Inequality and Risk-Taking Behaviour

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Abstract

This paper investigates social influences on attitudes to risk and offers an evolutionary explanation of risk-taking by young low-ranked males. Becker, Murphy and Werning (2005) found that individuals about to participate in a status tournament may take fair gambles even though they are risk averse in both wealth and status. Here their model is generalised by use of the insight of Hopkins and Kornienko (2010) that in a tournament or status competition one can consider equality in terms of the status or rewards available as well as in initial endowments. While Becker et al. found that risk-taking is increasing in the equality of initial endowments, it is found here that it is increasing in the *inequality* of rewards in the tournament. Further, it is shown that the poorest will be risk loving if the lowest level of status awarded is sufficiently low. Thus, the disadvantaged in society rationally engage in risky behavior when social rewards are sufficiently unequal. Finally, as greater inequality in terms of social status induces gambling, it can cause greater inequality of wealth.

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1 Introduction

Many people undertake highly risky activities. They engage in crime, go to war, participate in extreme sports, take drugs or start fights. To an evolutionary psychologist, the fact that young men are disproportionately represented in such activities is not a coincidence. Young men take risks in order to achieve social reputation or status which may improve mating success (Wilson and Daly, 1985). Such competition is more intense amongst men than women, as men face a greater variance in reproductive outcomes. To an economist, such behaviour is difficult to integrate into a tradition where decision makers are typically taken to be risk averse. Further, to my knowledge, economics has little to say about why risk attitudes should depend on either gender or age. If anything, the lower wealth of the young compared to older adults would be expected to make them relatively risk averse.[1]

A second question is the relation between risk-taking behaviour and inequality. Clearly, the traditional economic view of risk preferences being subjective and idiosyncratic says little of how such behaviour should vary with the wealth of others. Yet, there is evidence that such risk-taking behaviour is more common in more unequal societies. Crime has been found to be increasing in inequality by Kelly (2000) and Fajnzylber et al. (2002), among others. Wilkinson and Pickett (2009) find a positive relationship between inequality and a wider range of risky behaviours. To explain their empirical findings, they argue that violence and other forms of risk taking are provoked by unfavourable social comparisons, and inequality increases such "evaluation anxiety".

The problem is that existing formal models of social rivalry find results that seem to run in the opposite direction. Hopkins and Kornienko (2004) find in a model of status that competition decreases not increases with inequality. Becker, Murphy and Werning (2005) analyse a similar model but concentrate on the implications for risk taking. They find that such behaviour will only take place when the initial distribution of wealth is sufficiently equal. That is, there is more risk-taking behaviour in more equal societies. Further, gambling will be done at intermediate levels of wealth, so that the middle class should be the most risk-taking.

In this paper, I show how a simple change of perspective can provide very different results. A large population of individuals choose how much of their initial endowments to allocate to competition in a tournament. Performance in the tournament determines how a range of ranked rewards or status positions are allocated. The great insight of Becker et al. (2005) is that the anticipation of taking part in such a competition for status may induce individuals to take fair gambles, even though their underlying preferences over consumption and status are concave. This is integrated into the framework of Hopkins and Kornienko (2010) which

[1]The young are rich in physical health. However, this does not explain greater risk taking by men, nor why risk-taking should vary with social conditions, the subject of this paper. Finally, young men also adopt more risky investment strategies (Barber and Odean, 2001) and thus are apparently less risk averse in terms of financial as well as physical risk.

permits consideration of equality in terms of the status or rewards available as well as in terms of initial endowments. For example, the difference in return to occupying high versus low social position can and does vary across societies. Here, I find that risk-taking behaviour is increasing in inequality of final *rewards*, even if it is decreasing in the inequality of initial *endowments*.

Specifically, I find that if the value of the minimum status or reward approaches zero, the lowest-ranked in society will be risk-loving. This result holds even if the lowest rank have substantial wealth. Thus, in this model, it is the prospect of low status, independent of the affluence of society, that determines risk taking. Further, an increase in inequality of rewards will make low-ranked agents more risk loving. All agents after the tournament, in "old age", will be risk averse. Thus, this model implies, when combined with the observation from evolutionary psychology that men face more dispersed rewards than women, that risk-taking behaviour will be most common amongst young, low-ranked men, and that this behaviour will be more common in more socially unequal societies.

The basic intuition for these results is that social exclusion leads to desperation. More specifically, an individual who has an endowment that is low relatively to his rivals can expect only a low reward from participating in the tournament, even if his wealth is high in absolute terms. Further, if this reward is sufficiently low, the marginal value of doing better in the tournament can be arbitrarily high. For example, if low status means that marriage and children are unlikely to be attainable, then evolutionary considerations suggest that an individual in that situation should be desperate to change this outcome. This gives an incentive to gamble. In contrast, in a society that guarantees a relatively high minimum level of status, there would be less appetite for risky behaviour among the low-ranked.

Robson (1992) was the first to integrate relative concerns into risk preferences (see also Robson, 1996; Harbaugh and Kornienko, 2000; Hvide, 2002; Ray and Robson, 2012; Robson and Samuelson, 2010). The important difference in Robson (1992) is that there individual utility is directly assumed to be convex in relative wealth. This could be plausible in that it means that the difference between being first and second is more important than the difference between tenth and eleventh. However, here I explore an alternative idea. It is not the underlying preferences for high position that cause fierce competition for status. Rather it is the large objective difference in rewards to high and low position that is what is important. For example, a top tennis tournament typically has prize money that is highly convex in the ranking achieved, which will induce highly competitive behaviour by tennis professionals even though they may have utility that is concave in wealth. A further difference is that in Robson (1992), Ray and Robson (2012) and Becker et al. (2005) more gambling happens in more equal societies and it is more likely that those who gamble will have intermediate levels of wealth rather than being poor.

The previous study closest to the current work is Robson (1996). He considers a model where men care about relative wealth because of the possibility of polygyny: high relative wealth means that a man can attract multiple partners. This gives men an incentive to gamble. The effective difference here is that by varying the reward schedule, the rate at which relative wealth can be converted into marriage opportunities is altered. Thus, the incentive to gamble can itself be varied.

Becker et al. (2005) and Ray and Robson (2012) draw an important further conclusion from their analysis: there is an upper bound on the level of equality that can be supported in society. If the level of equality exceeds it, then some agents would have an incentive to gamble leading to a wider distribution of wealth.[2] In this paper, I find that the maximum level of equality in wealth is increasing in the equality of status. In fact, an arbitrarily equal distribution of wealth can be supported without gambling, if status is sufficiently equally distributed. Equally, an increase in the inequality of status can induce gambling. So, status inequality can create inequality in wealth.

Thus, this model provides an explicit theoretical mechanism which would support the apparent empirical relationship between inequality and risk-taking behaviour. But the causation flows in a different way than is normally assumed. If a society is relatively egalitarian in its treatment of its citizens, so that advantages of being rich beyond higher consumption are relatively minor, then individuals have little incentive to gamble and the wealth distribution will remain undispersed. However, where social advantage leads to highly differential mating opportunities, such as in polygamous societies, or where there is significant differences in treatment between the social classes, then this provides a strong incentive for the low ranked to take on risks. This in turn leads to a relatively dispersed distribution of wealth. Thus, unequal social arrangements can cause economic inequality.

2 A Status Tournament

The model is similar to that found in Frank (1985), Hopkins and Kornienko (2004, 2010) and Becker, Murphy and Werning (BMW) (2005). A large population of agents compete in a tournament with a range of ranked rewards that could represent either material outcomes, such as cash prizes, or non-material awards of status. Agents make a strategic decision over how to allocate their endowment between performance in the tournament and private consumption. As BMW first discovered, this situation can lead to individuals being willing to take fair gambles if they are offered before the tournament. This is because the utility function implied by equilibrium behaviour in the tournament can be convex in initial endowments, even though an individual has preferences that are concave in both consumption and rewards. The model is solved backwards. This section analyses the tournament stage of the game. The next section looks at the implied incentives to take gambles prior to the

[2] Importantly, Ray and Robson (2012) extend the analysis of concern for relative wealth to a dynamic setting and thereby endogenise the distribution of wealth. They find that in steady state there is persistent gambling.

tournament. Stable wealth distributions that give no incentive to gamble are characterised in Section 4. Section 5 shows how the tournament model can be extended to consider marriage matching under an uneven gender ratio and the effect on risk attitudes.

I assume a continuum of agents. Each has a different endowment of wealth z with endowments being allocated according to the publicly known distribution $G(z)$ on $[\underline{z}, \bar{z}]$ with $\underline{z} > 0$. The distribution $G(z)$ is twice differentiable with strictly positive density $g(z)$. The level of each contestant's endowment is her private information.

Next, and before the tournament, individuals may have an incentive to gamble with their wealth. It is assumed that a range of fair gambles are offered each in the form of a continuous density over a bounded interval. As Ray and Robson (2012) suggest, these gambles could be lotteries in the common meaning of the term or, more generally, entry into risky occupations or making risky investments. Then, one would expect that gambles are taken until the market clears in the sense that the distribution of wealth is such that no-one wishes to gamble further. It is shown in Section 4 that such stable wealth distributions exist.

For now, I consider the tournament taking place with the initial wealth distribution $G(z)$. If $G(z)$ is not itself stable, we would expect, by the time the tournament actually takes place, that gambling will have changed the wealth distribution to a stable distribution. However, the point is that it is the anticipation of taking part in a tournament when the distribution of wealth is not stable which gives the incentive to take risks. It is thus necessary to model the hypothetical possibility, of playing the tournament under the initial non-stable wealth distribution in order to understand risk attitudes.

In the tournament itself, agents make a simultaneous decision on how to divide their wealth z between performance x and consumption c, with $x + c = z$. Performance has no intrinsic utility, but rewards s are awarded on the basis of performance, with the best performer receiving the highest reward, and in general, one's rank in performance determining the rank of one's reward. A specific interpretation in BMW and Hopkins and Kornienko (2004) is that x represents expenditure on conspicuous consumption, and s is the resulting status. An alternative, first due to Cole, Mailath and Postlewaite (1992), is that s represents the quality of a marriage partner achieved. This scenario is considered explicitly in Section 5 below. Relating this to evolutionary considerations, the range of rewards in a society which permits a high degree of polygyny would be wider than in a society in which strict monogamy is enforced. What is important here is that there is a schedule of rewards or status positions available, which are assigned by performance, but are otherwise exogenous with respect to wealth.

In any case, it is assumed that all individuals have the same preferences over consumption c and status or rewards s,

$$U(c, s) \tag{1}$$

where U is a strictly increasing, strictly concave, three times differentiable function with

$U_c, U_s > 0$, and $U_{cc}, U_{ss} < 0$. So, agents are risk averse with respect to both consumption and status. I also assume that $U_{cs} \geq 0$, so that the case of additive separability $U_{cs} = 0$ and status and consumption being positive complements $U_{cs} > 0$ are both included. As BMW stress, it is when $U_{cs} > 0$, strict complementarity between rewards and consumption, that the results on risk taking are strongest. Note that x does not appear in the utility function and thus represents a pure cost to the individual. The amount spent on x could represent conspicuous consumption, labour effort or resources devoted to fighting or lobbying.

The order of moves is, therefore, the following:

1. Agents receive their endowments z.
2. Agents are offered fair gambles which they are free to accept or to reject.
3. Agents commit a part x of their after gambling wealth z to performance in the tournament.
4. Each agent receives a reward s, the value of which is determined by performance in the tournament.
5. Agents consume their remaining endowment $c = z - x$ and their reward s, receiving utility $U(c, s)$.

To this point, the model is identical to that of BMW (and very similar to that of Hopkins and Kornienko, 2004). However, here I follow Hopkins and Kornienko (2010) in assuming that the rewards or status positions of value s whose publicly known distribution has an arbitrary twice differentiable distribution function $H(s)$ on $[\underline{s}, \bar{s}]$, with $\underline{s} > 0$, and strictly positive density $h(s)$. BMW assume that $H(s)$ is fixed as a uniform distribution on [0,1]. As they point out, for the existence of equilibrium, this represents a harmless normalisation. However, this clearly prevents the major exercise here: identifying the change of behaviour arising from changes in the distribution of rewards.

Rewards or status are assigned assortatively according to rank in performance, with the highest performer receiving the highest reward and the lowest performer the lowest reward. Let $F(x)$ be the distribution of choices of performance. One's position in this distribution will determine the award achieved. Precisely, an individual who chooses a performance level x will receive a reward

$$S(x, F(\cdot)) = H^{-1}\left(\theta F(x) + (1-\theta) F^{-}(x)\right) \tag{2}$$

where and $F^{-}(x) = \lim_{\xi \uparrow x} F(\xi)$ and for some $\theta \in (0,1)$. The role of the parameter θ is to break potential ties that would occur if a mass of agents were to choose the same level of performance. For example, if a mass of agents chose the same performance, this rule would be consistent the corresponding range of rewards being equally distributed amongst those agents.[3] However, if all contestants choose according to a continuous strictly increasing strategy $x(z)$, then, first, $F(x) = F^{-}(x)$ for all x, and, second, $F(x(z)) = G(z)$.[4] Together, this implies, $H(s) = F(x) = G(z)$, one holds the same rank in wealth, performance and in reward achieved, or

$$S(x, F(x)) = H^{-1}(F(x)) = H^{-1}(G(z)) = S(z). \tag{3}$$

We can call $S(z)$ the reward or status function, as in a monotone equilibrium, it represents the relationship between initial endowment and the reward or status achieved.

Importantly, the reduced form equilibrium utility given a monotone equilibrium performance function $x(z)$ will then be

$$U(z) = U(z - x(z), S(z)). \tag{4}$$

We will see that this function $U(z)$ can be convex, even given our concavity assumptions on $U(c, s)$. Therefore, agents would accept a fair gamble over their endowment, if such a gamble was offered before the tournament.

If all agents follow a monotone strategy $x(z)$, then an individual with endowment z should choose $x(z)$. If she considers deviating to a different level of performance $x(\hat{z})$, she will have no incentive to do so if

$$-x'(\hat{z})U_c(z - x(\hat{z}), S(\hat{z})) + S'(\hat{z})U_s(z - x(\hat{z}), S(\hat{z})) = 0. \tag{5}$$

Setting $x(\hat{z}) = x(z)$ and rearranging, we have

$$x'(z) = \frac{U_s(z - x(z), S(z))S'(z)}{U_c(z - x(z), S(z))}. \tag{6}$$

The solution to the above differential equation with boundary condition,

$$x(\underline{z}) = 0 \tag{7}$$

will be our equilibrium strategy. This is shown in the next result, which is similar to results in Hopkins and Kornienko (2004, 2010) and BMW (2005).[5] Note that in the proof (in the

[3]Note that $F(x)$ and $F^{-}(x)$ are only distinct when a positive mass of agents choose the same performance $\hat{x}$. Denote $\bar{r} = F(\hat{x})$ and $\underline{r} = F^{-}(\hat{x})$ then the average value of rewards ranked between $\underline{r}$ and $\bar{r}$ is $v = \int_{\underline{r}}^{\bar{r}} H^{-1}(r)\ dr/(H(\bar{r}) - H(\underline{r}))$ and by the mean value theorem there is a $\theta \in (0, 1)$ such that $H^{-1}(\theta F(x) + (1 - \theta)F^{-}(x)) = v$.

[4]The probability that an individual i is placed higher than a randomly selected individual j is $F(x_i(z_i)) = \Pr[x_i(z_i) > x_j(z_j)] = \Pr[x_j^{-1}(x_i(z_i)) > z_j] = G(x_j^{-1}(x_i(z_i))) = G(z_i)$.

[5]In Hopkins and Kornienko (2004) and BMW (2005) individuals have an intrinsic desire for rank in performance (considered to be conspicuous consumption), information is complete and the equilibrium is unique. In Hopkins and Kornienko (2010) in contrast there is incomplete information, contestants signal their type by choice of performance. There is only one separating equilibrium, but other equilibria such as pooling equilibria may exist.

Appendix) it is shown that despite the possibility of the equilibrium utility function $U(z)$ being convex, individual utility is pseudoconcave in performance x so that the first order condition (5) above does represent a maximum.

Proposition 1. *There exists a unique solution $x(z)$ to differential equation (6) with boundary condition (7). This is the unique symmetric equilibrium to the tournament.*

Having established the framework of the tournament, the next step is to proceed in solving backwards. The next section considers the risk attitudes of agents who are about to participate in the tournament.

3 Implied Risk Attitudes

The main focus of this paper is to examine the risk attitudes implied by participation in the status tournament. As described in the previous section, an individual with wealth z will anticipate equilibrium utility $U(z) = U(z - x(z), S(z))$, where $x(z)$ is the equilibrium choice of performance and $S(z) = H^{-1}(G(z))$ is the reward function. If this function is convex for some range of wealth, then individuals with wealth on that range would take fair bets if such bets were offered to them prior to the tournament. The analysis in this section focuses on the question as to when in fact this function will be convex.

We have by the envelope theorem $U'(z) = U_c(z - x(z), S(z))$ and

$$U''(z) = U_{cc}(z - x(z), S(z))(1 - x'(z)) + U_{cs}(z - x(z), S(z))S'(z), \tag{8}$$

on $(\underline{z}, \bar{z})$. One can also look at the implied level of absolute risk aversion,

$$AR(z) = -\frac{U''(z)}{U'(z)} = \frac{-U_{cc}(z - x(z), S(z))(1 - x'(z)) - U_{cs}(z - x(z), S(z))S'(z)}{U_c(z - x(z), S(z))}. \tag{9}$$

Perhaps more usefully, to clarify the different potential effects on risk attitudes, one can decompose the expression (8) into (suppressing arguments)

$$U''(z) = U_{cc} + (\ U_{cs}S'(z) - x'(z)U_{cc}\ )$$

which separates the negative and positive elements but also the traditional and non-traditional parts. The first part U_{cc} is negative and reflects risk aversion towards regular consumption. The second, in brackets, gives the competitive part which is positive. By inspection one can immediately see that $U''(z)$ will be positive, even though $U_{cc} < 0$, if either $x'(z)$ or $S'(z)$ is sufficiently large. Note that $S'(z) = g(z)/h(S(z))$. Thus, BMW's result that equality in endowments would lead agents to be willing to accept lotteries follows quite directly. If the distribution of endowments $G(z)$ is strongly unimodal, then its density $g(z)$ will have a very high value at and around its mode.

If one wishes to consider the possibility of gambles that can give wealth outcomes outside the current wealth distribution, then it is necessary to extend the definition of $S(z)$. Thus, assume $S(z) = \underline{s}$ for any z below $\underline{z}$ and $S(z) = \bar{s}$ for z above $\bar{z}$. Let us further assumes that performance at these hypothetical wealth levels is zero for $z < \underline{z}$ and is $x(\bar{z})$ for $z > \bar{z}$. Then $U(z) = U(z, \underline{s})$ for z less than $\underline{z}$ and $U(z) = U(z - x(\bar{z}), \bar{s})$ for $z > \bar{z}$, which implies that utility is strictly concave for z outside $[\underline{z}, \bar{z}]$.[6]

The problem in obtaining unambiguous results on risk attitudes is that both status and wealth affect them but in differing directions. With low wealth (and hence low c) traditionally one would be risk averse. Nonetheless, one can find a sufficient condition for low status individuals to be risk loving. It is a condition on the marginal value of status such that low status leads to desperation.

Definition 1. Devil Take the Hindmost[7] (DTTH) condition: $\lim_{\underline{s}\to 0} U_s(c, \underline{s}) = \infty$ for any $c > 0$ and $U_c(c, s)$ is bounded above for $c > 0$.

For example, suppose $U = \log c + \log s$, then $U_s = 1/s$ so that as $\underline{s}$ tends to 0 then U_s tends to infinity. In general, since $U_{ss} < 0$ by assumption, as the lowest reward or level of status $\underline{s}$ decreases, it pushes its marginal value U_s higher. The DTTH assumption is simply that U_s is not bounded above. Thus, when the consequences of being last are sufficiently unattractive (for example, being seized by the devil), the value to the last-placed individual of moving up the field is arbitrarily high.

In other words, rewards s are in some way an essential good. Some have argued that social status only concerns the upper class, that in effect it is a luxury good. The examples from Wilson and Daly (1985) concern the willingness to use lethal violence in response to social slights. To the young men involved, social standing seems to be a matter of life and death.

I now show that given the DTTH condition, the poorest individuals in any society must be risk loving if their status is sufficiently low. Note the relative nature of this result: it holds whatever the minimum level of wealth $\underline{z}$ in society. That is, even in rich societies, the lowest ranked people can be risk loving. In the developed world, the poor may have consumption levels that are high by historic standards, but what this result shows is that if relative status is low, they still may be risk-taking.

The result is stated for the individual with the lowest possible status $\underline{s}$, but by continuity of the utility function, if the lowest ranked individual is risk loving, so will be an interval of others with higher wealth (see also Example 1 below).

[6]Thus $S(z)$ and hence $U'(z)$ may fail to be differentiable at $\underline{z}$. This causes some difficulties but is largely resolvable by looking at right derivatives at $\underline{z}$.

[7]On the origin of this phrase: "It is said when a class of students have made a certain progress in their mystic studies, they are obliged to run through a subterranean hall, and the last man is seized by the devil" (Brewer (2001)).

Proposition 2. *Assume the DTTH condition, fix the distribution of wealth with $\underline{z} > 0$, and consider a distribution of rewards such that $S'(\underline{s}) > 0$ and $\underline{s} > 0$. Then, there is an $s^* > 0$ such that if the minimum reward level $\underline{s}$ is less than s^* then the poorest individuals will be risk loving. That is, there will be a $\hat{z} \in (\underline{z}, \bar{z}]$ such that $U''(z) > 0$ on $(\underline{z}, \hat{z})$.*

Note that neither the DTTH assumption or the above result preclude consumption from also being essential. An individual who becomes risk loving as rewards fall to zero could become risk averse if his reward level was fixed, but his wealth approached zero. For simplicity, suppose there is additive separability so that $U_{cs} = 0$. Then, if U_c becomes large as $\underline{z}$ goes to zero, x' will go to zero, and U'' will be negative. So low wealth leads to risk aversion. It is low status that leads to risk taking.

3.1 Effects of an Increase in the Dispersion of Rewards

Let us now move to a principal question in this paper, the relationship between risk attitudes and inequality in rewards. Specifically, it is possible to show that making rewards more unequal leads to more risk taking behaviour. To do this, I will compare two distributions of rewards $H_a(s)$ and $H_p(s)$, a for ex ante and p for ex post. I suppose that the distribution of rewards changes from $H_a(s)$ to $H_p(s)$ for exogenous reasons. I then see how this affects risk attitudes.

To carry out this analysis, some notion of a distribution being more dispersed than another is needed. I use a strong version of the dispersive order. Specifically, I say that a distribution H_p is strictly larger in the dispersive order than a distribution H_a, or $H_p >_d H_a$ if

$$h_p(H_p^{-1}(r)) < h_a(H_a^{-1}(r)) \text{ for all } r \in [0, 1]. \tag{10}$$

The original definition of this stochastic order (Shaked and Shanthikumar, 2007, pp148-9) has the same condition but with a weak inequality, and on (0,1). A simple example of distributions satisfying this stronger condition would be any two uniform distributions where one distribution has support on a strictly longer interval than the other (see Hopkins and Kornienko (2010) for further examples and discussion).

Lemma 1. *Suppose that the distribution of rewards becomes strictly more dispersed in terms of the dispersive order $H_p >_d H_a$ and that the lowest reward $\underline{s}$ is unchanged then the poor become more risk loving. That is, there will be a $\hat{z} \in (\underline{z}, \bar{z}]$ such that $U_p''(z) > U_a''(z)$ and $AR_p(z) < AR_a(z)$ for all $z \in (\underline{z}, \hat{z})$.*

While one might think that a general increase in the dispersion of rewards would lead to a general increase in risk-taking, this may not be the case. This is because an increase in the dispersion of rewards makes the tournament more competitive, which will tend to raise performance and lower consumption. For example, Hopkins and Kornienko (2010) find simple sufficient conditions for all agents to increase performance and reduce consumption in response to more dispersed rewards. This matters as, other factors being equal, lower consumption typically increases risk aversion. The poor will nonetheless become more risk loving, as the consumption of the poorest is tied down by the boundary condition (7) which is unchanged.

A further result is that reducing the minimum reward level will increase risk-taking by the poorest in society.

Lemma 2. *Consider two distributions of rewards $H_a(s)$ and $H_p(s)$ which differ in terms of minimum status such that $\underline{s}_a > \underline{s}_p$, but $h_p(\underline{s}_p) = h_a(\underline{s}_a)$. Either (a) assume additive separability so that $U_{cs} = 0$; or (b) assume the DTTH condition and that $U_{css}, U_{ccs} \leq 0$ and that $U_a''(z) \leq 0$ on $(\underline{z}, \underline{z} + \epsilon)$ for some $\epsilon > 0$. Then, there will be a $\hat{z} \in (\underline{z}, \bar{z}]$ such that $U_p''(z) > U_a''(z)$ and $AR_p(z) < AR_a(z)$ for all $z \in (\underline{z}, \hat{z})$.*

Putting these two results together, it is possible to obtain the following result: greater inequality of rewards causes the poor to be more risk-taking. This case would include a form of mean preserving spread on rewards. For example, two uniform distributions having the same mean but with one H_p having a wider support would be suitable.

Proposition 3. *Suppose that the distribution of rewards becomes strictly more dispersed in terms of the dispersive order $H_p >_d H_a$, and the minimum reward decreases $\underline{s}_p \leq \underline{s}_a$. Either (a) assume additive separability so that $U_{cs} = 0$; or (b) assume the DTTH condition and that $U_{css}, U_{ccs} \leq 0$ and that $U_a''(z) \leq 0$ on $(\underline{z}, \underline{z} + \epsilon)$ for some $\epsilon > 0$. Then the poor become more risk loving. That is, there will be a $\hat{z} \in (\underline{z}, \bar{z}]$ such that $U_p''(z) > U_a''(z)$ and $AR_p(z) < AR_a(z)$ for all $z \in (\underline{z}, \hat{z})$.*

See Figure 1 for an illustration of this result. It also gives typical results on how performance and the level of utility responds to the greater level of competition implied by greater inequality of rewards. While there are no such results in this paper, Hopkins and Kornienko (2010) already have shown that greater dispersion of rewards lead to an increase in performance and a decrease in utility for most, and sometimes for all, individuals. See also Example 1 below.

3.2 Effects of an Increase in the Dispersion of Wealth

BMW argue that increase in the dispersion of wealth, such as produced by gambling over wealth, should reduce the desire to gamble. The basic thrust of their claims are supported in new results given below. However, the relationship between inequality of wealth and lower risk taking is not straightforward. For example, it is possible to show that certain mean preserving spreads in wealth will make the poorest less, not more, risk averse.

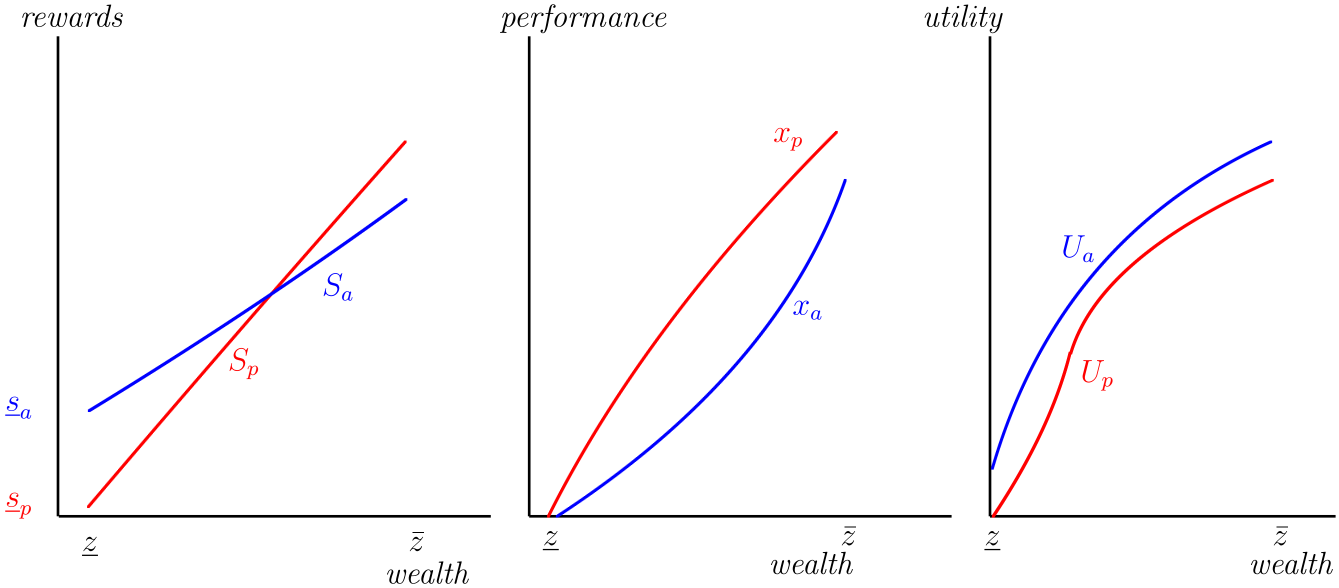

Figure 1: Illustration of Proposition 3: ex post status S_p is more dispersed than ex ante S_a leading to the slope of the equilibrium utility function becoming convex at low levels of wealth. Typically, ex post utility U_p is lower and ex post performance x_p is higher.

For this, I use second order stochastic dominance. Specifically, let us say a distribution F is more dispersed than a distribution G in terms of second order stochastic dominance with single crossing, and we write $F >_{sc} G$ if they have the same mean and

$$\int_{\underline{z}}^{z} F(t) - G(t)\ dt > 0 \tag{11}$$

for $z \in (\underline{z}, \bar{z})$ and are single crossing.[8] That is, $F(z) > G(z)$ on $(\underline{z}, \hat{z})$ and $F(z) < G(z)$ on $(\hat{z}, \bar{z})$ for some $\hat{z} \in (\underline{z}, \bar{z})$. This represents a refinement of the standard definition of second order stochastic dominance (see, for example, Wolfstetter, 1999, pp. 140-4), in which the inequality (11) hold weakly and there is no single crossing condition.

I now consider changes in the distribution of wealth, assuming that the distribution changes from some distribution $G_a(z)$ ex ante, to another distribution $G_p(z)$ ex post. One can think about this change occurring for two different reasons. First, there could be some exogenous change. Second, the distribution of wealth could become more dispersed due to the gambling activity by individuals. But whatever the reason for the change, we will see how this affects individual risk attitudes.

Proposition 4. *Suppose that the distribution of wealth becomes more dispersed in terms of second order stochastic dominance with single crossing $G_p >_{sc} G_a$, and minimum wealth $\underline{z}$*

[8]The use of the order $>_{sc}$ follows the convention in statistics and indicates that the distribution F is "bigger", i.e. more dispersed, than G. Note, however, that G second order stochastically dominates F.

is unchanged. Then the poor become less risk averse.That is, there will be a $\hat{z} \in (\underline{z}, \bar{z}]$ such that $U_p''(z) > U_a''(z)$ and $AR_p(z) < AR_a(z)$ for all $z \in (\underline{z}, \hat{z})$.

The above result is based on the assumption that the dispersion of wealth rises without the support of the distribution widening. So, the density of people rises at the top and bottom ends of the distribution. In general, as found by Hopkins and Kornienko (2004), a higher density means greater competitiveness, and here the higher density of poor people leads to a higher willingness to undertake risky behaviour.[9]

In contrast, to increase risk aversion at low incomes, it is necessary to disperse wealth over a greater range, and in particular to make the poorest poorer. Even then, to ensure that risk-taking decreases, one needs to impose strong conditions, such as requiring that the ratio U_{cc}/U_{cs} is increasing in c.[10] Nonetheless, the final result in this section establishes, as BMW supposed, that greater dispersion of wealth can lower individuals' willingness to gamble. Note that as the comparison is across wealth distributions with different supports, risk attitudes are compared at constant ranks. This method is discussed at much greater length in Hopkins and Kornienko (2010).

Proposition 5. *Suppose that the distribution of wealth becomes strictly more dispersed in terms of the dispersive order $G_p >_d G_a$, and minimum wealth decreases $\underline{z}_p \leq \underline{z}_a$. Assume that $U_{ccc} \geq 0$ and $U_a''(\underline{z}_a) \leq 0$ and assume either (a) that there is additive separability so that $U_{cs} = 0$; or (b) that $U_{ccs} \leq 0$ and U_{cc}/U_{cs} is increasing in c. Then the poor become more risk averse. That is, there will be a $\hat{r} \in (0, 1]$ such that $U_p''(G_p^{-1}(r)) < U_a''(G_a^{-1}(r))$ and $AR_p(G_p^{-1}(r)) > AR_a(G_a^{-1}(r))$ for all $r \in (0, \hat{r})$.*

3.3 Cobb-Douglas

In this section, for concreteness we look at Cobb-Douglas preferences, for which closed form solutions for equilibrium behaviour and preferences are possible. Suppose

$$U(c, s) = c^\alpha s^\beta = (z - x)^\alpha s^\beta \tag{12}$$

Let $\gamma = \beta/\alpha$. Then,

$$x'(z) = \gamma \frac{S'(z)}{S(z)}(z - x) \tag{13}$$

with again $x(\underline{z}) = 0$. This differential equation has the explicit solution

$$x(z) = z - \frac{\underline{s}^\gamma \underline{z} + \int_{\underline{z}}^{z} S^\gamma(t)dt}{S^\gamma(z)}, \; c(z) = \frac{\underline{s}^\gamma \underline{z} + \int_{\underline{z}}^{z} S^\gamma(t)dt}{S^\gamma(z)}.$$

[9]To be fair, similar problems would have arisen in the earlier analysis of the effect of greater inequality in rewards, if reward inequality had been increased on an unchanging support. The point is that one should be quite careful about what one means by greater inequality.

[10]This last condition holds for Cobb-Douglas and CES utility functions.

Thus

$$U(z) = \left(\underline{s}^{\gamma}\underline{z} + \int_{\underline{z}}^{z} S^{\gamma}(t)dt\right)^{\alpha} \tag{14}$$

and

$$U'(z) = \alpha S^{\gamma}(z)(\underline{s}^{\gamma}\underline{z} + \int_{\underline{z}}^{z} S^{\gamma}(t)dt)^{\alpha-1} = \alpha c^{\alpha-1}(z)S^{\beta}(z)$$

and

$$U''(z) = \alpha(\alpha-1)c^{\alpha-2}(z)S^{\beta}(z)(1-x'(z)) + \alpha\beta c^{\alpha-1}(z)S^{\beta-1}(z)S'(z).$$

With Cobb-Douglas preferences, the expression for absolute risk aversion is particularly neat,

$$AR(z) = -\frac{U''(z)}{U'(z)} = -\frac{\gamma S'(z)}{S(z)} + \frac{1-\alpha}{c(z)} \tag{15}$$

where again $\gamma = \beta/\alpha$. That is, changes in the ratio $S'(z)/S(z)$ clearly change risk preferences (though $c(z)$ will also change). But we could also define a form of relative risk aversion as $AR(z)c(z)$ which would give us

$$RR(z) = -\frac{c(z)U''(z)}{U'(z)} = -\frac{\gamma c(z)S'(z)}{S(z)} + (1-\alpha) = -x'(z) + (1-\alpha) \tag{16}$$

Example 1. Suppose rewards are uniform on $[\varepsilon, 1-\varepsilon]$ and wealth is uniform on $[1, 5]$ and $\alpha = \beta$ so that $\gamma = 1$. We have then

$$S(z) = \varepsilon + \frac{1-2\varepsilon}{4}(z-1)$$

and

$$U(z) = \left(\varepsilon + \int_{1}^{z} \varepsilon + \frac{1-2\varepsilon}{4}(t-1)dt\right)^{\alpha} = \left(\frac{(z-1)^2 + 2\varepsilon(-1+6z-z^2)}{8}\right)^{\alpha}$$

Take, for example, $\alpha = 0.4$. With a relatively equal distribution of rewards/status S_a, for example with $\varepsilon = 0.25$, all agents are risk averse. However, let us make rewards more unequal, $\varepsilon = .1$, label this new status function S_p. Then, $U_p(z)$ is convex on [1,2.44) and is concave on (2.44, 5]. See Figure 1. That is, take an individual with an endowment of about 2.5, then that individual will be risk loving with respect to losses and risk averse with respect to gains. Note that ex post equilibrium utility $U_p(z)$ is everywhere lower than ex ante $U_a(z)$ and ex post equilibrium performance $x_p(z)$ is everywhere higher ($x_a = (z-1)/2$ and $x_p = (z^2-1)/(2z-1)$). We can also verify that more dispersed wealth makes agents more risk averse. Keeping rewards dispersed with $\varepsilon = 0.1$ but making wealth also more dispersed, so for example wealth is now uniform on $[0.5, 5.5]$, utility will return to being concave at all wealth levels.

4 The Influence of the Distribution of Rewards on Stable Wealth Distributions

We have seen that anticipated participation in a status tournament can give individuals an incentive to take gambles. BMW, following Robson (1992), consider distributions of wealth that in contrast are *stable* in the sense that given such a distribution, no agent wishes to gamble and therefore the distribution of wealth does not change. Stable distributions can also be seen as clearing the market for gambling. If the initial wealth distribution was not stable, then there would be an incentive to gamble until the redistribution of wealth resulting from gambling made it stable.

Following Robson (1992) and BMW (2005), the exact process by which the market for gambling is cleared is not modelled. For example, there is no strategic analysis of the agents' simultaneous choice of gambles. Rather, simply the existence of stable distributions is shown. However, even if there are no available lotteries or the process of gambling does not converge to a stable distribution, this is not a problem for the main focus of the paper, which concerns the analysis of when is there an incentive to gamble, as covered in Section 3. Somewhat more problematic would be the possibility of gambling resulting in a discontinuous distribution of wealth. Fortunately, it is possible to demonstrate that any stable distribution is smooth and without gaps.

Note that there will potentially be many wealth distributions that are stable. Thus, BMW focus on the stable wealth distribution (which they call the "$*$ allocation") that induces risk neutrality at all levels of wealth.[11] The idea is that distributions that are less dispersed than the stable distribution will induce gambling (something that we formalise below). Thus, this stable distribution represents an upper bound on sustainable equality of wealth. So, let us call it the *most equal stable distribution* or MESD.

In this section, there are the following novel results. First, I prove the existence and uniqueness of the MESD and that it is smooth and without gaps. Second, I show that it is the most equal distribution consistent with stability in that any distribution that is even locally more equal is unstable. Third, I show that greater inequality in rewards implies that the MESD becomes more unequal. Fourth, a result that follows directly from the third, the maximum level of equality is as high as the degree of equality in status. Societies that offer a high degree of equality of esteem can support very equal distributions of wealth.

Further, suppose we take the MESD as a prediction of the actual distribution of wealth in society.[12] Then our second result implies that societies that are socially more unequal

[11]They show that marginal utility having a constant value λ, or $U'(z) = U_c(c(z), S(z)) = \lambda$ in current notation, is also a solution to the problem of a utilitarian social planner.

[12]One potential reason for focussing on the most equal stable distributions rather than other stable distributions is given in Ray and Robson (2012) who show that in dynamic model the incentive to gamble

cause more unequal wealth outcomes. For example, consider two societies that have the same initial distribution of wealth that is quite equal. The first society is relatively socially egalitarian with the status assigned to the poorest individual relatively high so that there is no incentive to gamble and the initial distribution of wealth is unchanged. However, the second society treats its citizens more unequally with a low minimum status. This we have seen can induce gambling, which will result in a greater dispersion of wealth. Thus, more unequal social conditions can produce wealth inequality.

Let us start with formal definitions of stability and the most equal stable distribution.

Definition 2. A stable wealth distribution is a distribution of wealth $G(z)$ such that, for a given distribution of rewards, equilibrium utility satisfies $U''(z) \leq 0$ at all positive wealth levels.

Definition 3. The most equal stable distribution MESD is a stable wealth distribution $G^*(z)$ such that $U''(z) = 0$ for all z in its support.

The aim in this section is to characterise the MESD. The next step is the following technical result.

Lemma 3. *Any MESD $G^*(z)$ is differentiable and has no gaps on its support.*

Now, if one sets the expression for $U''(z)$ in (8) to zero, this leads to the following differential equation (suppressing arguments)

$$S'(z) = \frac{U_c U_{cc}}{U_s U_{cc} - U_c U_{cs}} \tag{17}$$

with boundary condition

$$S(\underline{z}) = \underline{s} \tag{18}$$

(we know that $S(z) = H^{-1}(G(z))$ is differentiable because $H(s)$ is by assumption and $G(z)$ is shown to be by Lemma 3). Using this differential equation (17) and the differential equation (6) for equilibrium performance, one can write a new differential equation for equilibrium consumption,

$$c'(z) = \frac{U_c U_{cs}}{U_c U_{cs} - U_s U_{cc}}. \tag{19}$$

Given the boundary condition (7) for equilibrium performance, the boundary condition for the above equation will be $c(\underline{z}) = \underline{z}$. A solution of the two equations simultaneously will provide the MESD. Specifically, the MESD $G^*(z)$ is defined as $G^*(z) = H(S^*(z))$, where $S^*(z)$ is the solution to the equation (17).

provided by relative concerns prevents convergence to total equality. However, there is a tendency for wealth to converge over time as in standard growth models. Combining these insights, they find that steady-state wealth will be close to the most equal stable distribution.

Further, it is possible to prove the MESD is unique, for a given distribution of rewards and for a given mean wealth.

Proposition 6. *For a given distribution of rewards $H(s)$, there is a unique solution $(c^*(z), S^*(z))$ to the simultaneous differential equation system (17) and (19) with boundary conditions $c(\underline{z}) = \underline{z}$ and $S(\underline{z}) = \underline{s}$, such that $U(z) = U(c^*(z), S^*(z))$ is linear in z for all $z \in [\underline{z}, \bar{z}]$. Thus, $U''(z) = 0$ at for all wealth levels in $(\underline{z}, \bar{z})$. Further, assume that $U_{ccc} \geq 0, U_{ccs} \leq 0$ and U_{cc}/U_{cs} is increasing in c, then for fixed mean wealth m there is a unique distribution of wealth $G^*(z)$ such that $H^{-1}(G^*(z)) = S^*(z)$.*

Furthermore, this distribution $G^*(z)$ is the maximally equal stable distribution (MESD) in the following important sense. Any distribution that is even locally more equal than $G^*(z)$ is unstable. In contrast, any distribution such that $U''(z)$ is strictly negative could be subject to an increase to equality without becoming unstable.

Lemma 4. *Any distribution of wealth $G_p(z)$ that is locally less dispersed than the MESD $G^*(z)$ is not stable. That is, if $g_p(z) > g^*(z)$ on an interval $[z_1, z_2)$ but $g_p(z) = g^*(z)$ on $[\underline{z}, z_1)$ then $U_p''(z) > 0$ on $(z_1, z_1 + \epsilon)$ for some $\epsilon > 0$.*

Further, one can draw the following comparative statics result. The MESD moves with the distribution of rewards. If rewards become more (less) equal, the minimum level of wealth inequality falls (rises) in the sense of second order stochastic dominance with single crossing, a concept introduced in the previous section.

In what follows, it is assumed that there are ex ante and ex post distributions of rewards, $H_a(s)$ and $H_p(s)$ respectively. Under each distribution of rewards, we calculate $S_i^*(z)$ for $i = a, p$, the associated reward function that induces risk neutrality at all wealth levels. Further, by Proposition 6, this will also define $G_a^*(z)$ and $G_p^*(z)$ the ex ante and ex post most equal stable distributions of wealth. I find that a greater dispersion in rewards necessitates a greater dispersion in wealth in order to maintain risk neutrality. An example of this is illustrated in Figure 2.

Proposition 7. *Assume the ex post distribution of rewards H_p is more dispersed than the ex ante distribution H_a in terms of the dispersive order, $H_p >_d H_a$, that the minimum reward falls or $\underline{s}_p < \underline{s}_a$, that the maximum reward rises $\bar{s}_p > \bar{s}_a$ and the mean reward is unchanged. Assume further that $U_{css}, U_{ccs} \leq 0$, $U_{ccc} \geq 0$ and U_{cc}/U_{cs} is increasing in c. Then, the ex post MESD wealth distribution is more dispersed in terms of second order stochastic dominance with single crossing than ex ante. That is, $G_p^* >_{sc} G_a^*$.*

This has an important corollary. If we consider a sequence of distributions of rewards each progressively more equal than the previous, then the corresponding distributions of wealth would also become progressively more equal.

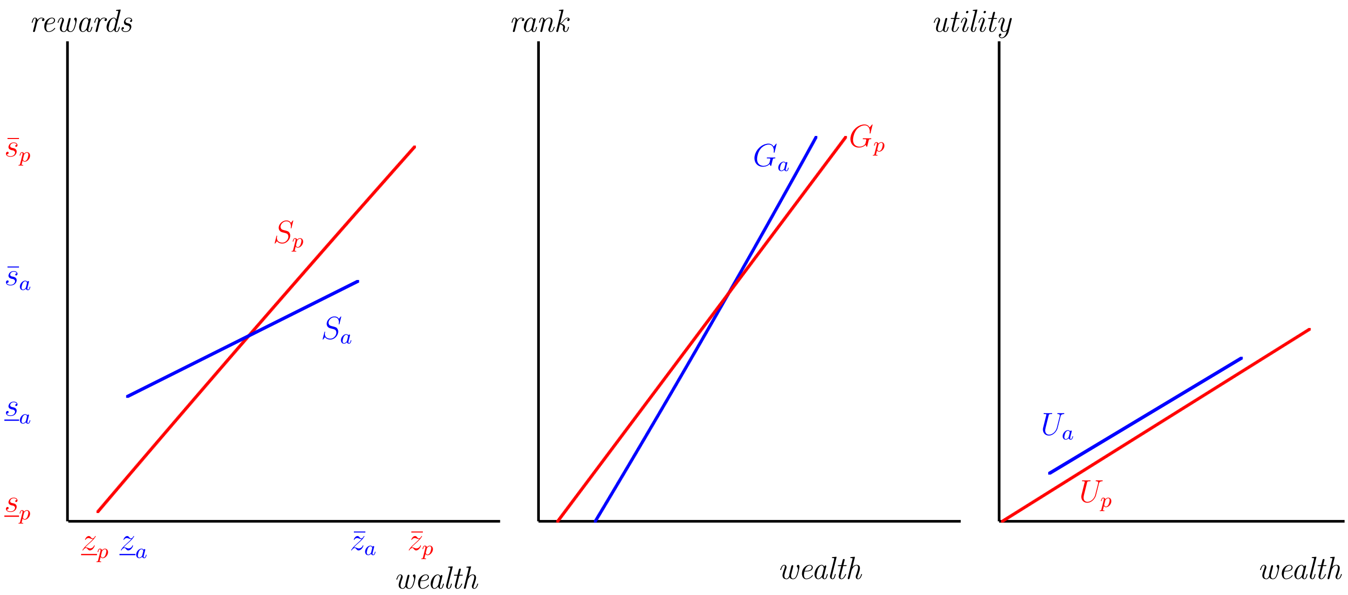

Figure 2: Illustration of Proposition 7: ex post rewards S_p are more dispersed than ex ante S_a. To maintain risk neutrality, ex post wealth G_p must be more dispersed than ex ante G_a, with ex post minimum wealth $\underline{z}_p$ being lower and maximum wealth $\bar{z}_p$ being higher than ex ante.

Corollary 1. *As the distribution of rewards approaches perfect equality, so does the Most Equal Stable Distribution of wealth.*

Despite the earlier results of BMW and Ray and Robson (2012), it is thus possible to sustain an equal society, even in the presence of status competition, provided there is an equality in terms of esteem.

4.1 Cobb-Douglas

Assume Cobb-Douglas preferences $U(c, s) = c^\alpha s^\beta$, then the differential equation (17) becomes

$$S'(z) = \frac{\alpha(1-\alpha)S(z)}{\beta c(z)}$$

and (19) becomes

$$c'(z) = \alpha.$$

This implies that performance and consumption are linear in wealth, specifically $x(z) = (1-\alpha)(z - \underline{z})$ and $c(z) = \alpha z + (1-\alpha)\underline{z}$. This in turn can be used to solve for $S^*(z)$:

$$S^*(z) = A[c(z)]^{(1-\alpha)/\beta} = A(\alpha z + (1-\alpha)\underline{z})^{(1-\alpha)/\beta},$$

where A is a constant of integration. One can check that this implies $U(z) = A^{\beta} c(z)$ which is linear as required. Of course, for strict concavity of the Cobb-Douglas utility function, one needs $\alpha + \beta < 1$, so that $S^*(z)$ is therefore convex. Thus, as $G^*(z)$, the MESD, is equal to $H(S^*(z))$, this minimum inequality wealth distribution will be more convex than the distribution of rewards.

Example 2. Assume that rewards are distributed uniformly on $[\varepsilon, 1-\varepsilon]$. Assume further that $\alpha = \beta = 1/2$ (of course, this means that the utility function is not strictly concave, but as we will see it makes everything conveniently linear). Then, given mean wealth of 1/2, the unique distribution $G^*(z)$ that solves for S^* is

$$G^*(z) = \frac{(1-\varepsilon)z - \varepsilon/2}{1-2\varepsilon}.$$

That is, it is uniform on $[\varepsilon/(2(1-\varepsilon)), (2-3\varepsilon)/(2(1-\varepsilon))]$. We have

$$S^*(z) = (1-\varepsilon)z + \varepsilon/2, \; U^*(z) = \frac{\varepsilon/2 + (1-\varepsilon)z}{\sqrt{2(1-\varepsilon)}}.$$

Clearly, a decrease in ε makes the distribution of rewards more dispersed. It will also make the equilibrium distribution of wealth $G^*(z)$ more dispersed. Equally, a more equal distribution of rewards, implies a more equal stable distribution of wealth. Indeed, as ε approaches 1/2, then both the distribution of rewards and the distribution of wealth become entirely concentrated at 1/2.

5 A Tournament of Particular Interest: Marriage Matching

In Section 2, a tournament model was introduced with a continuum of rewards that were specified in relatively abstract terms. In this section, the underlying institution is made more specific, drawing on the interpretation used in Cole et al. (1992), Peters and Siow (2002), Hoppe et al. (2009) and Bhaskar and Hopkins (2011). The important insight from this literature is that participation in a marriage market can provide an incentive for increased investment or signalling. Here I similarly analyse a tournament for marriage partners but focus on the implications for risk taking.

This is done in the particular context of where there is potentially an excess number of suitors on one side of the market. Wei and Zhang (2011a, b) have recently investigated the economic impact of the highly uneven sex ratio in China, finding evidence for this gender imbalance driving both investment and risk-taking behaviour. The implications for investment and signalling behaviour have been analysed theoretically by Bhaskar and Hopkins (2011) and Hoppe et al. (2009) respectively. However, the distinctive focus here is on risk-taking behaviour and in particular of those with low wealth who thus risk failing to find a marriage partner.

The crucial assumption will be that a measure $\mu \geq 1$ of men seek to match with a measure one of women - men are on the long side of the market.[13] It will be shown that an increase in μ will lead to an increase in risk taking by those men immediately above and below the margin of achieving marriage. As will be seen, it has a similar effect to an increase in inequality of the rewards faced by men.

As in the main model, a continuum of contestants (men) have utility (1), have wealth distributed according to $G(z)$ and must make a choice of performance x. However, s now denotes the quality of marriage partner achieved. In particular, there is a continuum of women that have quality distributed according to $H(s)$ and this is fully observable.[14] Suppose further that women's utility in a match is strictly increasing in the performance x of the man she is matched with. There are several reasons why this might be the case. As in Section 2, one can identify performance with a range of lobbying or fighting activities to displace other males. Alternatively it could be a signal of a man's underlying quality, his wealth z.[15]

In any case, given these preferences, the aim is to find a matching scheme that is stable in the classic sense of Gale-Shapley, and which is measure-preserving.[16] It is well-known that when each side of a matching problem have common and monotone preferences then typically there is a unique stable matching and this will be positive assortative. That is, the woman with the highest apparent quality will match with the highest performing man and so on down. However, since there are more men than women, a proportion of men are excess and will not marry. I assume that this is equivalent in match quality to achieving the lowest possible match $\underline{s}$. Let $r = \theta F(x) + (1-\theta)F^{-}(x)$. As in (2) the parameter $\theta \in (0,1)$ breaks any potential ties in performance. Let $\tilde{r} = (\mu - 1)/\mu$. The proposed matching is then,

$$S(x, F(\cdot); \mu) = H^{-1}(\mu r + 1 - \mu) \tag{20}$$

for $r \geq \tilde{r}$; men with rank $r < \tilde{r}$ are unmatched and receive $\underline{s}$. While matching is positive assortative, the different sizes of population of men and women means that the men's rank is rescaled to give the appropriate match. In the event of ties, θ can be chosen so that, first, the

[13]While the primary interpretation of the parameter μ is demographic, there are simply more men than women, another possibility is that μ represents the degree of polygamy in society in the following sense. In strict monogamy, only unmarried men enter the marriage market. Under polygamy, some married men will also participate and thus the total proportion of men entering the market, that is μ, will be higher.

[14]Here for simplicity only men compete. See Peters and Siow (2003), Bhaskar and Hopkins (2011) and Hoppe et al. (2009) for models with two sided investment or signalling.

[15]The current model is of complete information. However, the monotone equilibrium here is separating in the sense that a man's type can be inferred from his performance. The only substantial difference under incomplete information is the possibility of the existence of other equilibria such as pooling outcomes. See Hoppe et al. (2009), Hopkins (2012) and Hopkins and Kornienko (2010) for models with signalling.

[16]Matching being measure-preserving is the closest equivalent in continuum populations to matching being one-to-one in finite populations. See Cole et al. (1992) and Hopkins (2012) for further details.

above matching rule is equivalent to the ties being randomly broken; second, a proportion of exactly $1-\tilde{r}$ men match (and since we have μ men, the measure of men matched is $\mu(1-\tilde{r})=1$). We have the following result.

Lemma 5. *For a given distribution of performance $F(x)$ and for some $\mu \geq 1$, the matching (20) is the only stable measure-preserving matching.*

Further, assuming a strictly increasing equilibrium choice of performance (at least for men who are matched), we can replace $S(z)$ as defined in (3) with

$$S(z;\mu) = H^{-1}(\mu G(z) + 1 - \mu);\ \ S'(z;\mu) = \mu g(z)/h(S(z;\mu)). \tag{21}$$

for $z \in [\tilde{z}, \bar{z}]$; $S(z;\mu) = \underline{s}$ for $z \in [\underline{z}, \tilde{z})$.

The method of determining equilibrium behaviour will largely be the same as before for men with wealth such that they expect to marry. Their behaviour is determined by solution to a suitably modified differential equation,

$$x'(z) = \frac{U_s(z - x(z), S(z;\mu))S'(z;\mu)}{U_c(z - x(z), S(z;\mu))}. \tag{22}$$

However, men with wealth on $[\underline{z}, \tilde{z}]$ correctly anticipate that they will not marry and so do not compete.

Proposition 8. *When $\mu \geq 1$, there exists a unique symmetric equilibrium where $x(z) = 0$ on $[\underline{z}, \tilde{z})$ and, for $[\tilde{z}, \bar{z}]$, $x(z)$ is the unique solution to the differential equation (22) with initial condition $x(\tilde{z}) = 0$.*

Let us now consider risk attitudes implied by this equilibrium. As before, if one assumes the DTTH condition, there will be risk-taking at the lower end of the wealth distribution. In particular, equilibrium utility will convex for those with wealth around the cutoff level $\tilde{z}$. It is also possible to show that many of those men who in equilibrium will not marry will also be willing to take risks. This is true even though their utility function is locally concave. The point is that the convexity of the utility function at higher levels of wealth is sufficient to make gambling attractive (see Figure 3).

Proposition 9. *Assume DTTH, then there exist z_1, z_2, with $0 < z_1 < \tilde{z} < z_2$, and there exists an $s^* > 0$, such that if $\underline{s} < s^*$ then individuals with wealth in (z_1, z_2) would accept a fair gamble.*

Does an increase in the relative number of men increase the incentive to take risks? I show that under reasonable conditions an increase in men leads to an increase in the slope of the matching function similar to that resulting from an increase in inequality in rewards as analysed in Section 3.1. Specifically, a sufficient condition for this is that the distribution G

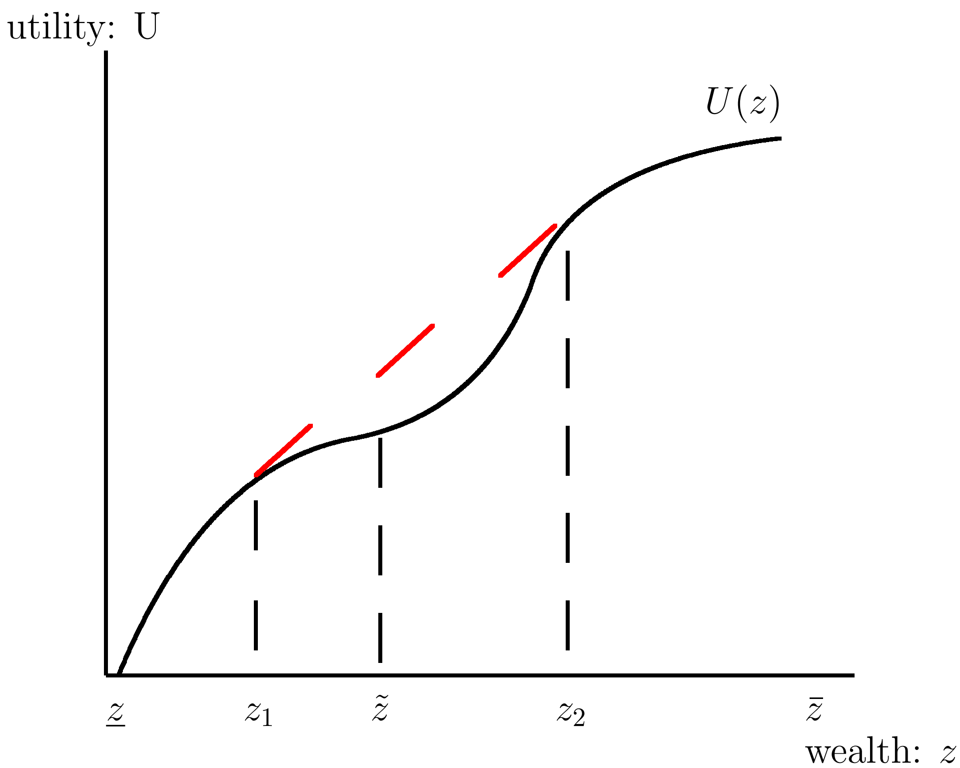

Figure 3: Although individuals with wealth $(z_1, \tilde{z})$ have utility that is locally concave, they would be willing to take gambles that have the possibility of resulting in wealth above the level $\tilde{z}$ that is critical for marriage.

has an increasing failure or hazard rate, where the hazard/failure rate is $\lambda = g(z)/(1-G(z))$. While a decreasing failure/hazard rate is possible if the density $g(z)$ decreases sufficiently quickly, most common distributions have a increasing failure rate.[17] Then, as for an increase in reward inequality, an increase in μ, the relative number of men, leads to greater risk taking.

Proposition 10. *Suppose that the relative number of men increases so that $\mu_p > \mu_a$. Assume that $G(z)$ has an increasing hazard rate, assume also the DTTH condition, that $U_{css}, U_{ccs} \leq 0$ and that $U_a''(z) \leq 0$ on $(\tilde{z}, \tilde{z} + \epsilon)$ for some $\epsilon > 0$. Then men with wealth close to the critical level $\tilde{z}$ become more risk loving. That is, there will be a $\hat{r} \in (\tilde{r}, 1]$ such that $U_p''(G^{-1}(r)) > U_a''(G^{-1}(r))$ and $AR_p(G^{-1}(r)) < AR_a(G^{-1}(r))$ for all $r \in (\tilde{r}, \hat{r})$.*

[17] See Hoppe et al. (2009) for further discussion of the role of failure rates in comparative statics in matching problems.

6 Discussion: Risk-Taking, Gender, Age and Empirical Implications

In this section, some further interpretation of the formal results is given, with some discussion of their possible empirical implications. For example, it is possible to draw some quite simple conclusions about risk taking by age and by gender. It follows quite directly that risk taking can be expected to be greater by young low-ranking males.

First, after the tournament, in "old age", all agents will be risk averse. Clearly, the model presented here is not a detailed model of the life-cycle. However, if we take the tournament to be largely concerned with competition for mating opportunities, then one would expect activity in the tournament to be concentrated in the early adult years. In any case, formally, in the final stage, after rewards have been assigned, an agent will have a reward s and will have her endowment less x, the amount spent on the tournament. She will have utility $U(z - x, s)$ if she goes ahead and consumes the remaining endowment and the reward. If offered a fair gamble over either, she will refuse as U is concave in both arguments by assumption. That is, gambling only occurs when young.

Second, from Proposition 3, one can see that a population facing a greater dispersion in rewards will have greater risk-taking by those with low endowments. So, if men as a population have more dispersed rewards than women, low-ranking men will be more risk-taking than low-ranking women.

It is well-recognised that, in an evolutionary sense, men's rewards are more variable than women's. As Wilson and Daly (1985, p60) write, "male fitness variance exceeds female fitness variance". This is because, while female fertility is limited by physiological constraints, male fertility can be much higher if access to multiple mates is possible. Wilson and Daly argue that therefore the effective degree of polygyny - the extent to which a single male can have multiple exclusive partners - determines the level of social competition amongst males and the "more intense this competition, the more we can expect males to be inclined to risky tactics" (p. 60).[18] That is, the current model provides formal support for Wilson and Daly's argument.

This paper does not make any claims with respect to welfare, for example, whether risk-taking behaviour should be encouraged or discouraged. There are two reasons. First, there are already results in this direction. Hopkins and Kornienko (2010) show in a more general model that greater equality in rewards can make most and possibly all individuals better off. Ray and Robson (2012) similarly show that a completely equal outcome where gambling is banned is Pareto superior to their equilibrium steady state. Second, the current model is too abstract to be much use in policy prescription. In particular, it does not distinguish between

[18] It has been suggested that the greater variability of male reproductive success is also the cause of greater male variability in a number of traits. That is, randomization by males can also be physiological.

an incentive to undertake risky activities such as crime, that carry negative externalities, and those that create wealth, such as entrepreneurship.

The idea that low ranked agents may have an incentive to gamble has an apparent similarity to the idea of "gambling for resurrection", in which agents who are near to bankruptcy have an incentive to gamble because any downside losses would be truncated. See, for example, Gollier et al. (1997). However, none of the results in this paper depend on any such mechanism. Here agents will take fair bets, even though they will have to suffer the downside in full.[19] Clearly, if limited liability were a possibility, then the incentive to gamble would be increased.

An evolutionary approach also suggests how the findings in this paper - risk-taking by the poor is increasing in inequality of status, but decreasing in the inequality of wealth - might be distinguished empirically. With existing models, such as Becker et al. (2005), there appears there is an endogeneity problem. If an equal wealth distribution causes gambling that causes the wealth distribution, then the effect of inequality on risk attitudes would be difficult to identify.

There are some grounds for hope, however. The current model specifies a distribution of rewards or status outcomes that is exogenous and independent of the distribution of wealth. Marriage arrangements are one example of how rewards could vary in this way. Some societies explicitly allow polygamy, others condone polygyny while others are strictly monogamous. Thus, the evolutionary return to high status would be quite different across these different societies. Further, while the underlying causation for these differing customs may be economic, such institutions change slowly and most individuals would plausibly take them as fixed, and thus an analyst can hope to treat them as exogenous. That is, if long run social arrangements ("culture") cause risk-taking that results in inequality, then there is a hope for meaningful empirical testing.

Indeed, broadly consistent with the hypotheses advanced here, existing cross-cultural studies (Bacon et al., 1963; Barber, 2000) find that polygamy is associated with higher levels of violent crime. Dreze and Khera (2000) and Edlund et al. (2007) find that crime is higher in provinces (in India and China respectively) where the male-female gender ratio is high and thus marriage opportunities for males are more fiercely competed. More recently, Wei and Zhang (2011) find that economic risk-taking behaviour such as entrepreneurship is also positively associated with the sex-ratio.

What can be said about the relation between risk-taking and growth, for example, whether one should expect more risk-taking in poor or rich countries? This is difficult to predict partly for the same reasons that risk attitudes in standard models are not necessarily monotone with respect to wealth. But suppose one imposes sufficient "prudence" (that is,

[19] It is true that the social reward level cannot fall below the minimum $\underline{s}$ and in that sense there is a form of limited social liability. However, none of the results depend on this.

the third derivative of utility with respect to consumption is positive), this would suggest, as in standard models, a general tendency toward risk neutrality as wealth rises. Nonetheless, what the main results of this paper suggest is that there could still be risk-taking at the bottom end of society depending on the form of social arrangements present.

Thus, the apparent empirical relationship between economic inequality and risk-taking behaviour might be misleading. Rather, as in the model presented here, it is unequal social relationships that can cause risk-taking behaviour. Inequality in wealth then follows as a result, as social inequality provides individuals with an incentive to gamble over wealth. This is a fascinating possibility which merits further empirical investigation.

Appendix

Proof of Proposition 1: This proof follows that of Proposition 1 of Hopkins and Kornienko (2004). A sketch is as follows. Given $U_{cs} \geq 0$, best replies are (weakly) increasing in z. Given the tie breaking rule (2), a symmetric equilibrium strategy must in fact be strictly increasing. If the equilibrium strategy is strictly increasing then it can be shown to be continuous and, furthermore, differentiable. Thus, it satisfies the differential equation (6). This has a unique solution by the fundamental theorem of differential equations.

It is also worth emphasising that the first order condition (5) is a maximum (despite the equilibrium utility function $U(z)$ potentially being convex), as if all others adopt the proposed equilibrium strategy, an agent's utility is pseudoconcave in x for each individual. That is, $U(z - x, H^{-1}(F(x)))$ is increasing in x for x less than the equilibrium choice $x(z)$ and is decreasing in x for x greater than $x(z)$. That is, utility $U(z - x, H^{-1}(F(x)))$ is pseudoconcave in performance x. To show this, if all agents adopt a strictly increasing strategy $x(z)$ then an individual's utility can be written as $U(z-x, H^{-1}(F(x)))$ and $\partial U/\partial x = -U_c(z-x, H^{-1}(F(x))) + U_s(z-x, H^{-1}(F(x)))f(x)/h(\cdot)$. Then, one has $\partial^2 U/\partial x \partial z = -U_{cc} + U_{cs} f(x)/h(\cdot) > 0$. Take $\breve{x} < x(z)$ and let $\breve{z}$ be such that $x(\breve{z}) = \breve{x}$, so that $\breve{z} < z$. Hence, for any $\breve{x} < x(z)$, $dU(z - \breve{x}, H^{-1}(F(\breve{x})))/dx \geq dU(\breve{z} - \breve{x}, H^{-1}(F(\breve{x})))/dx = 0$. Thus, utility is increasing in x for x below the equilibrium choice $x(z)$. A similar argument can establish that it is decreasing in x for x above $x(z)$.

Finally, the boundary condition (7) must hold as the agent with lowest wealth $\underline{z}$ in a symmetric equilibrium has status $S(\underline{z}) = \underline{s}$ and thus chooses performance x to maximise $U(\underline{z} - x, \underline{s})$. Clearly, the optimal choice of performance for the agent with wealth $\underline{z}$ is zero. $\square$

Proof of Proposition 2: One has from (7) that $c(\underline{z}) = \underline{z}$, so that from (8) it follows that

$$U''_+(\underline{z}) = U_{cc}(\underline{z}, \underline{s})(1 - x'_+(\underline{z})) + U_{cs}(\underline{z}, \underline{s})S'_+(\underline{z}) \tag{23}$$

where the subscript "+" indicates a right derivative (as described in footnote 6, $U''(\underline{z})$ may fail to exist) and from (6) that

$$x'_+(\underline{z}) = \frac{U_s(\underline{z},\underline{s})S'(\underline{z})}{U_c(\underline{z},\underline{s})}. \tag{24}$$

It can be calculated that

$$\frac{\partial x'_+(\underline{z})}{\partial \underline{s}} = \frac{U_{ss}U_c - U_{cs}U_s}{U_c^2}S'(\underline{z}) < 0.$$

But this implies that $x'_+(\underline{z})$ is monotone in $\underline{s}$. Further, applying the DTTH condition, one obtains $\lim_{\underline{s}\to 0} x'_+(\underline{z}) = \infty$ (note that as $U_{cs} \geq 0$ then $U_c(\underline{z},\underline{s})$ will not increase as $\underline{s}$ decreases). Putting these together, there is clearly an $s_0 > 0$ such that $x'_+(\underline{z}) = 1$ for $\underline{s} = s_0$. Therefore, given the continuity of $U''_+(\underline{z})$ in s, there exists s^* such that $U''(\underline{z})$ is strictly positive for $\underline{s} < s^*$ where $s^* \geq s_0 > 0$ (with $s^* = s_0$ only if $U_{cs} = 0$). The result then follows from the continuity of $U''(z)$ on $(\underline{z},\bar{z})$. □

Proof of Lemma 1: The second derivative (from the right) of the utility function risk aversion for the poorest agent is $U''_+(\underline{z}) = U_{cc}(\underline{z},\underline{s})(1 - x'_+(\underline{z})) + U_{cs}(\underline{z},\underline{s})S'_+(\underline{z})$ for $i = a, p$. That is, as $c(\underline{z}) = \underline{z}$ and $S(\underline{z}) = \underline{s}$ both ex ante and ex post, the only way that either $U''(\underline{z})$ or $AR(\underline{z})$ can change is in terms of S'_+ and x'_+. The dispersive order by its definition (10) implies that $h_p(H_p^{-1}(r)) < h_a(H_a^{-1}(r))$ for $r \in [0,1]$. Now, $S'(z) = g(z)/h(S(z)) = g(z)/h(H^{-1}(r))$. Thus, given $g(z)$ is unchanged, the dispersive order implies that $S'_p(z) > S'_a(z)$ for all $z \in (\underline{z},\bar{z})$. It is easy to verify that an increase in $S'_+(\underline{z})$ will also increase $x'_+(\underline{z})$ as given in (24). The result follows. □

Proof of Lemma 2: (a) Under additive separability, the second derivative (from the right) of the utility function for the poorest agent becomes $U''_+(\underline{z}) = (1 - x'_+(\underline{z}))U_{cc}(\underline{z})$. It is easy to verify that a decrease in $\underline{s}$ will increase $x'_+(\underline{z})$ as given in (24), but given separability will not affect U_c or U_{cc}. The result follows.
(b) When there is not additive separability, one has

$$\frac{\partial U''_+(\underline{z})}{\partial \underline{s}} = U_{ccs}(1 - x'_+(\underline{z})) + U_{css}S'_+(\underline{z}) - \frac{\partial x'_+(\underline{z})}{\partial \underline{s}}U_{cc},$$

which, given our assumptions, is certainly negative where $x'_+(\underline{z}) < 1$. In the proof of Proposition 2 it was shown that $x'_+(\underline{z})$ is monotone in $\underline{s}$. Thus, as noted, there must be a value s_0 such that if $\underline{s} = s_0$ then $x'_+(\underline{z}) = 1$. If, as assumed, $\underline{s}_a$ is such that $U''_+(\underline{z}) \leq 0$, then $\underline{s}_a > s_0$. If also $\underline{s}_p > s_0$, then it follows that ex post $U''_+(\underline{z})$ will be greater than ex ante, as $U''_+(\underline{z})$ is monotone in $\underline{s}$ on $(s_0, \underline{s}_a)$. If $\underline{s}_p \leq s_0$, then $U''_+(\underline{z})$ is greater ex post. Given that $U''_+(\underline{z})$ increases as a result of a decrease in $\underline{s}$, and the assumption that $U_{cs} > 0$, $AR_+(\underline{z})$ must decrease. □

Proof of Proposition 3: This follows directly from Lemma 1 and Lemma 2. □

Proof of Proposition 4: As wealth and status of the poorest agent is unchanged, the only effect on $U''_+(\underline{z})$ as given in (23) (and therefore $AR_+(\underline{z})$) is from a change in the density $S'_+(\underline{z}) = g(\underline{z})/h(\underline{s})$. Now, as second order stochastic dominance by definition requires $\int_{\underline{z}}^{z} G_p(t) - G_a(t)dt > 0$ on $(\underline{z}, \bar{z})$, we have (generically) $g_p(\underline{z}) > g_a(\underline{z})$ and so $U''_+(\underline{z})$ increases. One can see that $U'(\underline{z})$ is unchanged, so that $AR_+(\underline{z})$ must decrease. □

Proof of Proposition 5: We again consider $U''_+(\underline{z})$ as given in (23) and show that $U''_+(\underline{z}_p) < U''_+(\underline{z}_a)$. From (24) it can be calculated that

$$\frac{\partial x'_+(\underline{z})}{\partial \underline{z}} = \frac{U_{sc}U_c - U_{cc}U_s}{U_c^2} S'_+(\underline{z}) > 0,$$

and

$$\frac{\partial U''_+(\underline{z})}{\partial \underline{z}} = U_{ccc}(1 - x'_+(\underline{z})) + U_{ccs}S'_+(\underline{z}) - \frac{\partial x'_+(\underline{z})}{\partial \underline{z}} U_{cc}. \tag{25}$$

We return to the sign of the derivative below, but note that the result on $AR(z)$, in both (a) and (b), given that $\partial U'(\underline{z})/\partial \underline{z} = U_{cc} < 0$, and the assumption that $U''_+(\underline{z}_a) \leq 0$, is proven if $U''_+(\underline{z}_p) < U''_+(\underline{z}_a)$.

(a) Given additive separability, if $U''_+(\underline{z}) \leq 0$ then $x'_+(\underline{z}) \leq 1$. It follows that the derivative (25) is strictly positive. Thus, the decrease in minimum wealth considered by itself leads to a decrease in $U''_+(\underline{z})$. Further, by the dispersive order we have $g_p(\underline{z}_p) = g_p(G_p^{-1}(0)) < g_a(G_a^{-1}(0))$ and so $S'_+(\underline{z}_p) = g_p(\underline{z}_p)/h(\underline{s}) < g_a(\underline{z}_p)/h(\underline{s}) = S'_+(\underline{z}_a)$ and thus the greater dispersion also decreases $U''_+(\underline{z})$.

(b) If $U''_+(\underline{z}) \leq 0$ then $x'_+(\underline{z}) < 1$ and $S'_+(\underline{z}) \leq -(1 - x'_+(\underline{z}))U_{cc}/U_{cs}$. Thus, the derivative (25) satisfies

$$\frac{\partial U''_+(\underline{z})}{\partial \underline{z}} \geq (1 - x'_+(\underline{z}))(U_{ccc} - U_{ccs}\frac{U_{cc}}{U_{cs}}) - \frac{\partial x'_+(\underline{z})}{\partial \underline{z}} U_{cc}.$$

If U_{cc}/U_{cs} is increasing, then $U_{ccc} - U_{ccs}U_{cc}/U_{cs} \geq 0$ and the derivative (25) is positive. The result then follows as in part (a). □

Proof of Lemma 3: Suppose $G^*(z)$ is not differentiable at some point $\hat{z}$ then, without loss of generality, assume that $\lim_{z \downarrow \hat{z}} g(z) = g_+(\hat{z}) > g(z)$ for all z in some interval $(z_0, \hat{z})$. Then, $S'_+(\hat{z}) > S'(z)$ for z in $(z_0, \hat{z})$. Thus, because $U''(z)$ is increasing in $S'(z)$ (see (8)), we can find a z sufficiently close to $\hat{z}$ such that $U''(z) > U''_+(\hat{z})$. Clearly, it is not possible for both $U''_+(\hat{z}) = 0$ and $U''(z) = 0$ on the interval $(z_0, \hat{z})$. Thus, the first result is proved. Second, suppose that $g(z) = 0$ on some interval $[z_1, z_2]$ with $z_1 > \underline{z}$. Then, as $S'(z) = 0$ and thus $x'(z) = 0$ on this interval, one has $U''(z) < 0$ everywhere on $[z_1, z_2]$. Because it has been shown that $G^*(z)$ is differentiable, $S(z) = H^{-1}(G^*(z))$ will also be differentiable and thus $U''(z)$ is continuous. Thus, there is a $z > z_2$ sufficiently close to z_2 such that $U''(z) < 0$. Thus, this distribution does not induce $U''(z) = 0$ on its support. □

Proof of Proposition 6: By the definition of the differential equation (17), the solution $(c^*(z), S^*(z))$ implies that $U''(c^*(z), S^*(z)) = 0$. Thus, $U(z)$ is linear as $U'(z) = U_c > 0$.

Such a solution must exist by the fundamental theorem of differential equations because both (17) and (19) are continuously differentiable and bounded. The solution is unique for a given initial condition, that is, for a given minimum wealth level $\underline{z}$. That is, there is a family of distributions $\langle G_i \rangle$ that each satisfy $H^{-1}(G_i(z)) = S^*(z)$, each corresponding to a different level of minimum wealth $\underline{z}_i$. Finally, I prove that in this family, average wealth μ is strictly increasing in $\underline{z}$.

The equation system (c', S') as defined by (17) and (19) is autonomous, that is a function of c and S alone and only a function of z through c and S. It thus follows by fundamental theory of differential equations, that two solution curves $(c(z), S(z))$ cannot cross on the (c, S) plane. So, given two solutions with initial conditions $(\underline{z}_i, \underline{s})$ and $(\underline{z}_j, \underline{s})$ for some $\underline{z}_i < \underline{z}_j$, it follows that $c_j > c_i$ for any given value of S. Now consider the two associated solutions for rewards, $S_i(z)$ and $S_j(z)$ on the (z, S) plane. I claim there is no value of z such that $S_i(z) = S_j(z)$. Suppose not, then because $S_i(\underline{z}_i) = S_j(\underline{z}_j) = \underline{s}$ and $\underline{z}_j > \underline{z}_i$, at the first such crossing S_j must cross S_i from below. But as

$$\frac{\partial S'(c,s)}{\partial c} = \frac{-U_c U_{cs} U_{cc}^2 + U_{cc}^3 U_s - U_c^2 (U_{ccc} U_{cs} - U_{cc} U_{ccs})}{(U_s U_{cc} - U_c U_{cs})^2} < 0$$

(this follows from the assumptions on U_{ccc}, U_{ccs} and U_{cc}/U_{cs}) and as $c_j > c_i$, we have $S_i' > S_j'$ at such a point of crossing. Thus, such a crossing is not possible and so, given distinct initial values of endowments $\underline{z}_j > \underline{z}_i$, it must hold that $S_i(z) < S_j(z)$ for all z. Hence, for a fixed $H(s)$, we have $G_j(z) = H^{-1}(S_j(z)) < H^{-1}(S_i(z)) = G_i(z)$. That is, $G_j(z)$ stochastically dominates $G_i(z)$ and $\mu_j > \mu_i$. This implies, that for any given level of average wealth μ, there exists a unique $\underline{z}$ such that the mean of $G^*(z)$ is μ. □

Proof of Lemma 4: Start with a distribution of wealth $G^*(z) = G_a(z)$ such that $U''(z) = 0$ on its support $[\underline{z}, \bar{z}]$. Then suppose there is a local decrease in dispersion of wealth so that $g_p(z) > g_a(z)$ on an interval (z_1, z_2) but $g_p(z) = g_a(z)$ on $[\underline{z}, z_1]$. Thus because $g(z)$ is unchanged on $[\underline{z}, z_1]$ we have $x_a(z_1) = x_p(z_1)$. However, the increase in g_p on (z_1, z_2) implies that (generically) $G_p''(z_1) > G_a''(z_1)$ and thus $S_p''(z_1) > S_a''(z_1)$. This implies that $x_p''(z_1) > x_a''(z_1)$ (but $x'(z_1)$, $S'(z_1)$, $S(z_1)$ and $x(z_1)$ are unchanged). Then it is easy to verify that we have $U_p'''(z_1) > U_a'''(z_1) = 0$. So $U_p''(z) > 0$ on $(z_1, z_1 + \epsilon)$ for some $\epsilon > 0$. □

Proof of Proposition 7: By the dispersive order we have $h_p(H_p^{-1}(r)) < h_a(H_a^{-1}(r))$. Together with our other assumptions on minimum and maximum rewards, it implies that $H_p(s)$ and $H_p(s)$ are single crossing, with a unique reward $\hat{s}$ such that $H_p(\hat{s}) = H_a(\hat{s})$.

Let us assume that $\underline{z}_p < \underline{z}_a$, the minimum wealth level is lower under the new distribution (this later will be shown to hold). Given that solutions $(c(z), S(z))$ to the differential equation system cannot cross on the (c, S) plane, given our initial conditions $c_a(\underline{z}_a) = \underline{z}_a$, $S_a(\underline{z}_a) = \underline{s}_a$ and $c_p(\underline{z}_p) = \underline{z}_p$, $S_p(\underline{z}_p) = \underline{s}_p$ respectively, we have $c_p < c_a$ for a given level of S. Turning to solutions $S_a(z)$ and $S_p(z)$ graphed as a function of z alone, points of crossing of $S_a(z)$ and $S_p(z)$ are possible. However, because $c_a > c_p$ and $\partial S'(c, s)/\partial c < 0$ (as shown in the proof to the previous proposition), then $S'_p(z) > S'_a(z)$ at any such crossing. Thus, there is at most one crossing where $S_a(z) = S_p(z)$. There must be a crossing as otherwise clearly the mean reward could not be the same in both cases. Hence there is a unique crossing at endowment $\hat{z}$ where $S_a(\hat{z}) = S_p(\hat{z}) = \hat{s}$.

I now establish that $\underline{z}_p < \underline{z}_a$, the minimum wealth level is lower under the new distribution. Suppose not so that $\underline{z}_p \geq \underline{z}_a$. Then as solutions $(c(z), S(z))$ to the differential equation system cannot cross on the (c, S) plane, given our initial conditions that $c_p(\underline{z}_p) = \underline{z}_p \geq \underline{z}_a = c_a(\underline{z}_a)$ we have $c_p > c_a$ for a fixed level of S. Thus, given $\partial S'/\partial c < 0$ as established above, we would have $S'_p(z) < S'_a(z)$ at any potential point of crossing of $S_a(z)$ and $S_p(z)$ (graphed alone as a function of z). Since we have $S_p(\underline{z}_p) = \underline{s}_p < \underline{s}_a = S_a(\underline{z}_a)$, S_p would never in fact cross S_a. Thus, the average reward must be higher ex post than ex ante, which is not possible by assumption. Thus, in summary, $S_p(z)$ has a strictly larger support than $S_a(z)$, and $S_p(z)$ and $S_a(z)$ are single crossing, with $S_p(z)$ crossing from below.

But this also implies that the inverses of $S_p(z)$ and $S_a(z)$ are also single crossing. That is, the two functions $G_p^{-1}(H_p(s)) = S_p^{-1}(s)$ and $G_a^{-1}(H_a(s)) = S_a^{-1}(s)$ are single crossing, with $G_p^{-1}(H_p(\hat{s})) = \hat{z} = G_a^{-1}(H_a(\hat{s}))$. But if the inverse of the distribution functions are single-crossing then so are distribution functions $G_p(z)$ and $G_a(z)$ with clearly $G_p(z) > G_a(z)$ on $(\underline{z}_p, \hat{z})$ and $G_p(z) < G_a(z)$ on $(\hat{z}, \bar{z}_p)$. Single crossing of this form with an equal mean implies second order stochastic dominance (Wolfstetter, 1999, Proposition 4.6). □

Proof of Lemma 5: The matching that assigns a man with performance x to a woman as specified in (20) is stable as while any man with rank $F(x)$ would prefer a match with any woman with $s > S(F(x); \mu)$, such a woman would prefer her current match whose performance, say $\hat{x}$, would be greater than x. Suppose there is another matching $\tilde{S}$, such that a set of men X with positive measure are matched differently than under the positive assortive matching S. Then, there must exist $\hat{x} \in X$, such that $\tilde{S}(\hat{x}, F(\cdot); \mu) > S(\hat{x}, F(\cdot); \mu)$. In fact, there must be a positive measure of men who are matched strictly higher than under S. For this matching to be stable, all men with performance higher than $\hat{x}$ must be matched with women whose s is greater than $\tilde{S}(\hat{x}, F(\cdot); \mu)$. If not, then a woman $s = \tilde{S}(\hat{x}, F(); \mu)$ could propose a match with a man with performance $\tilde{x}$ where $\tilde{x} > \hat{x}$ and the man $\tilde{x}$ would find it acceptable. But the measure of men with x higher than $\hat{x}$, is strictly larger than the measure of women with s greater than $\tilde{S}(\hat{x}, F(\cdot); \mu)$. But this implies that $\tilde{S}$ is not measure-preserving. □

Proof of Proposition 8: The proof of Proposition 1 for the original game on $[\underline{z}, \bar{z}]$ applies directly to the one on $[\tilde{z}, \bar{z}]$. Thus, in the proposed equilibrium, an individual of wealth $\tilde{z}$ has no incentive to deviate upwards. It follows that agents with less wealth, that is with wealth on $[\underline{z}, \tilde{z})$, have no incentive to deviate upwards either. □

Proof of Proposition 9: The result that $U(z)$ is strictly convex on $(\tilde{z}, z_2)$ follows directly from Proposition 2. The utility of individuals on the interval $[\underline{z}, \tilde{z})$ is $U(z, \underline{s})$ which is clearly continuous and also concave in z, the latter simply by the assumption that $U_{cc} < 0$. Because

$U(z)$ is strictly convex on $(\tilde{z}, z_2)$, we have, for some $\theta \in (0,1)$, $\theta U(\tilde{z}) + (1-\theta)U(z_2) > U(\theta\tilde{z} + (1-\theta)(z_2))$. By the continuity of $U(z)$, there is an z_1 such that for $z \in (z_1, \tilde{z})$, one has $\theta U(z) + (1-\theta)U(z_2) > U(\theta z + (1-\theta)(z_2))$. $\square$

Proof of Proposition 10: First, note that as μ rises so does $\tilde{z}$, the lowest level of wealth at which matching is achieved. Specifically, $\tilde{z}_p = G^{-1}(1-1/\mu_p) > G^{-1}(1-1/\mu_a) = \tilde{z}_a$. Second, by assumption $S_a(\tilde{z}_a) = S_p(\tilde{z}_p) = \underline{s}$ and, by the initial condition, $x_a(\tilde{z}_a) = x_p(\tilde{z}_p) = 0$, and so both are unchanged. Thus from (25) in the proof of Proposition 5, the effect of higher wealth on $U''_+(\tilde{z})$ is positive.

Further, we have $S'(\tilde{z}) = \mu g(\tilde{z})/h(\underline{s}) = \mu g(G^{-1}(1-1/\mu))/h(\underline{s})$. Differentiating, this is increasing in μ if

$$g(\tilde{z}) + \frac{g'(\tilde{z})}{\mu g(\tilde{z})} = g(\tilde{z}) + \frac{g'(\tilde{z})(1-G(\tilde{z}))}{g(\tilde{z})} > 0.$$

It is easily verified that this inequality holds at $\tilde{z}$ (indeed at any z) if G has an increasing hazard rate. So we have $S'_p(\tilde{z}_p) > S'_a(\tilde{z}_a)$. So the effect on $U''_+(\tilde{z})$ from the change in the matching function $S(z)$ is also positive. Thus, it has been shown that $U''_+(\tilde{z}_p) > U''_+(\tilde{z}_a)$. By the continuity of $U''(z) = U''(G^{-1}(r))$ the main result follows. Finally, $U'(\tilde{z}) = U_c(\tilde{z}, \underline{z})$ decreases as $U_c c < 0$, so the result on $AR(z)$ follows. $\square$

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