)$, *where* $\omega_B = \pi_b\omega_H + (1-\pi_b)\omega_L$;
2. $c^G = (\omega_L, \omega_H)$.

5.1.2 Implementation of equilibrium

The following proposition shows that (c^B, c^G) can be implemented in equilibrium and, as in the baseline environment, is necessary to have more than one firm active in equilibrium with each of these firms offering latent contracts.

Proposition 11. *Let $\{\pi_g, \pi_b, \omega_h, \omega_l, u\}$ satisfy condition (65); then there exists an equilibrium of the menu game.*

[34] Note that, as in Rothschild and Stiglitz (1976), equilibrium might fail to exist for this environment.

[35] Suppose that $\pi_g u(\omega_H) + (1-\pi_g)u(\omega_L) < u(\omega_B)$. This implies that in the consumption space (c_l, c_h), the indifference curve for the high type passing through the endowment point (note that in such space, the indifference curves are a one-dimensional function, denoted by $U^g_{aut}(c_l t)$) passes below the point ω_B. Denote by c_{tb} the level of consumption so that $U^{g\prime}_{aut}(c_{tb}) = -\frac{1-\pi_b}{\pi_b}$. Formally, the indifference curve is a function of two variables $U^g(c_h, c_l) = \pi_g u(c_h) + (1-\pi_g)u(c_l)$; we then denote $U^{g\prime}_{aut}(c_{tb}) = \left.\frac{\partial U^g(c_h(c_l), c_l)}{\partial c_l}\right|_{U^g(c_h(c_l), c_l) = U^g(\omega_H, \omega_L)}$. By the contradicting assumption, this point must lie to the right of ω_L, which follows from the fact that at c_{tb} the indifference curve and the zero profit line for the bad type are at the maximum distance, while the indifference curve is below such line. Denote by c_{tm} the value of consumption in the low state so that $U^{g\prime}_{aut}(c_{tm}) = -\frac{1-\hat{p}_H}{\hat{p}_H}$. Since the slope of the indifference curve is decreasing in the consumption of the low state (keeping the level of utility constant), we have that $\omega_L > c_{tb} > c_{tm}$, which contradicts condition (65).

Proof. The following strategies for the firms implement the equilibrium. Let firms $i = 1, 2$ offer the menu: $C_i = \left\{ \left(\frac{\tau_L^B}{2}, \frac{\tau_H^B}{2} \right), (\tau_L^B, \tau_H^B), (0,0) \right\}$, where $\tau_L^B = \pi_b(\omega_H - \omega_L)$ and $\tau_H^B = (1-\pi_b)(\omega_L - \omega_H)$. Let all remaining firms $i \neq 1, 2$ offer the null menu: $C_i = \{(0,0)\}$. The agent's strategy given these menus is: at time zero accept the menu from both firms and at time 1, type B chooses $\left(\frac{\tau_L^B}{2}, \frac{\tau_H^B}{2} \right)$ from firms 1 and 2; type G chooses $(0,0)$ from all firms. In this equilibrium, all firms make zero profits and agents B and G get allocations c^B and c^G, respectively.

There are no profitable deviations by any firm (either entrant or incumbent). Given that there is always at least one firm offering a collection of menus that contains (τ_L^B, τ_H^B), there is no profitable deviation that an entrant or incumbent can make that makes the B agent better off.[36]

The only potentially profitable deviation that can attract G agents is if a firm offers additional insurance as the contract $\tilde{C} = \{(\varepsilon, -\alpha\varepsilon), (0,0)\}$ with α such that $\frac{1-\pi_g}{\pi_g} < \alpha < \frac{1-\pi_g}{\pi_g} \frac{u'(c^L)}{u'(c^H)}$. This deviation increases the utility of G agents and is profitable only if these agents accept it. However, given that the incumbent firms offer (τ_L^B, τ_H^B), if a firm (either one of the incumbents or an entrant) offers contract $\tilde{C}$, the B agent will choose the contract (τ_L^B, τ_H^B) together with $\tilde{c}$, since his indifference curve at c_B is $\frac{1-\pi_b}{\pi_b}$. In this case, this firm will make negative profits; hence, this deviation is not offered. Note that if there is only one active firm, this firm would deviate and offer additional insurance to the G agent as $\tilde{C}$. ∎

6 Conclusion

In this paper, we study a decentralized environment when firms compete for the provision of insurance. We focus on how the presence of non-exclusive trades endogenously limits the contracts offered, and consequently the amount of insurance implemented. We consider an environment in which consumers are privately informed about their skill shocks that evolve over time and can sign non-observable contracts with insurance providers. Our main results

[36] The allocation c^B is the unique solution of maximizing the B agent's utility subject to non-negative profits.

are that competition reduces the amount of insurance provided, the equilibrium is equivalent to a self-insurance economy, and only linear contracts are offered. Also, in equilibrium there is no redistribution.

To derive testable implications of the model, we extend the model and relax the assumption on the observability of contracts: firms can pay a cost to observe all the contracts an agent signs. Assuming agents are heterogeneous with respect to this cost, we find that agents with lower monitoring costs have access to the constrained efficient contract, while agents with higher monitoring costs have access to contracts that implement the self-insurance allocation. This implies that the first group of agents attains a higher level of lifetime utility. Considering education as a proxy for lifetime utility, we test the different intertemporal and intratemporal implications of this model using US data. We find that agents with a high level of education satisfy the optimality conditions of the constrained efficient model while the consumption and hours of agents with less education evolve according to the borrowing-savings economy.

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A Proofs of Section 2

Proof of Proposition 1

Proof. The proof is by construction. Starting from the equilibrium strategies of a general communication game, we construct strategies for a menu game and show that these strategies constitute an equilibrium.

Define as in (2) and (5) respectively the menus and the collection of menus that are compatible with message spaces $(\mathcal{M}, \mathcal{R})$. Define the strategy of firm i in this menu game as:

$$\hat{\mathcal{C}}^i = \{C^i \subseteq C^i \left(G^i\right) | G^i = \mathbf{Im}(\phi^{i,*})\}. \tag{66}$$

The collection of menus $\hat{\mathcal{C}}^i$ contains all the subsets of the allocation space that are consistent with the collection of allocation functions in the original equilibrium. Agents' strategies are defined as follows.

$$\hat{C}^i = \{\hat{C}^i_t \in \hat{\mathcal{C}}^i : \hat{C}^i_t = \mathbf{Im}(g^{i,*}_t | m^{i,t-1,*}) \text{ and } g^{i,*}_t = \phi^{i,*}(r^{i,*})\}$$

$$(\hat{b}^i(\theta^t), \hat{y}^i(\theta^t)) = g^{i,*}_t(m^{i,t,*}(\theta^t)).$$

Note that by construction $\hat{C}^i_t \in \hat{\mathcal{C}}^i$ and $(\hat{b}^i(\theta^t), \hat{y}^i(\theta^t)) \in \hat{C}^i_t$, $\forall \theta^t$, $\forall t$. The menu $\hat{C}^i_t$ is the subset of allocation space, $\mathbb{R}^2$, that corresponds to the allocation function chosen by the agent in the original equilibrium. Also $(\hat{b}^i, \hat{y}^i)$ corresponds to allocation determined by the allocation function given the equilibrium message sent by each type θ^t. If agents and firms follow these strategies, the equilibrium allocation in the menu game is the same as in the original equilibrium.

First let's show that the agents' strategies are an equilibrium. Suppose that at some time t, for some firm i $\exists$ $(b^i_t, y^i_t) \in \hat{C}^i_t$ such that:

$$u\left(\sum_{i=1}^{I}(b^i_t + y^i_t)\right) - v\left(\frac{\sum_{i=1}^{I} y^i_t}{\theta_t}\right) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(\hat{b}^{t-1}, b_t, \hat{y}^{t-1}, y_t, \theta_{t+1} | \hat{C}\right) >$$
$$u\left(\sum_{i=1}^{I}(\hat{b}^i_t + \hat{y}^i_t)\right) - v\left(\frac{\sum_{i=1}^{I} \hat{y}^i_t}{\theta_t}\right) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(\hat{b}^t, \hat{y}^t, \theta_{t+1} | \hat{C}\right).$$

Since $(b^i_t, y^i_t) \in \hat{C}^i_t$, there exists $m^i_t \in \mathcal{M}^i_t$ such that $(b^i_t, y^i_t) = g^{i,*}_t(m^{i,t})$. Replacing in the agents' payoff:

$$u\left(\sum_{i=1}^{I}\left(b^i(m^{i,t}) + y(m^{i,t})\right)\right) - v\left(\frac{\sum_{i=1}^{I} y(m^{i,t})}{\theta_t}\right) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(m^t, \theta_{t+1} | g^*\right) >$$
$$u\left(\sum_{i=1}^{I}\left(b^i(m^{i,t,*}) + y(m^{i,t,*})\right)\right) - v\left(\frac{\sum_{i=1}^{I} y(m^{i,t,*})}{\theta_t}\right) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(m^{t,*}, \theta_{t+1} | g^*\right).$$

But this contradicts $m^{i,*}$ being an equilibrium in the original game. Now suppose $\hat{C}^i$ is not an equilibrium for some i. There exists some $C^i \in \hat{\mathcal{C}}^i$ such that:

$$U(C^i, \hat{C}_{-i}) > U(\hat{C}).$$

Since $C^i \in \hat{\mathcal{C}}^i$, $\exists\ r^i \in \mathcal{R}^i$ such that $C^i = \mathbf{Im}(g^i)$ and $g^i = \phi^{i,*}(r^i)$. Replacing in the agents' payoff:

$$U\left(g^i, g^*_{-i},\right) > U\left(g^{i,*}, g^*_{-i}\right).$$

But this contradicts $r^{i,*}$ being an equilibrium in the original game.

Finally, we check that firms' strategies constitute an equilibrium. Suppose $\exists\ \mathcal{C}^i \in \mathcal{C}^i(\mathcal{R}^i, \mathcal{M}^i)$ such that $V^i(\mathcal{C}^i, \hat{\mathcal{C}}^{-i}) > V^i(\hat{\mathcal{C}}^i, \hat{\mathcal{C}}^{-i})$.

Since $\mathcal{C}^i \in \mathcal{C}^i(\mathcal{R}^i, \mathcal{M}^i)$, there exists ϕ^i such that $g^i = \phi^i(r^{i,*})$. Replacing in the firm's payoff in the original game $V^i(\phi^i, \phi^*_{-i}) > V^i(\phi^{i,*}, \phi^*_{-i})$. But this contradicts $\phi^{i,*}$ being an equilibrium in the original game. ∎

Proof of Proposition 2

Proof. We show the equivalence by construction. For a given firm i, by assumption there exists at least one $C^i \in \mathcal{C}^i$ which is not minimal. As notation let $C^i \times C^{-i} = C^1 \times C^i ... \times C^N$, and let $U(C^i, C^{-i})$ be the lifetime utility of a sequence of menus C as defined in equilibrium. Define the set

$$P\left(C^i\right) = \{C^{-i} \in \mathcal{C}^i \mid U(C^i, C^{-i}) \geq U(\tilde{C}^i, C^{-i}) \quad \forall \tilde{C}^i \in \mathcal{C}^i\}. \tag{67}$$

The set $P\left(C^i\right)$ contains all the menus C^{-i} offered by other firms $-i$ that resulted in C^i being chosen from firm i. Note that if C^i is the unique element of $\mathcal{C}^i$ the set $P\left(C^i\right) = \mathcal{C}^i$.
For each $C^{-i} \in P\left(C^i\right)$, construct the following sequence of menus:

$$\tilde{C}^i_t\left(C^{-i}|C^i_t\right) \equiv \left\{\left(b^{i,*}_t\left(\theta^t, C^i, C^{-i}\right), y^{i,*}_t\left(\theta^t, C^i, C^{-i}\right)\right) \in C^t_i, \quad \forall\ \theta_t \in \Theta\right\}, \quad \forall\ C^{-i}_t \in C^i, \forall t. \tag{68}$$

Each set $\tilde{C}^i_t\left(C^{-i}|C^i_t\right)$ contains the actual equilibrium choices of each type of agent and is a minimal menu. Let $\tilde{C}^i(C^{-i}) \equiv \{\tilde{C}^i_t\left(C^{-i}|C^i_t\right) \forall\ C^{-i}_t \in C^i, \forall t\}$. Finally let $\tilde{C}^i = \{\tilde{C}^i\left(C^{-i}\right)\ \forall\ C^{-i} \in P\left(C^i\right)\}$. We now replace the menu $C^i \in \mathcal{C}^i$ by $\tilde{C}^i$ and show that the equilibrium is the same. Let $\tilde{\mathcal{C}}^i = \{(\mathcal{C}^i \backslash C^i), \tilde{C}^i\}$. We prove the statement in two steps. We first show that each element of $\tilde{C}^i$ is chosen by the agent if and only if C^i was chosen in the original equilibrium. We then show that $\tilde{\mathcal{C}} = \{\tilde{\mathcal{C}}^i, \mathcal{C}^{-i}\}$ is an equilibrium of the menu game by showing that none of the firms $-i$ deviates to any $\tilde{\mathcal{C}}^{-i}$.

To show the first step, given that C^i was chosen in the original equilibrium $U(C^i, C^{-i}) \geq U(\hat{C}^i, C^{-i}) \quad \forall \hat{C}^i \in \mathcal{C}^i$. By construction, we have that $U(\tilde{C}^i(C^{-i}), C^{-i}) \geq U(C^i, C^{-i})$ and $U(\tilde{C}^i(C^{-i}), C^{-i}) \geq U(\hat{C}^i, C^{-i})$, for all $\hat{C}^i \in \tilde{C}^i$, so that $U(\tilde{C}^i(C^{-i}), C^{-i})) \geq U(C'^i, C^{-i})$ for all $C'^i \in \tilde{\mathcal{C}}^i$. To prove the reverse, suppose $\hat{C}^i \in \tilde{C}^i$ is chosen by the agent. By the definition of $\tilde{C}^i$, if $(b^i_t, y^i_t) \in \hat{C}^i_t \in \hat{C}^i$ then $(b^i_t, y^i_t) \in C^i_t \in C^i$ so $U(C^i, C^{-i}) \geq U(\hat{C}^i, C^{-i})$ and $U(C^i, C^{-i}) \geq U(C^{i'}, C^{-i})$ for all $C^{i'} \in \tilde{C}^i$. Given that $\hat{C}^i$ is chosen then $U(\hat{C}^i, C^{-i}) \geq U(C^{i'}, C^{-i})$ for all

$C^{i'} \in \mathcal{C}^i \backslash \tilde{C}^i$. Combining these inequalities, we get that $U(C^i, C^{-i}) \geq U(C^{i'}, C^{-i})$ for all $C^{i'} \in \mathcal{C}^i$, implying that C^i is chosen in the original equilibrium.

Suppose there exists a collection of menus $\tilde{\mathcal{C}}^{-i}$ so that $V^{-i}(\tilde{\mathcal{C}}^{-i}, \tilde{\mathcal{C}}^i) > V^{-i}(\mathcal{C}^{-i}, \tilde{\mathcal{C}}^i)$. Let C^* denote the equilibrium choice of the agent, such that $U\left(C^{i,*}, C^{-i,*}\right) \geq U(\hat{C}^i, \hat{C}^{-i})$, for all $(\hat{C}^i, \hat{C}^{-i}) \in \tilde{\mathcal{C}}^i \times \tilde{\mathcal{C}}^{-i}$. The first case is if $\tilde{\mathcal{C}}^{-i} \cap P\left(C^i\right) = 0$. If $C^{-i,*} \in \tilde{\mathcal{C}}^{-i} \cap \mathcal{C}^{-i}$, we immediately reach a contradiction since C^* was also chosen in the previous equilibrium so that profits must be equal. If $C^{-i,*} \notin \tilde{\mathcal{C}}^{-i} \cap \mathcal{C}^{-i}$, then we reach a contradiction with $\mathcal{C}$ being an equilibrium, since firm $-i$ would have deviated from offering the menu $\mathcal{C}^{-i} \backslash P\left(C^i\right) \cup C^{-i,*}$ and would make strictly greater profits.

The second case is if $\tilde{\mathcal{C}}^{-i} \cap P\left(C^i\right) \neq 0$. In this case, if $C^{-i,*} \in P\left(C^i\right)$ we immediately reach a contradiction since the agent chooses the same menu in both equilibrium so profits are the same. If $C^{-i,*} \notin P\left(C^i\right)$ then we contradict $\mathcal{C}$ being an equilibrium, since firm $-i$ would deviate from offering $\mathcal{C}^{-i} \backslash P\left(C^i\right) \cup C^{-i,*}$.

Repeating this procedure for every non-minimal menu C^i in the original $\mathcal{C}^i$, we construct a $\tilde{\mathcal{C}}$ where every menu is minimal. ∎

B No profitable deviation with redistribution.

We show that there is no profitable deviation at time T that implies some redistribution between agents.

We first show that any deviation, if chosen by agents, is such that transfers $\{b_T(\theta^T)\}$ satisfy the following ordering: for all i, j if $\theta_i > \theta_j$ then $b(\theta^{T-1}, \theta_i) > b(\theta^{T-1}, \theta_j)$. Suppose not, so there exists $\theta_i > \theta_j$ with $b(\theta^{T-1}, \theta_i) < b(\theta^{T-1}, \theta_j)$. Let $\{\hat{b}_T(\theta^{T-1}), \hat{y}(\theta^T)\}$ be the allocation chosen from the contract $\hat{C}$.[37] The agents' choices must satisfy the following, for all $\theta, \hat{\theta}$:

$$
\begin{aligned}
&u\left(\hat{b}_T(\theta^{T-1}) + b(\theta^T) + \hat{y}(\theta^T) + y(\theta^T)\right) - v\left(\frac{\hat{y}(\theta^T) + y(\theta^T)}{\theta}\right) \geq \\
&u\left(\hat{b}_T(\theta^{T-1}) + b(\theta^{T-1}, \hat{\theta}) + \hat{y}(\theta^{T-1}, \hat{\theta}) + y(\theta^{T-1}, \hat{\theta})\right) - v\left(\frac{\hat{y}(\theta^{T-1}, \hat{\theta}) + y(\theta^{T-1}, \hat{\theta})}{\theta}\right).
\end{aligned}
$$

Using this equation for θ_i and θ_j and from convexity of v, we have that $\hat{y}(\theta^T) + y(\theta^T) > \hat{y}(\theta^{T-1}, \hat{\theta}) + y(\theta^{T-1}, \hat{\theta})$. Agent θ_i is better-off with the following strategy: choosing the pairs $(\hat{b}_T(\theta^{T-1}), \hat{y}(\theta^{T-1}, \hat{\theta}))$ and $(b(\theta^{T-1}, \hat{\theta}), y(\theta^{T-1}, \hat{\theta}))$ and from menu C_T^S choosing $\delta_i = \hat{y}(\theta^T) + y(\theta^T) - (\hat{y}(\theta^{T-1}, \hat{\theta}) + y(\theta^{T-1}, \hat{\theta}))$. This allows him to have the same output requirements as in the original choice but higher consumption transfers.

We now show that any negative intratemporal transfers (transferring from less to more productive agents) induce a utility level lower than under self-insurance.

[37] From proposition (4), transfers in contract $\hat{C}$ do not depend on time T realization of the shock.

Let $N = |\Theta|$ be the number of possible shock realizations. From above, we focus on the case with transfers $\{b_T(\theta^T)\}$ ordered so that for all i, j if $\theta_i > \theta_j$ then $b(\theta^{T-1}, \theta_i) > b(\theta^{T-1}, \theta_j)$. Define the time T utility of an agent θ with level of transfers equal to b, that can optimally chose the amount to work by the following:

$$V(b, \theta) = \max_y u(b + y) - v\left(\frac{y}{\theta}\right). \tag{69}$$

Given the definition of V, the time T utility under menu $\hat{C}$ can be written as:

$$\bar{u}_T = \sum_{i=1}^{N} \pi(\theta_i) V(\hat{b}_T(\theta^{T-1}), \theta_i). \tag{70}$$

Let $\hat{V}$ be the time T utility derived from a deviation by an entrant.
Denote by $y^*(b, \theta)$ the solution of problem (69). The first order condition for (69) is:

$$u'(b + y^*(b, \theta)) = \frac{1}{\theta} v'\left(\frac{y^*(b, \theta)}{\theta}\right). \tag{71}$$

Notice that, for a given b, y^* is increasing in θ, since v is convex. The envelope condition for (69) implies:

$$\frac{\partial V(b, \theta)}{\partial b} = u'(b + y^*(b, \theta)) > 0. \tag{72}$$

Let $\bar{V}$ be the following:

$$\bar{V} = \sum_{i=1}^{N} \pi(\theta_i) V(b_i, \theta_i), \tag{73}$$

where each individual V is as in (69). We consider the most favorable case for the consumer and assume that the deviation incurs zero profits, so that $\sum_{i=1}^{N} \pi_i b_i = 0$. As in the case with two types we will show that $\hat{V}$, the utility under the deviation is such that $\hat{V} < V^N < \bar{u}_T$. As a first step we show the following

$$\sum_{i=1}^{N} \pi_i b_i V'(0, \theta_i) < 0, \tag{74}$$

this can be shown by multiplying and dividing the above by $V'(0, \underline{\theta})$ where $\underline{\theta}$ is the smallest θ_i. This implies that the sign of (74) is determined by the sign of the following

$$\sum_{i=1}^{N} \pi_i b_i V'(0, \theta_i) = V'(0, \underline{\theta}) \sum_{i=1}^{N} \pi_i b_i \frac{V'(0, \theta_i)}{V'(0, \underline{\theta})},$$

which is negative given the zero profit assumption and the fact that $V'(0, \theta_i)$ is decreasing in

θ_i. Define a scale parameter $g \in [0,1]$ for all the transfers, and define the following function of the scale parameter

$$G(g) = \sum_{i=1}^{N} \pi_i V(g \cdot b_i, \theta_i), \tag{75}$$

notice that $G(0) = \bar{u}$ and that $\hat{V} \leq G(1)$; we will show that G is monotonically decreasing in g. We have that

$$\frac{\partial G'(g)}{\partial g} = \sum_{i=1}^{N} \pi_i b_i V'(g \cdot b_i, \theta_i), \tag{76}$$

where $V'(g \cdot b_i, \theta_i) = u'(g \cdot b_i + y^*(g \cdot b_i, \theta_i))$. As in the previous case, we also have that

$$\begin{aligned} u'(g \cdot b_i + y^*(g \cdot b_i, \theta_i)) &< u'(y^*(0, \theta_i)), \qquad if\ b_i > 0, \\ u'(g \cdot b_i + y^*(g \cdot b_i, \theta_i)) &> u'(y^*(0, \theta_i)), \qquad if\ b_i < 0. \end{aligned}$$

This then implies that $G'(g) < G'(0)$ for all $g > 0$, from 74 we have that $G'(0) < 0$ so that $G(1) < G(0) = \bar{u}$.

C Proofs of Section 4

Proof of Lemma 2

Proof. Let $\{c^{BS}, y^{BS}\}$ and $\{c^M, y^M\}$ be the solution of (43) and (41), respectively. Since $\{c^{BS}, y^{BS}\}$ is in the constraint set of (41), $V(w) \geq \Pi(w)$ for all w. Suppose there exists w such that $V(w) = \Pi(w)$. This implies that $\{c^{BS}, y^{BS}\}$ is one of the solutions of (41) for this w. Let $\theta_l = \min_\theta \Theta$. A necessary first order condition for a solution of (41) is for all feasible w:

$$u'(c(\theta^{t-1}, \theta_l)) > \frac{1}{\theta_l} v'\left(\frac{y(\theta^{t-1}, \theta_l)}{\theta_l}\right), \quad \forall\ \theta^{t-1}. \tag{77}$$

However, since $\{c^{BS}, y^{BS}\}$ is a solution of (43), it must satisfy the following necessary first order condition:

$$u'(c(\theta^{t-1}, \theta_l)) = \frac{1}{\theta_l} v'\left(\frac{y(\theta^{t-1}, \theta_l)}{\theta_l}\right), \quad \forall\ \theta^{t-1}. \tag{78}$$

This contradicts $\{c^{BS}, y^{BS}\}$ being a solution of (41).[38] So no such w exists. ∎

Proof of Proposition 7

In order to prove the proposition, we show first the following two lemmas. For notation, let $U(c, y, \Theta)$ the life-time utility of any allocation $\left\{c(\theta^t), y(\theta^t)\right\}$ when shocks are in Θ.

[38] Note that (77) holds with equality only for the highest realization of utility.

Lemma 5. $w^{BS}(\lambda\theta) = \frac{1-\beta^T}{1-\beta}\log\lambda + w^L(\theta)$.

Proof. Let $\{c(\theta^t), y(\theta^t)\}$ be the solution of problem (45) when shocks are $\theta \in \Theta$. To prove the claim, we show that $\{\lambda c(\theta^t), \lambda y(\theta^t)\}$ solves the above problem if agents' shocks are $\lambda\theta$. Suppose not, then there exists an allocation $\{\hat{c}(\lambda\theta^t), \hat{y}(\lambda\theta^t)\}$ that delivers higher utility $U(\hat{c}, \hat{y}, \lambda\Theta)$. Consider the allocation $\left\{\frac{\hat{c}(\theta^t)}{\lambda}, \frac{\hat{y}(\theta^t)}{\lambda}\right\}$. This allocation is in the constraint set of problem (45) and delivers utility

$$U\left(\frac{\hat{c}}{\lambda}, \frac{\hat{y}}{\lambda}, \Theta\right) = U(\hat{c}, \hat{y}, \lambda\Theta) - \frac{1-\beta^T}{1-\beta}\log\lambda. \tag{79}$$

By the contradicting assumption, $U(\hat{c}, \hat{y}, \lambda\Theta) > U(\lambda c, \lambda y, \lambda\Theta) = U(c, y, \Theta) + \frac{1-\beta^T}{1-\beta}\log\lambda$, which implies $U(\hat{c}, \hat{y}, \lambda\Theta) - \frac{1-\beta^T}{1-\beta}\log\lambda > U(c, y, \Theta)$. Using (79), we get $U\left(\frac{\hat{c}}{\lambda}, \frac{\hat{y}}{\lambda}, \Theta\right) > U(c, y, \Theta)$, contradicting allocation $\{c(\theta^t), y(\theta^t)\}$ solving (45) for agents with shocks in Θ. ∎

Lemma 6. $w^M(\lambda\theta) > w^M(\theta) + \frac{1-\beta^T}{1-\beta}\log\lambda$.

Proof. Let $\{c(\theta^t), y(\theta^t)\}$ be the solution of problem (**??**). Consider the relaxed problem with the surplus constraint (**??**) holding as a weak inequality. Notice that the allocation $\{\lambda c(\lambda\theta^t), \lambda y(\lambda\theta^t)\}$ is in the constraint set of this relaxed problem when agents' shocks are $\lambda\theta$. Also, this constraint must hold with equality (otherwise the extra surplus can be distributed in an incentive compatible way, increasing agent's utility). This implies that the allocation that solves the problem must deliver strictly higher utility. This implies $w^M(\lambda\theta) > w^M(\theta) + \frac{1-\beta^T}{1-\beta}\log\lambda$. ∎

Proof of Proposition 7

Proof. Let $\gamma^* = V\left(w^L(\theta)\right)$, where the function V is the solution of the following problem:

$$V(w_0) = \max_{c,y} \sum_{\theta^t,t} q^t \pi(\theta^t)[y(\theta^t) - c(\theta^t)]$$

$$\sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u\left(c(\theta^t), 1 - \frac{y(\theta^t)}{\theta_t}\right)\right] = w_0$$

$$\sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u\left(c(\theta^t), 1 - \frac{y(\theta^t)}{\theta_t}\right)\right] \geq \sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u\left(c(\tilde{\theta}^t), 1 - \frac{y(\tilde{\theta}^t)}{\theta_t}\right)\right] \quad \forall\tilde{\theta}^t$$

This implies $w^M(\theta, \gamma^*) = w^L(\theta)$. Also, $w^M(\theta, \gamma^*) + \frac{1-\beta^T}{1-\beta}\log\lambda = w^L(\theta) + \frac{1-\beta^T}{1-\beta}\log\lambda$. Using the two previous lemmas,

$$w^M(\lambda\theta, \gamma^*) > w^M(\theta, \gamma^*) + \frac{1-\beta^T}{1-\beta}\log\lambda = w^L(\theta) + \frac{1-\beta^T}{1-\beta}\log\lambda = w^L(\lambda\theta).$$

Notice that if we want to break the indifference of the firms with respect to the θ agents, the result is also true for $\gamma = \gamma^* + \varepsilon$ for some ε small enough. Since in this case, $w^L(\theta) > w^M(\theta, \gamma^*)$ and $w^M(\theta, \gamma^*)$ is continuous on γ, so we can replicate the same steps. ∎

D Data

Table 8: Sample selection for CEX data

	CEX
Baseline sample	69,816
Hours restriction	46,559
Earnings<=0	46,002
Labor income<=0	45,745
Minimum wage restriction	43,802
Age >=25 and <=55	36,871
Final sample	36,871

Numbers indicate total observations remaining at each stage of the sample selection.

E Proofs of Section 5.1

Proof of Lemma 3

Proof. 1. Suppose (59) does not hold; this implies:

$$\frac{1-\pi_b}{\pi_b}\frac{u'(c_L)}{u'(c_H)} > \frac{1-\pi_b}{\pi_b}. \tag{80}$$

Consider the following menu offered by an entrant: $\hat{C} = \{(\varepsilon, -\alpha\varepsilon), (0,0)\}$ for some small ε and where α satisfies:

$$\frac{1-\pi_b}{\pi_b}\frac{u'(c_L)}{u'(c_H)} > \alpha > \frac{1-\pi_b}{\pi_b}. \tag{81}$$

Parameter α can be interpreted as the slope of a line passing between the zero profit line of the bad type and the slope of his indifference curve through c. This deviation

Table 10: Summary statistics for the CEX sample used.

	CEX (80-04)
Age	39.17 (8.74)
Education	
High school dropout	6.99
High school graduate	29.26
College	60.46
Race	
White	86.95
Black	9
Family composition	3.07 (1.58)
Average earnings ($)	30,340 (20,406)
Average annual consumption ($)	13,542 (6,842)
Food ($)	3,791 (1,965)
Rent ($)	262 (487)
Hours	2.123 (567)

Note - All dollar amounts in 1983 dollars.

is chosen by the B agents since

$$\begin{aligned} U^B(C+\hat{C}) &= \pi_b u\left(c_H - \alpha\varepsilon\right) + \left(1-\pi_b\right) u\left(c_L+\varepsilon\right) \\ &= \pi_b u\left(c_H\right) + \left(1-\pi_b\right) u\left(c_L\right) - \pi_b u'\left(c_H\right)\alpha + \left(1-\pi_b\right) u'\left(c_L\right) \\ &> U^B\left(C\right) - \pi_b u'\left(c_H\right)\frac{1-\pi_b}{\pi_b}\frac{u'\left(c_L\right)}{u'\left(c_H\right)} + \left(1-\pi_b\right) u'\left(c_L\right) > U^B\left(C\right) \end{aligned} \quad (82)$$

The minimum profits for the entrant occur when only the bad types accept this contract, since this increases the probability of a positive transfer from the insurance provider to the agent.[39] This deviation delivers strictly positive profits to the entrant even if only B agents accept it:

$$\Pi^B(\hat{C}) = \pi_b\alpha\varepsilon - \left(1-\pi_b\right)\varepsilon \quad \Rightarrow \quad \frac{1}{\pi_b\varepsilon}\Pi^B(\hat{C}) = \alpha - \frac{\left(1-\pi_b\right)}{\pi_b}.$$

[39] In general, given a contract of the type $\Gamma = (a\varepsilon, b\varepsilon)$ let $\pi^* = argmin_{\pi_a\in[\pi_b,\pi_g]}\{\pi_a\cdot a\varepsilon + (1-\pi_a)\cdot b\varepsilon\}$. The minimum profits are given when the type of agent has an accident probability equal to π^*; that is, $\pi^* = \pi_b$ if $a > b$ and $\pi^* = \pi_g$ if $a < b$.

The right-hand side of (81) then implies $\Pi^B(\hat{C}) > 0$. Hence, no equilibrium contract can prevent this deviation.

2. Suppose (60) does not hold; this implies:

$$\frac{1-\pi_g}{\pi_g}\frac{u'(c_L)}{u'(c_H)} < \frac{1-\hat{p}_H}{\hat{p}_H}. \tag{83}$$

Consider the following deviating menu $\hat{C} = \{(c_L - \varepsilon, c_H + \alpha\varepsilon), (0,0)\}$ where α satisfies:

$$\frac{1-\pi_g}{\pi_g}\frac{u'(c_L)}{u'(c_H)} < \alpha < \frac{1-\hat{p}_H}{\hat{p}_H}. \tag{84}$$

Differently from the previous case, this deviation will be accepted as a substitute policy by G agents, since:

$$\begin{aligned} U^G(\hat{C}) &= \pi_g u(c_H + \alpha\varepsilon) + (1-\pi_g)u(c_L - \varepsilon) \\ &= \pi_g u(c_H) + (1-\pi_g)u(c_L) + \pi_g u'(c_H)\alpha - (1-\pi_g)u'(c_L) \\ &> U^G(C) + \pi_g u'(c_H)\frac{1-\pi_g}{\pi_g}\frac{u'(c_L)}{u'(c_H)} - (1-\pi_g)u'(c_L) > U^G(C). \end{aligned} \tag{85}$$

This deviation delivers strictly positive profits to the entrant if all agents accept it, since:

$$\begin{aligned} \Pi(\hat{C}) &= \hat{p}_H(\omega_H - c_H - \alpha\varepsilon) + (1-\hat{p}_H)(\omega_L - c_L + \varepsilon) \\ &= \Pi(C) - \hat{p}_H\alpha\varepsilon + (1-\hat{p}_H)\varepsilon > \Pi(C) \geq 0. \end{aligned}$$

The deviation also delivers strictly positive profits to the entrant even if only G agents accept it, since:

$$\begin{aligned} \Pi^G(\hat{C}) &= \pi_g(\omega_H - c_H - \alpha\varepsilon) + (1-\pi_g)(\omega_L - c_L + \varepsilon) \\ &= \pi_g(\omega_H - c_H) + (1-\pi_g)(\omega_L - c_L)\varepsilon(\alpha\pi_g - (1-\pi_g)) \\ &> \pi_g(\omega_H - c_H) + (1-\pi_g)(\omega_L - c_L) - \varepsilon\left(\frac{(1-\hat{p}_H)}{\hat{p}_H}\pi_g - (1-\pi_g)\right), \end{aligned}$$

so that

$$\frac{\Pi(\hat{C})}{\pi_g} > (\omega_H - c_H) + \frac{(1-\pi_g)}{\pi_g}(\omega_L - c_L) - \varepsilon\left(\frac{(1-\hat{p}_H)}{\hat{p}_H} - \frac{(1-\pi_g)}{\pi_g}\right).$$

Since $\frac{(1-\hat{p}_H)}{\hat{p}_H} - \frac{(1-\pi_g)}{\pi_g} > 0$ if $(\omega_H - c_H) + \frac{(1-\pi_g)}{\pi_g}(\omega_L - c_L) > 0$ there exists ε small enough so that $\Pi(\hat{c}) > 0$. Suppose for contradiction that $\pi_g(\omega_H - c_H) + (1-\pi_g)(\omega_L - c_L) \leq$

0. Equation (59) implies that $c_L > c_G$ so:

$$\begin{aligned} (\omega_H - c_H) &> (\omega_L - c_L) \\ \pi_g(\omega_H - c_H) - \pi_g(\omega_H - c_H) &> \pi_b(\omega_H - c_H) - \pi_b(\omega_L - c_L) \\ \pi_g(\omega_H - c_H) + (1-\pi_g)(\omega_L - c_L) &> \pi_b(\omega_H - c_H) + (1-\pi_b)(\omega_L - c_L) \geq 0. \end{aligned}$$

The last inequality comes from the fact that total profits under c must be non-negative, so under the contradicting assumption, it must be that $\pi_b(\omega_H - c_H) + (1-\pi_b)(\omega_L - c_L) \geq 0$, reaching a contradiction. So it must be true that $\pi_g(\omega_H - c_H) + (1-\pi_g)(\omega_L - c_L) > 0$, and consequently condition (60) must hold.

∎

Proof of Lemma 4

Proof. The proof of condition (62) follows the proof of (59) in Lemma 3. Suppose that condition (63) is violated. If so, there exists an α so that

$$\frac{1-\pi_g}{\pi_g}\frac{u'(c_L^G)}{u'(c_H^G)} > \alpha > \frac{1-\hat{p}_H}{\hat{p}_H}. \tag{86}$$

Consider an entrant firm offering the menu $\hat{C} = \{(\varepsilon, -\alpha\varepsilon), (0,0)\}$. This menu is profitable even if all agents accept it, since

$$\Pi(\hat{C}) = \hat{p}_H\alpha\varepsilon - (1-\hat{p}_H)\varepsilon \Rightarrow \frac{\Pi(\hat{C})}{\varepsilon\hat{p}_H} = \alpha - \frac{(1-\hat{p}_H)}{\hat{p}_H} > 0,$$

where the last inequality follows from (86). In addition, $\hat{C}$ is accepted by the good type, since

$$U^G(C\cup\hat{C}) = \pi_g u(c_H^G - \alpha\varepsilon) + (1-\pi_g)u(c_L^G + \varepsilon) = U^G(C) - \alpha\pi_g u'(c_H^G) + (1-\pi_g)u'(c_L^G) > U^G(C),$$

where the last inequality follows from (86): $(1-\pi_g)u'(c_L^G) > \alpha\pi_g u'(c_H^G)$.

Suppose that condition (64) is violated. If so, there exists an α so that

$$\frac{1-\pi_g}{\pi_g}\frac{u'(c_L^G)}{u'(c_H^G)} < \alpha < \frac{1-\pi_g}{\pi_g}. \tag{87}$$

Consider an entrant firm offering the menu $\hat{C} = \{(-\varepsilon, \alpha\varepsilon), (0,0)\}$. This menu is profitable even if all types accept it:

$$\Pi(\hat{C}) = -\hat{p}_H\alpha\varepsilon + (1-\hat{p}_H)\varepsilon \Rightarrow \frac{\Pi(\hat{C})}{\varepsilon\hat{p}_H} = -\alpha + \frac{(1-\hat{p}_H)}{\hat{p}_H} > 0,$$

where the last inequality follows from (87): $\alpha < \frac{1-\pi_g}{\pi_g} < \frac{(1-\hat{p}_H)}{\hat{p}_H}$. Also $u^G(c^G+\tilde{c}) > u^G(\tilde{c})$, so that G agent is willing to accept it. ∎

Proof of Proposition 10

Proof.

Part 1. We first show that the allocation for the B agent generates 0 profits. Consider the Cartesian plane (c_L, c_H). For any allocation $x = (x_L, x_H)$, let $\sigma(x) = \left|\frac{\omega_H - x_H}{\omega_L - x_L}\right|$ be the slope of the line connecting the endowment point and allocation x. The allocation for the bad type must satisfy $\sigma(c^B) \geq \frac{1-\pi_b}{\pi_b}$; that is it cannot lie above the zero profit line for the bad type. Suppose it does; if so the profits originating from the bad type are negative (since $\pi_b < (\omega_H - c^B_H) < (1-\pi_b)(\omega_L - c^B_L)$). In this case, the incumbent firm can deviate to the following allocation $\tilde{c} = (c^B_H - \varepsilon, c^B_L - \varepsilon)$, and this allocation increases profits by ε. Since it delivers strictly lower utility than the original allocation, it will not be preferred by the good type if the original was not.[40] Similar arguments apply if the allocation is the result of transfers originating from multiple insurance providers, that is, $c^B = \left(\sum_{i=1}^N c^B_{i,L}, \sum_{i=1}^N c^B_{i,H}\right)$. In this case, if $\sigma(c^B) < \frac{1-\pi_b}{\pi_b}$, then there exists at least one insurance provider for which $\sigma(c^B_i) < \frac{1-\pi_b}{\pi_b}$.[41] From here onward we consider $\sigma(c^B) \geq \frac{1-\pi_b}{\pi_b}$. Suppose the previous relation holds with strict inequality: c^B is under the zero profit line for the B agent (delivering ε profits), then an entrant can offer the allocation $\tilde{C} = \{(0,0), (c^B_L - \omega_L + \varepsilon/2, c^B_H - \omega_H + \varepsilon/2)\}$. This allocation delivers strictly positive profits even if all agents accept it and is preferred by B agents, which is a contradiction.

If c^B is on the zero profit line and $c^B_L > c^B_H$ (the other case is excluded by (62)), there exists α such that $\frac{1-\pi_b}{\pi_b}\frac{u'(c^B_L)}{u'(c^B_H)} < \alpha < \frac{1-\pi_b}{\pi_b}$. An entrant can offer the contract $\tilde{c} = \{(0,0), (c^B_L - \varepsilon, c^B_H + \alpha\varepsilon)\}$. This contract is preferred by B agents and delivers positive profits even if all agents accept it. Hence, the equilibrium c^B must be on the intersection of the zero profit line for B agents and the 45° line.

Part 2. Given part 1 of this proof, $c^B = (\omega^B, \omega^B)$. This implies that the equilibrium allocation for the G agent must satisfy $U^G(c^G) \geq U^G(c^B)$ and $U^G(c^G) \geq U^G(\omega_L, \omega_H)$. This implies that in the (c_L, c_H) Cartesian plane, c^G must lie in the triangle delimited by the endowment point, the intersection of the indifference curve of the B agent at c^B with the

[40] Even if the G agent would accept the allocation, it would only increase profits, since a large ε can be chosen so that the allocation lies under the average zero profit line.

[41] To illustrate this, consider the case with $N = 2$ and denote with t the transfers from the insurance provider to the agent. Suppose that $\sigma(c^B) = \left|\frac{t^B_{1,H}+t^B_{2,H}}{t^B_{1,L}+t^B_{2,L}}\right| < \frac{1-\pi_b}{\pi_b}$ and $\left|\frac{t^B_{2,H}}{t^B_{2,L}}\right| > \left|\frac{t^B_{1,H}}{t^B_{1,L}}\right| > \frac{1-\pi_b}{\pi_b}$ (where without loss of generality $i=2$ is the firm with the highest slope for the transfers). If so $\left|\frac{t^B_{1,H}}{t^B_{1,L}}\right| > \left|\frac{t^B_{1,H}+t^B_{2,H}}{t^B_{1,L}+t^B_{2,L}}\right|$ which implies $\left|\frac{t^B_{1,H}}{t^B_{1,L}}\right| > \left|\frac{t^B_{2,H}}{t^B_{2,L}}\right|$, thus reaching a contradiction.

zero profit line for the G agent and the intersection of the indifference curve of the B agent at c^B with the intersection of indifference curve of the G agent at the endowment.

Suppose c^G is in this triangle and $c^G \neq (\omega_L, \omega_H)$. An entrant can offer the following contract: $\tilde{C} = \left\{(c_L^B - c_L^G + \varepsilon, c_H^B - c_H^G), (0,0)\right\}$ for a small $\varepsilon > 0$. This allocation will be chosen by the B type together with c^G since $\tilde{c} + c^G = c^B + (\varepsilon, 0)$. Also, this allocation is profitable for the entrant if any type chooses it. To see this, rewrite

$$\tilde{C} = (c_L^B - \omega_L + \omega_L - c_L^G + \varepsilon, c_H^B - \omega_H + \omega_H - c_H^G) = (t_L^B - t_L^G + \varepsilon, t_H^B - t_H^G),$$

where $t_L^B, t_L^G > 0$ and $t_H^B, t_H^G < 0$. Minimum profits occur only when the bad type accepts it. Suppose profits are negative: $-\pi_b(t_H^B - t_H^G) - (1-\pi_b)(t_L^B - t_L^G + \varepsilon) < 0$. Since $t_H^B = \frac{1-\pi_b}{\pi_b} t_L^B$, $\pi_b t_H^G + (1-\pi_b)(t_L^G - \varepsilon) < 0$. For sufficiently small ε this implies $-\frac{t_H^G}{t_L^G} > \frac{1-\pi_b}{\pi_b}$, since $t_L^G > 0$, $|\frac{t_H^G}{t_L^G}| > \frac{1-\pi_b}{\pi_b}$. This is a contradiction with $\left|\frac{\omega_H - c_H^G}{\omega_L - c_L^G}\right| \leq \frac{1-\hat{p_H}}{\hat{p_H}}$, since c^G lies above the average zero profit line given the parameter restriction. ∎